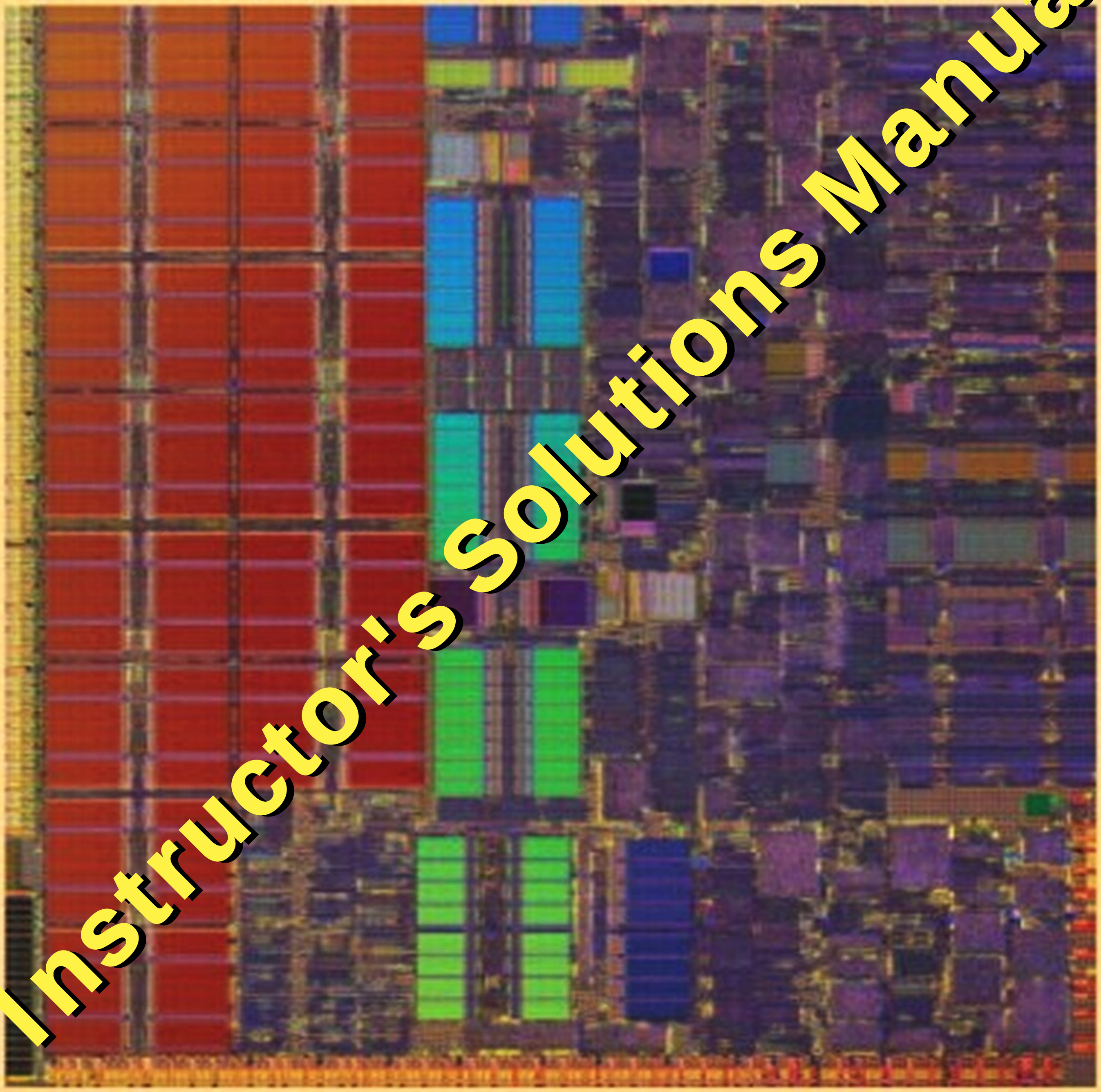
SOLID STATE ELECTRONIC DEVICES

SIXTH EDITION



Instructor's Solutions Manual

BEN G. STREETMAN ▪ SANJAY BANERJEE

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PEARSON

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Ben Streetman, *University of Texas, Austin*
Sanjay Banerjee, *University of Texas, Austin*

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Chapter 1 Solutions

Prob. 1.1

Which semiconductor in Table 1-1 has the largest E_g? the smallest? What is the corresponding λ? How is the column III component related to E_g?

largest E_g : ZnS, 3.6 eV

$$\lambda = \frac{1.24}{3.6} = 0.344\mu m$$

smallest E_g : InSb, 0.18 eV

$$\lambda = \frac{1.24}{0.18} = 6.89\mu m$$

Al compounds E_g > corresponding Ga compounds E_g > the corresponding In compounds E_g

Prob. 1.2

Find packing fraction of fcc unit cell.

$$\text{nearest atom separation} = \frac{5\sqrt{2}}{2}\text{Å} = 3.54\text{Å}$$

$$\text{tetrahedral radius} = 1.77\text{Å}$$

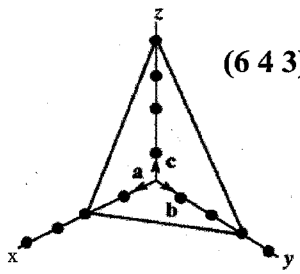
$$\text{volume of each atom} = 23.14\text{Å}^3$$

$$\text{number of atoms per cube} = 6\cdot\tfrac{1}{2} + 8\cdot\tfrac{1}{8} = 4 \text{ atoms}$$

$$\text{packing fraction} = \frac{23.1\text{Å}^3\cdot 4}{(5\text{Å})^3} = 0.74 = 74\%$$

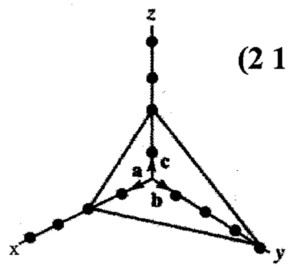
Prob. 1.3

Label planes.



(6 4 3)

x	y	z
2	3	4
1/2	1/3	1/4
6	4	3



(2 1 2)

x	y	z
2	4	2
1/2	1/4	1/2
2	1	2

Prob. 1.4

Calculate densities of Si and GaAs.

The atomic weights of Si, Ga, and As are 28.1, 69.7, and 74.9, respectively.

Si: $a = 5.43 \cdot 10^{-8}$ cm, 8 atoms/cell

$$\frac{8\text{ atoms}}{a^3} = \frac{8}{\left(5.43 \cdot 10^{-8}\text{cm}\right)^3} = 5 \cdot 10^{22}\,\tfrac{1}{\text{cm}^3}$$

$$\text{density} = \frac{5 \cdot 10^{22}\,\frac{1}{\text{cm}^3} \cdot 28.1\frac{\text{g}}{\text{mol}}}{6.02 \cdot 10^{23}\,\frac{1}{\text{mol}}} = 2.33\tfrac{\text{g}}{\text{cm}^3}$$

GaAs: $a = 5.65 \cdot 10^{-8}$ cm, 4 each Ga, As atoms/cell

$$\frac{4}{a^3} = \frac{4}{\left(5.65 \cdot 10^{-8}\text{cm}\right)^3} = 2.22 \cdot 10^{22}\,\tfrac{1}{\text{cm}^3}$$

$$\text{density} = \frac{2.22 \cdot 10^{22}\,\frac{1}{\text{cm}^3} \cdot \left(69.7 + 74.9\right)\frac{\text{g}}{\text{mol}}}{6.02 \cdot 10^{23}\,\frac{1}{\text{mol}}} = 5.33\tfrac{\text{g}}{\text{cm}^3}$$

Prob. 1.5

For InSb, find lattice constant, primitive cell volume, (110) atomic density.

$$\frac{\sqrt{3}a}{4} = 1.44\text{Å} + 1.36\text{Å} = 2.8\text{Å}$$

$$a = 6.47\text{Å}$$

FCC unit cell has 4 lattice points $\therefore$ volume of primitive cell $= \frac{a^3}{4} = 67.7\text{Å}^3$

area of (110) plane $= \sqrt{2}a^2$

density of In atoms $= \frac{4\cdot\frac{1}{4}+2\cdot\frac{1}{2}}{\sqrt{2}\,a^2} = \frac{\sqrt{2}}{a^2} = 3.37\cdot10^{14}\,\frac{1}{\text{cm}^2}$

same number of Sb atoms $= 3.37\cdot10^{14}\,\frac{1}{\text{cm}^2}$

Prob. 1.6

Find density of sc unit cell.

nearest atom separation $= 2\cdot 2.5\text{Å} = 5\text{Å}$

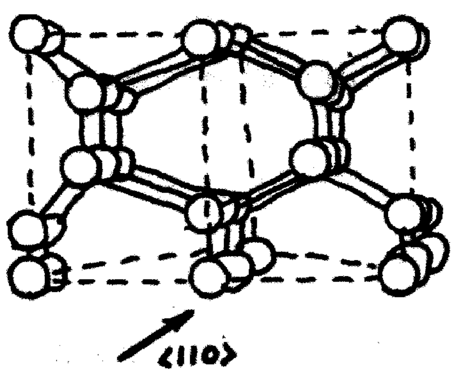
number of atoms per cube $= 8\cdot\frac{1}{8} = 1$ atom

mass of one atom $= \dfrac{5.42\,\frac{\text{g}}{\text{mol}}}{6.02\cdot10^{23}\,\frac{\text{atom}}{\text{mol}}} = 9\cdot10^{-24}\,\frac{\text{g}}{\text{atom}}$

density $= \dfrac{1\text{atom}\cdot 9\cdot10^{-24}\,\frac{\text{g}}{\text{atom}}}{(5\text{Å})^3} = 0.072\,\frac{\text{g}}{\text{cm}^3}$

Prob. 1.7

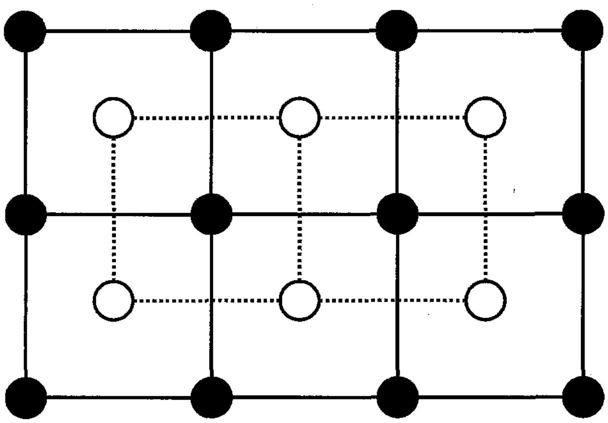
Draw <110> direction of diamond lattice.

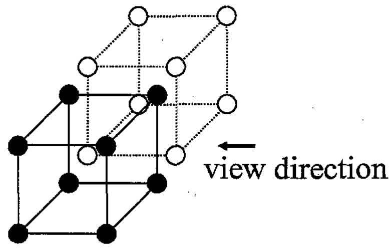


This view is tilted slightly from (110) to show the alignment of atoms. The open channels are hexagonal along this direction.

Prob. 1.8

Show bcc lattice as interpenetrating sc lattices.

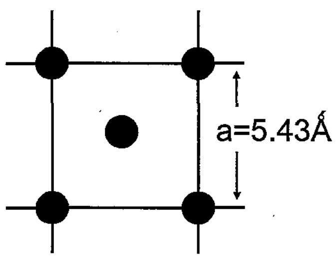




The shaded points are one sc lattice. The open points are the interpenetrating sc lattice located a/2 behind the plane of the front shaded points.

Prob. 1.9

(a) Find number of Si atoms/cm^2 on (100) surface.

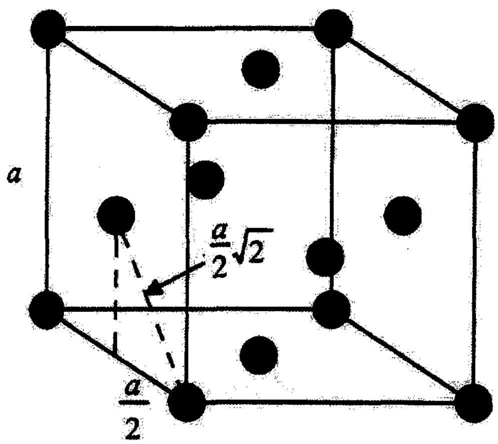


fcc lattice with $a = 5.43\text{Å}$

number atoms per (100) surface $= 4\cdot\frac{1}{4} + 1 = 2$ atoms

atoms per (100) surface area $= \dfrac{2}{(5.43\text{Å})^2} = 6.78\cdot10^{14}\,\frac{1}{\text{cm}^2}$

(b) Find the nearest neighbor distance in InP.



fcc lattice with $a = 5.87\text{Å}$

nearest neighbor distance $= \dfrac{a}{2}\cdot\sqrt{2} = \dfrac{5.87\text{Å}}{2}\cdot\sqrt{2} = 4.15\text{Å}$

Prob. 1.10

Find NaCl density.
Na^+: atomic weight 23g/mol, radius 1Å
Cl^-: atomic weight 35.5g/mol, radius 1.8Å

unit cell with $a = 2.8\text{Å}$ by hard sphere approximation

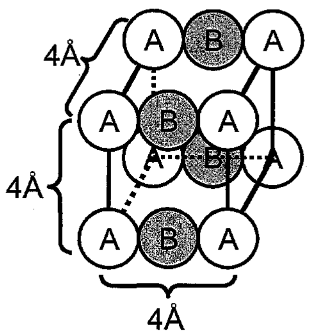
$$\text{½ Na and ½ Cl atoms per unit cell} = \frac{\frac{1}{2}\frac{\text{atoms}}{\text{cell}}\cdot 23\frac{\text{g}}{\text{mol}} + \frac{1}{2}\frac{\text{atoms}}{\text{cell}}\cdot 35.5\frac{\text{g}}{\text{mol}}}{6.02\cdot 10^{23}\frac{\text{atoms}}{\text{mol}}} = 4.86\cdot 10^{-23}\frac{\text{g}}{\text{cell}}$$

$$\text{density} = \frac{4.86\cdot 10^{-23}\frac{\text{g}}{\text{cell}}}{(2.8\cdot 10^{-8}\text{cm})^3\frac{1}{\text{cell}}} = 2.2\frac{\text{g}}{\text{cm}^3}$$

The hard sphere approximation is comparable with the measured $2.17\frac{\text{g}}{\text{cm}^3}$ density.

Prob. 1.11

Find packing fraction, B atoms per unit volume, and A atoms per unit area.



Note: The atoms are the same size and touch each other by the hard sphere approximation.

radii of A and B atoms are then 1Å

number of A atoms per unit cell $= 8\cdot\frac{1}{8} = 1$

number of B atoms per unit cell $= 1$

volume of atoms per unit cell $= 1\cdot\frac{4\pi}{3}\cdot(1\text{Å})^3 + 1\cdot\frac{4\pi}{3}\cdot(1\text{Å})^3 = \frac{8\pi}{3}\text{Å}^3$

volume of unit cell $= (4\text{Å})^3 = 64\text{Å}^3$

$$\text{packing fraction} = \frac{\frac{8\pi}{3}\text{Å}^3}{64\text{Å}^3} = \frac{\pi}{24} = 0.13 = 13\%$$

$$\text{B atoms volume density} = \frac{1\text{ atom}}{64\text{Å}^3} = 1.56\cdot 10^{22}\frac{1}{\text{cm}^3}$$

number of A atoms on (100) plane $= 4\cdot\frac{1}{4} = 1$

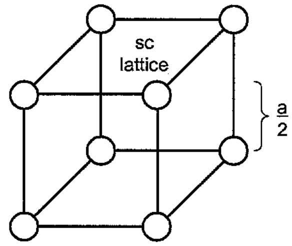
$$\text{A atoms (100) aerial density} = \frac{1\text{ atom}}{(4\text{Å})^2} = 6.25\cdot 10^{14}\frac{1}{\text{cm}^2}$$

Prob. 1.12

Find atoms/cell and nearest neighbor distance for sc, bcc, and fcc lattices.

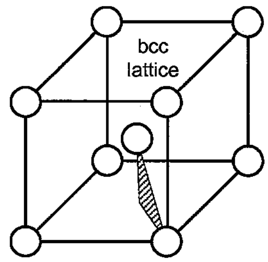
sc: atoms/cell $= 8 \cdot \frac{1}{8} = 1$

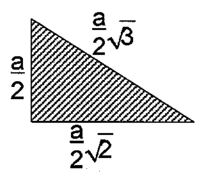
nearest neighbor distance $= a$



bcc: atoms/cell $= 8 \cdot \frac{1}{8} + 1 = 2$

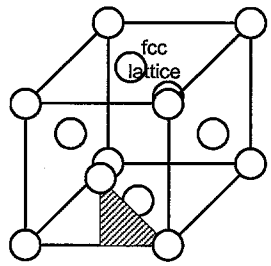
nearest neighbor distance $= \dfrac{a \cdot \sqrt{3}}{2}$

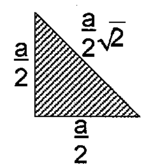




fcc: atoms/cell $= 8 \cdot \frac{1}{8} + 6 \cdot \frac{1}{2} = 4$

nearest neighbor distance $= \dfrac{a \cdot \sqrt{2}}{2}$

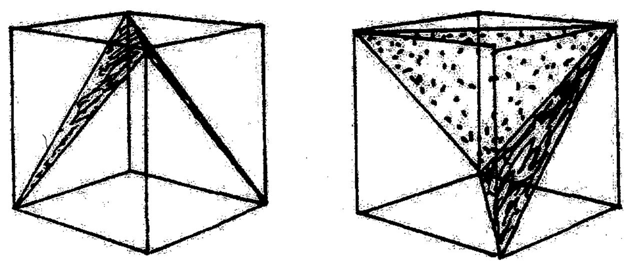




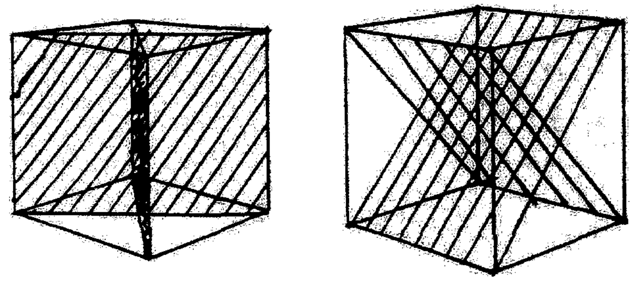
Prob. 1.13

Draw cubes showing four {111} planes and four {110} planes.

{111} planes



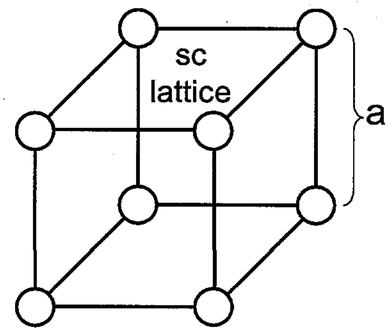
{110} planes



Prob. 1.14

Find fraction occupied for sc, bcc, and diamond lattices.

sc: atoms/cell $= 8 \cdot \frac{1}{8} = 1$



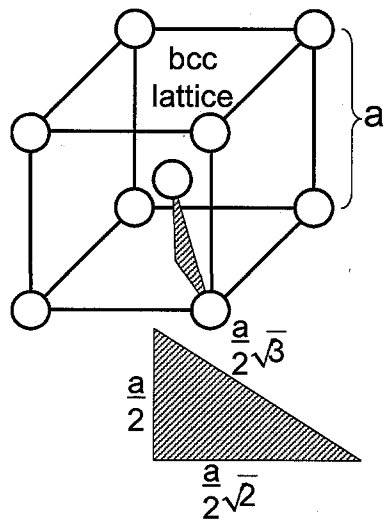
nearest neighbour $= a \rightarrow$ radius $= \frac{a}{2}$

atom sphere volume $= \frac{4\pi}{3} \cdot \left(\frac{a}{2}\right)^3 = \frac{\pi \cdot a^3}{6}$

unit cell volume $= a^3$

fraction occupied $= \frac{1 \cdot \frac{\pi \cdot a^3}{6}}{a^3} = \frac{\pi}{6} = 0.52$

bcc: atoms/cell $= 8 \cdot \frac{1}{8} + 1 = 2$



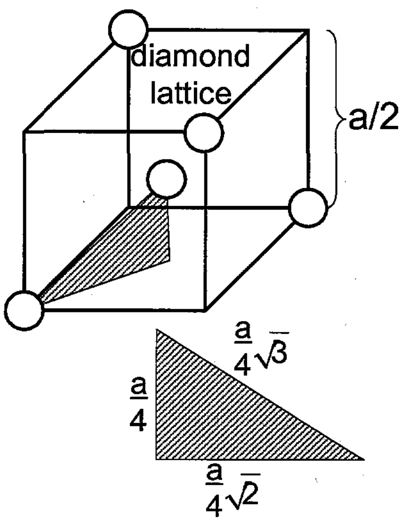
nearest neighbour $= \frac{a \cdot \sqrt{3}}{2} \rightarrow$ radius $= \frac{a \cdot \sqrt{3}}{4}$

atom sphere volume $= \frac{4\pi}{3} \cdot \left(\frac{a \cdot \sqrt{3}}{4}\right)^3 = \frac{\pi \cdot \sqrt{3} \cdot a^3}{16}$

unit cell volume $= a^3$

fraction occupied $= \frac{2 \cdot \frac{\pi \cdot \sqrt{3} \cdot a^3}{16}}{a^3} = \frac{\pi \cdot \sqrt{3}}{8} = 0.68$

diamond: atoms/cell $= 4$ (fcc) $+ 4$ (offset fcc) $= 8$



nearest neighbour $= \frac{a \cdot \sqrt{3}}{4} \rightarrow$ atom radius $= \frac{a \cdot \sqrt{3}}{8}$

atom sphere volume $= \frac{4\pi}{3} \cdot \left(\frac{a \cdot \sqrt{3}}{8}\right)^3 = \frac{\pi \cdot \sqrt{3} \cdot a^3}{128}$

unit cell volume $= a^3$

fraction occupied $= \frac{8 \cdot \frac{\pi \cdot \sqrt{3} \cdot a^3}{128}}{a^3} = \frac{\pi \cdot \sqrt{3}}{16} = 0.34$

Prob. 1.15

Calculate densities of Ge and InP.

The atomic weights of Ge, In, and P are 72.6, 114.8, and 31, respectively.

Ge: $a = 5.66 \cdot 10^{-8}$ cm, 8 atoms/cell

$$\frac{8 \text{ atoms}}{a^3} = \frac{8}{\left(5.66 \cdot 10^{-8}\text{cm}\right)^3} = 4.41 \cdot 10^{22} \tfrac{1}{\text{cm}^3}$$

$$\text{density} = \frac{4.41 \cdot 10^{22} \frac{1}{\text{cm}^3} \cdot 72.6 \frac{\text{g}}{\text{mol}}}{6.02 \cdot 10^{23} \frac{1}{\text{mol}}} = 5.32 \tfrac{\text{g}}{\text{cm}^3}$$

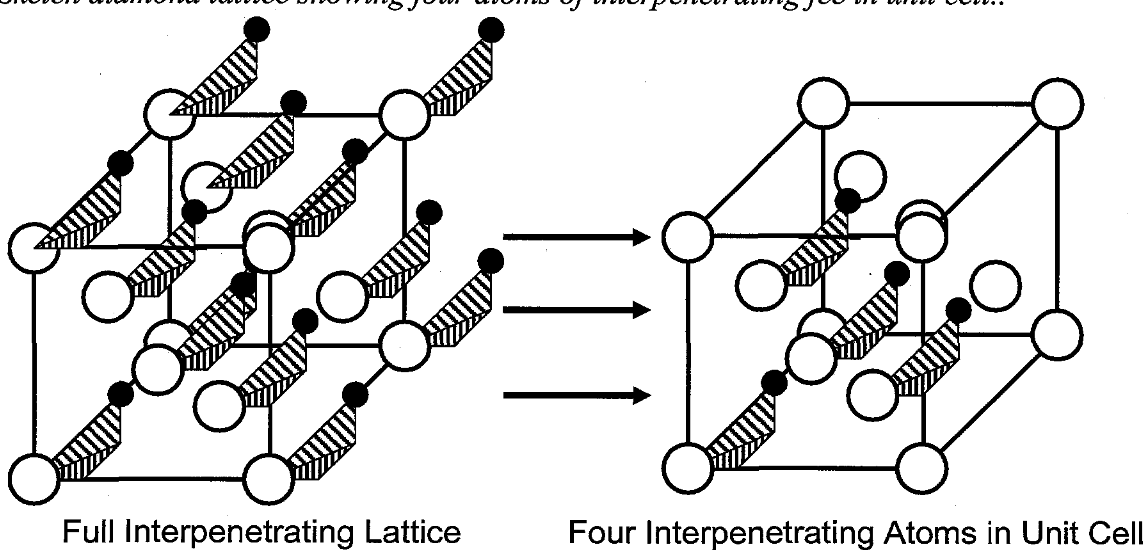
GaAs: $a = 5.87 \cdot 10^{-8}$ cm, 4 each In, P atoms/cell

$$\frac{4}{a^3} = \frac{4}{\left(5.87 \cdot 10^{-8}\text{cm}\right)^3} = 1.98 \cdot 10^{22} \tfrac{1}{\text{cm}^3}$$

$$\text{density} = \frac{1.98 \cdot 10^{22} \frac{1}{\text{cm}^3} \cdot (114.8+31) \frac{\text{g}}{\text{mol}}}{6.02 \cdot 10^{23} \frac{1}{\text{mol}}} = 4.79 \tfrac{\text{g}}{\text{cm}^3}$$

Prob. 1.16

Sketch diamond lattice showing four atoms of interpenetrating fcc in unit cell..



Prob. 1.17

Find $AlSb_xAs_{1-x}$ to lattice match InP and give band gap.

Lattice constants of AlSb, AlAs, and InP are 6.14Å, 5.66Å, and 5.87Å, respectively from Appendix III. Using Vegard's Law,

$$6.14\text{Å}\cdot x + 5.66\text{Å}\cdot(1-x) = 5.87\text{Å} \rightarrow x = 0.44$$

$AlSb_{0.44}As_{0.56}$ lattice matches InP and has E_g=1.9eV from Figure 1-13.

Find $In_xGa_{1-x}P$ to lattice match GaAs and give band gap.

Lattice constant of InP, GaP, and GaAs are 5.87Å, 5.45Å, and 5.65Å, respectively from Appendix III. Using Vegard's Law,

$$5.87\text{Å}\cdot x + 5.45\text{Å}\cdot(1-x) = 5.65\text{Å} \rightarrow x = 0.48$$

$In_{0.48}Ga_{0.52}P$ lattice matches GaAs and has E_g=2.0eV from Figure 1-13.

Prob. 1.18

Find weight of As (k_d=0.3) added to 1kg Si in Czochralski growth for $10^{15}cm^{-3}$ doping.

atomic weight of As $= 74.9\frac{g}{mol}$

$$C_s = k_d \cdot C_L = 10^{15}\frac{1}{cm^3} \rightarrow C_L = \frac{10^{15}\frac{1}{cm^3}}{0.3} = 3.33\cdot 10^{15}\frac{1}{cm^3}$$

assume As may be neglected for overall melt weight and volume

$$\frac{1000\text{g Si}}{2.33\frac{g}{cm^3}} = 429.2\text{cm}^3 \text{ Si}$$

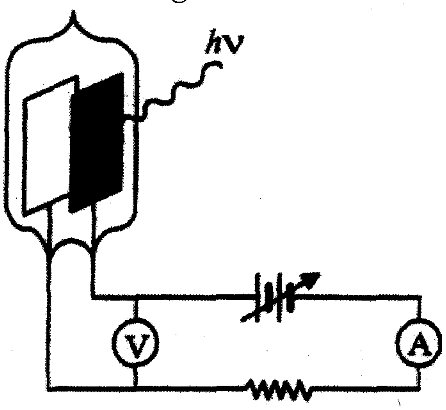
$$3.33\cdot 10^{15}\frac{1}{cm^3} \cdot 429.2\text{cm}^3 = 1.43\cdot 10^{18}\text{As atoms}$$

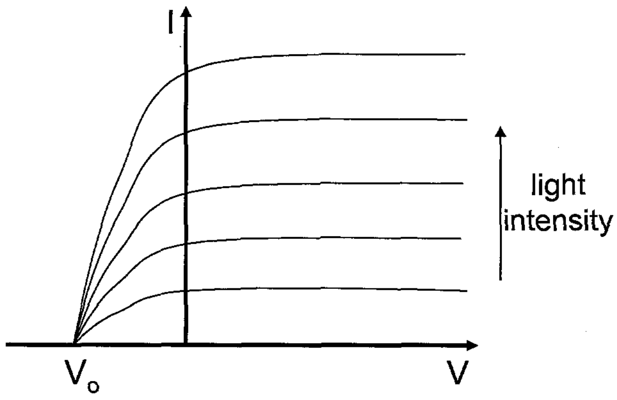
$$\frac{1.43\cdot 10^{18}\text{atoms} \cdot 74.9\frac{g}{mol}}{6.02\cdot 10^{23}\frac{atoms}{mol}} = 1.8\cdot 10^{-4}\text{g As} = 1.8\cdot 10^{-7}\text{kg As}$$

Chapter 2 Solutions

Prob. 2.1

(a&b) Sketch a vacuum tube device. Graph photocurrent I versus retarding voltage V for several light intensities.





Note that V_o remains same for all intensities.

(c) Find retarding potential.

λ=2440Å=0.244µm $\qquad$ Φ=4.09eV

$$V_o = h\nu - \Phi = \frac{1.24\text{eV}\cdot\mu\text{m}}{\lambda(\mu\text{m})} - \Phi = \frac{1.24\text{eV}\cdot\mu\text{m}}{0.244\mu\text{m}} - 4.09\text{eV} = 5.08\text{eV} - 4.09\text{eV} \approx 1\text{eV}$$

Prob. 2.2

Show third Bohr postulate equates to integer number of DeBroglie waves fitting within circumference of a Bohr circular orbit.

$$r_n = \frac{4\pi\epsilon_o n^2\hbar^2}{mq^2} \text{ and } \frac{q^2}{4\pi\epsilon_o r^2} = \frac{mv^2}{r} \text{ and } p_\theta = mvr$$

$$r_n = \frac{4\pi\epsilon_o n^2\hbar^2}{mq^2} = \frac{n^2\hbar^2}{mr_B^{\,2}}\frac{4\pi\epsilon_o r_n^{\,2}}{q^2} = \frac{n^2\hbar^2}{mr_n^{\,2}}\cdot\frac{r_n}{mv^2} = \frac{n^2\hbar^2}{m^2v^2r_n}$$

$$m^2v^2r_n^{\,2} = n^2\hbar^2$$

$$mvr_n = n\hbar$$

$p_\theta = n\hbar$ is the third Bohr postulate

Prob. 2.3

(a) Find generic equation for Lyman, Balmer, and Paschen series.

$$\Delta E = \frac{hc}{\lambda} = \frac{mq^4}{32\pi^2\epsilon_o^2 n_1^2\hbar^2} - \frac{mq^4}{32\pi^2\epsilon_o^2 n_2^2\hbar^2}$$

$$\frac{hc}{\lambda} = \frac{mq^4(n_2^2 - n_1^2)}{32\epsilon_o^2 n_1^2 n_2^2\hbar^2\pi^2} = \frac{mq^4(n_2^2 - n_1^2)}{8\epsilon_o^2 n_1^2 n_2^2 h^2}$$

$$\lambda = \frac{8\epsilon_o^2 n_1^2 n_2^2 h^2 \cdot hc}{mq^4(n_2^2 - n_1^2)} = \frac{8\varepsilon_o^2 h^3 c}{mq^4}\cdot\frac{n_1^2 n_2^2}{n_2^2 - n_1^2}$$

$$\lambda = \frac{8(8.85\cdot10^{-12}\,\tfrac{F}{m})^2\cdot(6.63\cdot10^{-34}\,J{\cdot}s)^3\cdot 2.998\cdot10^8\,\tfrac{m}{s}}{9.11\cdot10^{-31}kg\cdot(1.60\cdot10^{-19}C)^4}\cdot\frac{n_1^2 n_2^2}{n_2^2 - n_1^2}$$

$$\lambda = 9.11\cdot10^8 m\cdot\frac{n_1^2 n_2^2}{n_2^2 - n_1^2} = 9.11\text{Å}\cdot\frac{n_1^2 n_2^2}{n_2^2 - n_1^2}$$

n_1=1 for Lyman, 2 for Balmer, and 3 for Paschen

(b) Plot wavelength versus n for Lyman, Balmer, and Paschen series.

LYMAN SERIES				
n	n^2	n^2-1	n^2/(n^2-1)	911*n^2/(n^2-1)
2	4	3	1.33	1215
3	9	8	1.13	1025
4	16	15	1.07	972
5	25	24	1.04	949

LYMAN LIMIT 911Å

BALMER SERIES				
n	n^2	n^2-4	4n^2/(n^2-4)	911*4*n^2/(n^2-4)
3	9	5	7.20	6559
4	16	12	5.33	4859
5	25	21	4.76	4338
6	36	32	4.50	4100
7	49	45	4.36	3968

BALMER LIMIT 3644Å

PASCHEN SERIES				
n	n^2	n^2-9	9*n^2/(n^2-9)	911*9*n^2/(n^2-9)
4	16	7	20.57	18741
5	25	16	14.06	12811
6	36	27	12.00	10932
7	49	40	11.03	10044
8	64	55	10.47	9541
9	81	72	10.13	9224
10	100	91	9.89	9010

PASCHEN LIMIT 8199Å

Prob. 2.4

Show equation 2-17 corresponds to equation 2-3. That is show $c\cdot R = \dfrac{m\cdot q^4}{2\cdot K^2\cdot\hbar^2\cdot h}$.

From 2-17 and solution to 2.3,

$$\upsilon_{21} = \frac{c}{\lambda} = \frac{2.998\cdot10^8\,\frac{m}{s}}{9.11\cdot10^{-8}m\cdot\dfrac{n_1^2 n_2^2}{n_2^2 - n_1^2}} = 3.29\cdot10^{15}\text{Hz}\cdot\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

From 2-3,

$$\upsilon_{21} = c\cdot R\cdot\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) = 2.998\cdot10^8\,\tfrac{m}{s}\cdot1.097\cdot10^7\,\tfrac{1}{m}\cdot\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) = 3.29\cdot10^{15}\text{Hz}\cdot\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

Prob. 2.5

(a) Find Δpx for Δx=1Å.

$$\Delta p_x \cdot \Delta x = \frac{h}{4\pi} \rightarrow \Delta p_x = \frac{h}{4\pi \cdot \Delta x} = \frac{6.63 \cdot 10^{-34}\,\mathrm{J \cdot s}}{4\pi \cdot 10^{-10}\mathrm{m}} = 5.03 \cdot 10^{-25}\,\tfrac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

(b) Find Δt for ΔE=1eV.

$$\Delta E \cdot \Delta t = \frac{h}{4\pi} \rightarrow \Delta t = \frac{h}{4\pi \cdot \Delta E} = \frac{4.14 \cdot 10^{-15}\mathrm{eV \cdot s}}{4\pi \cdot 1\mathrm{eV}} = 3.30 \cdot 10^{-16}\mathrm{s}$$

Prob. 2.6

Find wavelength of 100eV and 12keV electrons. Comment on electron microscopes compared to visible light microscopes.

$$E = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2 \cdot E}{m}}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2 \cdot E \cdot m}} = \frac{6.63 \cdot 10^{-34}\mathrm{J \cdot s}}{\sqrt{2 \cdot 9.11 \cdot 10^{-31}\mathrm{kg}}} \cdot E^{-\frac{1}{2}} = E^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19}\mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m}$$

For 100eV,

$$\lambda = E^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19}\mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = (100\mathrm{eV} \cdot 1.602 \cdot 10^{-19}\,\tfrac{\mathrm{J}}{\mathrm{eV}})^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19}\mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = 1.23 \cdot 10^{-10}\mathrm{m} = 1.23\text{Å}$$

For 12keV,

$$\lambda = E^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19}\mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = (1.2 \cdot 10^{4}\mathrm{eV} \cdot 1.602 \cdot 10^{-19}\,\tfrac{\mathrm{J}}{\mathrm{eV}})^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19}\mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = 1.12 \cdot 10^{-11}\mathrm{m} = 0.112\text{Å}$$

The resolution on a visible microscope is dependent on the wavelength of the light which is around 5000Å; so, the much smaller electron wavelengths provide much better resolution.

Prob. 2.7

Show that τ is the average lifetime in exponential radioactive decay.

The probability of finding an atom in the stable state at time t is $N(t)=N_o \cdot e^{-\frac{t}{\tau}}$. This is analogous to the probability of finding a particle at position x for finding the average.

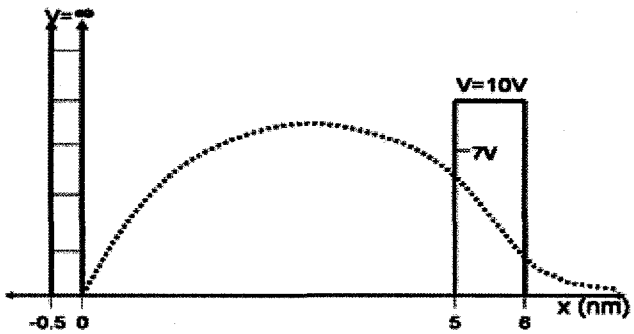
$$\langle t \rangle = \frac{\int_0^\infty t e^{-\frac{t}{\tau}} dt}{\int_0^\infty e^{-\frac{t}{\tau}} dt} = \frac{\tau^2}{\tau} = \tau$$

This may also be found by mimicking the diffusion length calculation (Equations 4-37 to 4-39).

Prob. 2.8

Find the probability of finding an electron at x<0. Is the probability of finding an electron at x>0 zero or non-zero? Is the classical probability of finding an electron at x>6 zero or non?

The energy barrier at x=0 is infinite; so, there is zero probability of finding an electron at x<0 ($|\psi|^2=0$). However, it is possible for electrons to tunnel through the barrier at 5<x<6; so, the probability of finding an electron at x>6 would be quantum mechanically greater than zero ($|\psi|^2>0$) and classical mechanically zero.



Prob. 2.9

Find $4\cdot p_x^{\,2}+2\cdot p_z^{\,2}+\dfrac{7\cdot E}{m}$ *for* $\Psi(x,y,z,t) = A\cdot e^{j(10\cdot x+3\cdot y-4\cdot t)}$.

$$\left\langle p_x^{\,2}\right\rangle = \frac{\int_{-\infty}^{\infty} A^*\cdot e^{-j(10\cdot x+3\cdot y-4\cdot t)}\left(\frac{\hbar}{j}\frac{\partial}{\partial x}\right)^2 A\cdot e^{j(10\cdot x+3\cdot y-4\cdot t)}dx}{\int_{-\infty}^{\infty}|A|^2\, e^{-j(10\cdot x+3\cdot y-4\cdot t)}e^{j(10\cdot x+3\cdot y-4\cdot t)}dx} = 100\cdot\hbar^2$$

$$\left\langle p_z^{\,2}\right\rangle = \frac{\int_{-\infty}^{\infty} A^*\cdot e^{-j(10\cdot x+3\cdot y-4\cdot t)}\left(\frac{\hbar}{j}\frac{\partial}{\partial z}\right)^2 A\cdot e^{j(10\cdot x+3\cdot y-4\cdot t)}dz}{\int_{-\infty}^{\infty}|A|^2\, e^{-j(10\cdot x+3\cdot y-4\cdot t)}e^{j(10\cdot x+3\cdot y-4\cdot t)}dz} = 0$$

$$\left\langle E\right\rangle = \frac{\int_{-\infty}^{\infty} A^*\cdot e^{-j(10\cdot x+3\cdot y-4\cdot t)}\left(-\frac{\hbar}{j}\frac{\partial}{\partial t}\right) A\cdot e^{j(10\cdot x+3\cdot y-4\cdot t)}dt}{\int_{-\infty}^{\infty}|A|^2\, e^{-j(10\cdot x+3\cdot y-4\cdot t)}e^{j(10\cdot x+3\cdot y-4\cdot t)}dt} = 4\cdot\hbar$$

$$4\cdot p_x^{\,2}+2\cdot p_z^{\,2}+\frac{7\cdot E}{m} = 400\hbar^2+\frac{28\hbar}{9.11\cdot 10^{-31}\text{kg}}$$

Prob. 2.10

Find the uncertainty in position (Δx) and momentum (Δp).

$$\Psi(x,t) = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{\pi x}{L}\right) \cdot e^{-2\pi jEt/h} \quad \text{and} \quad \int_0^L \Psi^* \cdot \Psi dx = 1$$

$$\langle x \rangle = \int_0^L \Psi^* \cdot x \cdot \Psi dx = \frac{2}{L}\int_0^L x \cdot \sin^2\left(\frac{\pi x}{L}\right) dx = 0.5L \quad \text{(from problem note)}$$

$$\langle x^2 \rangle = \int_0^L \Psi^* \cdot x \cdot \Psi dx = \frac{2}{L}\int_0^L x^2 \cdot \sin^2\left(\frac{\pi x}{L}\right) dx = 0.28L^2 \quad \text{(from problem note)}$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{0.28L^2 - (0.5L)^2} = 0.17L$$

$$\Delta p \geq \frac{h}{4\pi \cdot \Delta x} = 0.47 \cdot \frac{h}{L}$$

Prob. 2.11

Calculate the first three energy levels for a 10Å quantum well with infinite walls.

$$E_n = \frac{n^2 \cdot \pi^2 \cdot \hbar^2}{2 \cdot m \cdot L^2} = \frac{(6.63 \cdot 10^{-34})^2}{8 \cdot 9.11 \cdot 10^{-31} \cdot (10^{-9})^2} \cdot n^2 = 6.03 \cdot 10^{-20} \cdot n^2$$

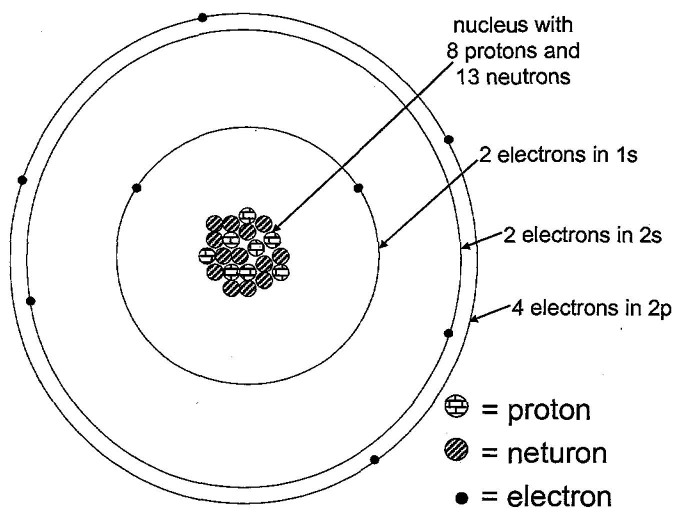
$$E_1 = 6.03 \cdot 10^{-20} J = 0.377 eV$$

$$E_2 = 4 \cdot 0.377 eV = 1.508 eV$$

$$E_3 = 9 \cdot 0.377 eV = 3.393 eV$$

Prob. 2.12

Show schematic of atom with $1s^2 2s^2 2p^4$ and atomic weight 21. Comment on its reactivity.



This atom is chemically reactive because the outer 2p shell is not full. It will tend to try to add two electrons to that outer shell.

Chapter 3 Solutions

Prob. 3.1

Calculate the approximate donor binding energy for GaAs ($\in_r = 13.2$, $m_n^* = 0.067m_0$).

From Equation 3-8 and Appendix II,

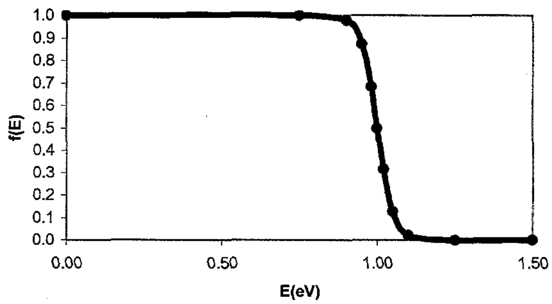
$$E = \frac{m_n^* \cdot q^4}{8 \cdot (\in_0 \in_r)^2 \cdot h^2} = \frac{0.067 \cdot (9.11 \cdot 10^{-31}\text{kg}) \cdot (1.6 \cdot 10^{-19}\text{C})^4}{8 \cdot (8.85 \cdot 10^{-12}\,\frac{\text{F}}{\text{m}} \cdot 13.2)^2 \cdot (6.63 \cdot 10^{-34}\text{J} \cdot \text{s})^2} = 8.34 \cdot 10^{-22}\text{J} = 5.2\text{ meV}$$

Prob. 3.2

Plot Fermi function for E_F=1eV and show the probability of an occupied state ΔE above E_F is equal to the probability of an empty state ΔE below E_F so $f(E_F + \Delta E) = 1 - f(E_F - \Delta E)$.

use $f(E) = \dfrac{1}{1+e^{\frac{E-E_F}{k_B T}}}$ and kT=0.0259eV

E(eV)	(E-EF)/kT	f(E)
0.75	-9.6525	0.99994
0.90	-3.8610	0.97939
0.95	-1.9305	0.87330
0.98	-0.7722	0.68399
1.00	0.0000	0.50000
1.02	0.7722	0.31600
1.05	1.9305	0.12669
1.10	3.8610	0.02061
1.25	9.6525	0.00006



occupation probability above $E_F = f(E_F+\Delta E) = \dfrac{1}{1+e^{\frac{\Delta E}{kT}}}$

empty probability below $E_F = 1 - f(E_F - \Delta E) = 1 - \dfrac{1}{1+e^{\frac{-\Delta E}{kT}}}$

$$1 - f(E_F - \Delta E) = 1 - \frac{1}{1+e^{\frac{-\Delta E}{kT}}} = \frac{e^{\frac{-\Delta E}{kT}}}{1+e^{\frac{-\Delta E}{kT}}} = \frac{1}{e^{\frac{\Delta E}{kT}}+1} = \frac{1}{1+e^{\frac{\Delta E}{kT}}} = f(E_F+\Delta E)$$

This shows that the probability of an occupied state ΔE above E_F is equal to the probability of an empty state ΔE below E_F.

Prob. 3.3

Calculate electron, hole, and intrinsic carrier concentrations.

E_g=1.1eV $N_C=N_V$ $n=10^{15}\,\frac{1}{cm^3}$ E_C-E_d=0.2eV E_C-E_F=0.25eV   T=300K

$n=10^{15}\,\frac{1}{cm^3}$

$$n = N_C \cdot e^{-\frac{E_C-E_F}{kT}} \rightarrow N_C = n \cdot e^{\frac{E_C-E_F}{kT}} = 10^{15}\,\frac{1}{cm^3} \cdot e^{\frac{0.25eV}{0.0259eV}} = 1.56 \cdot 10^{19}\,\frac{1}{cm^3}$$

$$N_V = 1.56 \cdot 10^{19}\,\frac{1}{cm^3}$$

$$p = N_V \cdot e^{-\frac{E_F-E_V}{kT}} = 1.56 \cdot 10^{19}\,\frac{1}{cm^3} \cdot e^{-\frac{0.85eV}{0.0259eV}} = 8.71 \cdot 10^{4}\,\frac{1}{cm^3}$$

$$n_i = \sqrt{n \cdot p} = 9.35 \cdot 10^{9}\,\frac{1}{cm^3} \quad \left(\text{note: } n_i = \sqrt{N_C \cdot N_V} \cdot e^{-\frac{Eg}{2kT}} \text{ may also be used}\right)$$

Prob 3.4

Find temperature at which number of electrons in Γ and X minima are equal.

$$\frac{n_X}{n_\Gamma} = \frac{N_{CX}}{N_{C\Gamma}} e^{-\frac{0.35}{kT}} \text{ from Equation 3-15}$$

Since there are 6 X minima along the <1 0 0> directions, Equation 3-16b gives:

$$N_{cX} \propto 6 \cdot (m_{eX})^{\frac{3}{2}} \propto 6 \cdot (0.30)^{\frac{3}{2}}$$

$$N_{c\Gamma} \propto (m_{e\Gamma})^{\frac{3}{2}} \propto (0.065)^{\frac{3}{2}}$$

$$\frac{n_X}{n_\Gamma} = \frac{6 \cdot (0.30)^{\frac{3}{2}}}{(0.065)^{\frac{3}{2}}} e^{-\frac{0.35}{kT}} = 1 \text{ for } n_\Gamma = n_X$$

$$e^{-\frac{0.35}{kT}} = \frac{(0.065)^{\frac{3}{2}}}{6 \cdot (0.30)^{\frac{3}{2}}} \rightarrow e^{\frac{0.35}{kT}} = \frac{6 \cdot (0.30)^{\frac{3}{2}}}{(0.065)^{\frac{3}{2}}} = 59.4$$

$$\frac{0.35eV}{kT} = \ln(59.4) = 4.09 \rightarrow kT = \frac{0.35eV}{4.09} = 0.0857eV$$

$$T = 988K$$

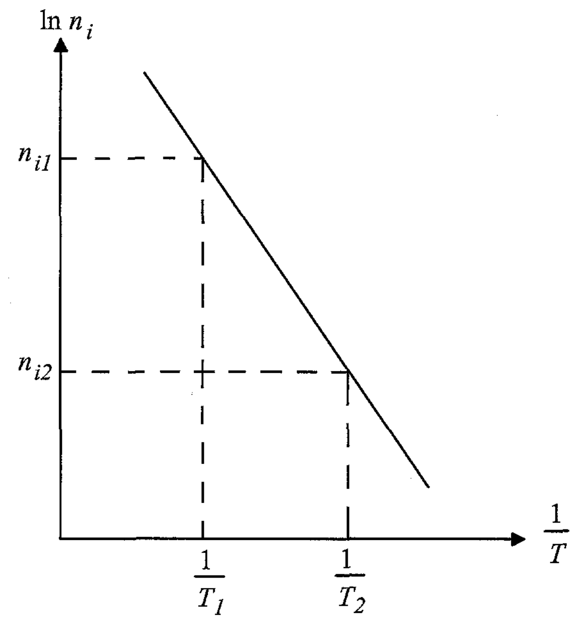
Prob. 3.5

Discuss m for GaAs and GaP. What happens if a Γ valley electron moves to the L valley?*

From Figure 3.10, the curvature of the Γ valley is much greater than L or X. Thus Γ valley electrons have much smaller mass. The light mass Γ electrons in GaAs (μ_n=8500) have higher mobility than the heavy mass X electrons (μ_n=300) in GaP since μ_n is inversely proportional to m*. If light mass electrons in Γ were transferred to the heavier mass L valley at constant energy, they would slow down. The conductivity would decrease (see discussion in Section 10.3).

Prob. 3.6

Find Eg for Si from Figure 3-17.



for n_{i1} and n_{i2} on graph

$$n_{i1} = 3\cdot 10^{14} \qquad \frac{1}{T_1} = 2\cdot 10^{-3}\,\tfrac{1}{K}$$

$$n_{i2} = 10^{8} \qquad \frac{1}{T_2} = 4\cdot 10^{-3}\,\tfrac{1}{K}$$

This result is approximate because the temperature dependences of N_C, N_V, and E_g are neglected.

$$n_i = \sqrt{N_C N_V}\cdot e^{-\frac{E_g}{2kT}} \rightarrow E_g = -2kT\cdot \ln\frac{n_i}{\sqrt{N_C N_V}} \rightarrow \ln n_i = -\frac{E_g}{2kT} + \ln\sqrt{N_C N_V}$$

$$\ln\frac{n_{i1}}{n_{i2}} = \ln n_{i1} - \ln n_{i2} = \left(-\frac{E_g}{2kT_1} + \ln\sqrt{N_C N_V}\right) - \left(-\frac{E_g}{2kT_2} + \ln\sqrt{N_C N_V}\right) = \frac{E_g}{2k}\cdot\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$\text{for Si (see above)} \rightarrow E_g = 2k\cdot\left(\frac{\ln\frac{n_{i1}}{n_{i2}}}{\frac{1}{T_2}-\frac{1}{T_1}}\right) = 2\cdot 8.62\cdot 10^{14}\cdot\left(\frac{\ln\frac{3\cdot 10^{14}}{10^{8}}}{4\cdot 10^{-3}\,\frac{1}{K} - 2\cdot 10^{-3}\,\frac{1}{K}}\right) = 1.3\text{eV}$$

Prob. 3.7

(a) Find N_d for Si with $10^{16} cm^{-3}$ boron atoms and a certain number of donors so E_F-E_i=0.36eV.

$$n_o = n_i e^{\frac{E_F-E_i}{kT}}$$

$$n_o = N_d - N_a \;\rightarrow\; N_d = n_o + N_a = n_i e^{\frac{E_F-E_i}{kT}} + N_a = 1.5\cdot 10^{10}\,\tfrac{1}{cm^3}\cdot e^{\frac{0.36eV}{0.0259eV}} + 10^{16}\,\tfrac{1}{cm^3} = 2.63\cdot 10^{16}\,\tfrac{1}{cm^3}$$

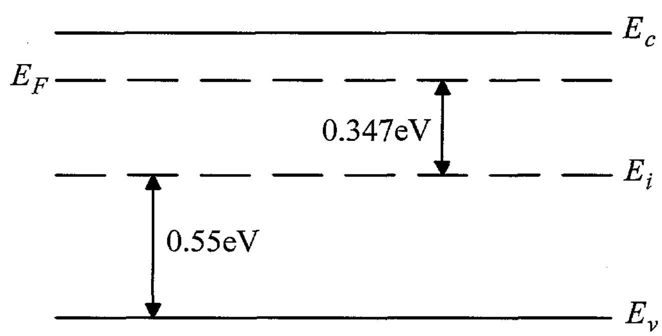
(b) Si with $10^{16} cm^{-3}$ In and a certain number of donors has E_F-E_V=0.26eV. How many In atoms are unionized (i.e.: neutral)?

$$\text{fraction of } E_a \text{ states filled} = f(E_a) = \frac{1}{1+e^{\frac{E_a-E_F}{kT}}} = \frac{1}{1+e^{\frac{0.26eV-0.36eV}{.0259eV}}} = 0.979$$

$$\text{unionized In} = \left[1\text{-}f(E_a)\right]\cdot N_{In} = 0.021\cdot 10^{16}\,\tfrac{1}{cm^3} = 2.1\cdot 10^{14}\,\tfrac{1}{cm^3}$$

Prob. 3.8

Show that Equation 3-25 results from Equation 3-15 and Equation 3-19. Find the position of the Fermi level relative to E_i at 300K for n_o=10^{16} cm^{-3}.



Equation 3-15 → $n_0 = N_C \cdot e^{-\frac{(E_C-E_F)}{kT}}$

$$n_0 = N_C\cdot e^{-\frac{E_C-E_F}{kT}} = N_C\cdot e^{-\frac{E_C-E_i}{kT}}\cdot e^{\frac{E_F-E_i}{kT}} = n_i\cdot e^{\frac{E_F-E_i}{kT}} \quad \text{using 3-21 yields Equation 3-25a}$$

Equation 3-19 → $p_0 = N_V \cdot e^{-\frac{E_F-E_V}{kT}}$

$$p_0 = N_V\cdot e^{-\frac{E_F-E_V}{kT}} = N_V\cdot e^{-\frac{E_i-E_V}{kT}}\cdot e^{\frac{E_i-E_F}{kT}} = n_i\cdot e^{\frac{E_i-E_F}{kT}} \quad \text{using 3-21 yields Equation 3-25b}$$

for Fermi level relative to E_i at 300K for n_o=10^{16} cm^{-3}

$$E_F\text{-}E_i = 0.0259eV\cdot \ln\frac{1.5\cdot 10^{10}}{10^{16}} = 0.347eV$$

Prob. 3.9

Find the displacement of E_i from the middle of E_g for Si at 300K with m_n=1.1m_o and m_p=0.56m_o.

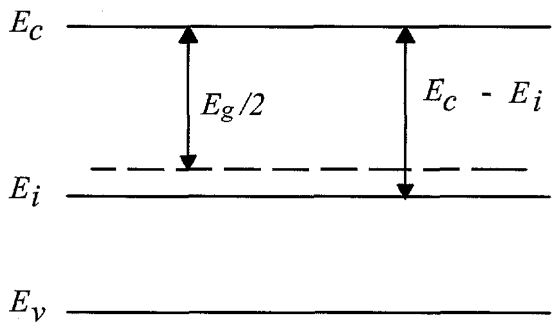
E_i is not exactly in the middle of the gap because the density of states N_C and N_V differ.

$$N_C \cdot e^{-\frac{E_C - E_i}{kT}} = \sqrt{N_C \cdot N_V} \cdot e^{-\frac{E_g}{2kT}}$$ since each equal to n_i in Equation 3-21 and Equation 3-23

$$e^{\frac{-E_C - E_i + \frac{E_g}{2}}{kT}} = \sqrt{\frac{N_V}{N_C}} = \left(\frac{m_p^*}{m_n^*}\right)^{\frac{3}{4}}$$

$$\frac{E_g}{2} - \left(E_C + E_i\right) = kT \cdot \frac{3}{4} \cdot \ln\frac{m_p^*}{m_n^*} = 0.0259\text{eV} \cdot \frac{3}{4} \cdot \ln\frac{0.56}{1.1} = -0.013\text{eV for Si at 300K}$$

So, E_i is about one half kT below the center of the band gap.



Prob. 3.10

Is Si doped with 10^{15} donors per cm^3 n-type at 400K? Is Ge?

At T=400K, Figure 3-17 indicates that $n \gg n_i$ for Si doped with $N_d = 10^{15}$ cm^{-3}; so, the Si would be n-type. At T=400K, Figure 3-17 indicates that $n \approx n_i \approx 10^{15}$ cm^{-3} for Ge doped with $N_d = 10^{15}$ cm^{-3}; so, the Ge would require more donors for useful n-type doping.

Prob. 3.11

Calculate electron, hole, and intrinsic carrier concentrations. Sketch band diagram.

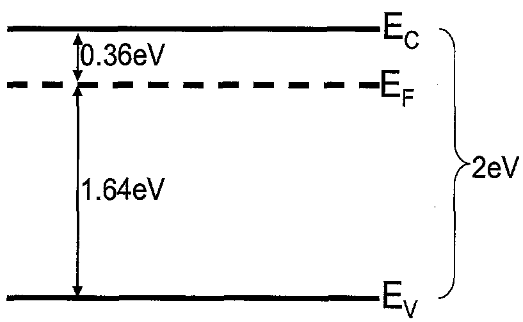
$N_C=10^{19}\frac{1}{cm^3}$ $\quad N_V=5\cdot10^{18}\frac{1}{cm^3}$ $\quad E_g=2eV$ $\quad T=627°C=900K$ $\quad n=10^{17}\frac{1}{cm^3}$

$$n=N_C\cdot e^{-\frac{E_C-E_F}{kT}} \rightarrow E_C\text{-}E_F=-k\cdot T\cdot\ln\left(\frac{n}{N_C}\right)=-0.078eV\cdot\ln\left(\frac{10^{17}}{10^{19}}\right)=0.36eV$$

$$E_F-E_V=\left[(E_C-E_V)\text{-}(E_C-E_F)\right]=\left[E_g\text{-}(E_C-E_F)\right]=\left[2eV\text{-}0.36eV\right]=1.64eV$$

$$p=N_V\cdot e^{-\frac{E_F-E_V}{kT}}=5\cdot10^{18}\frac{1}{cm^3}\cdot e^{-\frac{1.64eV}{0.078eV}}=3.7\cdot10^{9}\frac{1}{cm^3}$$

$$n_i=\sqrt{n\cdot p}=1.9\cdot10^{13}\frac{1}{cm^3}\quad\left(\text{note: } n_i=\sqrt{N_C\cdot N_V}\cdot e^{-\frac{Eg}{2kT}}\text{ may also be used}\right)$$



Prob. 3.12

(a) Show that the minimum conductivity of a semiconductor occurs when $n_0 = n_i\sqrt{\mu_p / \mu_n}$.

$$\sigma = q\cdot(n\cdot\mu_n + p\cdot\mu_p) = q\cdot\left(n\cdot\mu_n + \frac{n_i^2}{n}\cdot\mu_p\right)$$

$$\frac{\partial\sigma}{\partial n} = q\cdot\left(\mu_n - \frac{n_i^2}{n^2}\cdot\mu_p\right) = 0 \text{ for minimum conductivity at electron concentration } n_{min}$$

$$n_{min}^2 = n_i^2\cdot\frac{\mu_p}{\mu_n} \rightarrow n_{min} = n_i\cdot\sqrt{\frac{\mu_p}{\mu_n}}$$

(b) What is σ_{min}.

$$\sigma_{min} = q\cdot\left(n_{min}\cdot\mu_n + \frac{n_i^2}{n_{min}}\cdot\mu_p\right) = q\cdot\left(n_i\cdot\sqrt{\frac{\mu_p}{\mu_n}}\cdot\mu_n + \frac{n_i^2}{n_i\cdot\sqrt{\frac{\mu_p}{\mu_n}}}\cdot\mu_p\right) = 2\cdot q\cdot n_i\cdot\sqrt{\mu_n\cdot\mu_p}$$

(c) Calculate σ_{min} and σ_i for Si.

$$\sigma_{min} = 2\cdot q\cdot n_i\cdot\sqrt{\mu_n\cdot\mu_p} = 2\cdot 1.6\cdot 10^{-19}\text{C}\cdot 1.5\cdot 10^{10}\,\tfrac{1}{\text{cm}^3}\cdot\sqrt{1350\,\tfrac{\text{cm}^2}{\text{V·s}}\cdot 480\,\tfrac{\text{cm}^2}{\text{V·s}}} = 3.9\cdot 10^{-6}\,\tfrac{1}{\Omega\cdot\text{cm}}$$

$$\sigma_i = q\cdot(n_i\cdot\mu_n + n_i\cdot\mu_p) = 2\cdot 1.6\cdot 10^{-19}\text{C}\cdot 1.5\cdot 10^{10}\,\tfrac{1}{\text{cm}^3}\cdot(1350\,\tfrac{\text{cm}^2}{\text{V·s}}\cdot 480\,\tfrac{\text{cm}^2}{\text{V·s}}) = 4.4\cdot 10^{-6}\,\tfrac{1}{\Omega\cdot\text{cm}}$$

or the reciprocal of ρ_i in Appendix III may be taken

Prob. 3.13

(a) Find the current at 300K with 10V applied for a Si bar 1 μm long, 100 μm² in cross sectional area, and doped with 10^{17} cm^{-3} antimony.

With , $\mathcal{E} = \dfrac{10\text{V}}{10^{-4}\text{cm}} = 10^5 \frac{\text{V}}{\text{cm}}$ the sample is in the velocity saturation regime.

From Figure 3-24, $v_s = 10^7 \frac{\text{cm}}{\text{s}}$.

$$I = q \cdot A \cdot n \cdot v_s = 1.6 \cdot 10^{-19}\text{C} \cdot 10^{-6}\text{cm}^2 \cdot 10^{17} \tfrac{1}{\text{cm}^3} \cdot 10^7 \tfrac{\text{cm}}{\text{s}} = 0.16\text{A}$$

(b) In pure Si, find time for an electron to drift 1μm in an electric field of $100 \frac{V}{cm}$? For $10^5 \frac{V}{cm}$?

from Appendix III, $\mu_n = 1350 \frac{\text{cm}^2}{\text{V}\cdot\text{s}}$

$$v_d = \mu_n \cdot \mathcal{E} = 1350 \tfrac{\text{cm}^2}{\text{V}\cdot\text{s}} \cdot 100 \tfrac{\text{V}}{\text{cm}} = 1.35 \cdot 10^5 \tfrac{\text{cm}}{\text{s}}$$

low field: $$t = \frac{L}{v_d} = \frac{10^{-4}\text{cm}}{1.35 \cdot 10^5 \frac{\text{cm}}{\text{s}}} = 7.4 \cdot 10^{-10}\text{s} = 0.74\text{ ns}$$

scattering limited velocity $v_s = 10^7 \frac{\text{cm}}{\text{s}}$ from Figure 3-24

high field: $$t = \frac{L}{v_d} = \frac{10^{-4}\text{cm}}{10^7 \frac{\text{cm}}{\text{s}}} = 10^{-11}\text{s} = 10\text{ ps}$$

Prob. 3.14

(a) Find n_o and ρ for Si doped with 10^{17} cm^{-3} boron.

$N_a \gg n_i$ so $p_o = N_a = 10^{17} \frac{1}{\text{cm}^3}$ may be assumed

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \cdot 10^{10} \frac{1}{\text{cm}^3})^2}{10^{17} \frac{1}{\text{cm}^3}} = 2.25 \cdot 10^3 \tfrac{1}{\text{cm}^3}$$

$N_a = 10^{17} \frac{1}{\text{cm}^3}$ gives $\mu_p = 250 \frac{\text{cm}^2}{\text{V}\cdot\text{s}}$ from Figure 3-23

$$\sigma = q \cdot \mu_p \cdot p_o = 1.6 \cdot 10^{-19}\text{C} \cdot 250 \tfrac{\text{cm}^2}{\text{V}\cdot\text{s}} \cdot 10^{17} \tfrac{1}{\text{cm}^3} = 4.0 \tfrac{1}{\Omega\cdot\text{cm}}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{4.0 \frac{1}{\Omega\cdot\text{cm}}} = 0.25\ \Omega\cdot\text{cm}$$

(b) Find n_o for Ge doped with $3 \cdot 10^{13}$ Sb atoms per cm³.

Assuming N_a is zero and using Equation 3-28 gives $n_o = \dfrac{n_i^2}{n_o} + N_d$ or $n_o^2 - N_d \cdot n_o - n_i^2 = 0$.

By quadratic formula,

$$n_o = \frac{N_d \pm \sqrt{N_d^2 + 4 \cdot n_i^2}}{2} = \frac{3 \cdot 10^{13} \frac{1}{\text{cm}^3} \pm \sqrt{(3 \cdot 10^{13} \frac{1}{\text{cm}^3})^2 + 4 \cdot (3 \cdot 10^{13} \frac{1}{\text{cm}^3})^2}}{2} = 4.4 \cdot 10^{13} \tfrac{1}{\text{cm}^3}$$

Prob. 3.15

Find the current density for applied voltages 2.5V and 2500V respectively.

For 2.5V,

$$\sigma = q \cdot \mu_n \cdot n_0 \text{ (since } n_o \gg n_i) = 1.6 \cdot 10^{-19}\text{C} \cdot 1500 \tfrac{\text{cm}^2}{\text{V}\cdot\text{s}} \cdot 10^{15} \tfrac{1}{\text{cm}^3} = 0.24 \tfrac{1}{\Omega\cdot\text{cm}}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{0.24 \frac{1}{\Omega\cdot\text{cm}}} = 4.17\ \Omega\cdot\text{cm}$$

$$R = \frac{\rho \cdot L}{A} = \frac{4.17\ \Omega\cdot\text{cm} \cdot 5\cdot10^{-4}\text{cm}}{A} = \frac{2.83\cdot10^{3}\Omega\cdot\text{cm}^2}{A}$$

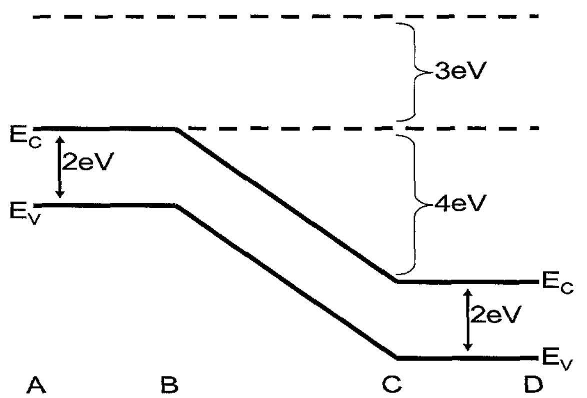
$$\frac{I}{A} = \frac{V}{R\cdot A} = \frac{2.5\text{V}}{2.83\cdot10^{3}\Omega\cdot\text{cm}^2} = 8.82\cdot10^{-2}\tfrac{\text{A}}{\text{cm}^2} \text{ for 2.5V}$$

For 2500V, $\mathcal{E} = \dfrac{2500\text{V}}{5\cdot10^{-4}\text{cm}} = 5\cdot10^{6}\tfrac{\text{V}}{\text{cm}}$ which is in the velocity saturation regime.

$$\frac{I}{A} = q\cdot n\cdot v_s = 1.6\cdot10^{-19}\text{C}\cdot10^{15}\tfrac{1}{\text{cm}^3}\cdot10^{7}\tfrac{\text{cm}}{\text{s}} = 1.6\cdot10^{3}\tfrac{\text{A}}{\text{cm}^2}$$

Prob. 3.16

Draw a band diagram and give the wave function at D in terms of the normalization constant.



General Wavefunction: $\Psi(x,t) = \alpha \cdot e^{i(kx-\omega t)}$

$$\text{Energy at D} = \hbar\cdot\omega = \frac{\hbar^2\cdot k^2}{2\cdot m_o} = 3\text{eV} + 4\text{eV} = 7\text{eV} = 7\text{eV}\cdot1.6\cdot10^{-19}\tfrac{\text{J}}{\text{eV}} = 1.12\cdot10^{-18}\text{J}$$

$$\omega = \frac{1.12\cdot10^{-18}\text{J}}{\hbar} = \frac{1.12\cdot10^{-18}\text{J}}{1.06\cdot10^{-34}\text{J}\cdot\text{s}} = 1.06\cdot10^{16}\text{ Hz}$$

$$k = \sqrt{\frac{1.12\cdot10^{-18}\text{J}\cdot2\cdot m_o}{\hbar^2}} = \sqrt{\frac{1.12\cdot10^{-18}\text{J}\cdot2\cdot9.11\cdot10^{-31}\text{kg}}{(1.06\cdot10^{-34}\text{J}\cdot\text{s})^2}} = 1.35\cdot10^{10}\tfrac{1}{\text{m}}$$

Wavefunction at D: $\Psi(x,t) = \alpha\cdot e^{i(1.35\cdot10^{10}\frac{1}{\text{m}}\cdot x\ -\ 1.06\cdot10^{16}\text{Hz}\cdot t)}$ where α is the normalization constant

Prob. 3.17

Show the electron drift velocity in pure Si for $100\frac{V}{cm}$ *is less than* v_{th}. *Comment on the electron drift velocity for* $10^4\frac{V}{cm}$.

$$v_d = \mathcal{E}\cdot\mu_n = 100\tfrac{V}{cm}\cdot 1350\tfrac{cm^2}{V\cdot s} = 1.35\cdot 10^5\tfrac{cm}{s}$$

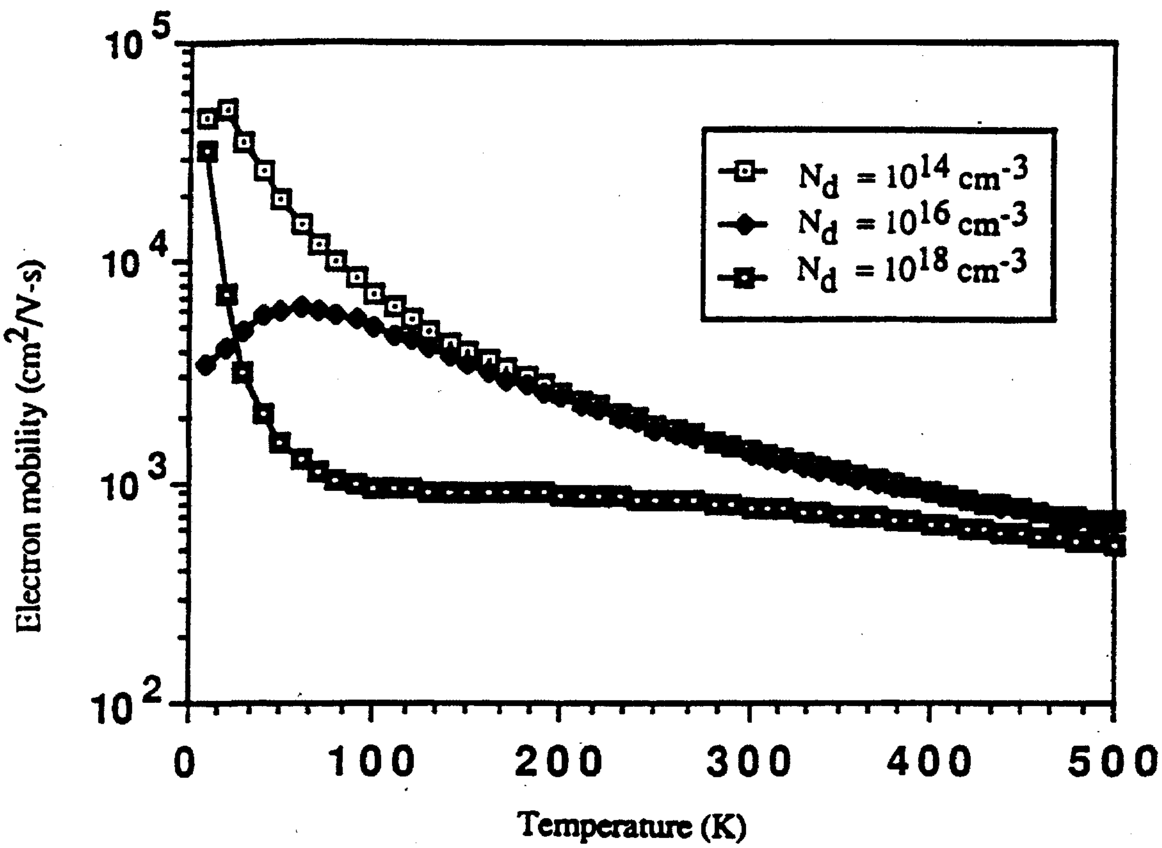
$$\tfrac{1}{2}m_o v_{th}^2 = kT \;\rightarrow\; v_{th} = \sqrt{\frac{2kT}{m_o}} = 9.54\cdot 10^6\tfrac{cm}{s}$$

so, $v_d < v_{th}$ for $100\frac{V}{cm}$

For $10^4\frac{V}{cm}$, the equivalent calculation for drift velocity assuming constant μ_n gives $1.35\cdot 10^7\frac{cm}{s}$ which is larger than the thermal velocity. The device is in velocity saturation.

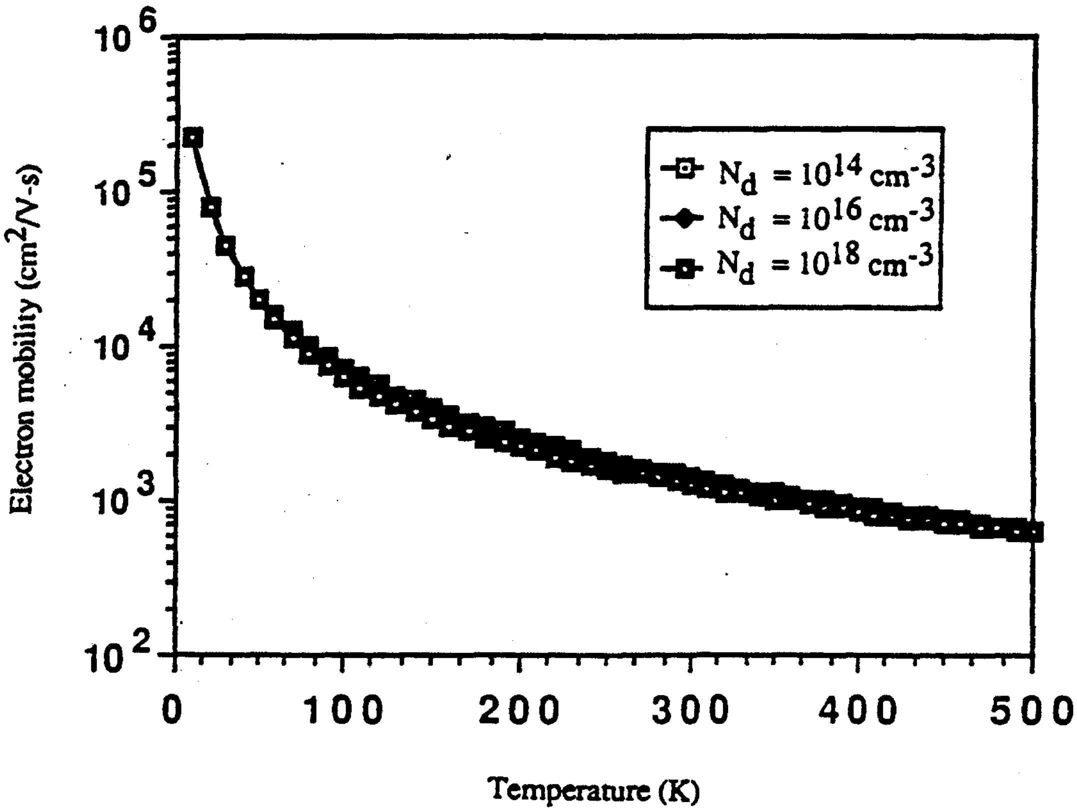
Prob. 3.18

Plot mobility versus temperature.



Prob. 3.19

Repeat plot of mobility versus temperature in 3.18 considering carrier freeze out.



When freeze-out occurs, ionized impurity scattering disappears, and only the phonon scattering remains. In Si, other mechanisms, including neutral impurity scattering, contribute to mobility.

Prob. 3.20

Find the hole concentration and mobility with Hall measurement on a p-type semiconductor bar.

The voltage measured is the Hall voltage plus the ohmic drop.
The sign of V_H changes with the magnetic field, but the ohmic voltage does not.

$$V_{Hall} = \frac{V_{H1} - V_{H2}}{2} = \frac{3.2\text{mV} - (-2.8\text{mV})}{2} = 3.0\text{mV}$$

ohmic drop = 3.2mV-3.0mV = 0.2mV

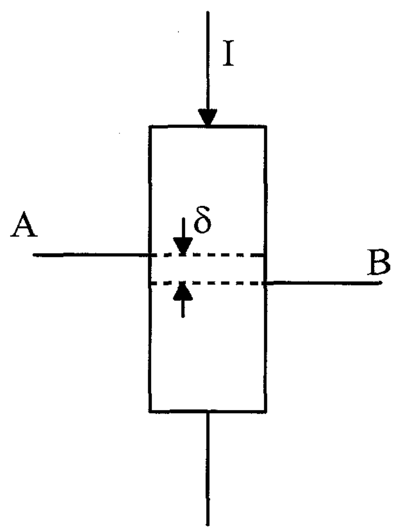
$$p_o = \frac{I_x \cdot B_z}{q \cdot t \cdot V_{AB}} \text{ (Equation 3-50)} = \frac{3 \cdot 10^{-3}\text{A} \cdot 10^{-4}\,\frac{\text{Wb}}{\text{cm}^2}}{1.6 \cdot 10^{-19}\text{C} \cdot 2 \cdot 10^{-3}\text{cm} \cdot 3 \cdot 10^{-3}\text{V}} = 3.125 \cdot 10^{17}\,\frac{1}{\text{cm}^3}$$

$$\rho = \frac{V_{CD} \cdot w \cdot t}{I_x \cdot L} \text{ (Equation 3-51)} = \frac{2 \cdot 10^{-4}\text{V} \cdot 5 \cdot 10^{-2}\text{cm} \cdot 2 \cdot 10^{-3}\text{cm}}{3 \cdot 10^{-3}\text{A} \cdot 2 \cdot 10^{-4}\text{cm}} = 0.033\Omega \cdot \text{cm}$$

$$\mu_p = \frac{\sigma}{q \cdot p_0} = \frac{1}{q \cdot \rho \cdot p_0} = \frac{1}{1.6 \cdot 10^{-19}\text{C} \cdot 0.033\Omega \cdot \text{cm} \cdot 3.125 \cdot 10^{17}\,\frac{1}{\text{cm}^3}} = 600\,\frac{\text{cm}^2}{\text{V}\cdot\text{s}}$$

Prob. 3.21

Find V_H with Hall probes misaligned.



Displacement of the probes by an amount δ give a small IR drop V_δ in addition to V_H. The Hall voltage reverses when the magnetic field is reversed; however, V_δ does not depend on the direction of the magnetic field.

for positive magnetic field: $V_{AB}^{+} = V_H + V_\delta$

for negative magnetic field: $V_{AB}^{-} = -V_H + V_\delta$

$$V_{AB}^{+} - V_{AB}^{-} = 2 \cdot V_H$$

$$V_H = \frac{V_{AB}^{+} - V_{AB}^{-}}{2}$$

So, the true Hall voltage may be obtained by subtracting the voltage with a negative magnetic field from the voltage with a positive magnetic field and dividing by 2.

Prob. 3.22

Find expected resistivity and Hall voltage.

$\mu_n = 700 \frac{\text{cm}^2}{\text{V·s}}$ from Figure 3-23

$\sigma = q \cdot \mu_n \cdot n_o$ (p_0 is negligible) $= 1.6 \cdot 10^{-19}\text{C} \cdot 700 \frac{\text{cm}^2}{\text{V·s}} \cdot 10^{17} \frac{1}{\text{cm}^3} = 11.2 \frac{1}{\Omega\cdot\text{cm}}$

$$\rho = \frac{1}{\sigma} = \frac{1}{11.2 \frac{1}{\Omega\cdot\text{cm}}} = 0.0893\ \Omega \cdot \text{cm}$$

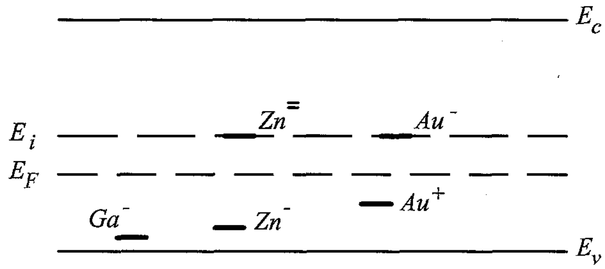
$$R_H = -\frac{1}{q \cdot n_o} = -\frac{1}{1.6 \cdot 10^{-19}\text{C} \cdot 10^{17} \frac{1}{\text{cm}^3}} = -62.5 \frac{\text{cm}^3}{\text{C}}$$

From Equations 3-49 and 3-52, $V_{AB} = \frac{I_x \cdot B_z \cdot R_H}{t} = \frac{10^{-3}\text{A} \cdot 10^{-5} \frac{\text{Wb}}{\text{cm}^2} \cdot (-62.5 \frac{\text{cm}^3}{\text{C}})}{10^{-2}\text{cm}} = -62.5 \mu\text{V}$

Chapter 4 Solutions

Prob. 4.1

State expected charge state for Ga, Zn, and Au in Si sample with E_F 0.4eV above valence band.



From Figure 4-9,
Ga^-: $E_F >$ Ga^- so singly negative
Zn^-: $E_F > Zn^-$ but $E_F < Zn^=$ so singly negative
Au^o: $Au^+ < E_F < Au^-$ so neutral

Prob. 4.2

Find the separation of the quasi-Fermi levels and the change of conductivity when shining light.

The light induced electron-hole pair concentration is determined by:

$$\delta n = \delta p = g_{op} \cdot \tau = 10^{19} \tfrac{1}{cm^3 \cdot s} \cdot 10^{-5} s = 10^{14} \tfrac{1}{cm^3}$$

$\delta n \ll$ dopant concentration of $n_o = 10^{16} \tfrac{1}{cm^3}$ so low level

$$n = n_o + \delta n = 10^{16} \tfrac{1}{cm^3} + 10^{14} \tfrac{1}{cm^3} \approx 10^{16} \tfrac{1}{cm^3}$$

$$p = p_o + \delta p = \frac{n_i^2}{n_o} + \delta p = \frac{(1.5 \cdot 10^{10} \tfrac{1}{cm^3})^2}{10^{16} \tfrac{1}{cm^3}} + 10^{14} \tfrac{1}{cm^3} \approx 10^{14} \tfrac{1}{cm^3}$$

$$kT \text{ for } 450K = 0.0259eV \cdot \frac{450K}{300K} = 0.039eV$$

$$F_n - E_i = kT \cdot \ln\left(\frac{n}{n_i}\right) = 0.039eV \cdot \ln\left(\frac{10^{16} \tfrac{1}{cm^3}}{10^{14} \tfrac{1}{cm^3}}\right) = 0.18eV$$

$$E_i - F_p = kT \cdot \ln\left(\frac{p}{n_i}\right) = 0.039eV \cdot \ln\left(\frac{10^{14} \tfrac{1}{cm^3}}{10^{14} \tfrac{1}{cm^3}}\right) = 0eV$$

$$F_n - F_p = 0.18eV$$

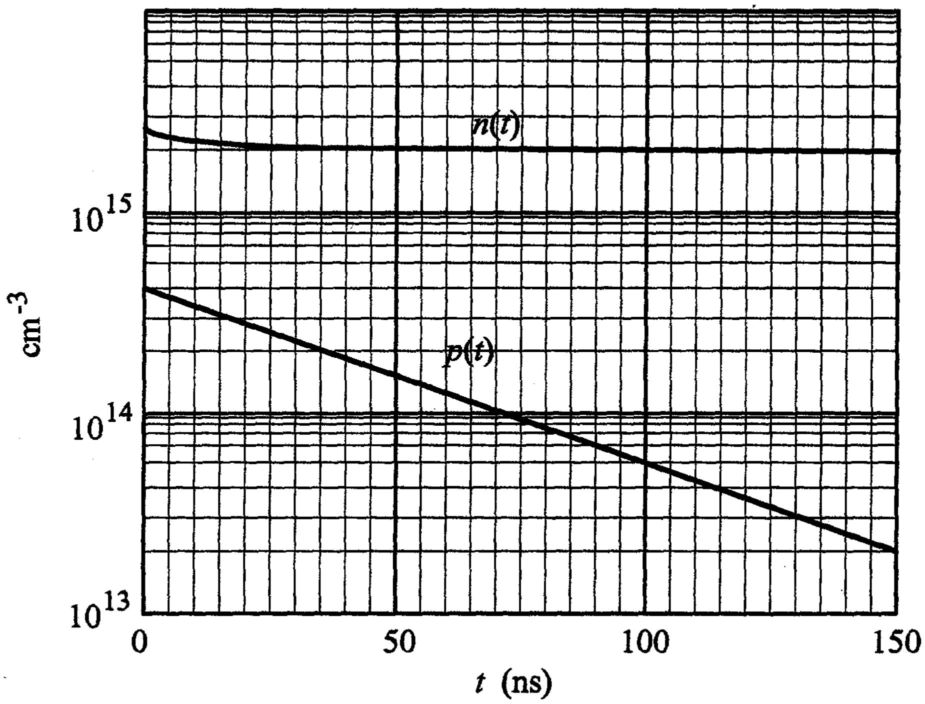
$$\mu_n = \frac{D_n}{\frac{kT}{q}} = \frac{36 \tfrac{cm^2}{s}}{0.039V} = 927 \tfrac{cm^2}{V \cdot s}$$

$$\mu_p = \frac{D_p}{\frac{kT}{q}} = \frac{12 \tfrac{cm^2}{s}}{0.039V} = 309 \tfrac{cm^2}{V \cdot s}$$

$$\Delta\sigma = q \cdot (\mu_n \cdot \delta n + \mu_p \cdot \delta p) = 1.6 \cdot 10^{-19} C \cdot (927 \tfrac{cm^2}{V \cdot s} \cdot 10^{14} \tfrac{1}{cm^3} + 309 \tfrac{cm^2}{V \cdot s} \cdot 10^{14} \tfrac{1}{cm^3}) = 0.0198 \tfrac{1}{\Omega \cdot cm}$$

Prob. 4.3

Plot n(t) and p(t) for τ=5μs Si with τ=5μs, $n_0 = 2 \cdot 10^{15} \frac{1}{cm^3}$, *and* $\Delta n = \Delta p = 4 \cdot 10^{14} \frac{1}{cm^3}$ *at t=0.*



Prob. 4.4

For the sample in 4.3, find α_r *and use it to find Δn for* $g_{op} = 10^{19} \frac{EHP}{cm^3 \cdot s}$.

$$\alpha_r = \frac{1}{\tau \cdot n_o} = \frac{1}{5 \cdot 10^{-6} s \cdot 2 \cdot 10^{15} \frac{1}{cm^3}} = 10^{-10} \frac{cm^3}{s}$$

$$g_{op} = \alpha_r \cdot (n_o \cdot \delta n + \delta n^2) = 10^{-10} \frac{cm^3}{s} \cdot (2 \cdot 10^{15} \frac{1}{cm^3} \cdot \delta n + \delta n^2) = 10^{19} \frac{1}{cm^3 \cdot s}$$

$$\delta n^2 + 2 \cdot 10^{15} \frac{1}{cm^3} \cdot \delta n - 10^{29} \frac{s}{cm^6} = 0 \rightarrow \delta n = 5 \cdot 10^{13} \frac{1}{cm^3} \approx \Delta n$$

Prob. 4.5

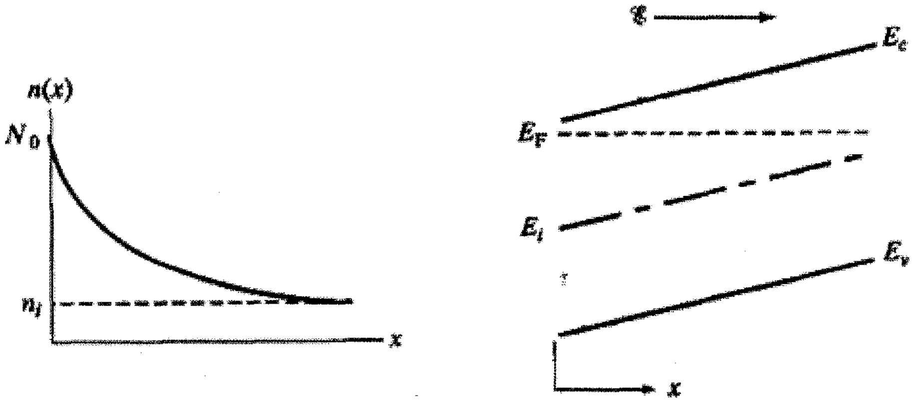
Find expression for $\mathcal{E}(x)$, solve at $a = \frac{1}{\mu m}$, and sketch the band diagram indicating $\mathcal{E}$.

(a) $\mathcal{E}(x) = -\dfrac{D_n}{\mu_n} \cdot \dfrac{\frac{\partial n}{\partial x}}{n} = -\dfrac{kT}{q} \cdot \dfrac{N_o \cdot (-a) \cdot e^{-ax}}{N_o \cdot e^{-ax}} = \dfrac{kT \cdot a}{q}$

$\mathcal{E}$ depends on a but not x or N_o

(b) $\mathcal{E}(1\frac{1}{\mu m}) = 0.0259V \cdot 10^4 \frac{1}{cm} = 259 \frac{V}{cm}$

(c)



Prob. 4.6

Find the separation of the quasi-Fermi levels and the change of conductivity when shining light.

The light induced electron-hole pair concentration is determined by:

$\delta n = \delta p = g_{op} \cdot \tau = 10^{19} \frac{1}{cm^3 \cdot s} \cdot 10^{-5} s = 10^{14} \frac{1}{cm^3}$

$\delta n \ll$ dopant concentration of $n_o = 10^{15} \frac{1}{cm^3}$ so low level

$n = n_o + \delta n = 10^{15} \frac{1}{cm^3} + 10^{14} \frac{1}{cm^3} = 1.1 \cdot 10^{15} \frac{1}{cm^3}$

$p = p_o + \delta p = \dfrac{n_i^2}{n_o} + \delta p = \dfrac{(1.5 \cdot 10^{10} \frac{1}{cm^3})^2}{10^{15} \frac{1}{cm^3}} + 10^{14} \frac{1}{cm^3} \approx 10^{14} \frac{1}{cm^3}$

$\mu_n = 1300 \frac{cm^2}{V \cdot s}$ from Figure 3-23

$\mu_p = \dfrac{D_p}{\frac{kT}{q}} = \dfrac{12 \frac{cm^2}{s}}{0.0259V} = 463 \frac{cm^2}{V \cdot s}$

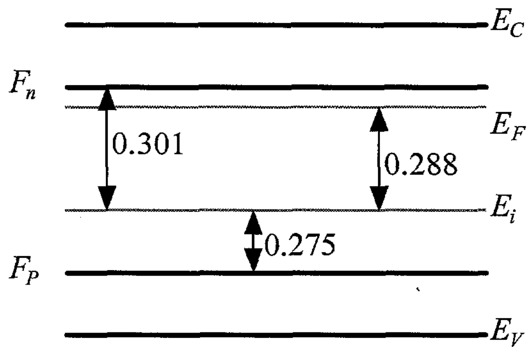
quasi-Fermi level separation $= F_n - F_p = kT \cdot \ln\left(\dfrac{n \cdot p}{n_i^2}\right) = 0.0259eV \cdot \ln\left(\dfrac{1.1 \cdot 10^{15} \frac{1}{cm^3} \cdot 10^{14} \frac{1}{cm^3}}{(1.5 \cdot 10^{10} \frac{1}{cm^3})^2}\right) = 0.518eV$

$\Delta\sigma = q \cdot (\mu_n \cdot \delta n + \mu_p \cdot \delta p) = 1.6 \cdot 10^{-19} C \cdot (1300 \frac{cm^2}{V \cdot s} \cdot 10^{14} \frac{1}{cm^3} + 463 \frac{cm^2}{V \cdot s} \cdot 10^{14} \frac{1}{cm^3}) = 0.0282 \frac{1}{\Omega \cdot cm}$

Prob. 4.7

Calculate the quasi-Fermi level separation and draw a band diagram for steadily illuminated n-type Si .

The light induced electron-hole pair concentration is determined by:



$\delta n = \delta p = g_{op} \cdot \tau = 10^{21} \frac{1}{cm^3 \cdot s} \cdot 10^{-6} s = 10^{15} \frac{1}{cm^3}$

$\delta n \approx$ dopant concentration of $n_o = 10^{15} \frac{1}{cm^3}$ which is comparable with N_d

$\rightarrow$ so NOT low level and δn^2 cannot be neglected

$$\alpha_r = \frac{1}{\tau_n \cdot n_o} = \frac{1}{10^{-6} s \cdot 10^{15} \frac{1}{cm^3}} = 10^{-9} \frac{cm^3}{s}$$

$g_{op} = \alpha_r \cdot n_o \cdot \delta n + \alpha_r \cdot \delta n^2$

$10^{21} = 10^{-9} \frac{cm^3}{s} \cdot 10^{15} \frac{1}{cm^3} \cdot \delta n + 10^{-9} \frac{cm^3}{s} \cdot \delta n^2$

solve for $\delta n = 6.18 \cdot 10^{14} \frac{1}{cm^3} = \delta p$

$$F_n - E_i = kT \cdot \ln \frac{n_o + \delta n}{n_i} = 0.0259 eV \cdot \ln \frac{10^{15} \frac{1}{cm^3} + 6.18 \cdot 10^{14} \frac{1}{cm^3}}{1.5 \cdot 10^{10} \frac{1}{cm^3}} = 0.301 eV$$

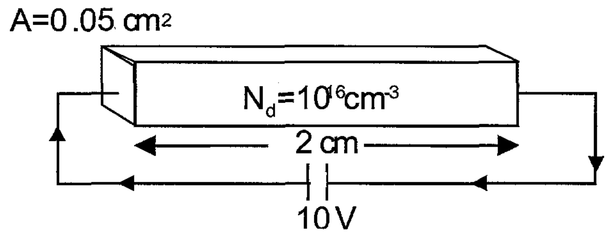
$$E_i - F_P = kT \cdot \ln \frac{\delta p}{n_i} = 0.0259 eV \cdot \ln \frac{6.18 \cdot 10^{14} \frac{1}{cm^3}}{1.5 \cdot 10^{10} \frac{1}{cm^3}} = 0.276 eV$$

$$E_F - E_i = kT \cdot \ln \frac{n_o}{n_i} = 0.0259 eV \cdot \ln \frac{10^{15} \frac{1}{cm^3}}{1.5 \cdot 10^{10} \frac{1}{cm^3}} = 0.288 eV$$

quasi-Fermi level separation = $F_n - F_p = 0.301 eV + 0.276 eV = 0.577 eV$

Prob. 4.8

Find the current with 10V with no light applied. 10V with light applied, and 100,000V with light applied for the doped Si bar.



$$\alpha_r = \frac{1}{\tau \cdot n_o} = \frac{1}{10^{-4}s \cdot 10^{16}\frac{1}{cm^3}} = 10^{-12}\,\frac{cm^3}{s}$$

$$g_{op} = \alpha_r \cdot n_o \cdot \delta n + \alpha_r \cdot \delta n^2 \;\rightarrow\; 10^{20}\frac{1}{cm^3 \cdot s} = 10^{-12}\frac{cm^3}{s}\left(10^{16}\frac{1}{cm^3}\cdot \delta n + \delta n^2\right)$$

$$\delta n^2 + 10^{16}\frac{cm^3}{s}\cdot \delta n - 10^{32}\frac{1}{cm^3 \cdot s} = 0$$

$$\delta n = 10^{16}\frac{1}{cm^3}\left(\frac{-1 \pm \sqrt{1+4}}{2}\right)$$

$$\delta n = 6.18 \cdot 10^{15}\frac{1}{cm^3} = \delta p$$

10V and no light:

$$\mathcal{E} = \frac{10V}{2cm} = 5\frac{V}{cm}$$

$\mu_n = 1070\frac{cm^2}{V \cdot s}$ from Fig 3-23

$$I = A \cdot q \cdot n_0 \cdot \mu_n \cdot \mathcal{E} = 0.05cm^2 \cdot 1.609 \cdot 10^{-19}C \cdot 10^{15}\frac{1}{cm^3}\cdot 1070\frac{cm^2}{V \cdot s}\cdot 5\frac{V}{cm} = 0.428A$$

10V and light:

$$I = A \cdot q \cdot \left[(n_0 + \delta n)\cdot \mu_n + \delta p \cdot \mu_p\right]\cdot \mathcal{E}$$

$$I = 0.05cm^2 \cdot 1.609 \cdot 10^{-19}C \cdot \left[\left(10^{15}\frac{1}{cm^3} + 6.18 \cdot 10^{15}\frac{1}{cm^3}\right)\cdot 1070\frac{cm^2}{V \cdot s} + 6.18 \cdot 10^{15}\frac{1}{cm^3}\cdot 550\frac{cm^2}{V \cdot s}\right]\cdot 5\frac{V}{cm}$$

$$I = 0.816A$$

100,000V and light:

$$v_s = 10^7\frac{cm}{s} \text{ and } \mathcal{E} = \frac{100{,}000V}{2cm} = 50{,}000\frac{V}{cm}$$

$$I = A \cdot q \cdot \left[(n_0 + \delta n)\cdot v_s + \delta p \cdot \mu_p \cdot \mathcal{E}\right]$$

$$I = 0.05cm^2 \cdot 1.609 \cdot 10^{-19}C \cdot \left[\left(10^{15}\frac{1}{cm^3} + 6.18 \cdot 10^{15}\frac{1}{cm^3}\right)\cdot 10^7\frac{cm}{s} + 6.18 \cdot 10^{15}\frac{1}{cm^3}\cdot 550\frac{cm^2}{V \cdot s}\cdot 5\frac{V}{cm}\right]$$

$$I = 2.53 \cdot 10^3 A$$

Prob. 4.9

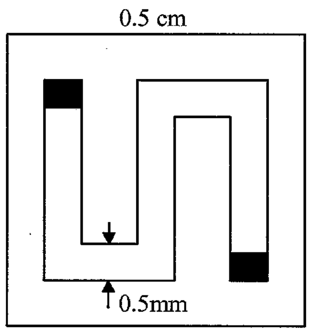
Design a 5μm CdS photoconductor with 10MΩ dark resistance in a 0.5cm square.

In the dark neglecting p_o,

$$\rho = \frac{1}{\sigma} = \frac{1}{q\cdot\mu_n\cdot n_o} = \frac{1}{1.609\cdot10^{-19}\text{C}\cdot 250\,\frac{\text{cm}^2}{\text{V}\cdot\text{s}}\cdot 10^{14}\,\frac{1}{\text{cm}^3}} = 250\Omega\cdot\text{cm}$$

$$R = \frac{\rho\cdot L}{w\cdot t} \rightarrow L = \frac{R\cdot w\cdot t}{\rho} = \frac{10^7\Omega\cdot w\cdot 5\cdot10^{-4}\text{cm}}{250\Omega\cdot\text{cm}} = 20\cdot w$$

a number of solutions fulfill this L-w relation including that shown below with w=0.5mm and L=1cm



$$\rho = \frac{1}{\sigma} = \frac{1}{q\cdot\left[\mu_n\cdot(n_o+\delta n) + \mu_n\cdot\delta p\right]} = \frac{1}{1.609\cdot10^{-19}\text{C}\cdot\left[250\,\frac{\text{cm}^2}{\text{V}\cdot\text{s}}\cdot(10^{14}\,\frac{1}{\text{cm}^3}+10^{15}\,\frac{1}{\text{cm}^3}) + 15\,\frac{\text{cm}^2}{\text{V}\cdot\text{s}}\cdot10^{15}\,\frac{1}{\text{cm}^3}\right]} = 21.6\Omega\cdot\text{c}$$

$$R = \frac{\rho\cdot L}{w\cdot t} = \frac{21.6\Omega\cdot\text{cm}\cdot 1\text{cm}}{5\cdot10^{-2}\text{cm}\cdot5\cdot10^{-4}\text{cm}} = 8.62\cdot10^5\Omega$$

$$\Delta R = 10^7\Omega - 8.62\cdot10^5\Omega = 9.14\text{M}\Omega$$

Prob. 4.10

A 100mW laser (λ=632.8nm) is focused on a 100μm thick GaAs sample (α=$3\cdot10^4\,\frac{1}{cm}$). Find the photons emitted per second and the power to heat.

$$I_t = I_0\cdot e^{-\alpha l} = 100\text{mA}\cdot e^{-3\cdot10^4\frac{1}{\text{cm}}\cdot10^{-2}\text{cm}} \approx 0\text{mA so absorbed power is full } 100\text{mW} = 0.1\tfrac{\text{J}}{\text{s}}$$

$$\text{energy of one photon} = \frac{1.24\text{eV}\cdot\mu\text{m}}{0.6328\mu\text{m}} = 1.96\text{eV}$$

$$\text{power converted to heat} = \frac{1.96\text{eV}-1.43\text{eV}}{1.96\text{eV}}\cdot100\text{mW} = 2.7\cdot10^{-2}\,\tfrac{\text{J}}{\text{s}}$$

$$\text{photons per second} = \frac{0.1\frac{\text{J}}{\text{s}}}{1.609\cdot10^{-19}\,\frac{\text{J}}{\text{eV}}\cdot1.96\text{eV}} = 3.19\cdot10^{17}\,\frac{\text{photons}}{\text{second}}$$

or

$$\text{photons per second} = \frac{\text{power to photons}}{1.609\cdot10^{-19}\,\frac{\text{J}}{\text{eV}}\cdot\text{photon energy}} = \frac{0.073\frac{\text{J}}{\text{s}}}{1.609\cdot10^{-19}\,\frac{\text{J}}{\text{eV}}\cdot1.43\frac{\text{eV}}{\text{photon}}} = 3.19\cdot10^{17}\,\frac{\text{photons}}{\text{second}}$$

Prob. 4.11

Find the photocurrent ΔI in terms of τ_n and τ_t for a sample dominate by μ_n.

$$\Delta\sigma \approx q\cdot\mu_n\cdot\Delta n = q\cdot\mu_n\cdot g_{op}\cdot\tau_n$$

$$\text{transit time} = \tau_t = \frac{L}{v_d} = \frac{L}{\frac{V\cdot\mu_n}{L}} = \frac{L^2}{V\cdot\mu_n}$$

$$\Delta I = \frac{V}{\Delta R} = \frac{V\cdot A\cdot\Delta\sigma}{L} = \frac{V\cdot A\cdot q\cdot\mu_n\cdot g_{op}\cdot\tau_n}{L} = \frac{A\cdot L\cdot q\cdot g_{op}\cdot\tau_n}{\tau_t}$$

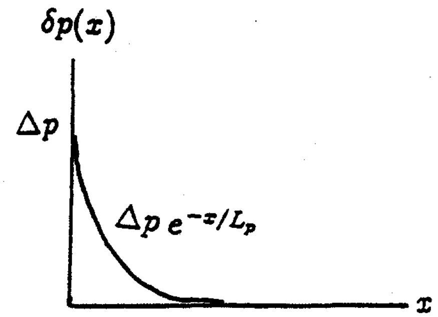
Prob. 4.12

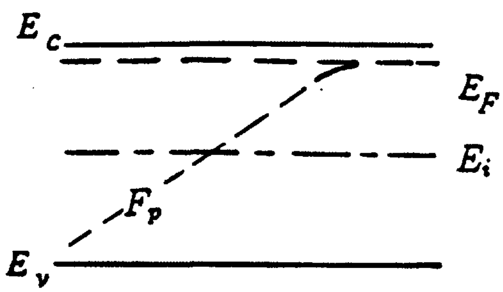
Find $F_p(x)$ for an exponential excess hole distribution.

for $\delta p \gg p_0$

$$p(x) \simeq \delta p(x) = \Delta p\cdot e^{-\frac{x}{L_p}} = n_i\cdot e^{\frac{E_i-F_p}{kT}}$$

$$E_i - F_p = kT\cdot\ln\left(\frac{\Delta p\cdot e^{-\frac{x}{L_p}}}{n_i}\right) = kT\cdot\left[\ln\left(\frac{\Delta p}{n_i}\right) - \frac{x}{L_p}\right]$$

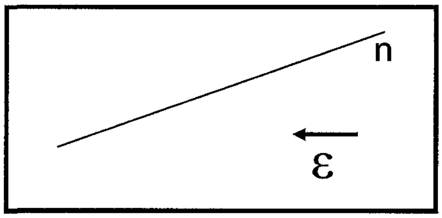




Since the excess minority hole concentration is assumed to be small compared to n_o throughout, so no band bending is observable on this scale.

Prob. 4.13

Show current flow in the n-type bar and describe the effects of doubling the electron concentration or adding a constant concentration of electrons uniformally.



← electron diffusion (high to low concentration)
→ current density (J_n) for diffusion
→ electron drift
← current density (J_n) for drift

note: currents are opposite electron flow because of negative charge

initially:

$$J_n \text{ diffusion} = q \cdot D_n \cdot \tfrac{dn}{dx}$$

$$J_n \text{ drift} = q \cdot n \cdot \mu_n \cdot \mathcal{E}$$

double electron concentration:

$$J_n \text{ diffusion} = q \cdot D_n \cdot 2\tfrac{dn}{dx} \quad \rightarrow \quad \text{doubles}$$

$$J_n \text{ drift} = q \cdot 2n \cdot \mu_n \cdot \mathcal{E} \rightarrow \text{doubles}$$

add constant concentration (n_+):

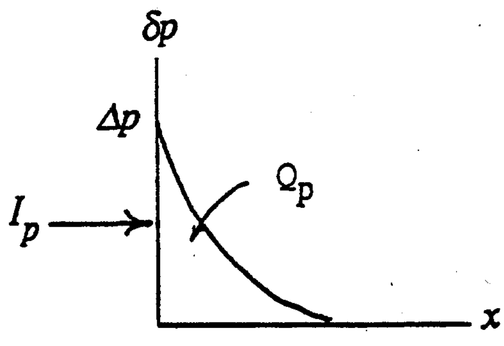
$$J_n \text{ diffusion} = q \cdot D_n \cdot \tfrac{dn}{dx} \quad \rightarrow \quad \text{does not change}$$

$$J_n \text{ drift} = q \cdot (n + n_+) \cdot \mu_n \cdot \mathcal{E} \rightarrow \text{increases by } q \cdot n_+ \cdot \mu_n \cdot \mathcal{E}$$

Prob. 4.14

Show the hole current feeding an exponential $\delta p(x)$ may be found from Q_p/τ_p.

The charge distribution, Q_p, disappears by recombination and must be replaced by injection an average of τ_p seconds.
Thus, the current injected must be Q_p/τ_p.

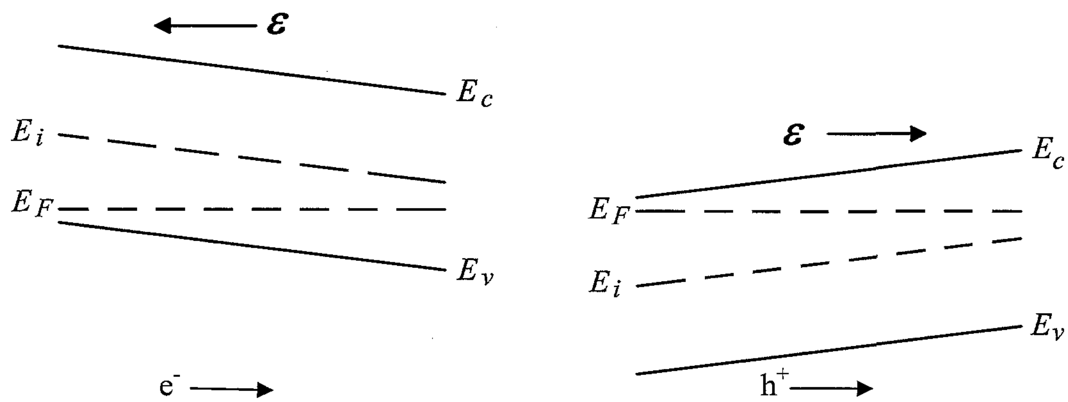


$$Q_p = q \cdot A \cdot \int_0^\infty \delta p \cdot dx = q \cdot A \cdot \int_0^\infty \Delta p \cdot e^{-\frac{x}{L_p}} \cdot dx = q \cdot A \cdot L_p \cdot \Delta p$$

$$I_p = \frac{Q_p}{\tau_p} = \frac{q \cdot A \cdot L_p \cdot \Delta p}{\tau_p} = \frac{q \cdot A \cdot D_p \cdot \Delta p}{L_p}$$

Prob. 4.15

Draw band diagrams for exponential donor and acceptor dopings. Show field directions and direction of drift of minority carriers.



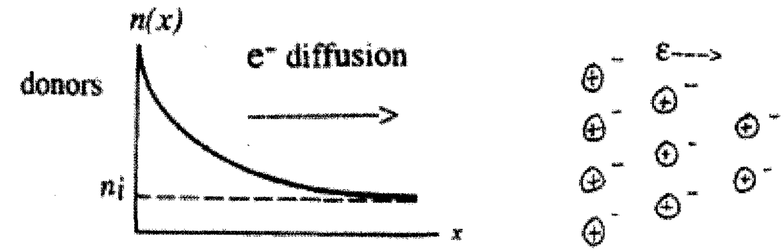
acceptor doping acceptor doping

Note: Both minority carriers are accelerated "downhill" in the doping gradient.

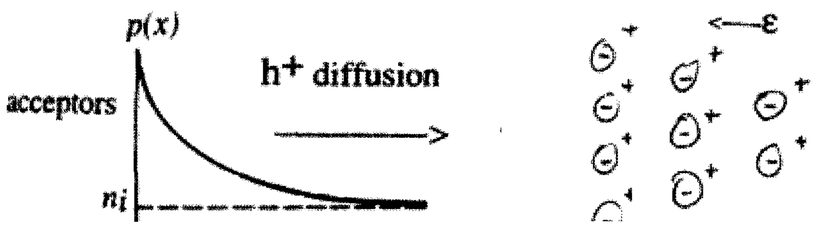
Prob. 4.16

Explain how a built-in field results from a doping gradient.

The donor doping gradient results in a tendency for electrons to diffuse to the lower concentration areas. As the electrons diffuse, the positively charged ionized donors pull the electrons back due to the coulombic force. This force corresponds to an electric field oriented in the same direction as the concentration gradient. The electric field results in drift current equal and opposite to the diffusion current, and the sample remains in equilibrium with a built-in electric field.



The acceptor doping gradient results in a tendency for holes to diffuse to the lower concentration areas. As the holes diffuse, the negatively charged acceptor ions pull the holes back due to the coulombic force. This force corresponds to an electric field oriented in the direction opposite the concentration gradient. The electric field results in drift current equal and opposite to the diffusion current, and the sample remains in equilibrium with a built-in electric field.



Prob. 4.17

Include recombination in the Haynes-Shockley experiment and find τ_p.

To include recombination, let the peak value vary as $e^{-\frac{t}{\tau_p}}$.

$$\delta p = \frac{\Delta p \cdot e^{-\frac{t}{\tau_p}}}{\sqrt{4\pi \cdot D_p \cdot t}} \cdot e^{\frac{-x^2}{4 \cdot D_p \cdot t}}$$

peak V_p at x=0 $= B \cdot \frac{\Delta p \cdot e^{-\frac{t}{\tau_p}}}{\sqrt{4\pi \cdot D_p \cdot t}}$ where B is a proportionality constant

$$\frac{V_{p1}}{V_{p2}} = \frac{B \cdot \frac{\Delta p \cdot e^{-\frac{t}{\tau_p}}}{\sqrt{4\pi \cdot D_p \cdot t_1}}}{B \cdot \frac{\Delta p \cdot e^{-\frac{t}{\tau_p}}}{\sqrt{4\pi \cdot D_p \cdot t_2}}} = \sqrt{\frac{t_2}{t_1}} \cdot \frac{e^{-\frac{t_1}{\tau_p}}}{e^{-\frac{t_2}{\tau_p}}} = \sqrt{\frac{t_2}{t_1}} \cdot e^{\frac{t_2 - t_1}{\tau_p}}$$

$$\frac{80\text{mV}}{20\text{mV}} = \sqrt{\frac{200\mu\text{s}}{50\mu\text{s}}} \cdot e^{\frac{200\mu\text{s} - 50\mu\text{s}}{\tau_p}} \rightarrow \frac{150\mu\text{s}}{\tau_p} = \ln\frac{4}{\sqrt{4}} \rightarrow \tau_p = \frac{150\mu\text{s}}{\ln 2} = 216.4\mu\text{s}$$

Chapter 5 Solutions

Prob. 5.1

Find the time that it takes to grow the first 200nm, the next 300nm, and the final 400nm. Draw and calculate step heights after reoxidation.

Time for first 200nm = 0.13 hours from Appendix VI at 1000°C

Time for next 300nm = 0.6 hours for 500nm – 0.13 hours for 200nm = 0.47 hours

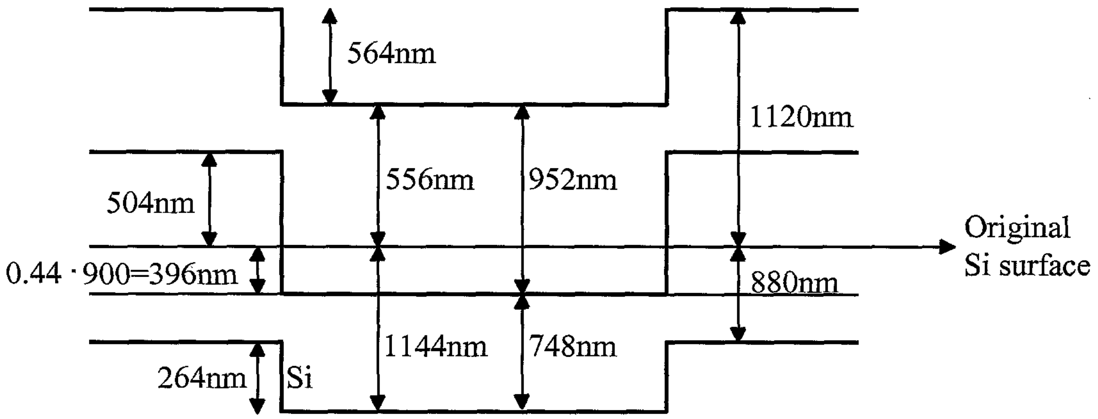
Time for final 400nm = 1.8 hours for 900nm – 0.6 hours for 500 nm = 1.2 hours

Oxidation Time After Etch = 6.0 hours for 2000nm – 1.4 hours for 900nm at 1100°C
= 4.6 hours

Oxide Growth Inside Window = 1700nm

Step in oxide = 564 nm

Step in Si = 264 nm



Prob. 5.2

Plot the distributions for B diffused into Si (N_d=$5\cdot10^{16}\,\frac{1}{cm^3}$) at 1000°C for 30min ($D$=$3\cdot10^{-14}\,\frac{cm^2}{s}$) with (a) constant source N_o=$5\cdot10^{20}\,\frac{1}{cm^3}$ and (b) limited source N_o=$5\cdot10^{13}\,\frac{1}{cm^3}$ on the surface prior to diffusion.

The Gaussian distribution differs from Equation 4-44 because all atoms are assumed to diffuse into the sample (i.e. there is no diffusion in the –x direction).

$$\sqrt{D\cdot t}=\sqrt{3\cdot10^{-14}\,\tfrac{cm^2}{s}\cdot30\text{min}\cdot60\tfrac{s}{min}}=\sqrt{5.4\cdot10^{-10}cm^2}=0.0735\mu m$$

$$\text{(a) } N=N_o\cdot\operatorname{erfc}\left(\frac{x}{2\sqrt{D\cdot t}}\right)=N_o\cdot\operatorname{erfc}\left(\frac{x}{0.147\mu m}\right)$$

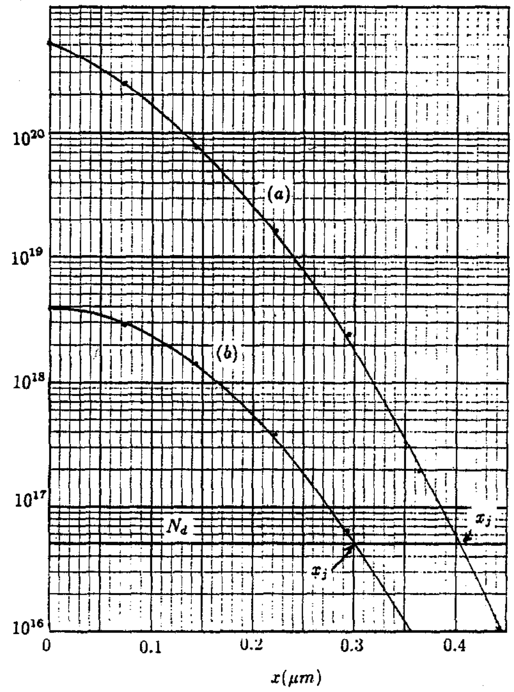
$x(\mu m)$	u	erfc u	$N(x)=N_0$ erfc u
0.0735	0.5	0.47	2.4×10^{20}
0.1470	1.0	0.16	8.0×10^{19}
0.2205	1.5	0.033	1.7×10^{19}
0.2940	2.0	0.0048	2.4×10^{18}
0.3675	2.5	0.0004	2.0×10^{17}
0.4410	3.0	0.000023	1.2×10^{16}

x_j=0.4μm

$$\text{(b) } N=\frac{N_s}{\sqrt{\pi}\cdot\sqrt{D\cdot t}}\cdot e^{-\left(\frac{x}{2\sqrt{D\cdot t}}\right)^2}=\frac{N_s}{0.1302\mu m}\cdot e^{-\left(\frac{x}{0.147\mu m}\right)^2}$$

$x(\mu m)$	u	$\exp(-u^2)$	$N(x)$
0.0735	0.5	0.78	3.0×10^{18}
0.1470	1.0	0.37	1.4×10^{18}
0.2205	1.5	0.105	4.0×10^{17}
0.2940	2.0	0.018	6.9×10^{16}
0.3675	2.5	0.0019	7.3×10^{15}

x_j=0.3μm



Prob. 5.3

For the unlimited source in Problem 5.2, calculate the time to achieve a junction depth of 1 micron.

Use N_o from Appendix VII and D from Appendix VIII.

$$N=N_o\cdot\operatorname{erfc}\left(\frac{x}{2\sqrt{D\cdot t}}\right)=10^{21}\,\tfrac{1}{cm^3}\cdot\operatorname{erfc}\left(\frac{1\mu m}{2\sqrt{3\cdot10^{-14}\,\frac{cm^2}{s}\cdot t}}\right)=2\cdot10^{16}\,\tfrac{1}{cm^3}$$

$$\operatorname{erfc}\left(\frac{1\mu m}{2\sqrt{3\cdot10^{-14}\,\frac{cm^2}{s}\cdot t}}\right)=\frac{2\cdot10^{16}\,\frac{1}{cm^3}}{10^{21}\,\frac{1}{cm^3}}=2\cdot10^{-5}$$

$$\frac{1\mu m}{2\sqrt{3\cdot10^{-14}\,\frac{cm^2}{s}\cdot t}}=\frac{10^{-4}cm}{2\sqrt{3\cdot10^{-14}\,\frac{cm^2}{s}\cdot t}}=3.0$$

$t = 9260s = 2$ hrs 34 minutes 20 seconds

Prob. 5.4

Find the implant parameters for an As implant into Si with the peak at the interface.

$R_p = 0.1\mu m \rightarrow$ Energy $= 180$keV from Appendix IX

Straggle $= \Delta R_p = 0.035\mu m$

Ion Distribution from Equation 5-1a $= N(x) = \dfrac{\phi}{\sqrt{2\pi}\cdot\Delta R_p}\cdot e^{-\frac{1}{2}\left(\frac{x-R_p}{\Delta R_p}\right)^2}$

$$N_{peak} = 5\cdot10^{19}\,\tfrac{1}{cm^3} = \frac{\phi}{\sqrt{2\pi}\cdot\Delta R_p} = \frac{\phi}{\sqrt{2\pi}\cdot3.5\cdot10^{-6}cm} = \frac{\phi}{8.77\cdot10^{-6}cm}$$

$$\phi = 5\cdot10^{19}\,\tfrac{1}{cm^3}\cdot8.77\cdot10^{-6}cm = 4.39\cdot10^{14}\,\tfrac{1}{cm^2}$$

$$\phi = \frac{I\cdot t}{q\cdot A} = \frac{I\cdot20s}{1.6\cdot10^{-19}C\cdot200cm^2} = 4.39\cdot10^{14}\,\tfrac{1}{cm^2}$$

Beam Current $= I = 0.7$ mA

Prob. 5.5

Calculate and plot the P distribution.

Energy = 200keV $\rightarrow$ $R_p = 0.255\mu m$, $\Delta R_p = 0.0837\mu m$ from Appendix IX

Dose = $\phi = 2.1 \cdot 10^{14} \frac{1}{cm^2}$

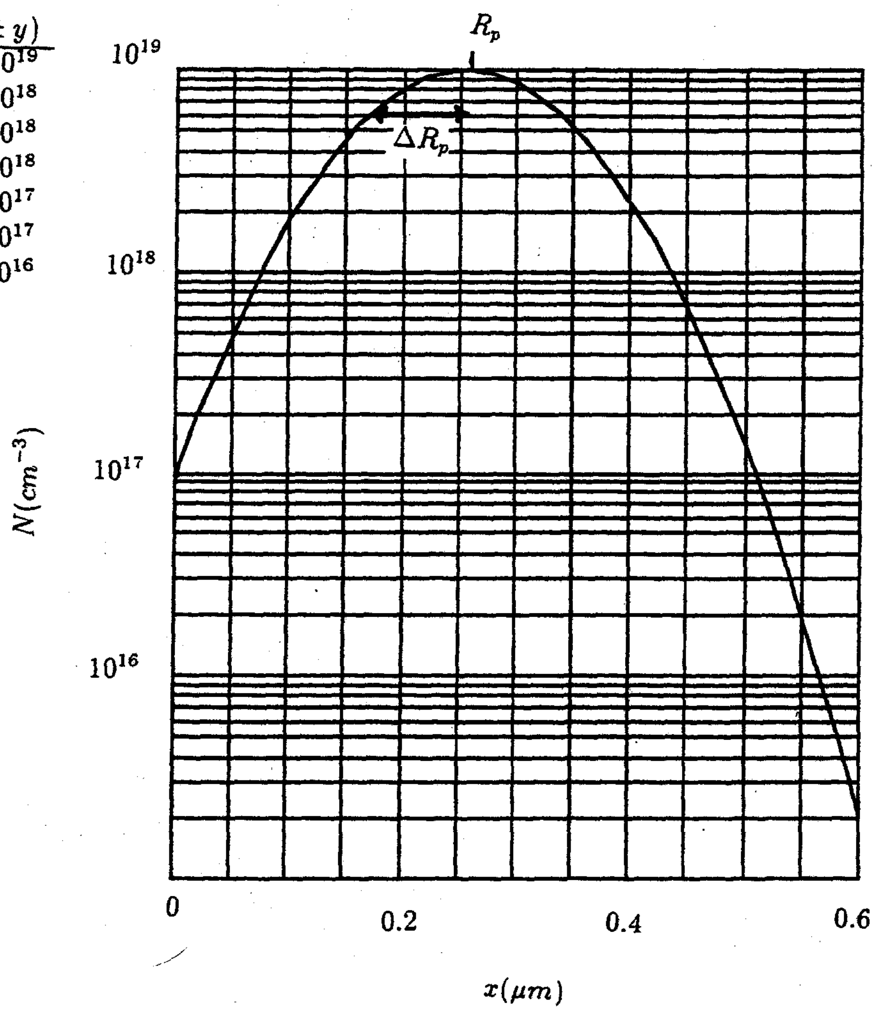
From Equation 5-1a, $N(x) = \dfrac{\phi}{\sqrt{2\pi} \cdot \Delta R_p} \cdot e^{-\frac{1}{2}\left(\frac{x - R_p}{\Delta R_p}\right)^2} = \dfrac{2.1 \cdot 10^{14} \frac{1}{cm^2}}{\sqrt{2\pi} \cdot 8.37 \cdot 10^{-6} cm} \cdot e^{-\frac{1}{2}\left(\frac{x - 0.255\mu m}{0.1\mu m}\right)^2}$

Peak $N = \dfrac{2.1 \cdot 10^{14} \frac{1}{cm^2}}{\sqrt{2\pi} \cdot 8.37 \cdot 10^{-6} cm} = 10^{19} \frac{1}{cm^3}$

Let y be the distance (μm) on either side of R_p.

$N(R_p \pm y) = 10^{19} \frac{1}{cm^3} \cdot e^{-71.37 \cdot y^2}$

y (μm)	$N(R_p \pm y)$
0	1×10^{19}
0.05	8.4×10^{18}
0.1	4.9×10^{18}
0.15	2×10^{18}
0.2	5.8×10^{17}
0.25	1.2×10^{17}
0.3	1.6×10^{16}



Prob. 5.6

In patterning the structure shown in the question, design the mask aligner optics in terms of numerical aperture of the lens and the wavelength of the source.

$$l_{min} = \frac{0.8 \cdot \lambda}{NA} = 1\mu m \qquad DOF = \frac{\lambda}{2 \cdot (NA)^2} = 2\mu m$$

$$\left(\frac{1\mu m \cdot NA}{0.8}\right) \cdot \left(\frac{1}{2 \cdot (NA)^2}\right) = 2\mu m \rightarrow NA = 0.3125 \quad \lambda = 0.39\mu m$$

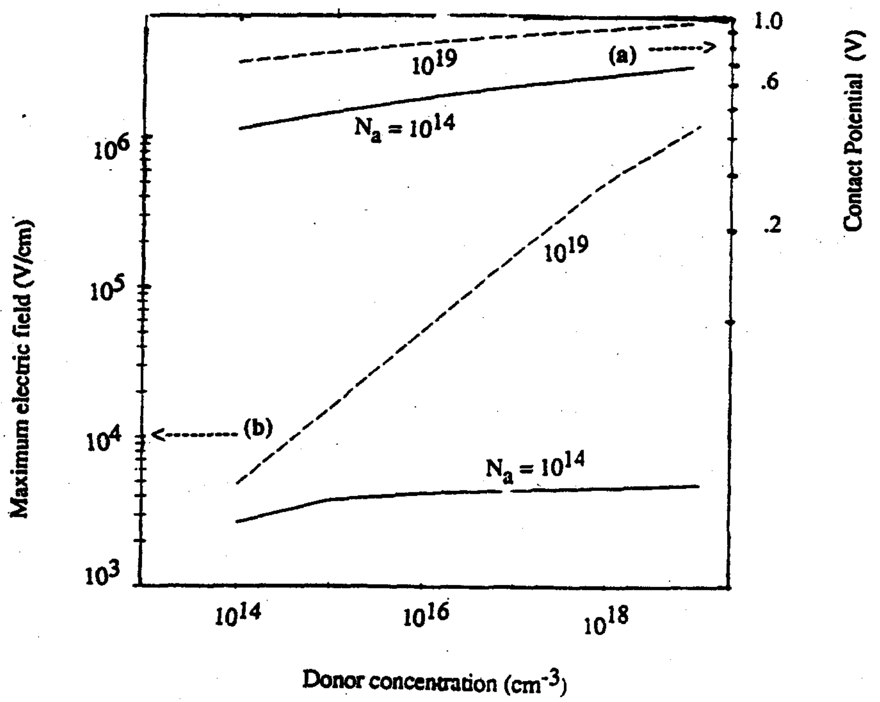
Prob. 5.7

a) Calculate contact potential V_0, in a Si p-n junction at 300 K.

$$V_0 = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$$

$$V_0 = 0.0259 \ln \frac{N_a N_d}{(1.5 \cdot 10^{10})^2}$$

b) Plot E_0 vs. N_d



Prob. 5.8

Find the electron diffusion and drift currents at x_n in a p^+-n junction.

$$I_p(x_n) = q \cdot A \cdot \frac{D_p}{L_p} \cdot p_n \cdot e^{\frac{q \cdot V}{kT}} \cdot e^{-\frac{x_n}{L_p}} \text{ for } V \gg \frac{kT}{q}$$

$$I = I_p(x_n = 0) = q \cdot A \cdot \frac{D_p}{L_p} \cdot p_n \cdot e^{\frac{q \cdot V}{kT}}$$

Assuming space charge neutrality, the excess hole distribution is equal to the excess electron distribution $\delta n(x_n) = \delta p(x_n)$

$$I_n(x_n)_{diff} = q \cdot A \cdot D_n \cdot \frac{d\delta p}{dx_n} = -q \cdot A \cdot \frac{D_n}{L_p} \cdot p_n \cdot e^{\frac{q \cdot V}{kT}} \cdot e^{-\frac{x_n}{L_p}}$$

$$I_n(x_n)_{drift} = I - I_n(x_n)_{diff} - I_p(x_n) = q \cdot A \cdot \frac{p_n}{L_p} \cdot \left[D_n \cdot \left(1 - e^{-\frac{x_n}{L_p}}\right) + D_n \cdot e^{-\frac{x_n}{L_p}} \right] \cdot e^{\frac{q \cdot V}{kT}}$$

Prob. 5.9

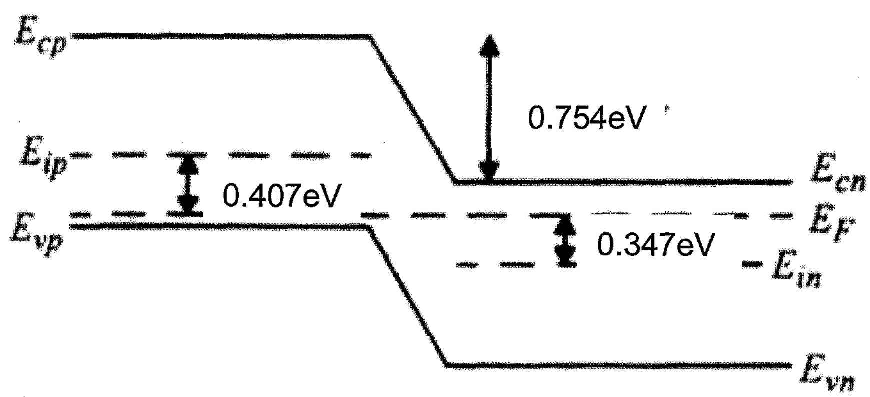
A Si junction has $N_a = 10^{17} \frac{1}{cm^3}$ and $N_d = 10^{16} \frac{1}{cm^3}$. Find (a) E_F, V_o, and band diagram and (b) compare this value of V_o to that from Equation 5-8.

(a)

$$E_{ip} - E_F = kT \cdot \ln \frac{p_p}{n_i} = 0.0259\text{eV} \cdot \ln \frac{10^{17} \frac{1}{cm^3}}{1.5 \cdot 10^{10} \frac{1}{cm^3}} = 0.407\text{eV}$$

$$E_F - E_{in} = kT \cdot \ln \frac{n_n}{n_i} = 0.0259\text{eV} \cdot \ln \frac{10^{16} \frac{1}{cm^3}}{1.5 \cdot 10^{10} \frac{1}{cm^3}} = 0.347\text{eV}$$

$$q \cdot V_o = 0.407\text{eV} + 0.347\text{eV} = 0.754\text{eV}$$



$$\text{(b) } q \cdot V_o = kT \cdot \ln \frac{N_a N_d}{n_i^2} = 0.0259\text{eV} \cdot \ln \frac{10^{17} \frac{1}{cm^3} \cdot 10^{16} \frac{1}{cm^3}}{\left(1.5 \cdot 10^{10} \frac{1}{cm^3}\right)^2} = 0.754\text{eV}$$

Prob. 5.10

Find V_0, x_{n0}, x_{p0}, Q_+, and $\mathcal{E}_o$ in Si at 300 K.

$$V_0 = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$$

$$V_0 = 0.0259\text{V} \ln \frac{4 \cdot 10^{18} \frac{1}{\text{cm}^3} \cdot 10^{16} \frac{1}{\text{cm}^3}}{(1.5 \cdot 10^{10} \frac{1}{\text{cm}^3})^2}$$

$$V_0 = 0.8498 \text{ V}$$

$$W = \sqrt{\frac{2\,\epsilon_{Si}\,V_0}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

$$W = 0.334\ \mu\text{m}$$

$$x_{n0} = \frac{W}{1 + \frac{N_d}{N_a}} = 0.333\ \mu\text{m}$$

$$x_{p0} = \frac{W}{1 + \frac{N_a}{N_d}} = 0.83\ \text{nm}$$

$$Q_+ = -Q_- = q\,A\,x_{n0}\,N_d = 0.107\ \text{nC}$$

$$\mathcal{E}_0 = \frac{-q\,N_d\,x_{n0}}{\epsilon_{Si}} = -5.1 \cdot 10^4\ \text{V/cm}$$

Prob. 5.11

Describe the effect on the hole diffusion current of doubling the p^+ doping.

The depletion edge and electron diffusion current on the p^+ side may be ignored and

$$L_p = \sqrt{D_p \cdot \tau_p} = \sqrt{20\tfrac{cm^2}{s} \cdot 50 \cdot 10^{-9}s} = 10^{-3}cm = 10\mu m$$

$$\delta p = \frac{n_i^2}{N_d} \cdot \left(e^{\frac{qV}{kT}} - 1\right) \cdot e^{-\frac{x}{L_p}}$$

$$\frac{d(\delta p)}{dx} = -\frac{1}{L_p} \cdot \frac{n_i^2}{N_d} \cdot \left(e^{\frac{qV}{kT}} - 1\right) \cdot e^{-\frac{x}{L_p}} = -\frac{1}{10^{-3}cm} \cdot \frac{\left(10^{10}\frac{1}{cm^3}\right)^2}{10^{16}\frac{1}{cm^3}} \cdot \left(e^{\frac{0.6}{0.026}} - 1\right) \cdot e^{-\frac{2\mu m}{10\mu m}} = -8.6 \cdot 10^{16}\tfrac{1}{cm^4}$$

$$J_p(\text{diffusion}) = -q \cdot D_p \cdot \frac{d(\delta p)}{dx} = 1.609 \cdot 10^{-19}C \cdot 20\tfrac{cm^2}{s} \cdot 8.6 \cdot 10^{16}\tfrac{1}{cm^4} = 0.277\tfrac{A}{cm^2}$$

Since this is independent of the p^+ doping, there will be no change.

Prob. 5.12

For the Si p^+-n junction, find I for V_f=0.5V.

$$I = q \cdot A \cdot \frac{D_p}{L_p} \cdot p_n \cdot e^{\frac{q \cdot V}{kT}} = q \cdot A \cdot \frac{D_p}{\sqrt{D_p \cdot \tau_p}} \cdot \frac{n_i^2}{n_n} \cdot e^{\frac{q \cdot V}{kT}}$$

$$I = 1.6 \cdot 10^{-19}C \cdot 10^{-3}cm^2 \cdot \frac{10\frac{cm^2}{s}}{\sqrt{10\frac{cm^2}{s} \cdot 10^{-6}s}} \cdot \frac{(1.5 \cdot 10^{10}\frac{1}{cm^3})^2}{5 \cdot 10^{16}\frac{1}{cm^3}} \cdot e^{\frac{0.5eV}{0.0259eV}} = 0.55\mu A$$

Prob. 5.13

(a) Why is C_s negligible in reverse bias?

For reverse bias of more than a few tenths of a volt, $\Delta p_n \approx -p_n$. Changes in the reverse bias do not appreciably alter the (negative) excess hole distribution. The primary variation is in the width of the depletion region, giving rise to the junction capacitance.

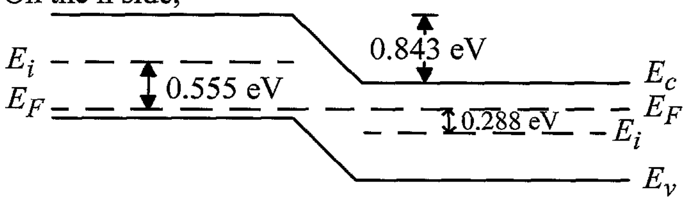
(b) With equal doping, which carrier dominates injection in a GaAs junction?

Electron injection dominates since $\mu_n \gg \mu_p$. With $n_n = p_p$ it is clear that a carrier with higher mobility will determine the injection.

Prob. 5.14

(a) Find C_j for V=-10V for a Si p^+-n junction $10^{-2}\,cm^2$ *in area with* $N_d = 10^{15}\,\frac{1}{cm^3}$.

On the n side,



$$E_F - E_{in} = kT\cdot\ln\frac{N_d}{n_i} = 0.0259eV\cdot\ln\frac{10^{15}\,\frac{1}{cm^3}}{1.5\cdot10^{10}\,\frac{1}{cm^3}} = 0.288eV$$

On the p-side, $E_{ip} - E_F = \frac{1}{2}\cdot E_g \cong 0.555eV$

$$q\cdot V_o = 0.555eV + 0.288eV = 0.834eV$$

$$C_j = \frac{A}{2}\cdot\left(\frac{2\cdot q\cdot\in\cdot N_d}{V_o - V}\right)^{\frac{1}{2}} = \frac{10^{-2}cm^2}{2}\cdot\left(\frac{2\cdot1.6\cdot10^{-19}C\cdot11.8\cdot8.85\cdot10^{-14}\,\frac{F}{cm}\cdot10^{15}\,\frac{1}{cm^3}}{0.834eV - (-10eV)}\right)^{\frac{1}{2}} = 2.78\cdot10^{-11}F$$

(b) What is W just prior to avalanche?

From Figure 5-22, V_{br}=300V from Figure 5-22

$$W = \left(\frac{2\cdot\in_s\cdot V_{br}}{q\cdot N_d}\right)^{\frac{1}{2}} = \left(\frac{2\cdot11.8\cdot8.85\cdot10^{-14}\,\frac{F}{cm}\cdot300V}{1.6\cdot10^{-19}C\cdot10^{15}\,\frac{1}{cm^3}}\right)^{\frac{1}{2}} = 1.98\cdot10^{-3}cm = 20\mu m$$

Prob. 5.15

Show $\mathcal{E}_o$ depends on doping of a lightly doped substrate.

$$\mathcal{E}_o = -\frac{q}{\in}\cdot N_d\cdot x_{n_o} = -\left[\frac{2\cdot q\cdot V_o}{\in}\cdot\left(\frac{N_a\cdot N_d}{N_a + N_d}\right)\right]^{\frac{1}{2}} = -\left[\frac{2\cdot q\cdot V_o}{\in}\cdot\left(\frac{1}{N_a} + \frac{1}{N_d}\right)^{-1}\right]^{\frac{1}{2}}$$

the lightly doped side dominates so the doping variation of V_o has only a minor effect

$$\mathcal{E}_o = -\frac{q}{\in}\cdot N_d\cdot x_{n_o} = -\left[\frac{2\cdot q\cdot V_o\cdot N_d}{\in}\right]^{\frac{1}{2}}$$

Prob. 5.16

For the abrupt Si n^+-p junction, calculate the peak electric field and depletion capacitance for reverse bias. Find the total excess stored electric charge and the electric field far from the depletion region on the p side.

Depletion region is mostly on the p-side.

$$V_{total} = V_r + V_o \simeq V_r = 100\text{V}$$

$$W = \sqrt{\frac{2\epsilon_{Si} V_{total}}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

Since N_d is very high, $\frac{1}{N_d}$ may be neglected.

$$W = \sqrt{\frac{2\epsilon_{Si} V_{total}}{q \cdot N_a}} = \sqrt{\frac{2 \cdot 11.8 \cdot 8.85 \cdot 10^{-14} \frac{\text{F}}{\text{cm}} \cdot 100\text{V}}{1.6 \cdot 10^{-19}\text{C} \cdot 10^{17} \frac{1}{\text{cm}^3}}} = 1.14\mu\text{m}$$

$$\mathcal{E} = \frac{2 \cdot V_{total}}{W} = \frac{2 \cdot 100\text{V}}{1.14 \cdot 10^{-4}\text{cm}} = 1.75 \cdot 10^6 \frac{\text{V}}{\text{cm}}$$

$$C_j = \frac{\epsilon_{Si} A}{W} = \frac{11.8 \cdot 8.85 \cdot 10^{-14} \frac{\text{F}}{\text{cm}} \cdot 0.0001\text{cm}^2}{1.14 \cdot 10^{-4}\text{cm}} = 0.916\text{pF}$$

$$I = 20 \cdot 10^{-3}\text{A} = \frac{Q_n}{\tau_n}$$

$$Q_n = 20 \cdot 10^{-3}\text{A} \cdot 0.1 \cdot 10^{-6}\text{s} = 2 \cdot 10^{-9}\text{C}$$

Far from the junction on the p-side, the current is only hole drift.

$$I = 20 \cdot 10^{-3}\text{A} = q \cdot A \cdot p \cdot \mu_p \cdot \mathcal{E} = 1.6 \cdot 10^{-19}\text{C} \cdot 0.0001\text{cm}^2 \cdot 10^{17} \frac{1}{\text{cm}^3} \cdot 200 \frac{\text{cm}^2}{\text{V} \cdot \text{s}} \cdot \mathcal{E}$$

$$\mathcal{E} = 62.5 \frac{\text{V}}{\text{cm}}$$

Prob. 5.17

Find the electron injection efficiency I_n/I.

(a) $$\frac{I_n}{I} = \frac{q\cdot A\cdot \frac{D_n}{L_n}\cdot n_p\cdot\left(e^{\frac{q\cdot V}{kT}}-1\right)}{q\cdot A\cdot\left(\frac{D_p}{L_p}\cdot p_n+\frac{D_n}{L_n}\cdot n_p\right)\cdot\left(e^{\frac{q\cdot V}{kT}}-1\right)} = \frac{1}{1+\frac{D_pL_n}{D_nL_p}\cdot\frac{p_n}{n_p}}$$

(b) $$\frac{D_p^n}{D_n^p} = \frac{\mu_p^n}{\mu_n^p} \text{ and } \frac{p_n}{n_p} = \frac{p_p}{n_n} \text{ gives } \frac{D_p^n\cdot p_n}{D_n^p\cdot n_p} = \frac{\mu_p^n\cdot p_p}{\mu_n^p\cdot n_n}$$

$$\frac{I_n}{I} = \frac{1}{1+\frac{L_n^p\cdot\mu_p^n\cdot p_p}{L_p^n\cdot\mu_n^p\cdot n_n}}$$ so making $n_n \gg p_p$ (i.e. using n^+- p) increases $\frac{I_n}{I}$

Prob. 5.18

Find $I_p(x_p)$ when $p_p=n_n$.

$$I = q\cdot A\cdot\left(\frac{D_p}{L_p}\cdot p_n+\frac{D_n}{L_n}\cdot n_p\right)\cdot\left(e^{\frac{qV}{kT}}-1\right)$$

is composed of

$$I_n(x_p) = q\cdot A\cdot\frac{D_n}{L_n}\cdot n_p\cdot e^{-\frac{x_p}{L_n}}\cdot\left(e^{\frac{qV}{kT}}-1\right)$$

$$I_p(x_p) = I - I_n(x_p) = q\cdot A\cdot\left[\frac{D_n}{L_n}\cdot n_p\cdot\left(1-e^{-\frac{x_p}{L_n}}\right)+\frac{D_p}{L_p}\cdot p_n\right]\cdot\left(e^{\frac{qV}{kT}}-1\right)$$

Since $N_a=N_d$, $n_p=p_n=\frac{n_i^2}{N_a}$ giving

$$I_p(x_p) = q\cdot A\cdot\left[\frac{D_n}{L_n}\cdot\left(1-e^{-\frac{x_p}{L_n}}\right)+\frac{D_p}{L_p}\right]\cdot\frac{n_i^2}{N_a}\cdot\left(e^{\frac{qV}{kT}}-1\right)$$

Prob. 5.19

For the given p-n junction, calculate the built-in potential, the zero-bias space charge width, and the current for a 0.5V forward bias.

(a) Calculate built-in potential:

for $N_a = 10^{15}\,\frac{1}{cm^3}$ and $N_d = 10^{17}\,\frac{1}{cm^3}$

$$V_o = \phi_{FP} + \phi_{FN} = \frac{kT}{q}\cdot\left[\ln\frac{N_a}{n_i} + \ln\frac{N_d}{n_i}\right]$$

$$V_o = 0.026V\cdot\left[\ln\frac{10^{15}}{1.5\cdot10^{10}} + \ln\frac{10^{17}}{1.5\cdot10^{10}}\right]$$

$$V_o = 0.026V\cdot[11.1 + 15.7] = 0.70V$$

(b) Calculate total width of space change region

$$W = \sqrt{\frac{2\epsilon_s}{q}\cdot\left(\frac{N_a + N_d}{N_a N_d}\right)\cdot V_o}$$

Thermal equilibrium means total potential ϕ_T across the P-N junction equals V_o

$$W = \sqrt{\frac{2\epsilon_s}{q\cdot N_a}\cdot\left(\frac{N_a + N_d}{N_a N_d}\right)\cdot V_o}$$

$$W = \sqrt{\frac{2\epsilon_s}{q\cdot N_a}\cdot\left[1 + \frac{N_a}{N_d}\right]\cdot V_o}$$

$$W = \sqrt{\frac{2\cdot\left(8.85\cdot10^{-14}\,\frac{F}{cm}\cdot11.8\right)}{1.6\cdot10^{-19}C}\cdot\frac{1}{10^{15}\,\frac{1}{cm^3}}\left[1 + \frac{10^{15}\,\frac{1}{cm^3}}{10^{17}\,\frac{1}{cm^3}}\right]\cdot0.70V}$$

$$W = \sqrt{1.3\cdot10^{-8}\,\frac{cm^2}{V}\cdot[1 + .01]\cdot0.70V} = 9.6\cdot10^{-5}\,cm = 0.96\text{ microns}$$

(c) Forward bias current:

$$\mu_n = 1500\frac{cm^2}{V\cdot s} \qquad \mu_p = 450\frac{cm^2}{V\cdot s} \qquad \tau = 2.5\cdot10^{-3}s \qquad n_i = 1.5\cdot10^{10}\,\tfrac{1}{cm^3}$$

$$D_n = \mu_n\left(\frac{kT}{q}\right) = 1500\frac{cm^2}{V\cdot s}\cdot 0.026V = 38.9\frac{cm^2}{s}$$

$$D_p = \mu_p\left(\frac{kT}{q}\right) = 450\frac{cm^2}{V\cdot s}\cdot 0.026V = 11.7\frac{cm^2}{s}$$

$$L_n = \sqrt{D_n\cdot\tau} = 0.31cm \qquad L_p = \sqrt{D_p\cdot\tau} = 0.17cm$$

$$J_0 = \left(q\cdot n_i^2\right)\cdot\left(\frac{D_P}{N_d\cdot L_P} + \frac{D_n}{N_a\cdot L_n}\right)$$

$$J_0 = \left(1.6\cdot10^{-19}C\right)\cdot\left(1.5\cdot10^{10}\,\tfrac{1}{cm^3}\right)^2\cdot\left(6.88\cdot10^{-18}\,\tfrac{cm^4}{s} + 1.25\cdot10^{-15}\,\tfrac{cm^4}{s}\right) = 4.5\cdot10^{-12}\,\tfrac{C}{cm^2\cdot s}$$

$$I = A\cdot J_0\cdot\left(e^{\frac{q\cdot V}{kT}} - 1\right) = \left(0.001cm^2\right)\cdot\left(4.5\cdot10^{-12}\,\tfrac{C}{cm^2\cdot s}\right)\cdot\left[e^{\left(\frac{0.7V}{0.026V}\right)} - 1\right] = 2.2\cdot10^{-3}A$$

Most of the current is carried by electrons because N_a is less than N_d. To double the electron current, halve the acceptor doping.

Prob. 5.20

Find the total forward bias junction capacitance and reverse bias electric field.

For n^+ - p in reverse bias,

$$C_j = \frac{A\cdot\epsilon_s}{W} = \frac{A}{2}\cdot\sqrt{\frac{2\cdot q\cdot\epsilon_s}{V_o - V}\cdot N_a} = \frac{25\mu m^2}{2}\cdot\sqrt{\frac{2\cdot1.6\cdot10^{-19}C\cdot11.8\cdot8.85\cdot10^{-14}\,\frac{F}{cm}\cdot10^{16}\,\frac{1}{cm^3}}{0.026V\cdot\ln\left(\frac{10^{16}\,\frac{1}{cm^3}\cdot10^{20}\,\frac{1}{cm^3}}{\left(1.5\cdot10^{10}\,\frac{1}{cm^3}\right)^2}\right) - (-2V)}} = 4.2\cdot10$$

For n^+ - p in forward bias,

$$J = J_o\cdot\left(e^{\frac{qV}{kT}} - 1\right) = 10^{-9}\,\tfrac{A}{cm^2}\cdot\left(e^{\frac{0.5V}{0.026}} - 1\right) = 0.225\,\tfrac{A}{cm^2} \text{ because only drift current}$$

$$J = q\cdot\mu_p\cdot N_a\cdot\mathcal{E} \text{ in p region far from junction}$$

$$\mathcal{E} = \frac{0.225\,\frac{A}{cm^2}}{1.6\cdot10^{-19}C\cdot250\,\frac{cm^2}{V\cdot s}\cdot10^{16}\,\frac{1}{cm^3}} = 0.56\frac{V}{cm}$$

Prob. 5.21

In a p^+- n junction with n-doping changed from N_d to $2N_d$, describe the changes in junction capacitance, built-in potential, breakdown voltage, and ohmic losses.

a) junction capacitance increases
b) built-in potential increases
c) breakdown voltage decreases
d) ohmic losses decrease

Prob. 5.22

Sketch the equilibrium band diagram with precise values for the bar.

on right side

$$p = 10^{18}\frac{1}{cm^3} = 10^{16}\frac{1}{cm^3}\cdot e^{\frac{E_i-E_F}{kT}}$$

$$E_i\text{-}E_F = 0.026eV\cdot\frac{600K}{300K}\cdot\ln\left(\frac{10^{18}\frac{1}{cm^3}}{10^{16}\frac{1}{cm^3}}\right) = 0.24eV$$

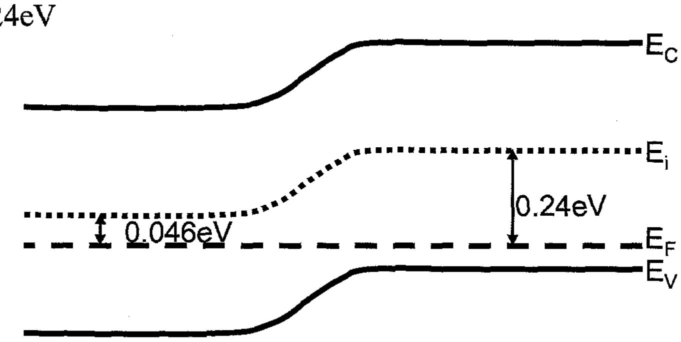
on left side

$$N_a \sim n_i$$

$$p = N_a + \frac{n_i^2}{p} = 2\cdot10^{16}\frac{1}{cm^3} + \frac{10^{32}\frac{1}{cm^6}}{p}$$

$$p = 2.4\cdot10^{16}\frac{1}{cm^3} = 10^{16}\frac{1}{cm^3}\cdot e^{\frac{E_i-E_F}{kT}}$$

$$E_i\text{-}E_F = 0.026eV\cdot\frac{600K}{300K}\cdot\ln(2.4) = 0.046eV$$



Prob. 5.23

Plot I_p and I_n versus distance.

Assume that the minority carrier mobilities are the same as the majority carrier mobilities given in Figure 3-23a.

$\mu_n = 700 \frac{cm^2}{V \cdot s}$

$\mu_p = 250 \frac{cm^2}{V \cdot s}$

$D_n = 0.0259V \cdot 700 \frac{cm^2}{V \cdot s} = 18.13 \frac{cm^2}{s}$

$D_p = 0.0259V \cdot 250 \frac{cm^2}{V \cdot s} = 6.475 \frac{cm^2}{s}$

$L_n = \sqrt{D_n \tau_n} = \sqrt{18.13 \frac{cm^2}{s} \cdot 10^{-6} s} = 2.54 \cdot 10^{-3} cm$

$L_p = \sqrt{D_p \tau_p} = \sqrt{6.475 \frac{cm^2}{s} \cdot 10^{-6} s} = 4.26 \cdot 10^{-3} cm$

$$p_n = \frac{n_i^2}{N_a} = \frac{2.25 \cdot 10^{20} \frac{1}{cm^6}}{10^{17} \frac{1}{cm^3}} = 2.25 \cdot 10^3 \frac{1}{cm^3}$$

$n_p = p_n = 2.25 \cdot 10^3 \frac{1}{cm^3}$

$\Delta p_n = p_n \cdot e^{\frac{qV}{kT}} = 2.25 \cdot 10^3 \frac{1}{cm^3} \cdot e^{\frac{0.7}{0.0259}} = 1.23 \cdot 10^{15} \frac{1}{cm^3}$

$\Delta n_p = \Delta p_n = 1.23 \cdot 10^{15} \frac{1}{cm^3}$

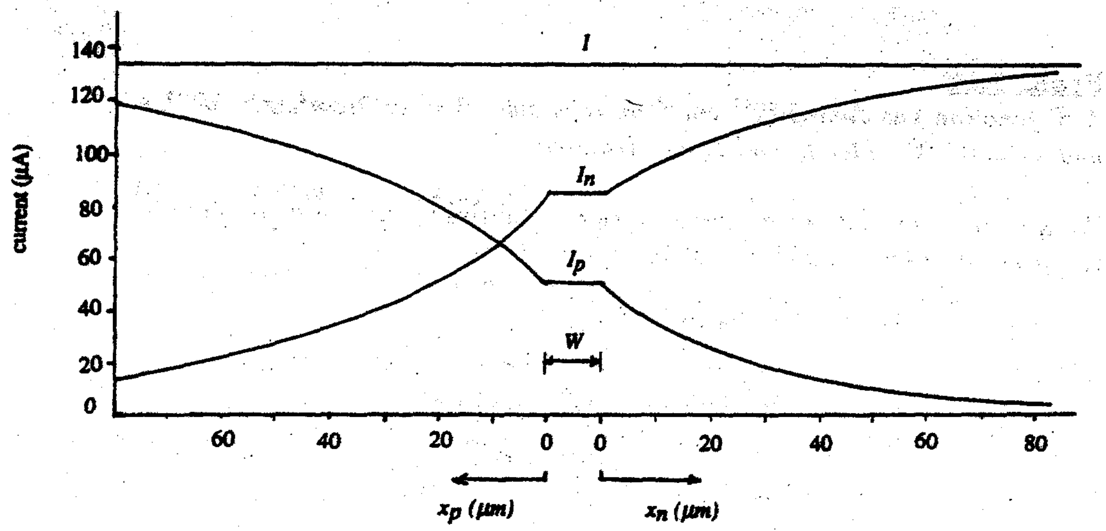
$$I_n(x_p) = qA \frac{L_n}{\tau_n} e^{-\frac{x_p}{L_n}} = 5 \cdot 10^{-5} A \cdot e^{-\frac{x_p}{2.54 \cdot 10^{-3} cm}}$$

$$I_p(x_n) = qA \frac{L_p}{\tau_p} e^{-\frac{x_n}{L_p}} = 8.38 \cdot 10^{-5} A \cdot e^{-\frac{x_n}{4.26 \cdot 10^{-3} cm}}$$

$I = 5 \cdot 10^{-5} A + 8.38 \cdot 10^{-5} A = 133.8 \mu A$

$I_n(x_n) = I - I_p(x_n)$

$I_p(x_p) = I - I_n(x_p)$



Prob. 5.24

Find the new junction capacitance for the given changes.

$$C_j \propto \left(\frac{N_d}{V_r}\right)^{\frac{1}{2}} \qquad C_{j,original} = 10pF$$

$$C_{j,new} \propto \left(\frac{2 \cdot N_d}{8 \cdot V_r}\right)^{\frac{1}{2}} = \tfrac{1}{2} \cdot \left(\frac{N_d}{V_r}\right)^{\frac{1}{2}} \qquad C_{j,new} = \tfrac{1}{2} \cdot C_{j,original} = 5pF$$

Prob. 5.25

Find the minimum width to ensure avalanche breakdown.

$V_{br} = 300V$ from Figure 5-22 and Equation 5-23b

$V_r = V\text{-}V_o \simeq 300V$ and $N_a \gg N_d$

$$x_{n_o} \simeq W = \left[\frac{2 \cdot \epsilon \cdot V_r}{q \cdot N_d}\right]^{\frac{1}{2}} = \left[\frac{2 \cdot 11.8 \cdot 8.85 \cdot 10^{-14} \cdot 300V}{1.6 \cdot 10^{-19} C \cdot 10^{15} \frac{1}{cm^3}}\right]^{\frac{1}{2}} = 2 \cdot 10^{-3} cm = 20 \mu m$$

Prob. 5.26

Calculate the capacitance and relate to N_a.

$$C = \frac{\epsilon_S}{W} \cdot A = A \cdot \sqrt{\frac{q \cdot N_a \cdot \epsilon_S}{2(V_O + V_R)}}$$

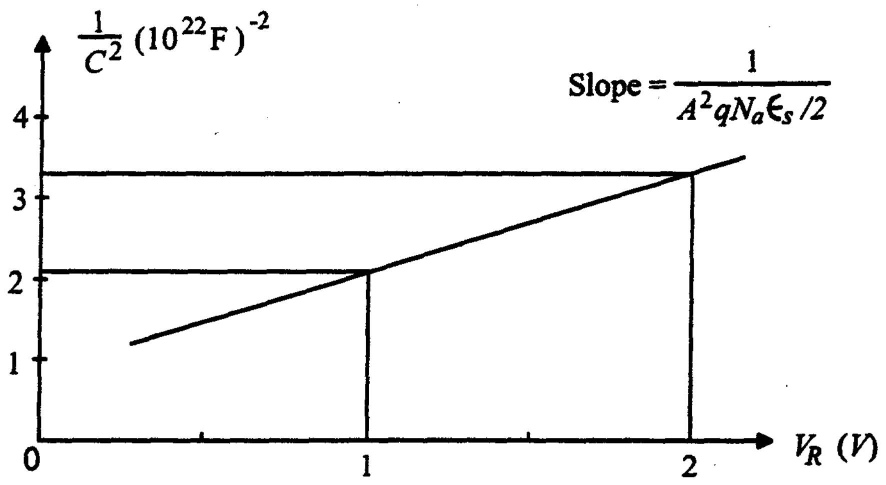
$$V_o = 0.55\text{eV} + .0259\text{eV} \cdot \ln \frac{N_a}{n_i}$$

$$\frac{1}{C^2} = \frac{1}{A^2} \cdot \frac{V_o + V_R}{q \cdot N_a \cdot \frac{\epsilon_s}{2}}$$

This means the slope of $\frac{1}{C^2}$ versus V_R is $\frac{1}{A^2 \cdot q \cdot N_a \cdot \frac{\epsilon_s}{2}}$ so knowing the area and material type allows N_a to be found.

For $N_a = 10^{15} \frac{1}{\text{cm}^3}$, $V_o = 0.84\text{eV} \rightarrow \frac{1}{C^2} = 1.197 \cdot 10^{22} \frac{V}{C^2} \cdot (0.84\text{eV} + V_R)$

For $N_a = 10^{17} \frac{1}{\text{cm}^3}$, $V_o = 0.94\text{eV} \rightarrow \frac{1}{C^2} = 1.197 \cdot 10^{22} \frac{V}{C^2} \cdot (0.94\text{eV} + V_R)$



Prob. 5.27

Calculate the Debye length for a Si p-n junction on the p-side with $N_a = 10^{18}$ cm^{-3} and $N_d = 10^{14}$, 10^{16}, and 10^{18} cm^{-3}. Compare with the calculated value of W for each case.

Find L_D and W, leaving N_d as a variable.

$$L_D = \left[\frac{\epsilon_s \cdot kT}{q^2 \cdot N_d}\right]^{\frac{1}{2}} = \left[\frac{\epsilon_S}{q \cdot N_d} \cdot \frac{kT}{q}\right]^{\frac{1}{2}} = \left[\frac{11.8 \cdot 8.85 \cdot 10^{-14} \frac{F}{cm}}{1.6 \cdot 10^{-19} C} \cdot 0.0259 eV\right]^{\frac{1}{2}} N_d^{-\frac{1}{2}} = \frac{411}{N_d^{\frac{1}{2}}}$$

$$W = \left[\frac{2 \cdot \epsilon_s \cdot kT}{q^2} \cdot \left(\ln \frac{N_a \cdot N_d}{n_i^2}\right) \cdot \left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{\frac{1}{2}} = 581 \frac{1}{cm^2} \cdot \left[\left(\ln \frac{N_d}{225 \frac{1}{cm^3}}\right) \cdot \left(10^{-18} cm^3 + \frac{1}{N_d}\right)\right]^{\frac{1}{2}}$$

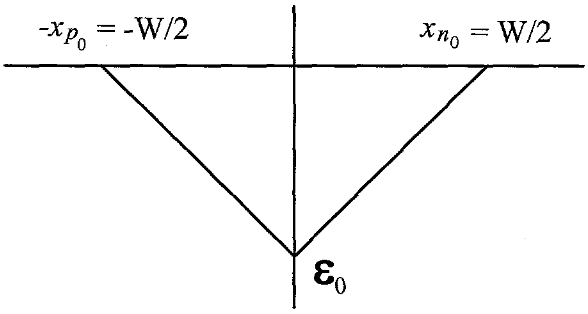
Plugging the doping values into these equations:

N_d (cm^{-3})	L_D (cm)	W (cm)	L_D / W (%)
10^{14}	4.11×10^{-5}	3.01×10^{-4}	7.3
10^{16}	4.11×10^{-6}	3.27×10^{-5}	8.0
10^{18}	4.11×10^{-7}	4.93×10^{-6}	12.0

The Debye length varies from 7 to 12 percent of W across this doping range.

Prob. 5.28

For the given symmetric p-n Si junction, find the reverse breakdown voltage.



$$N_a = N_d \quad \rightarrow \quad x_{n0} = x_{p0} = \frac{W}{2}$$

$$\mathcal{E}_o = \frac{q \cdot N_a}{\in} \cdot x_{n0} = \frac{q \cdot N_a}{\in} \cdot \frac{W}{2}$$

The breakdown voltage plus built-in voltage is:

$$\int_{-x_{p0}}^{x_{n0}} \mathcal{E} \cdot dx = \text{the area under the triangle}$$

$$V_{br} + V_o = \mathcal{E}_o \cdot x_{n0} = \mathcal{E}_o \cdot \frac{W}{2} = \mathcal{E}_o \cdot \frac{1}{2} \cdot \left(\frac{2 \cdot \in \cdot \mathcal{E}_o}{q \cdot N_a} \right) = \frac{\in \cdot \mathcal{E}_o^2}{q \cdot N_a}$$

$$= \frac{11.8 \cdot 8.85 \cdot 10^{-14} \frac{F}{cm} \cdot (5 \cdot 10^5 \frac{V}{cm})^2}{1.6 \cdot 10^{-19} C \cdot 10^{17} \frac{1}{cm^3}} = 16.32V$$

Prob. 5.29

The current in a long p^+-n diode is tripled at $t = 0$.

(a) *What is the slope of $\delta p(x_n = 0)$?*

The slope triples at $t = 0$:

$$3I = -qAD_p \left.\frac{d\delta p}{dx_n}\right|_{x_n=0}$$

The slope is

$$\left.\frac{d\delta_p}{dx_n}\right|_{x_n=0} = -3I/qAD_p$$

(b) *Relate $V(t = \infty)$ to $V(t = 0^-)$.*

Call V^- the voltage before $t = 0$.

Call V^∞ the voltage at $t = \infty$.
at $t = 0^-$:

$$I = \frac{qAD_p}{L_p} p_n \, e^{qV^-/kT}$$

at $t = \infty$:

$$3I = \frac{qAD_p}{L_p} p_n \, e^{qV^\infty/kT}$$

Taking the ratio :

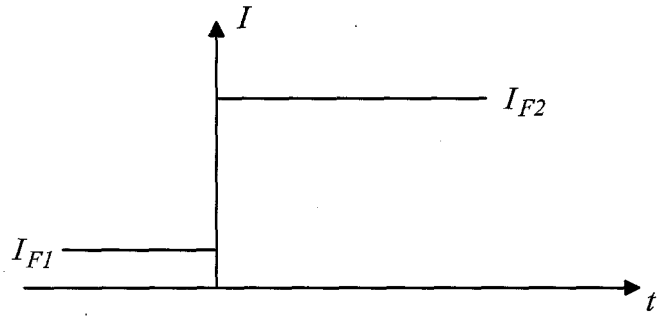
$$3 = e^{q(V^\infty - V^-)/kT}$$

$$V^\infty = V^- + \frac{kT}{q} \ln 3$$

or $V^- + 0.0285$

Prob. 5.30

Find the stored charge Q_p as a function of time in the n-region if a long p^+-n forward bias current is switched from I_{F1} to I_{F2} at $t = 0$.



$$Q_p(0) = I_{F1} \cdot \tau_p$$

$$Q_p(\infty) = I_{F2} \cdot \tau_p$$

$$I_{F2} = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p}{dt}$$

Taking the Laplace transform

$$\frac{I_{F2}}{s} = \frac{Q_p(s)}{\tau_p} + s \cdot Q_p(s) - I_{F1} \cdot \tau_p$$

$$Q_p(s) = \left(\frac{I_{F2}}{s} + I_{F1} \cdot \tau_p \right) \frac{1}{s + \frac{1}{\tau_p}}$$

Transforming back to time domain

$$Q_p(t) = I_{F2} \cdot \tau_p \cdot \left(1 - e^{-\frac{t}{\tau_p}} \right) + I_{F1} \cdot \tau_p \cdot e^{-\frac{t}{\tau_p}}$$

Prob. 5.31

The forward current in a long p^+-n diode is suddenly raised from 0 to I at t = 0.

(a) *Find and sketch $Q_p(t)$.*

$$I = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p(t)}{dt}$$

$$Q_p(t) = A + Be^{-\frac{t}{\tau_p}}$$

at t = 0, $Q_p(0) = A + B = 0$

at t -> infinity, $Q_p(\infty) = A = I\,\tau_p$

thus, $B = -I\,\tau_p$

$$Q_p(t) = I \cdot \tau_p \left(1 - e^{-\frac{t}{\tau_p}}\right)$$

(b) *Find $\Delta p_n(t)$ and v(t) in the quasi-steady state approximation.*

If $$Q_p = q \cdot A \cdot \int_0^{\infty} \Delta p_n \cdot e^{-\frac{x_n}{L_p}} \cdot dx_n = q \cdot A \cdot L_p \cdot \Delta p_n$$

Then $$\Delta p_n(t) = \frac{Q_p(t)}{q \cdot A \cdot L_p}$$

$$\Delta p_n(t) = p_n \cdot e^{\frac{qv}{kT}} = \frac{I \cdot \tau_p \cdot \left(1 - e^{-\frac{t}{\tau_p}}\right)}{q \cdot A \cdot L_p}$$

Thus $$v(t) = \frac{kT}{q} \cdot \ln\left(\frac{\Delta p_n(t)}{p_n}\right)$$

$$v(t) = \frac{kT}{q} \cdot \ln\left(\frac{I \cdot \tau_p \cdot \left(1 - e^{\frac{-t}{\tau_p}}\right)}{q \cdot A \cdot L_p \cdot p_n}\right)$$

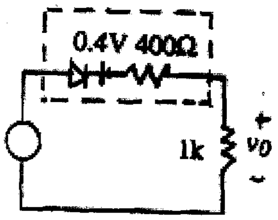
Prob. 5.32

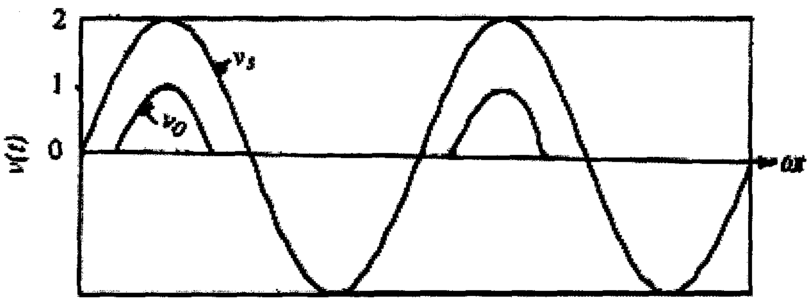
Sketch the voltage across a 1 kΩ resistor in series with a diode (offset 0.4 V, resistance 400 Ω) and a voltage source of 2 sin ωt.

An ideal diode is a perfect short (resistance is zero) when the voltage across the diode is greater than the offset voltage. If the voltage is less, the diode is a perfect open (resistance is infinite). Thus,

$$v_s < 0.4, v_0 = 0$$

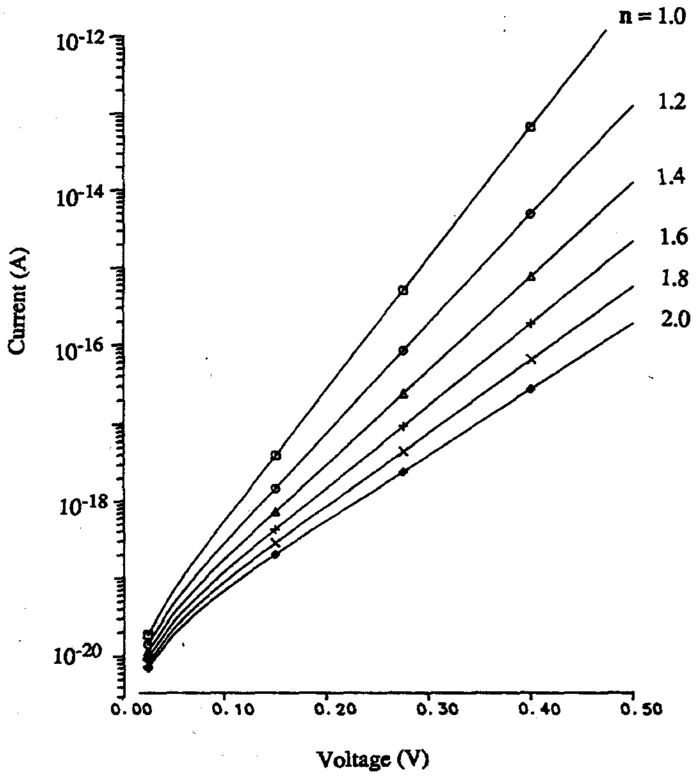
$$v_s > 0.4, v_0 = \frac{(v_s - 0.4)\cdot 1k\Omega}{1k\Omega + 400\Omega}$$





Prob. 5.33

Plot current versus voltage.

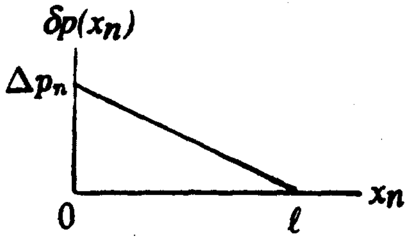


Prob. 5.34

Find Q_p and I when holes are injected from p_+ into a short n-region of length l, if δp varies linearly.

$$Q_p = q \cdot A \cdot \int_0^{\ell} \partial p \cdot dx_n = q \cdot A \cdot \frac{\ell \cdot \Delta p_n}{2}$$

$$I = I_p\left(x_n = 0\right) = -\,q \cdot \frac{A \cdot D_p \cdot \Delta p_n}{\ell}$$

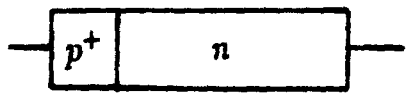


Prob. 5.35

Find the hole distribution and the total current in a narrow-base diode.

$$\frac{d^2\partial p(x_n)}{dx_n^2}=\frac{\partial p(x_n)}{L_p^2}$$

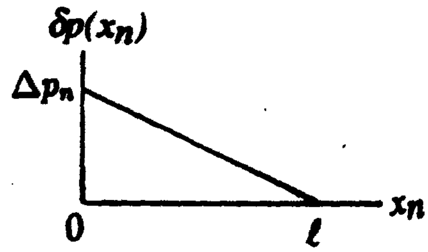
$$\partial p(x_n)=Ce^{-\frac{x_n}{L_p}}+De^{\frac{x_n}{L_p}}$$



Boundary conditions:

When $x_n=0,\ \partial p=\Delta p_n=C+D$

When $x_n=l,\ \partial p=0=Ce^{-\frac{l}{L_p}}+De^{\frac{l}{L_p}}$



Thus,

$$C=\frac{\Delta p_n e^{\frac{l}{L_p}}}{e^{\frac{l}{L_p}}-e^{-\frac{l}{L_p}}}$$

$$D=\Delta p_n-C=\frac{-\Delta p_n e^{-\frac{l}{L_p}}}{e^{\frac{l}{L_p}}-e^{-\frac{l}{L_p}}}$$

Plugging C and D back into the solution

a) $$\partial p(x_n)=\frac{\Delta p_n\left[e^{\frac{l-x_n}{L_p}}-e^{-\frac{x_n-l}{L_p}}\right]}{e^{\frac{l}{L_p}}-e^{-\frac{l}{L_p}}}$$

$$I=\left[-q\cdot A\cdot D_p\cdot\frac{d\partial p(x_n)}{dx_n}\right]_{x_n\to 0}=\frac{-q\cdot A\cdot D_p\cdot\Delta p_n}{L_p}\cdot\left[-\frac{e^{\frac{l}{L_p}}+e^{-\frac{l}{L_p}}}{e^{\frac{l}{L_p}}-e^{-\frac{l}{L_p}}}\right]$$

b) $$I=\left[\frac{q\cdot A\cdot D_p}{L_p}\cdot p_n\cdot \mathrm{ctnh}\frac{l}{L_p}\right]\cdot\left(e^{\frac{qV}{kT}}-1\right)$$

Prob. 5.36

For the narrow-base diode, find the current components due to recombination in n, and recombination at the ohmic contact.

The steady-state charge stored in the excess hole distribution is

$$Q_p = q\cdot A\int_0^l \delta p(x_n)\cdot dx_n = q\cdot A\int_0^l\left[C\cdot e^{-\frac{x_n}{L_p}} + D\cdot e^{\frac{x_n}{L_p}}\right]\cdot dx_n$$

$$Q_p = q\cdot A\cdot L_p\left[-C\left(e^{-\frac{l}{L_p}}-1\right)+D\left(e^{\frac{l}{L_p}}-1\right)\right]$$

$$Q_p = q\cdot A\cdot L_p\cdot \Delta p_n\cdot\frac{e^{\frac{l}{L_p}}+e^{-\frac{l}{L_p}}-2}{e^{\frac{l}{L_p}}-e^{-\frac{l}{L_p}}}$$

The current due to recombination in n is

$$\frac{Q_p}{\tau_p} = \frac{qAL_pp_n}{\tau_p}\left[\operatorname{ctnh}\frac{l}{L_p}-\operatorname{csch}\frac{l}{L_p}\right]\left(e^{\frac{qV}{kT}}-1\right)$$

using $\dfrac{L_p}{\tau_p} = \dfrac{D_p}{L_p}$

$$\frac{Q_p}{\tau_p} = \frac{qAD_pp_n}{L_p}\left[\tanh\frac{l}{L_p}\right]\left(e^{\frac{qV}{kT}}-1\right)$$

The current due to recombination at $x_n = l$,

$$I-\frac{Q_p}{\tau_p} = \left[\frac{qAL_pp_n}{\tau_p}\operatorname{csch}\frac{l}{L_p}\right]\left(e^{\frac{qV}{kT}}-1\right)$$

These correspond to the base recombination and collector currents in the p-n-p BJT with $V_{CB} = 0$ given in Equation 7-20.

Prob. 5.37

If the n region of a graded p^+-n has $N_d=Gx^m$, find $\mathcal{E}_o$, $\mathcal{E}(x)$, Q, and C_j.

a) $$\frac{d\mathcal{E}}{dx} = \frac{q}{\in}N_d = \frac{q}{\in}Gx^m \rightarrow \int_{\mathcal{E}_o}^{0} d\mathcal{E} = \frac{q}{\in}G\int_0^W x^m dx$$

$$-\mathcal{E}_o = \frac{q}{\in}G\frac{W^{m+1}}{m+1}$$

$$\mathcal{E}_o = -\frac{q}{\in}G\frac{W^{m+1}}{m+1}$$

b) $$\int_0^{\mathcal{E}} d\mathcal{E} = \frac{q}{\in}G\int_W^x x^m dx \rightarrow \mathcal{E}(x) = \frac{qG}{\in(m+1)}\left(x^{m+1} - W^{m+1}\right)$$

$$\mathcal{E} = -\frac{dV}{dx} \rightarrow -\int_V^{V_o} dV = \int_0^W \mathcal{E}\cdot dx$$

$$-(V_o - V) = \frac{qG}{\in(m+1)}\left(\frac{x^{m+2}}{m+2} - W^{m+1}\cdot x\right)_0^W$$

$$-(V_o - V) = \frac{qG}{\in(m+1)}\left(\frac{W^{m+2}}{m+2} - \frac{(m+2)\cdot W^{m+2}}{m+2}\right) = \frac{qG}{\in(m+1)}\cdot\frac{-(m+1)W^{m+2}}{m+2}$$

$$V_o - V = \frac{qG\cdot W^{m+2}}{\in(m+2)}$$

c) $$Q = qA\int_0^W Gx^m dx = \frac{qAGW^{m+1}}{m+1}$$

$$W^{m+1} = \left(W^{m+2}\right)^{\frac{m+1}{m+2}} = \left(\frac{(V_o-V)\cdot\in\cdot(m+2)}{qG}\right)^{\frac{m+1}{m+2}}$$

$$Q = \frac{qAG}{m+1}\cdot\left(\frac{(V_o-V)\cdot\in\cdot(m+2)}{qG}\right)^{\frac{m+1}{m+2}}$$

d) $$Q = \frac{qAG}{m+1} \cdot \left(\frac{(V_o - V) \cdot \in \cdot (m+2)}{qG} \right)^{\frac{m+1}{m+2}}$$

$$\frac{dQ}{d(V_o - V)} = \frac{qAG}{m+1} \cdot \left(\frac{(V_o - V) \cdot \in \cdot (m+2)}{qG} \right)^{\frac{m+1}{m+2} - 1} \cdot \frac{\in \cdot (m+2)}{qG}$$

$$\frac{dQ}{d(V_o - V)} = \frac{A \cdot \in \cdot (m+2)}{m+1} \cdot \left(\frac{(V_o - V) \cdot \in \cdot (m+2)}{qG} \right)^{\frac{-1}{m+2}}$$

$$\frac{dQ}{d(V_o - V)} = A \cdot \left(\in^{m+2} \right)^{\frac{1}{m+2}} \cdot \left(\frac{qG}{(V_o - V) \cdot \in \cdot (m+2)} \right)^{\frac{1}{m+2}}$$

$$\frac{dQ}{d(V_o - V)} = A \cdot \left(\frac{qG \cdot \in^{m+1}}{(V_o - V) \cdot (m+2)} \right)^{\frac{1}{m+2}}$$

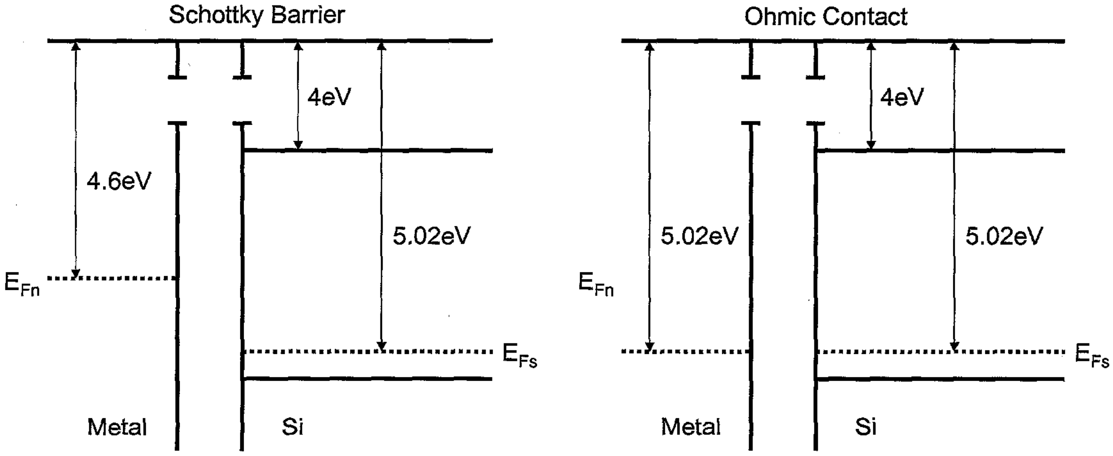
Prob. 5.38

Draw the equilibrium band diagram and explain whether the junction is a Schottky or ohmic contact. Describe how to change the metal work function to switch the contact type.

$$\phi_s = \chi + 0.55\text{eV} + kT \cdot \ln \frac{N_a}{n_i} = 4\text{eV} + 0.55\text{eV} + 0.0259\text{eV} \cdot \ln \frac{10^{18} \frac{1}{cm^3}}{1.5 \cdot 10^{10} \frac{1}{cm^3}} = 5.02\text{eV}$$

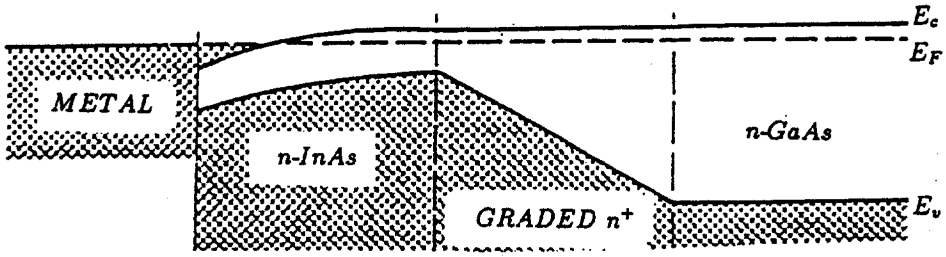
For this p-type semiconductor, ($\Phi_m = 4.6$eV) < ($\Phi_s = 5.02$eV); so, the junction is a Schottky barrier.

The junction becomes an ohmic contact at $\Phi_m > \Phi_s$. The metal work function must be raised to 5.02eV to make this an ohmic contact.



Prob. 5.39

Use InAs to make an ohmic contact to GaAs.



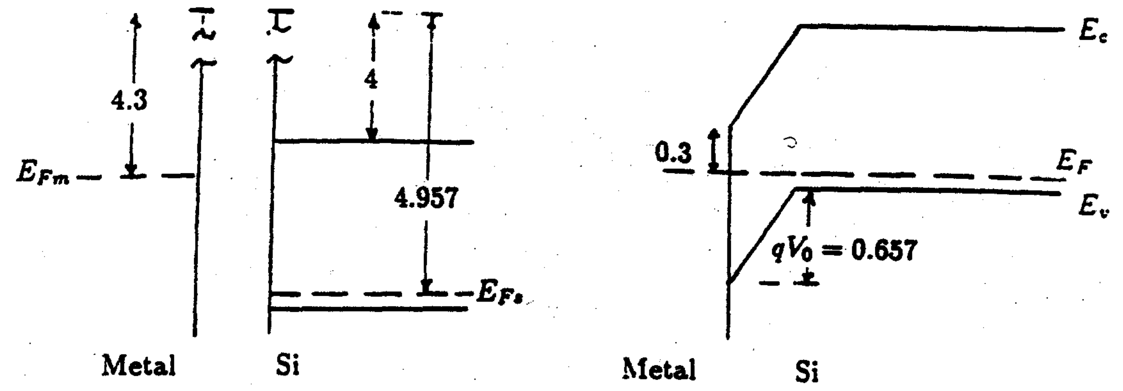
For further discussion, see *Woodall, et al., J. Vac. Sci. Technol. 19. 626(1981).*

Prob. 5.40

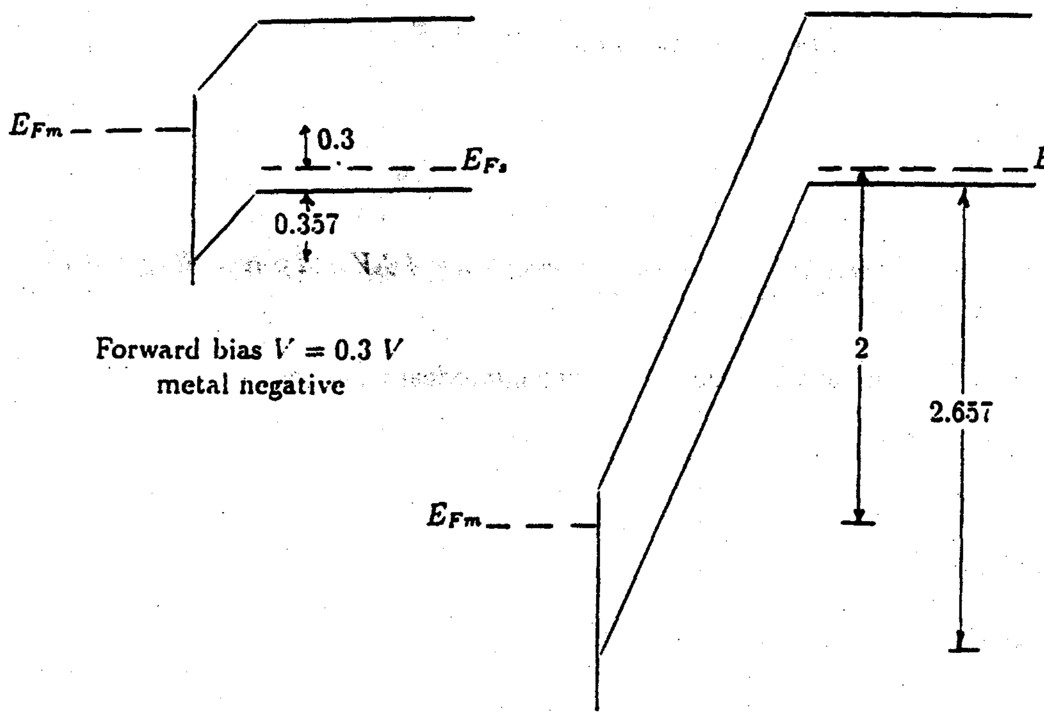
Draw the equilibrium band diagram (a) and the band diagrams for 0.3V forward bias and 2.0V reverse bias (b).

a) $E_i - E_F = kT \cdot \ln\frac{p_o}{n_i} = 0.0259\text{eV} \cdot \ln\frac{10^{17}\frac{1}{cm^3}}{1.5 \cdot 10^{10}\frac{1}{cm^3}} = 0.407\text{eV}$

$\Phi_s = 4 + 0.55 + 0.407 = \mathbf{4.957\ eV}$



b)



Reverse bias $V = -2\ V$
metal positive

Chapter 6 Solutions

Prob. 6.1

Find V_O, V_P, and V_T. Find $V_{D,sat}$ for V_G =-3V.

$$V_O = \frac{kT}{q} \cdot \ln \frac{N_a \cdot N_d}{n_i^2} = 0.0259\text{eV} \cdot \ln \frac{10^{18}\frac{1}{\text{cm}^3} \cdot 10^{16}\frac{1}{\text{cm}^3}}{\left(1.5 \cdot 10^{10}\frac{1}{\text{cm}^3}\right)^2} = 0.814\text{V}$$

$$V_P = \frac{q \cdot a^2 \cdot N_d}{2 \cdot \in} = \frac{1.6 \cdot 10^{-19}\text{C} \cdot (10^{-4}\text{cm})^2 \cdot 10^{16}\frac{1}{\text{cm}^3}}{2 \cdot 11.8 \cdot 8.85 \cdot 10^{16}\frac{\text{F}}{\text{cm}}} = 7.66\text{V}$$

$$V_T = V_P - V_O = 6.85\text{V}$$

$$V_{D,sat} = V_T + V_G = 6.85\text{V} - 3.00\text{V} = 3.85\text{V}$$

Prob. 6.2

Find $I_{D,sat}$ for V_G=0V, -2V, -4V, and -6V and plot $I_{D,sat}$ versus $V_{D,sat}$ for JFET in 6.1.

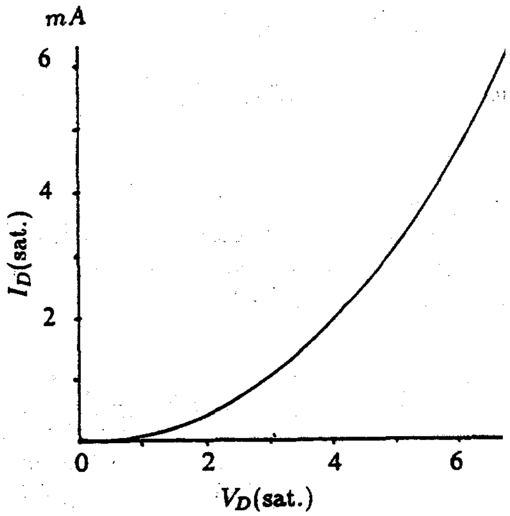
$$G_O = 2 \cdot a \cdot q \cdot \mu_n \cdot n \cdot \frac{Z}{L} = 2 \cdot 10^{-4}\text{cm} \cdot 1.6 \cdot 10^{-19}\text{V} \cdot 10^3 \frac{\text{cm}^2}{\text{V} \cdot \text{s}} \cdot 10^{16}\frac{1}{\text{cm}^3} \cdot 10 = 3.2 \cdot 10^{-3}\text{S}$$

$$I_{D,sat} = G_O \cdot V_P \cdot \left[\frac{V_G - V_O}{V_P} + \frac{2}{3} \cdot \left(\frac{V_G - V_O}{V_P} \right)^{\frac{3}{2}} + \frac{1}{3} \right]$$

$$I_{D,sat} = 3.2 \cdot 10^{-3}\text{S} \cdot 7.66\text{V} \cdot \left[\frac{V_G - 0.814\text{V}}{7.66\text{V}} + \frac{2}{3} \cdot \left(\frac{V_G - 0.814\text{V}}{7.66\text{V}} \right)^{\frac{3}{2}} + \frac{1}{3} \right]$$

We can plot this vs. V_D (sat.) = 6.85 + V_G

V_G	V_D(sat.)	I_D(sat.)
0 V	6.85 V	6.13 mA
−1	5.85	4.25
−2	4.85	2.80
−3	3.85	1.707
−4	2.85	0.907
−5	1.85	0.372
−6	0.85	0.076

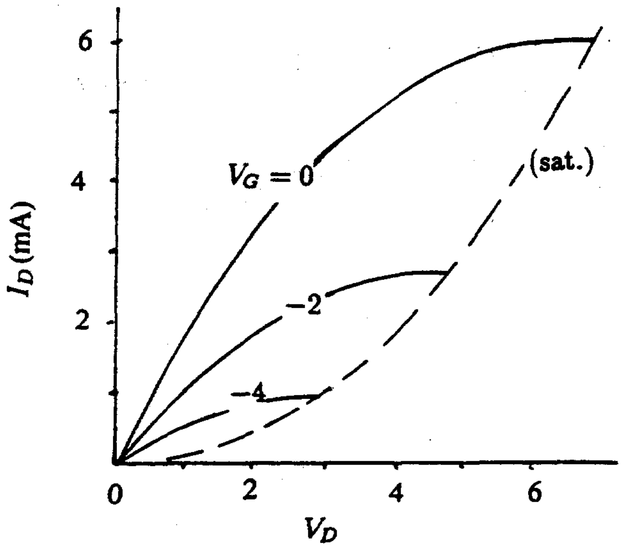


Prob. 6.3

Graph I_D versus V_D for V_G=0V, -2V, -4V, and -6V for JFET in 6.1.

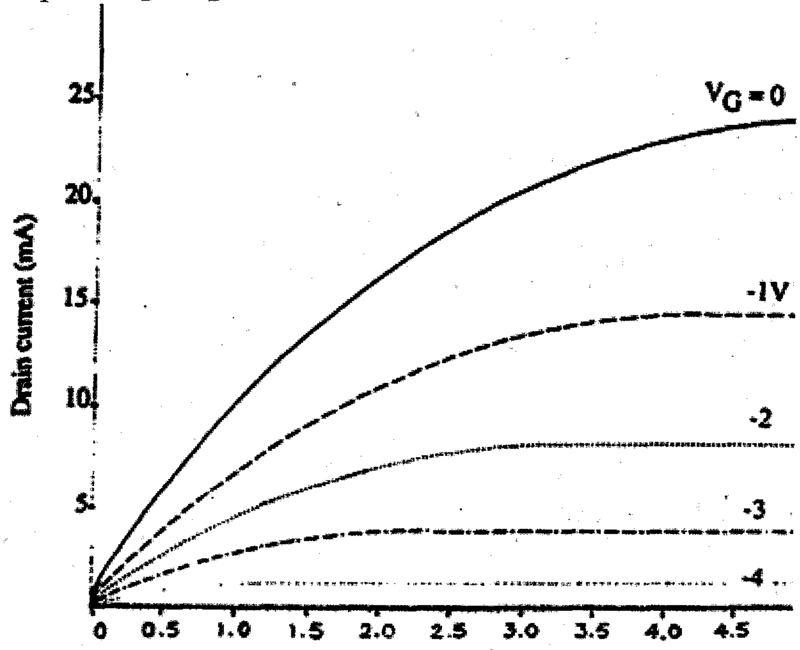
$$I_D = G_O \cdot V_P \cdot \left[\frac{V_D}{V_P} + \frac{2}{3} \cdot \left(\frac{V_O - V_G}{V_P} \right)^{\frac{3}{2}} - \frac{2}{3} \cdot \left(\frac{V_O + V_D - V_G}{V_P} \right)^{\frac{3}{2}} \right]$$

$$I_D = 3.2 \cdot 10^{-3} S \cdot 7.66V \cdot \left[\frac{V_D}{7.66V} + \frac{2}{3} \cdot \left(\frac{0.814V - V_G}{7.66V} \right)^{\frac{3}{2}} - \frac{2}{3} \cdot \left(\frac{0.814V + V_D - V_G}{7.66V} \right)^{\frac{3}{2}} \right]$$



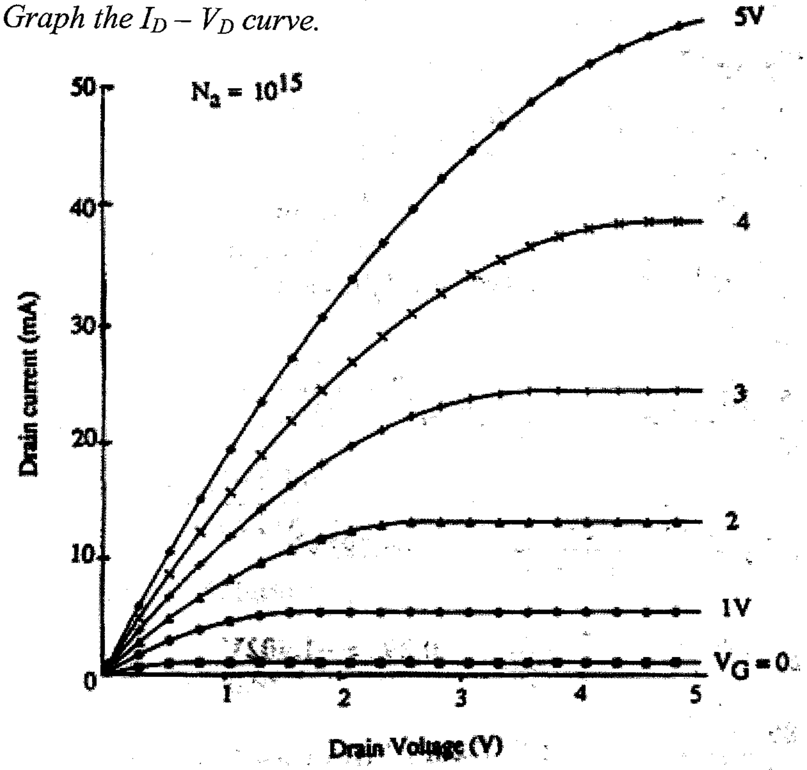
Prob. 6.4

Graph the $I_D - V_D$ curve.



Prob. 6.5

Graph the $I_D - V_D$ curve.



Prob. 6.6

For current I_D varying linearly with V_D at low values of V_D for a JFET,
(a) use the binomial expansion to rewrite Equation 6-9
Equation 6-9 may be rewritten as

$$I_D \approx G_O \cdot V_P \cdot \left[\frac{V_D}{V_P} + \frac{2}{3} \cdot \left(\frac{-V_G}{V_P} \right)^{\frac{3}{2}} - \frac{2}{3} \cdot \left(\frac{V_D - V_G}{V_P} \right)^{\frac{3}{2}} \right]$$

$$I_D = G_O \cdot \left[V_D + \frac{2}{3} \cdot \frac{(-V_G)^{\frac{3}{2}}}{V_P^{\frac{1}{2}}} - \frac{2}{3} \cdot \frac{(-V_G)^{\frac{3}{2}}}{V_P^{\frac{1}{2}}} \cdot \left(\frac{V_D}{-V_G} + 1 \right)^{\frac{3}{2}} \right]$$

use the binomial approximation $(1+x)^{\frac{3}{2}} \approx 1 + \frac{3}{2} \cdot x$

$$I_D = G_O \cdot \left[V_D + \frac{2}{3} \cdot \frac{(-V_G)^{\frac{3}{2}}}{V_P^{\frac{1}{2}}} - \frac{2}{3} \cdot \frac{(-V_G)^{\frac{3}{2}}}{V_P^{\frac{1}{2}}} \cdot \left(1 + \frac{3}{2} \cdot \left(-\frac{V_D}{V_G} \right) \right) \right]$$

$$I_D = G_O \cdot V_D \cdot \left[1 - \left(\frac{-V_G}{V_P} \right)^{\frac{1}{2}} \right]$$

(b) show that I_D/V_D in the linear range is the same as g_m (sat),

$$\frac{I_D}{V_D} = G_O \cdot \left[1 - \left(\frac{-V_G}{V_P} \right)^{\frac{1}{2}} \right] = g_m(\text{sat})$$

(c) and find the value of V_G for device turn off.

$V_G = -V_P$

Prob. 6.7

Show that the width of the depletion region in Figure 6-15 is given by Equation 6-30.

Use the mathematics leading to Equation 5-23b with Φ_s for the potential difference across the depletion region contained in x_{po}=W.

Prob. 6.8

Find the maximum depletion width, minimum capacitance, and threshold voltage.

$$\Phi_F = \frac{kT}{q}\ln\frac{N_a}{n_i} = 0.0259\text{V}\ln\frac{10^{16}\frac{1}{cm^3}}{1.5\cdot10^{10}\frac{1}{cm^3}} = 0.347\text{V}$$

$$W_m = 2\sqrt{\frac{\epsilon_s\cdot\Phi_F}{q\cdot N_a}} = 2\left[\frac{11.8\cdot8.85\cdot10^{-14}\frac{F}{cm}\cdot0.347\text{V}}{1.6\cdot10^{-19}\text{C}\cdot10^{16}\frac{1}{cm^3}}\right]^{1/2} = 3.01\cdot10^{-5}\,\text{cm} = 0.301\,\mu\text{m}$$

$$C_i = \frac{\epsilon_i}{d} = \frac{3.9\cdot8.85\cdot10^{-14}\frac{F}{cm}}{10^{-6}\text{cm}} = 3.45\cdot10^{-7}\frac{F}{cm^2}$$

$$Q_d = -q\cdot N_a\cdot W_m = -1.6\cdot10^{-19}\text{C}\cdot10^{16}\frac{1}{cm^3}0.301\cdot10^{-4}\text{cm} = -4.82\cdot10^{-8}\frac{C}{cm^2}$$

$$V_T = -\frac{Q_d}{C_i}+2\cdot\Phi_F = \frac{4.82\cdot10^{-8}\frac{C}{cm^2}}{34.5\cdot10^{-8}\frac{F}{cm^2}}+2\cdot0.347\text{V} = 0.834\,\text{V}$$

At maximum depletion

$$C_d = \frac{\epsilon_s}{W_m} = \frac{11.8\cdot8.85\cdot10^{-14}\frac{F}{cm}}{0.301\cdot10^{-4}\text{cm}} = 3.47\cdot10^{-8}\frac{F}{cm^2}$$

$$C_{min} = \frac{C_iC_d}{C_i+C_d} = \frac{3.45\cdot10^{-8}\frac{F}{cm^2}\cdot3.47\cdot10^{-8}\frac{F}{cm^2}}{3.45\cdot10^{-8}\frac{F}{cm^2}+3.47\cdot10^{-8}\frac{F}{cm^2}} = 3.15\cdot10^{-8}\frac{F}{cm^2}$$

Prob. 6.9

Find W_m, V_{FB}, and V_T. Sketch the C-V curve.

$$\Phi_F = -kT\cdot\ln\frac{N_d}{n_i} = -0.0259V\cdot\ln\frac{5\cdot10^{17}\frac{1}{cm^3}}{1.5\cdot10^{10}\frac{1}{cm^3}} = -0.449V$$

$$W_m = 2\cdot\left[\frac{\epsilon_s\cdot(-\Phi_F)}{q\cdot N_d}\right]^{\frac{1}{2}} = 2\cdot\left[\frac{11.8\cdot8.85\cdot10^{-14}\frac{F}{cm}\cdot(0.449V)}{1.6\cdot10^{-19}C\cdot5\cdot10^{17}\frac{1}{cm^3}}\right]^{\frac{1}{2}} = 0.049\mu m$$

$$Q_d = q\cdot N_d\cdot W_m = 1.6\cdot10^{-19}C\cdot5\cdot10^{17}\frac{1}{cm^3}\cdot0.049\cdot10^{-4}cm = 3.92\cdot10^{-7}\frac{C}{cm^2}$$

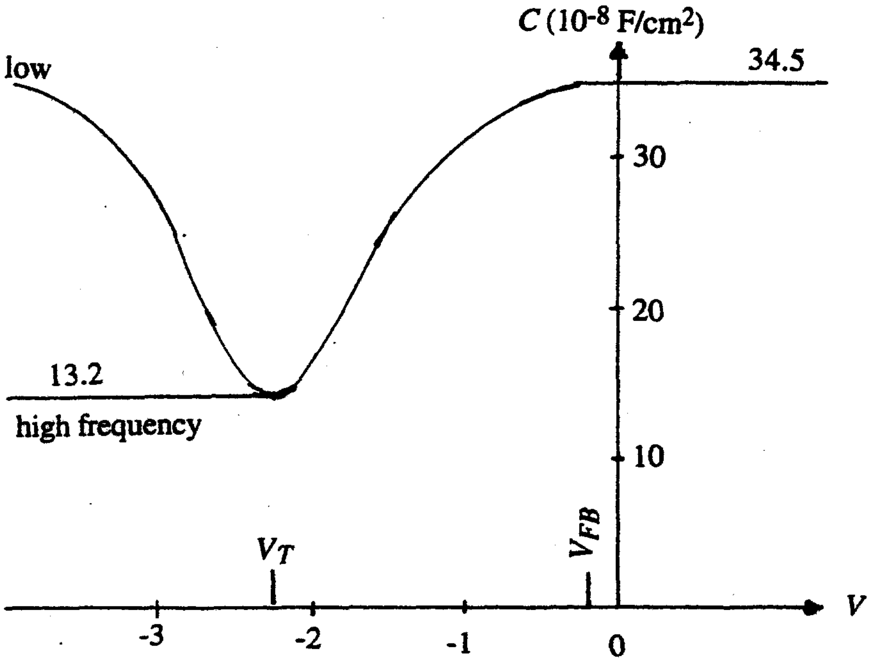
$$C_i = \frac{\epsilon_i}{d} = \frac{3.9\cdot8.85\cdot10^{-14}\frac{F}{cm}}{10^{-6}cm} = 3.45\cdot10^{-7}\frac{F}{cm^2}$$

$$V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i} = -0.15V - \frac{1.6\cdot10^{-19}C\cdot5\cdot10^{10}\frac{1}{cm^3}}{3.45\cdot10^{-7}\frac{F}{cm^2}} = -0.173V$$

$$V_T = 2\cdot\Phi_F - \frac{Q_d}{C_i} + V_{FB} = -0.898V - 1.136V - 0.173V = -2.2V$$

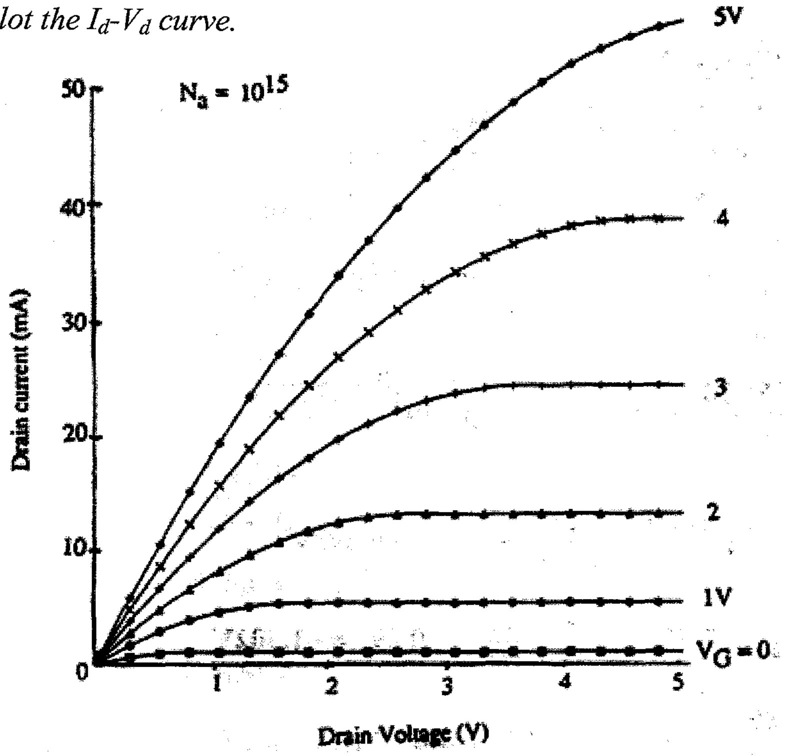
$$C_d = \frac{\epsilon_s}{W_m} = \frac{11.8\cdot8.85\cdot10^{-14}\frac{F}{cm}}{0.049\cdot10^{-4}cm} = 2.13\cdot10^{-7}\frac{F}{cm^2}$$

$$C_{min} = \frac{C_i\cdot C_d}{C_i + C_d} = \frac{3.45\cdot10^{-7}\frac{F}{cm^2}\cdot2.13\cdot10^{-7}\frac{F}{cm^2}}{3.45\cdot10^{-7}\frac{F}{cm^2}+2.13\cdot10^{-7}\frac{F}{cm^2}} = 1.32\cdot10^{-7}\frac{F}{cm^2}$$



Prob. 6.10

Plot the I_d-V_d curve.



Prob. 6.11

Calculate the V_T and find the B dose necessary to change the V_T to zero.

From Figure 6-17, $\Phi_{ms} = -0.1V$

$$C_i = \frac{\epsilon_i}{d} = \frac{3.9 \cdot 8.85 \cdot 10^{-14}\,\frac{F}{cm}}{50 \cdot 10^{-8}\,cm} = 6.90 \cdot 10^{-7}\,\frac{F}{cm^2}$$

$$\Phi_F = -kT \cdot \ln\frac{N_d}{n_i} = -0.0259V \cdot \ln\frac{10^{18}\,\frac{1}{cm^3}}{1.5 \cdot 10^{10}\,\frac{1}{cm^3}} = -0.467V$$

$$W = 2 \cdot \left[\frac{\epsilon_s \cdot (-\Phi_F)}{q \cdot N_d}\right]^{\frac{1}{2}} = 2 \cdot \left[\frac{11.8 \cdot 8.85 \cdot 10^{-14}\,\frac{F}{cm} \cdot (0.467V)}{1.6 \cdot 10^{-19} C \cdot 10^{18}\,\frac{1}{cm^3}}\right]^{\frac{1}{2}} = 3.49 \cdot 10^{-6} cm$$

$$V_T = 2 \cdot \Phi_F - \frac{Q_d}{C_i} + \Phi_{ms} - \frac{Q_i}{C_i} = -0.934V - \frac{1.6 \cdot 10^{-19} C \cdot 3.49 \cdot 10^{12}\,\frac{1}{cm^2}}{6.90 \cdot 10^{-7}\,\frac{F}{cm^2}} - 0.1V - \frac{1.6 \cdot 10^{-19} C \cdot 2 \cdot 10^{10}\,\frac{1}{cm^2}}{6.90 \cdot 10^{-7}\,\frac{F}{cm^2}} = -1.85V$$

Enhancement Mode P-Channel Device

To achieve $V_T = 0V$ a $\Delta V_T = 1.85V$ is required

$$Q_{Boron} = \Delta V_T \cdot C_i = 1.85V \cdot 6.90 \cdot 10^{-7}\,\frac{F}{cm^2} = 1.28 \cdot 10^{-6}\,\frac{C}{cm^2}$$

$$\text{Boron Dose} = \frac{Q_{Boron}}{q} = \frac{1.28 \cdot 10^{-6}\,\frac{C}{cm^2}}{1.6 \cdot 10^{-19} C} = 7.98 \cdot 10^{12}\,\frac{1}{cm^2}$$

Prob. 6.12

Find V_{FB} for a positive oxide charge Q_{ox} located x' below the metal. Repeat for an arbitrary distribution ρ(x').

(a) At flat band, the induced charge in the semiconductor is zero.

$$\mathcal{E} = -\frac{Q_{ox}}{\epsilon_i}$$

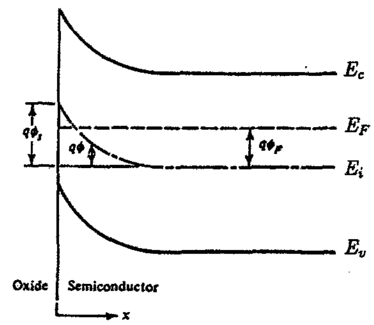
$$V = V_{FB} = x' \cdot \mathcal{E} = -\frac{x' \cdot Q_{ox}}{\epsilon_i}$$

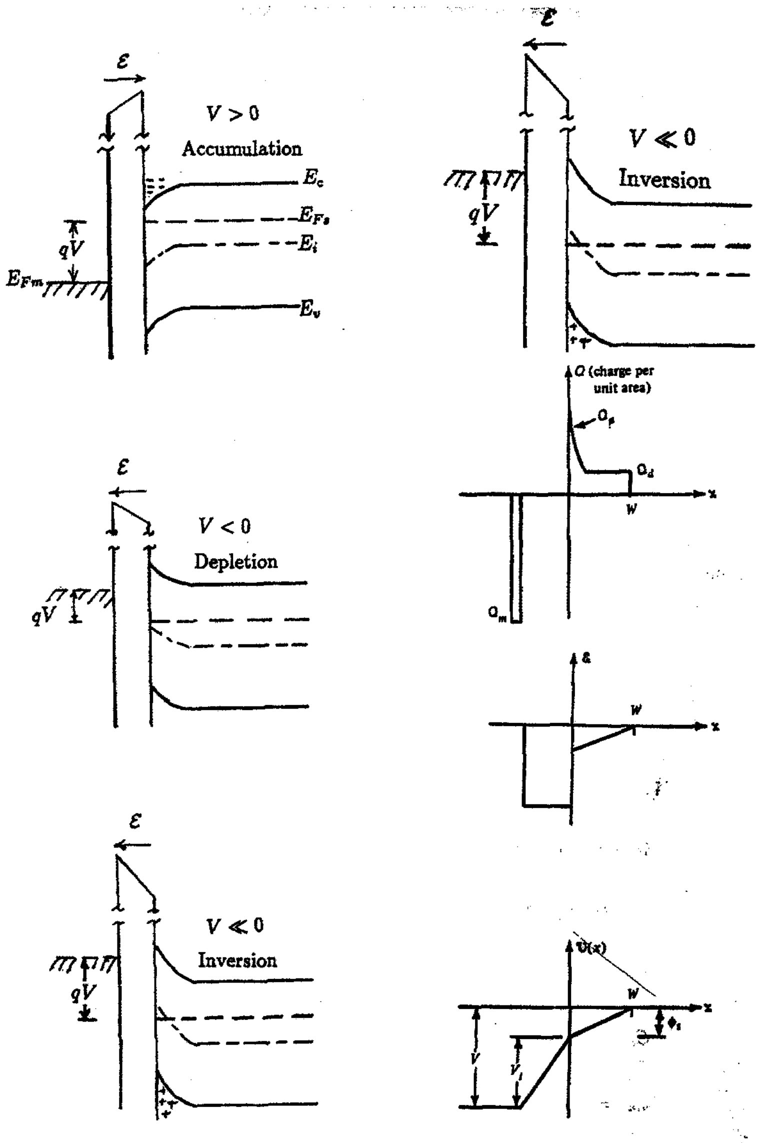
(b) A distributed charge will act as a sum of discrete charges over the entire oxide.

$$V = V_{FB} = -\frac{1}{\epsilon_i}\int_0^d x' \cdot \rho(x') \cdot dx = -\frac{1}{C_i}\int_0^d \frac{x'}{d} \cdot \rho(x') \cdot dx'$$

Prob. 6.13

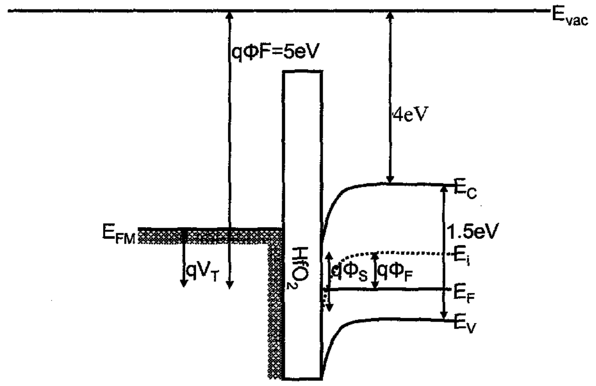
Draw figures for an n-type substrate.





Prob. 6.14

Sketch the band diagram and CV characteristics and calculate the appropriate values.



$$\Phi_{ms} = \Phi_m - \Phi_s = 5\text{eV} - \left(4\text{eV} + \frac{1.5\text{eV}}{2} + 0.026\text{eV}\cdot\ln\left(\frac{10^{18}\frac{1}{\text{cm}^3}}{10^{12}\frac{1}{\text{cm}^3}}\right)\right) = -0.11\text{eV}$$

$$C_i = \frac{\epsilon_i}{d} = \frac{25\cdot 8.85\cdot 10^{-14}\frac{\text{F}}{\text{cm}}}{100\cdot 10^{-8}\text{cm}} = 2.21\cdot 10^{-6}\frac{\text{F}}{\text{cm}^2}$$

$$W_m = \left(\frac{2\cdot 2\Phi_F\cdot\epsilon_s}{q\cdot N_a}\right)^{\frac{1}{2}} = \left(\frac{2\cdot 2\cdot 0.36\text{eV}\cdot 10\cdot 8.85\cdot 10^{-14}\frac{\text{F}}{\text{cm}}}{1.6\cdot 10^{-19}\text{C}\cdot 10^{18}\frac{1}{\text{cm}^3}}\right)^{\frac{1}{2}} = 2.8\cdot 10^{-6}\text{cm}$$

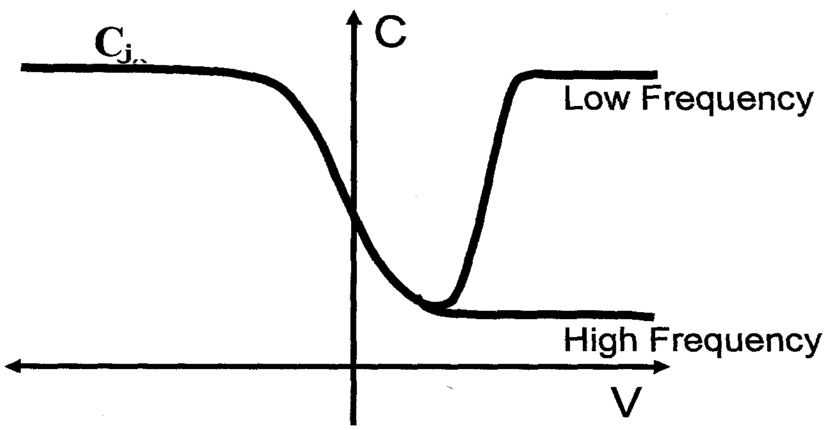
$$V_T = \Phi_{ms} - \frac{Q_{ox}}{C_i} - \frac{Q_d}{C_i} + 2\cdot\Phi_F = 0.81\text{V}$$

At V_T, at interface, $n=10^{18}\text{cm}^{-3}$ and $p=n_i^2/n=10^6\text{cm}^{-3}$
Deep in substrate, $n=10^6\text{cm}^{-3}$ and $p=10^{18}\text{cm}^{-3}$

At high frequency, inversion electrons do not respond while at low frequency, they do

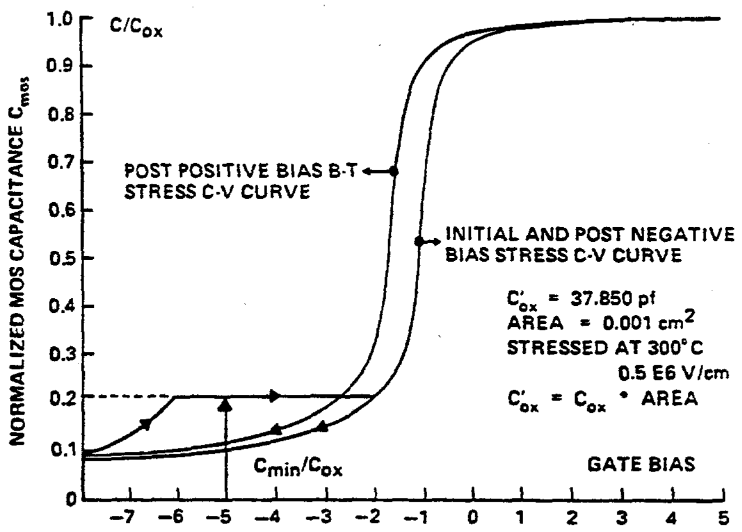
At large negative bias, doubling the oxide thickness reduces C_i by ½

At large negative bias, doubling the substrate doping does not change C_i but would affect the depletion capacitances



Prob. 6.15

Find the oxide thickness and substrate doping.



EXPERIMENTAL C-V CURVES SHOWING INITIAL, POST POSITIVE BIAS B-T STRESS AND POST NEGATIVE BIAS B-T STRESS RESULTS • CAPACITANCE HAS BEEN NORMALIZED TO CAPACITANCE IN STRONG ACCUMULATION

C_i = measured capacitance / area in strong accumulation

$$C_i = \frac{37.85\cdot 10^{-12}\text{F}}{.001\,\text{cm}^2} = 37.85\cdot 10^{-9}\,\tfrac{\text{F}}{\text{cm}^2}$$

$$d = \frac{\epsilon_i}{C_i} = \frac{3.85\cdot 8.85\cdot 10^{-14}\,\frac{\text{F}}{\text{cm}}}{37.85\cdot 10^{-9}\,\frac{\text{F}}{\text{cm}^2}} = 9.0\cdot 10^{-6}\text{cm} = 900\text{Å}$$

Use the normalized minimum capacitance corresponding to quasi-equilibrium

$$C'_{min} = \frac{C_{min}}{C_i} = 0.2$$

$$C_{min} = \frac{C_i C_{d_{min}}}{C_i + C_{d_{min}}} \text{ where } C_{min} \text{ is the total series capacitance}$$

$$C_{d_{min}} = C_i\cdot\left[\frac{C_{min}}{C_i - C_{min}}\right] = C_i\left[\frac{C'_{min}}{1 - C'_{min}}\right] = \frac{37.85\cdot 10^{-12}\text{F}}{.001\text{cm}^2}\cdot\left[\frac{0.2}{1-0.2}\right] = 9.46\cdot 10^{-9}\,\frac{\text{F}}{\text{cm}^2}$$

$$N_d = 10^{\left[30.388 + 1.683\cdot\log\left(C_{d_{min}}\right) - 0.03177\cdot\left(\log\left(C_{d_{min}}\right)\right)^2\right]}$$

$$N_d = 10^{\left[30.388 + 1.683\cdot(-8.02) - 0.03177\cdot(64.39)\right]} = 10^{14.83} = 6.88\cdot 10^{14}\,\tfrac{1}{\text{cm}^3}$$

Prob. 6.16

Determine the initial flatband voltage.

To determine the initial (pre-stressed) flatband voltage V_{FB_0}
First calculate C_{FB} from the previously determined doping density

$$C_{FB}=\frac{C_i \cdot C_{debye}}{C_i+C_{debye}}$$

$$\frac{C_{FB}}{C_i}=\frac{C_{debye}}{C_i+C_{debye}}=C'_{FB} \text{ (normalized flatband capacitance)}$$

$$C_{debye}=\frac{\epsilon_s}{L_D}=\frac{\epsilon_s}{\sqrt{\frac{kT}{q}\cdot\frac{\epsilon_s}{qN_A}}}=\sqrt{\frac{q^2}{kT}\cdot\epsilon_S\cdot N_A}$$

$$C_{debye}=\sqrt{\frac{1.6\cdot10^{-19}C}{0.0259V}\cdot\left(11.8\cdot8.85\cdot10^{-14}F\right)6.9\cdot10^{-14}\tfrac{1}{cm^3}}=6.7\cdot10^{-8}\tfrac{F}{cm^2}$$

$$C'_{FB}=\frac{C_{debye}}{C_i+C_{debye}}=\frac{6.7\cdot10^{-8}\frac{F}{cm^2}}{3.785\cdot10^{-8}\frac{F}{cm^2}+4.71\cdot10^{-8}\frac{F}{cm^2}}=0.78$$

V_{FB}= -1.0V from plot in Figure 6-15

Prob. 6.17

Determine the field oxide charge and the mobile ion content.

$$V_{FB}=\Phi_{ms}-q\frac{Q_i}{C_i}=-1.0V \text{ (from 6.16) where } \Phi_{ms}=-0.35V$$

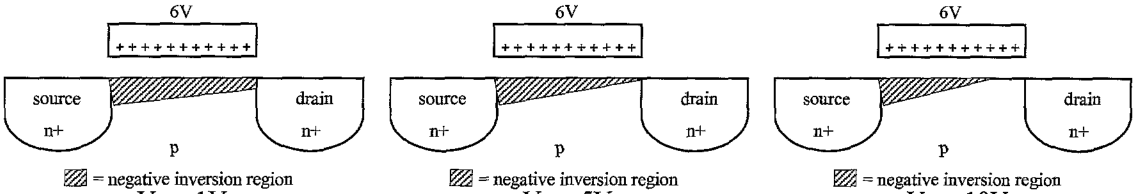
$$Q_i=\left(\Phi_{ms}-V_{FB}\right)\cdot\frac{C_i}{q}=\left(-0.35V-(-1.0V)\right)\cdot\frac{3.785\cdot10^{-8}\frac{F}{cm^2}}{1.6\cdot10^{-19}C}=1.53\cdot10^{11}\tfrac{1}{cm^2}$$

To determine mobile ion concentration, compare the positive and negative bias temperature flatband voltages.

$$Q_{ion}=\left(V_{FB-}-V_{FB+}\right)\frac{C_i}{q}=\left[-1.0V-(-1.5V)\right]\cdot\frac{3.785\cdot10^{-8}\frac{F}{cm^2}}{1.6\cdot10^{-19}C}=1.2\cdot10^{11}\tfrac{ions}{cm^2}$$

Prob. 6.18

Calculate V_G, output conductance, transconductance, and the new I_D.



$V_D = 1V$ | $V_D = 5V$ pinch off | $V_D = 10V$ pinch off

$$V_{D,sat} = V_G - V_T \rightarrow V_G = V_{D,sat} + V_T = 5V + 1V = 6V$$

$$g = \frac{\partial I_D}{\partial V_D} = 0 \text{ since device is in saturation}$$

$$I_{D,sat} = \frac{\mu C_i Z}{2L} \cdot (V_G - V_T)^2 = k \cdot (V_G - V_T)^2$$

$$I_{D,sat} = k \cdot (V_G - V_T)^2 \rightarrow 10^{-4} A = k \cdot (6V - 1V)^2 \rightarrow k = \frac{10^{-4} A}{25V^2} = 4 \cdot 10^{-6} \tfrac{A}{V^2}$$

$$g_m = \frac{\partial I_D}{\partial V_G} = 2 \cdot k \cdot (V_G - V_T) = 2 \cdot 4 \cdot 10^{-6} \tfrac{A}{V^2} \cdot 5V = 4 \cdot 10^{-5} \tfrac{A}{V}$$

For $V_G - V_T = 3V$ and $V_D = 4V$ device is in saturation

$$I_{D,sat} = k \cdot (V_G - V_T)^2 = 4 \cdot 10^{-5} \tfrac{A}{V^2} \cdot (3V)^2 = 3.6 \cdot 10^{-4} A$$

Prob. 6.19

Determine the initial flatband voltage.

$$V_T = V_{FB} + 2\Phi_F - \frac{Q_D}{C_i}$$

$$V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i}$$

$$C_i = \frac{\epsilon_i}{d} = \frac{8.854 \cdot 10^{-14}\,\frac{F}{cm} \cdot 3.9}{100 \cdot 10^{-8}\,cm} = 3.45 \cdot 10^{-7}\,\frac{F}{cm^2}$$

Note: here use dielectric constant of oxide

$$V_{FB} = \Phi_{ms} - \frac{Q_{ox}}{C_i} = -1.5 - \frac{5 \cdot 10^{10} \cdot 1.6 \cdot 10^{-19} C}{3.45 \cdot 10^{-7} F} = -1.523V$$

$$\Phi_F = \frac{kT}{q} \cdot \ln\frac{N_A}{n_i} = 0.026V \cdot \ln\left(\frac{10^{18}\,\frac{1}{cm^3}}{1.5 \cdot 10^{10}\,\frac{1}{cm^3}}\right) = 0.467V$$

$$W_m = \sqrt{\frac{2\epsilon_s\left(2\Phi_F + V_B\right)}{qN_a}} = 6.695 \cdot 10^{-6} cm$$

Note: here use dielectric constant of silicon

$V_T = -0.223V + 0.698V + 0.140V = 0.615V$

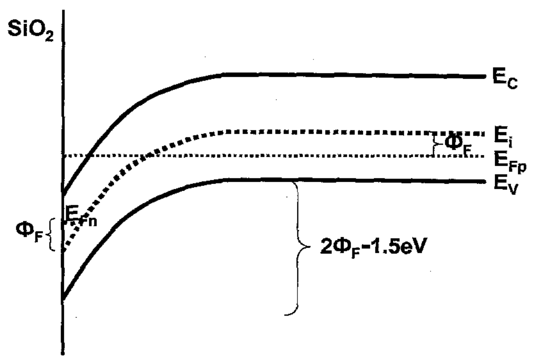
With $V_{SUB} = -2.5V$, depletion charge increases. Instead of bandbending of $2\phi_F$, now have bandbending of $(2\phi_F + V_B)$.

$$\frac{Q_D}{C_i} = \frac{1.071 \cdot 10^{-6} C}{3.45 \cdot 10^{-7} F} = 3.103V$$

$$V_T = -1.533V + 0.934V + 3.103V = 2.514V$$

At V_T near the interface, $n = N_a = 10^{18}\,\frac{1}{cm^3}$ and $p = \frac{n_i^2}{N_a} = 2.25 \cdot 10^2\,\frac{1}{cm^3}$

In bulk, $p = 10^{18}\,\frac{1}{cm^3}$ and $n = \frac{n_i^2}{p} = 2.25 \cdot 10^2\,\frac{1}{cm^3}$



Prob. 6.20

$$V_T = V_{FB} + 2\phi_F - \frac{Q_d}{C_i}$$

$$V_{FB} = 2\phi_F - \frac{Q_i}{C_i}$$

$$C_i = \frac{\varepsilon_i}{d} = \frac{8.85\times10^{-14}\times3.9}{100\times10^{-8}} = \mathbf{3.452\times10^{-7}\,F/cm^2}$$

Note: Here we use dielectric constant of oxide.

According to Fig. (6-17) in the textbook, for $N_a = 10^{18}\,\text{cm}^{-3}$

$$\Rightarrow \quad \Phi_{ms} = -1.1\text{V}$$

$$V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i} = -1.1 - \frac{5\times10^{10}\times1.6\times10^{-19}}{3.452\times10^{-7}} = -1.12\text{V}$$

$$\phi_F = \frac{kT}{q}\ln\frac{N_a}{n_i} = 0.0259\cdot\ln\left(\frac{10^{18}}{1.5\times10^{10}}\right) = 0.467\text{V}$$

$$W = \sqrt{\frac{2\varepsilon_s(2\phi_F)}{qN_a}} = \sqrt{\frac{2(11.8)(8.85\times10^{-14})(2\times0.347)}{1.6\times10^{-19}\times10^{18}}} = 3.49\times10^{-6}\text{cm}$$

Note: Here we used dielectric constant of Si.

$$Q_d = -qN_aW_m$$

$$V_T = V_{FB} + 2\phi_F - \frac{Q_d}{C_i} = -1.12 + 2(0.467) + \frac{1.6\times10^{-19}\times10^{18}\times3.49\times10^{-6}}{3.452\times10^{-7}} = 1.43\mathbf{V}$$

Prob. 6.21

For the MOSFET, calculate the drain current at $V_G = 5V$, $V_D = 0.1V$. Repeat for $V_G = 3V$, $V_D = 5V$.

For $V_G = 5\text{V}$, $V_D = 0.1\text{V}$, since $V_T = 1\text{V}$

$V_D < (V_G - V_T) \rightarrow$ linear region

$$I_D = \frac{Z}{L}\cdot\bar{\mu}_n\cdot C_i\cdot\left[(V_G - V_T)\cdot V_D - \frac{1}{2}\cdot V_D^2\right]$$

$$= \frac{50\mu\text{m}}{2\mu\text{m}}\cdot 200\tfrac{\text{cm}^2}{\text{V}\cdot\text{s}}\cdot 3.452\cdot10^{-7}\text{F}\cdot\left[(5\text{V}-1\text{V})\cdot 0.1\text{V} - \frac{1}{2}(0.1\text{V})^2\right] = 6.82\cdot10^{-4}\text{A}$$

For $V_G = 3\text{V}$, $V_D = 5\text{V}$, $V_D(\text{sat}) = V_G - V_T = 3\text{V} - 1\text{V} = 2\text{V}$

$$I_D = \frac{Z}{L}\cdot\bar{\mu}_n\cdot C_i\left[(V_G - V_T)\cdot V_D(\text{sat}) - \frac{1}{2}\cdot V_D^2(\text{sat})\right]$$

$$= \frac{50\mu\text{m}}{2\mu\text{m}}\cdot 200\tfrac{\text{cm}^2}{\text{V}\cdot\text{s}}\cdot 3.452\cdot10^{-7}\text{F}\cdot\left[(2\text{V})^2 - \frac{1}{2}(2\text{V})^2\right] = 3.45\cdot10^{-3}\text{A}$$

Prob. 6.22

For the given MOSFET, calculate the linear V_T and k_N, saturation V_T and k_N.

1. Choose $V_D \ll V_D(sat)$ to ensure that I_D-V_D curve is in the linear regime
   e.g., choose $V_D = 0.2V$

(1)	$V_G = 4$ V	$V_D = 0.2$ V	$I_D = 0.35$ mA
(2)	$V_G = 5$ V	$V_D = 0.2$ V	$I_D = 0.62$ mA

In linear regime

(3) $I_D = k_N[(V_G - V_T)V_D - V_D^2/2]$

From equation (3), inserting the values from (1) and (2)

$0.35 \cdot 10^{-3} = k_N\,[(4 - V_T)(0.2)]$
$0.62 \cdot 10^{-3} = k_N\,[(5 - V_T)(0.2)]$
$0.35/0.62 = (4 - V_T)\,/\,(5 - V_T)$
$1.75 - 0.35V_T = 2.48 - 0.62V_T$
$V_T = 2.71V$, therefore, $k_N = 1.36 \cdot 10^{-3}$ A/V^2

2. Choose $V_D \gg V_D(sat)$ to ensure that I_D-V_D curve is in the saturation regime
   e.g. choose $V_D = 3V$

(4)	$V_G = 4$ V	$V_D = 3$ V	$I_D = 0.74$ mA
(5)	$V_G = 5$ V	$V_D = 3$ V	$I_D = 1.59$ mA

In saturation regime

(6) $I_D = (1/2)\,k_N\,(V_G - V_T)^2$

$$0.74 \cdot 10^{-3} = \frac{k_N}{2}(4V - V_T)^2$$

$$1.59 \cdot 10^{-3} = \frac{k_N}{2} \cdot (5V - V_T)^2$$

$$\frac{0.74}{1.59} = \frac{(4V - V_T)^2}{(5V - V_T)^2}$$

$V_T = 1.85V$, $k_N = 3.20 \cdot 10^{-4}$A/V^2

Prob. 6.23

For Problem 6.22, calculate the gate oxide thickness and the substrate doping either graphically or iteratively.

(a) $k_N = \frac{Z}{L} \cdot \bar{\mu}_n \cdot C_i$

use k_N from Problem 6.22 and $\bar{\mu}_n = 500 \frac{cm^2}{V \cdot s}$

$$1.36 \cdot 10^{-3} \frac{A}{V^2} = \frac{100\mu m}{2\mu m} \cdot 500 \frac{cm^2}{V \cdot s} \cdot C_i$$

$$C_i = 5.42 \cdot 10^{-8} \frac{F}{cm^2} = \frac{\epsilon_i}{d} = \frac{3.9 \cdot 8.85 \cdot 10^{-14} \frac{F}{cm}}{d}$$

$d = 6.36 \cdot 10^{-6} cm = 636Å$

(b) $V_T = V_{FB} + 2 \cdot \phi_F - \frac{Q_d}{C_i}$

$$2.71V = 2 \cdot \phi_F - \frac{Q_d}{C_i} = 2 \cdot \phi_F - \frac{\sqrt{q \cdot \epsilon_s \cdot N_a \cdot \phi_F}}{C_i}$$

start from $\phi_F = 0.3V$ (note: since $V_T = 2.71$ V, it cannot be PMOS)

Step 1:

$$2.71V = 0.6V + \frac{2 \cdot \sqrt{1.6 \cdot 10^{-19} C \cdot 11.8 \cdot 8.85 \cdot 10^{-14} \frac{F}{cm} \cdot N_a \cdot 0.3V}}{5.42 \cdot 10^{-8} F}$$

$N_a = 6.523 \cdot 10^{16} \frac{1}{cm^3}$

$$\phi_F = \frac{kT}{q} \cdot \ln \frac{N_a}{n_i} = 0.0259V \cdot \ln \frac{6.37 \cdot 10^{16} \frac{1}{cm^3}}{1.5 \cdot 10^{10} \frac{1}{cm^3}} = 0.395V$$

Step 2:

$$2.71V = 0.792V + \frac{2 \cdot \sqrt{1.6 \cdot 10^{-19} C \cdot 11.8 \cdot 8.85 \cdot 10^{-14} \frac{F}{cm} \cdot N_a \cdot 0.395V}}{5.42 \cdot 10^{-8} F}$$

$N_a = 4.08 \cdot 10^{16} cm^{-3}$

$$\phi_F = \frac{kT}{q} \cdot \ln \frac{N_a}{n_i} = 0.0259V \cdot \ln \frac{4.08 \cdot 10^{16} \frac{1}{cm^3}}{1.5 \cdot 10^{10} \frac{1}{cm^3}} = 0.384V$$

Step 3:

$$2.71V = 0.767 + \frac{2\sqrt{q \cdot \epsilon_s \cdot N_a \cdot \phi_F}}{C_{ox}}$$

$N_a = 4.22 \cdot 10^{16} \frac{1}{cm^3} \rightarrow \phi_F = 0.385$ V

gives a self-consistent set of values

n-channel MOSFET, $N_a = 4.22 \cdot 10^{16} \frac{1}{cm^3}$

Prob. 6.24

For the given Si MOSFET, calculate the inversion charge per unit area. Also sketch the dispersion relation for the first three sub-bands.

For 2-D situation, the density of states is given by

$$N(E)=\frac{m^*}{\pi h^2} \quad \text{for x and y plane.}$$

in k space:

$$E=\frac{h^2}{2m^*}\cdot(k_x^2+k_y^2+k_z^2) \text{ where } k_z=\frac{n\cdot\pi}{L_z}$$

$$=\frac{h^2}{2m^*}(k_x^2+k_y^2)+\underbrace{\frac{h^2\pi^2n^2}{2m^*L_z^2}}_{E_n}$$

$$E_n=\frac{h^2\pi^2n^2}{2m^*L_z^2}=\frac{\left(\frac{6.63\cdot10^{-34}J\cdot s}{2\pi}\right)^2\cdot\pi^2\cdot n^2}{2\cdot0.2\cdot9.11\cdot10^{-31}kg\cdot(10^{-8}m)^2}$$

$$=3.016\cdot10^{-21}J\cdot n^2=0.01885eV\cdot n^2=18.85\cdot n^2 \text{ meV}$$

$$E_1=18.85meV,\ E_2=75.4meV,\ E_3=170meV$$

Let units be $\frac{m^*}{\pi h^2}$

The number of electrons per unit area is given by:

$$n=\int_{E_1}^{E_F}N(E)\cdot f(E)\cdot dE=\frac{m^*}{\pi h^2}\cdot(E_2-E_1)+\frac{2m^*}{\pi h^2}\cdot\left(\frac{E_3-E_2}{2}\right)$$

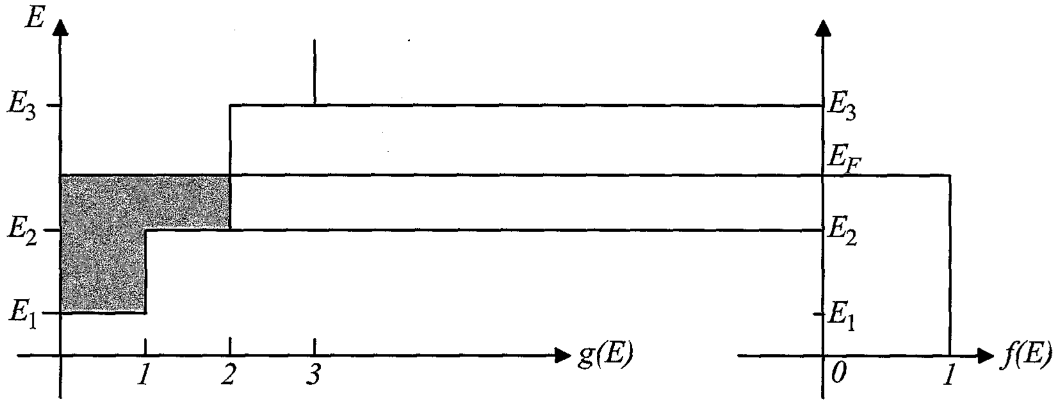
(since the Fermi probability is 1 below E_F and E_F is in the middle between E_2 and E_3.)

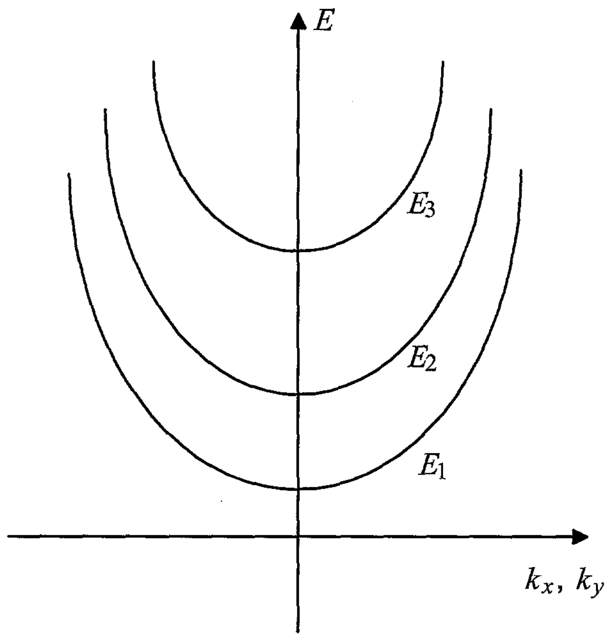
charge per unit area by simplifying the above expression:

$$qn=\frac{q\cdot m^*}{\pi h^2}\cdot(E_3-E_1)=4\pi\cdot\frac{q\cdot m^*}{h\cdot h}\cdot[(9\text{-}1)\cdot18.85\cdot10^{-3}eV]$$

$$=4\pi\cdot\frac{1.6\cdot10^{-19}C\cdot0.2\cdot9.11\cdot10^{-31}kg}{4.14\cdot10^{-15}\frac{eV}{s}\cdot6.63\cdot10^{-34}\frac{J}{s}}\cdot[(9\text{-}1)\cdot18.85\cdot10^{-3}eV]$$

$$=20.12\frac{C}{m^2}$$





Prob. 6.25

Choose a species, energy, dose, and beam current.

$$\Delta V_T = -2V = \frac{\Delta Q_{ox}}{C_i} \rightarrow \frac{\Delta Q_{ox}}{q} = \left(\frac{\varepsilon_i}{d}\right)\cdot\left(\frac{2}{q}\right) = 1.08\cdot10^{12}\,\frac{\text{ions}}{\text{cm}^2}$$

Any n-type ion okay, but based on projected range, use Phosphorus

$R_p = 400\,\mathring{A} \rightarrow E = 33\text{keV}$ from Appendix IX

Half the dose is wasted in oxide so full dose $= 2.16 \times 10^{12}\,\frac{1}{\text{cm}^2}$

$$\frac{I\cdot t}{q\cdot A} = 2.16\cdot10^{12}\,\frac{1}{\text{cm}^2} \rightarrow \frac{I\cdot 20\text{s}}{1.6\cdot10^{-19}\text{C}\cdot200\text{cm}^2} = 2.16\cdot10^{12}\,\frac{1}{\text{cm}^2} \rightarrow I = 3.46\mu\text{A}$$

Prob. 6.26

Plot the drain characteristics for an n^+-polysilicon-SiO_2-Si p-channel transistor with $N_d = 10^{16}\,\frac{1}{cm^3}$, $Q_i = 5\cdot10^{10}q$, $d = 100\,\mathring{A}$, $\mu_p = 200\,\frac{cm^2}{V\cdot s}$, *and* $Z = 10L$.

$$-I_D = \frac{\mu_p Z C_i}{L}\left[(V_G - V_T) - \frac{1}{2}V_D\right]V_D$$

where $V_T = -1.1\,V$ and $\mu_p Z C_i/L = (200)(10)(3.45\times10^{-7}) = 6.9\times10^{-4}$

For $V_G = -3$, $V_D(\text{sat.}) = -1.9$

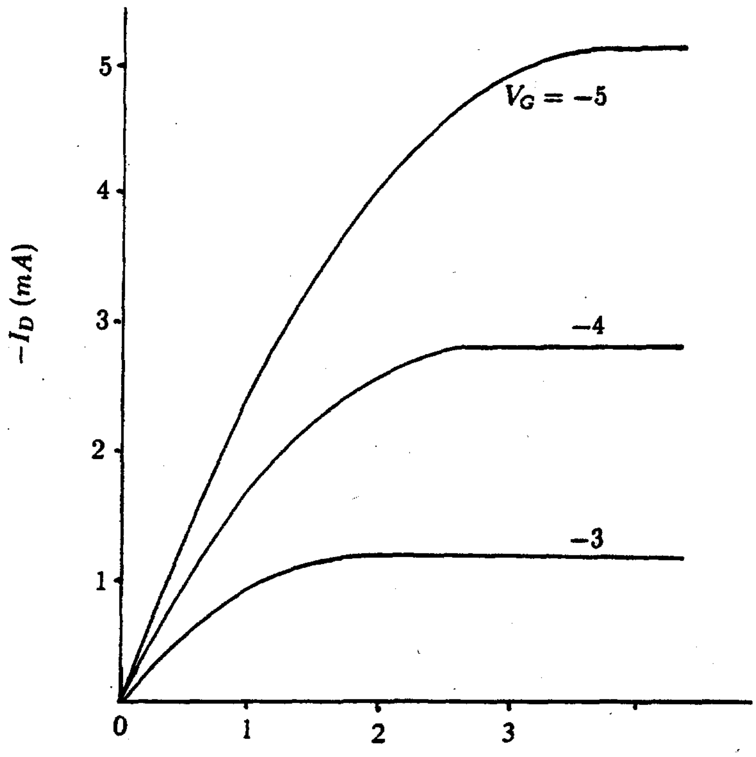
$-V_D$:	0.3	0.5	0.8	1.0	1.5
$-I_D\ (mA)$:	0.36	0.57	0.83	0.96	1.2

For $V_G = -4$, $V_D(\text{sat.}) = -2.9$

$-V_D$:	0.5	1	1.5	2	2.5
$-I_D\ (mA)$:	0.9	1.7	2.2	2.6	2.8

For $V_G = -5$, $V_D(\text{sat.}) = -3.9$

$-V_D$:	1	1.5	2	2.5	3	3.9
$-I_D\ (mA)$:	2.3	3.3	4	4.6	5	5.2



Prob. 6.27

For the transistor in Problem 6-26 with L=1μm, calculate the cutoff frequency above pinch-off.

$$f_c \simeq \frac{g_m}{2\pi C_i LZ}$$

For p-channel, we must include a minus sign in Equation 6-54 for positive g_m.

$$f_c = \frac{1}{2\pi C_i LZ} \cdot \frac{Z}{L} \cdot \mu_p \cdot C_i \cdot (V_T - V_G) = \frac{\mu_p}{2\pi L^2} \cdot (V_T - V_G)$$

$$\text{For } V_G = -5V, \; f_c = \frac{200 \frac{cm^2}{V \cdot s}}{2\pi \cdot (10^{-4} cm)^2} \cdot (-1.1 - -5V) = 12.4GHz$$

$$\text{For } V_G = -3V, \; f_c = \frac{200 \frac{cm^2}{V \cdot s}}{2\pi \cdot (10^{-4} cm)^2} \cdot (-1.1 - -3V) = 6GHz$$

Prob. 6.28

Derive the drain conductance $g_D' = \frac{\partial I_D'}{\partial V_D}$ *beyond saturation in terms of the effective channel length L-ΔL and then in terms of* V_D.

Using L′ in Equation 6-53,

$$I'_D = \tfrac{1}{2}\mu_n C_i \frac{Z}{L'}(V_G - V_T)^2 = I_{D,SAT}\cdot\frac{L}{L'} = I_{D,SAT}\cdot\frac{L}{L-\Delta L}$$

$$g'_D = \frac{\partial I'_D}{\partial V_D} = I_{D,SAT}\cdot\frac{\partial}{\partial V_D}\frac{L}{L-\Delta L} = I_{D,SAT}\cdot\frac{\partial}{\partial V_D}\left(1-\frac{\Delta L}{L}\right)^{-1}$$

$$\frac{\partial}{\partial V_D}\left(1-\frac{\Delta L}{L}\right)^{-1} = -1\cdot\left(1-\frac{\Delta L}{L}\right)^{-2}\cdot\left(-\frac{1}{L}\right)\cdot\frac{\partial \Delta L}{\partial V_D}$$

$$= -1\cdot\left(1-\frac{\Delta L}{L}\right)^{-2}\cdot\left(-\frac{1}{L}\right)\cdot\frac{\partial}{\partial V_D}\left[\frac{2\epsilon_s(V_D - V_{D,SAT})}{qN_a}\right]^{\frac{1}{2}}$$

$$= -1\cdot\left(1-\frac{\Delta L}{L}\right)^{-2}\cdot\left(-\frac{1}{L}\right)\cdot\frac{1}{2}\cdot\frac{2\epsilon_s}{qN_a}\left[\frac{2\epsilon_s(V_D - V_{D,SAT})}{qN_a}\right]^{-\frac{1}{2}}$$

$$g'_D = \frac{\partial I'_D}{\partial V_D} = I_{D,SAT}\cdot -1\cdot L^2\left(L-\left[\frac{2\epsilon_s(V_D - V_{D,SAT})}{qN_a}\right]^{\frac{1}{2}}\right)^{-2}\cdot\left(-\frac{1}{L}\right)\cdot\frac{1}{2}\cdot\frac{2\epsilon_s}{qN_a}\left[\frac{2\epsilon_s(V_D - V_{D,SAT})}{qN_a}\right]^{-\frac{1}{2}}$$

$$g'_D = \frac{\partial I'_D}{\partial V_D} = \frac{I_{D,SAT}\cdot L\cdot\left(\frac{2\epsilon_s}{qN_a}\right)^{\frac{1}{2}}}{2\cdot\left(L-\left[\frac{2\epsilon_s(V_D - V_{D,SAT})}{qN_a}\right]^{\frac{1}{2}}\right)^2\cdot(V_D - V_{D,SAT})^{\frac{1}{2}}}$$

Prob. 6.29

An n-channel MOSFET has a 1 μm long channel with $N_a = 10^{16}\frac{1}{cm^3}$ and $N_d = 10^{20}\frac{1}{cm^3}$ in the source and drain. Find the V_D which causes punch-through.

$$V_o = kT\cdot\ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.0259V\cdot\ln\left(\frac{10^{16}\frac{1}{cm^3}\cdot 10^{20}\frac{1}{cm^3}}{(1.5\cdot 10^{10}\frac{1}{cm^3})^2}\right) = 0.933V$$

There are two depletion regions, one at the source end and one at the drain end of the channel. $N_d \gg N_a$ so most of W is in the p-side (channel).

At the (zero-bias) source end,

$$x_{pS} = \left(\frac{2\epsilon_s V_o}{q\cdot N_a}\right)^{\frac{1}{2}} = \left(\frac{2\cdot 11.8\cdot 8.85\cdot 10^{-14}\frac{F}{cm^2}\cdot 0.993V}{1.609\cdot 10^{-19}C\cdot 10^{16}\frac{1}{cm^3}}\right)^{\frac{1}{2}} = 0.35\mu m$$

In the drain end, $x_{pD} = \left(\frac{2\epsilon_s (V_o + V_D)}{q\cdot N_a}\right)^{\frac{1}{2}}$

Punch-through occurs when $x_{pD} = L - x_{pS} = 0.65\mu m$

$$0.933V + V_D = \frac{(0.65\cdot 10^{-4}cm)^2\cdot 1.609\cdot 10^{-19}C\cdot 10^{16}\frac{1}{cm^3}}{2\cdot 11.8\cdot 8.85\cdot 10^{-14}\frac{F}{cm^2}} \rightarrow V_D = 2.3V$$

Chapter 7 Solutions

Prob. 7.1

Plot the doping profile.

For the base diffusion,

$$D \cdot t = 3 \cdot 10^{-13}\,\tfrac{cm^2}{s} \cdot 3600s = 10.8 \cdot 10^{-10} cm^2$$

$$2\sqrt{D \cdot t} = 6.58 \cdot 10^{-5} cm \qquad \sqrt{\pi \cdot D \cdot t} = 5.82 \cdot 10^{-5} cm$$

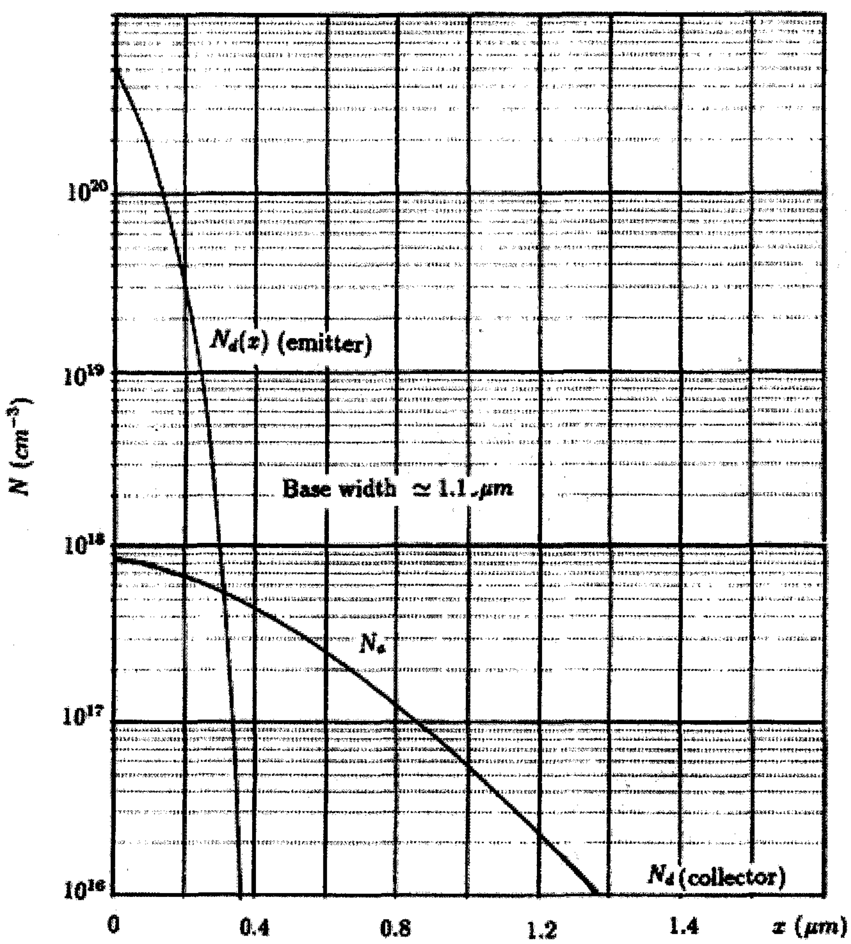
$$N_a(x) = \frac{N_s}{\sqrt{\pi \cdot D \cdot t}} \cdot e^{-\left(\frac{x}{2\sqrt{D \cdot t}}\right)^2} = 8.6 \cdot 10^{17}\,\tfrac{1}{cm^4} \cdot e^{-\frac{x^2}{4.3 \cdot 10^{-9} cm^2}}$$

For the emitter diffusion,

$$D \cdot t = 3 \cdot 10^{-14}\,\tfrac{cm^2}{s} \cdot 900s = 27.0 \cdot 10^{-12} cm^2$$

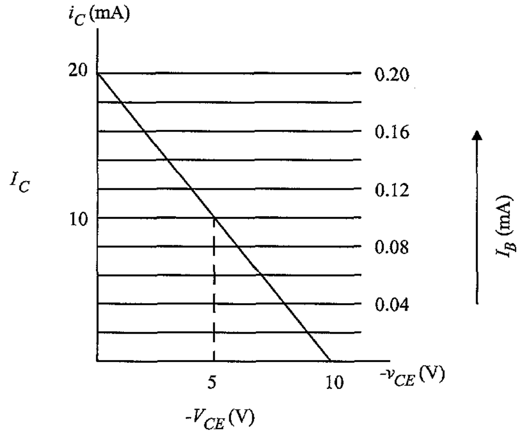
$$2\sqrt{D \cdot t} = 1.04 \cdot 10^{-5} cm \qquad \sqrt{\pi \cdot D \cdot t} = 9.21 \cdot 10^{-6} cm$$

$$N_d(x) = N_s \cdot \mathrm{erfc}\left(\frac{x}{2\sqrt{D \cdot t}}\right) = 5 \cdot 10^{20}\,\tfrac{1}{cm^3} \cdot \mathrm{erfc}\left(\frac{x}{1.04 \cdot 10^{-5} cm}\right)$$



Prob. 7.2

Sketch I_C versus $-V_{CE}$ for the BJT and find $-V_{CE}$ for I_B=0.1mA.



$-V_{CE}$=5V for I_B=0.1mA

Prob. 7.3

Plot δp across the base of a p-n-p with W_b/L_p=0.5.

$M_1 = 1.58$ and $M_2 = 0.58$ from Equation 7-14

$e^{\frac{1}{2}} = 1.58$ and $e^{-\frac{1}{2}} = 0.606$

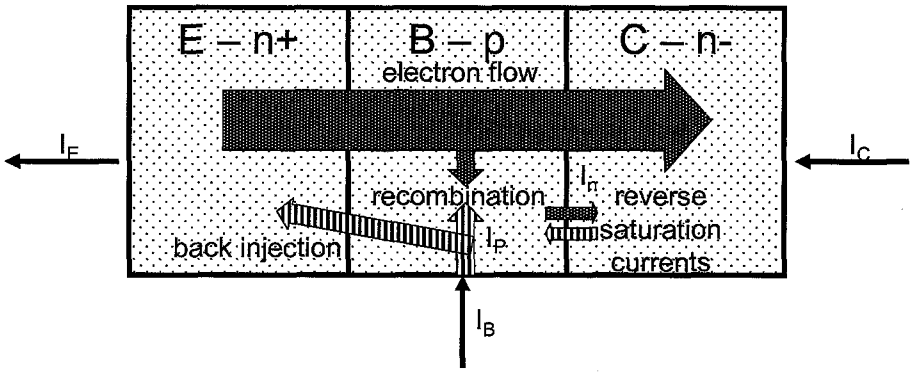
These values may be filled in to obtain a plot such as Figure 7-7 with normalized axis.

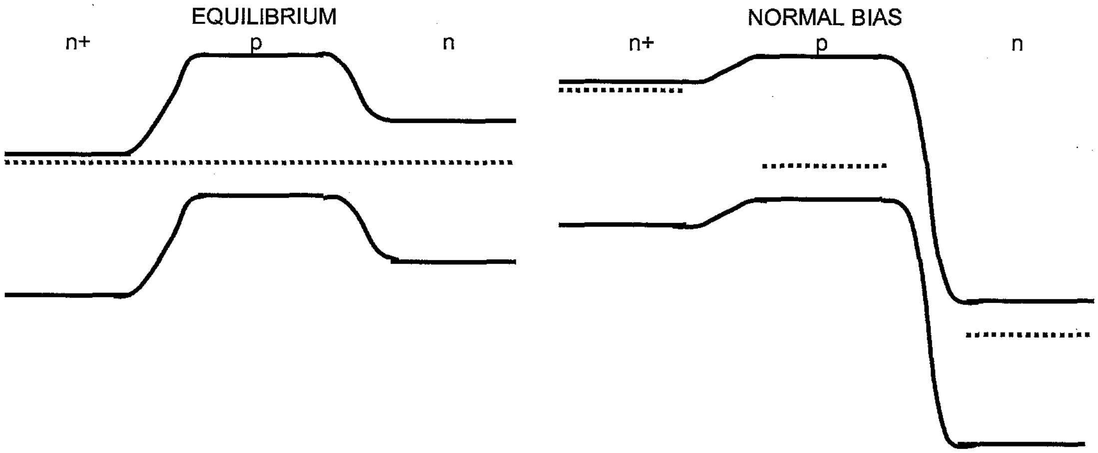
$$\frac{\delta p}{\Delta p_E} = M_1 e^{-\frac{x_n}{L_p}} - M_2 e^{\frac{x_n}{L_p}} = 1.58e^{-\frac{1}{2}} - 0.58e^{\frac{1}{2}} = 0 \text{ for } \tfrac{x_n}{L_p}=0.5$$

$$\frac{\delta p}{\Delta p_E} = M_1 e^{-\frac{x_n}{L_p}} - M_2 e^{\frac{x_n}{L_p}} = 1.58e^{0} - 0.58e^{0} = 1.0 \text{ for } \tfrac{x_n}{L_p}=0$$

Prob. 7.4

For a n+—p—n- BJT, show the current contributions, and band diagram.





Prob. 7.5

Find α and β for the given BJT characteristics.

$$p_p = 10 \cdot n_n \qquad \mu_{nE} = 0.5 \cdot \mu_{pB} \qquad W_b = 0.1 \cdot L_p \qquad \tau_p = \tau_n$$

$$\frac{L_p^n}{L_n^p} = \frac{\sqrt{D_p^n \cdot \tau_p}}{\sqrt{D_n^p \cdot \tau_n}} = \frac{\sqrt{\frac{kT}{q} \cdot \mu_p^n \cdot \tau_p}}{\sqrt{\frac{kT}{q} \cdot \mu_n^p \cdot \tau_n}} = \sqrt{2}$$

$$\alpha = B \cdot \gamma = \frac{1}{\cosh\left(\frac{W_b}{L_p}\right) + \frac{L_p^n \cdot n_n \cdot \mu_n^p}{L_n^p \cdot p_p \cdot \mu_p^n} \sinh\left(\frac{W_b}{L_p}\right)} = \frac{1}{\cosh(0.1) + \sqrt{2} \cdot 0.1 \cdot 0.5 \sinh(0.1)} = 0.988$$

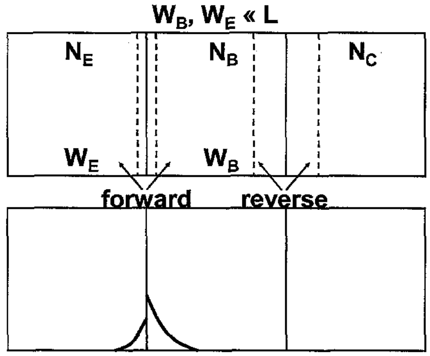
$$\beta = \frac{\alpha}{1 - \alpha} = 82$$

Prob. 7.6

Calculate the desired values for each BJT modification.

Let us assume a PNP BJT; the results are the same for NPN.

a) find the change in the collector current



$W_B, W_E >> L$

$$I_C = \frac{q \cdot A \cdot D_p \cdot p_n}{W} = \frac{q \cdot A \cdot D_p \cdot n_i^2}{N_B W_B} \cdot e^{\frac{q \cdot V_{BE}}{kT}}$$

If $W_B' = 0.5 \cdot W_B$ and $N_B' = 10 \cdot N_B$

then $I_C' = \frac{I_C}{10 \cdot 0.5} = \frac{I_C}{5}$

b) find γ and B

Since we assume that the base and the emitter are much shorter than the diffusion length, the carrier concentration profiles vary linearly with distance in the top figure.

If $N_E = 100 \cdot N_B$ and $W_E = 0.1 \cdot W_B$

then using $I_{E_n} \alpha \frac{n_i^2}{N_E \cdot W_E} \cdot e^{\frac{q \cdot V_{BE}}{kT}}$ and $I_{E_p} \alpha \frac{n_i^2}{N_B \cdot W_B} \cdot e^{\frac{q \cdot V_{BE}}{kT}}$

$$\gamma = \frac{I_{E_p}}{I_{E_p} + I_{E_n}} = \frac{\frac{1}{N_B \cdot W_B}}{\frac{1}{N_B \cdot W_B} + \frac{1}{N_E \cdot W_E}} = \frac{1}{1 + \frac{N_B \cdot W_B}{N_E \cdot W_E}} = \frac{1}{1 + \frac{N_B \cdot W_B}{100 \cdot N_B \cdot 0.1 \cdot W_B}} = \frac{1}{1 + 0.1} = 0.91$$

Base carrier profile is linear so B = 1

c) find B and γ

Here, since we assume that the base and emitter regions are much longer than the diffusion lengths, the carrier concentrations decay exponentially with distance, as shown in the lower figure.

Base carrier profile for long diode exponentially decays to 0; so B = 0

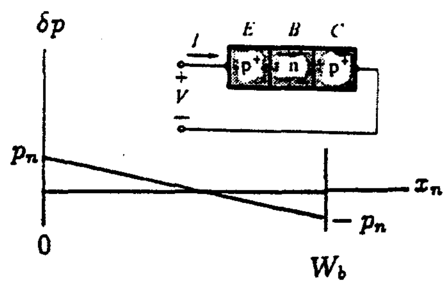
If $N_E = 100 \cdot N_B$ and $L_n = L_p = L$

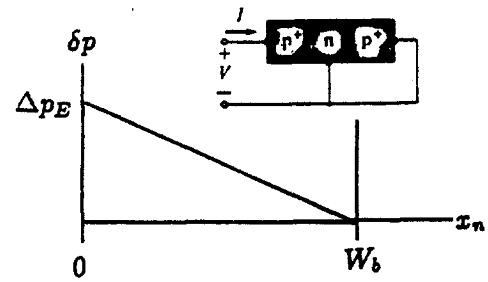
then using $I_{E_n} \alpha \frac{n_i^2}{N_E \cdot L} \cdot e^{\frac{q \cdot V_{BE}}{kT}}$ and $I_{E_p} \alpha \frac{n_i^2}{N_B \cdot L} \cdot e^{\frac{q \cdot V_{BE}}{kT}}$

$$\gamma = \frac{I_{E_p}}{I_{E_p} + I_{E_n}} = \frac{\frac{1}{N_B \cdot L}}{\frac{1}{N_B \cdot L} + \frac{1}{N_E \cdot L}} = \frac{1}{1 + \frac{N_B}{N_E}} = \frac{1}{1 + \frac{N_B}{100 \cdot N_B}} = \frac{1}{1 + 0.01} = 0.99$$

Prob. 7.7

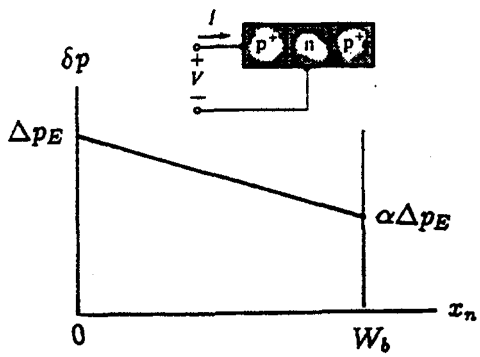
Identify which gives the best diode characteristics.

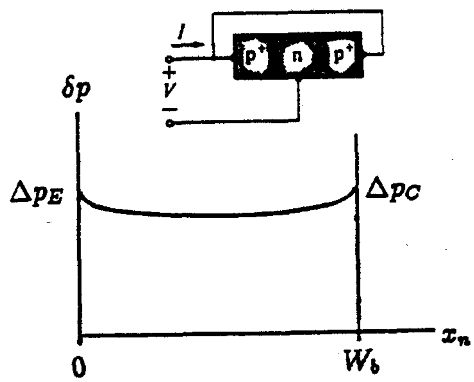




(a) $I_E = I_C$, $I_B = 0$. Since V is large, the collector is strongly reverse biased, $\Delta p_C = -p_n$. Since $I_E = I_C$, $\Delta p_E = -\Delta p_C$ $= p_n$ from Eq.(7-34). The area under $\delta p(x_n)$ is zero.

(b) $V_{CB} = 0$, thus $\Delta p_C = 0$. Notice that this is the narrow-base diode distribution.





(c) *Since* $I_C = 0$, $\Delta p_C = \alpha \Delta p_E$ *from Eq.(7-34b).*

(d) $V_{EB} = V_{CB} = V$. *Thus* $\Delta p_C = \Delta p_E$

Connection (b) gives the best diode since the stored charge is least and the current is good. Connection (a) is not a good diode since the current is small and symmetrical about V=0. Connections (c) and (d) are not good diodes because the stored charge is high.

Prob. 7.8

Derive I_B and use the charge control approach to find B and I_B.

$$Q_p = q\cdot A\cdot \int_0^{W_b} \delta p(x_n)\cdot dx_n = q\cdot A\cdot L_p\cdot\left[C_1\left(e^{\frac{W_b}{L_p}}-1\right)-C_2\left(e^{-\frac{W_b}{L_p}}-1\right)\right]$$

$$I_B = \frac{Q_p}{\tau_p} = \frac{q\cdot A\cdot L_p}{\tau_p}\cdot\left[\frac{\left(\Delta p_C - \Delta p_E\cdot e^{-\frac{W_b}{L_p}}\right)\cdot\left(e^{\frac{W_b}{L_p}}-1\right)-\left(\Delta p_E\cdot e^{\frac{W_b}{L_p}}-\Delta p_C\right)\cdot\left(e^{-\frac{W_b}{L_p}}-1\right)}{e^{\frac{W_b}{L_p}}-e^{-\frac{W_b}{L_p}}}\right]$$

$$I_B = \frac{q\cdot A\cdot D_p}{L_p}\cdot\left[\frac{\left(\Delta p_C+\Delta p_E\right)\cdot\left[\left(e^{\frac{W_b}{L_p}}+e^{-\frac{W_b}{L_p}}\right)-2\right]}{e^{\frac{W_b}{L_p}}-e^{-\frac{W_b}{L_p}}}\right]$$

$$I_B = \frac{q\cdot A\cdot D_p}{L_p}\cdot\left[\left(\Delta p_C+\Delta p_E\right)\cdot\left(\operatorname{ctnh}\left(\frac{W_b}{L_p}\right)-\operatorname{csch}\left(\frac{W_b}{L_p}\right)\right)\right]$$

$$I_B = \frac{q\cdot A\cdot D_p}{L_p}\cdot\left[\left(\Delta p_C+\Delta p_E\right)\cdot\tanh\left(\frac{W_b}{2\cdot L_p}\right)\right]$$

$$I_{Ep} = \frac{Q_p}{\tau_t}+\frac{Q_p}{\tau_p} \qquad I_C = \frac{Q_p}{\tau_t} \qquad I_B = \frac{Q_p}{\tau_p}$$

$$B = \frac{I_C}{I_{Ep}} = \frac{\tau_p}{\tau_t+\tau_p} = \frac{1}{1+\frac{\tau_t}{\tau_p}} = \frac{1}{1+\frac{W_b^2}{2\cdot D_p\cdot\tau_p}} = \frac{1}{1+\frac{1}{2}\cdot\left(\frac{W_b}{L_p}\right)^2}$$

make use of $\frac{1}{1+x}\approx 1 - x$ for small x

$B \approx 1-\frac{1}{2}\cdot\left(\frac{W_b}{L_p}\right)^2$ which matches equation (7-26) $B = \operatorname{sech}\left(\frac{W_b}{L_p}\right) = 1-\frac{1}{2}\cdot\left(\frac{W_b}{L_p}\right)^2$

Prob. 7.9

(a) Find I for the transistor connection

$$\Delta p_C = 0 \text{ so } I = \frac{q \cdot A \cdot D_p \cdot \Delta p_E}{L_p} \cdot \mathrm{ctnh}\left(\frac{W_b}{L_p}\right)$$

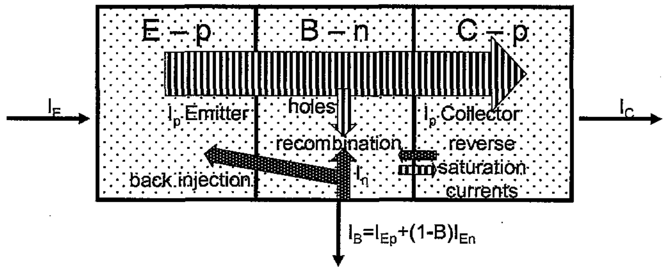
(b) Describe how I divides between the base and collector leads.

$$I_C = \frac{q \cdot A \cdot D_p \cdot \Delta p_E}{L_p} \cdot \mathrm{csch}\left(\frac{W_b}{L_p}\right)$$

$$I_B = \frac{q \cdot A \cdot D_p \cdot \Delta p_E}{L_p} \cdot \tanh\left(\frac{W_b}{2 \cdot L_p}\right)$$

Prob. 7.10

Show currents and find the current transfer ratio, the emitter junction efficiency, the common base current gain, the common emitter current gain, I_{CBO}, the base transit time, and the lifetime.



$$\text{current transfer ratio} = B = \frac{I_{C_p}}{I_{E_p}} = \frac{9.8\text{mA}}{10\text{mA}} = 0.9800$$

$$\text{emitter efficiency} = \gamma = \frac{I_{E_p}}{I_{E_p} + I_{E_n}} = \frac{10\text{mA}}{10\text{mA}+0.1\text{mA}} = 0.9901$$

$$\text{common base current gain} = \alpha = B \cdot \gamma = 0.9703$$

$$\text{common emitter current gain} = \beta = \frac{\alpha}{1\text{-}\alpha} = 32.7$$

$$I_{CBO} = I_{C_n} = 1\mu\text{A}$$

$$\text{base transit time} = \tau_t = \frac{Q_B}{I_{C_p}} = \frac{4.9 \cdot 10^{-11}\text{C}}{9.8 \cdot 10^{-3}\text{A}} = 5 \cdot 10^{-9}\text{s} = 5\text{ns}$$

$$\text{lifetime} = \tau_p = \frac{Q_B}{(1\text{-}B) \cdot I_{E_p}} = \frac{4.9 \cdot 10^{-11}\text{C}}{0.02 \cdot 10 \cdot 10^{-3}\text{A}} = 2.45 \cdot 10^{-8}\text{s} = 24.5\text{ns}$$

If I_{C_p} decreases then B decreases as do α and β.

If I_{E_n} decreases then B decreases as do α and β.

Prob. 7.11

Find the peak electric field, depletion capacitance, and the neutral base width narrowing.

$$V_{CB} = \frac{1}{2}\cdot\mathcal{E}_o\cdot W = \frac{1}{2}\cdot\frac{q\cdot W}{\epsilon_o\cdot\left(\frac{1}{N_d}+\frac{1}{N_a}\right)}\cdot W = 50V$$

$$W = \left[\frac{2\cdot 50V\cdot 11.8\cdot 8.85\cdot 10^{-14}\,\frac{F}{cm}}{1.6\cdot 10^{-19}C}\cdot\left(\frac{1}{10^{18}\,\frac{1}{cm^3}}+\frac{1}{10^{17}\,\frac{1}{cm^3}}\right)\right]^{\frac{1}{2}} = 0.851\mu m = 8.51\cdot 10^{-5}cm$$

$$\mathcal{E}_o = \frac{2\cdot V_{CB}}{W} = \frac{2\cdot 50V}{8.51\cdot 10^{-5}cm} = 1.18\cdot 10^{6}\,\frac{V}{cm}$$

$$C_j = \frac{\epsilon}{W} = \frac{11.8\cdot 8.85\cdot 10^{-14}\,\frac{F}{cm}}{8.51\cdot 10^{-5}cm} = 12.4\frac{nF}{cm^2}$$

$$W = x_{n_o} + x_{p_o} = 8.51\cdot 10^{-5}cm$$

$$10^{18}\,\frac{1}{cm^3}\cdot x_{n_o} = 10^{17}\,\frac{1}{cm^3}\cdot x_{p_o}$$

$$x_{n_o} = \frac{8.51\cdot 10^{-5}cm}{11} = 7.74\cdot 10^{-6}cm = 0.077\mu m$$

This base width narrowing causes the collector current to increase with increasing base-collector reverse bias.

This effect is called the Early effect.

Prob. 7.12

(a) Show that I_{EO} and I_{CO} are the saturation currents of the emitter and collector.

For $I_C = 0$, equation 7-32b gives $I_{CS} \cdot \left(e^{\frac{q \cdot V_{CB}}{kT}} - 1 \right) = \alpha_n \cdot I_{ES} \cdot \left(e^{\frac{q \cdot V_{EB}}{kT}} - 1 \right)$

Substituting in 7-32a gives $I_E = I_{ES} \cdot (1 - \alpha_I \cdot \alpha_N) \cdot \left(e^{\frac{q \cdot V_{EB}}{kT}} - 1 \right) = I_{EO} \cdot \left(e^{\frac{q \cdot V_{CB}}{kT}} - 1 \right)$

For $I_E = 0$, equation 7-32a gives $I_{ES} \cdot \left(e^{\frac{q \cdot V_{EB}}{kT}} - 1 \right) = \alpha_I \cdot I_{CS} \cdot \left(e^{\frac{q \cdot V_{CB}}{kT}} - 1 \right)$

Substituting in 7-32b gives $-I_C = I_{CS} \cdot (1 - \alpha_I \cdot \alpha_N) \cdot \left(e^{\frac{q \cdot V_{CB}}{kT}} - 1 \right) = I_{CO} \cdot \left(e^{\frac{q \cdot V_{CB}}{kT}} - 1 \right)$

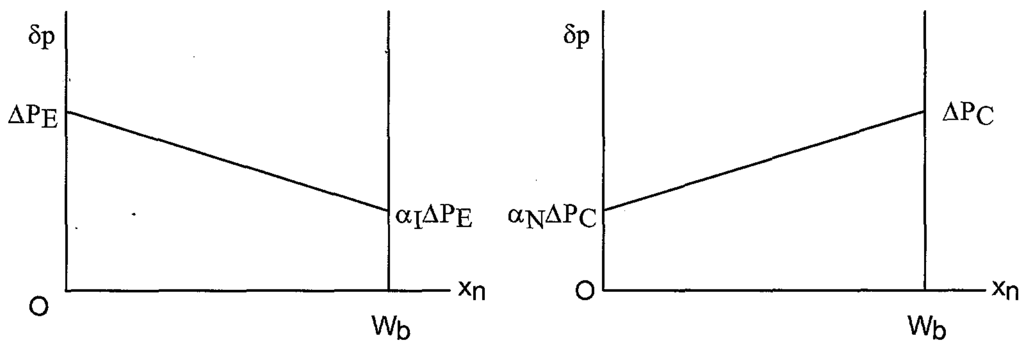
where the minus sign arises from the choice of I_C defined in the reverse direction through the collector junction.

(b) Find Δp_C with emitter forward biased and collector open and with emitter open and collector forward biased. Find Δp_E with collector forward biased and emitter open.

$I_C = 0 \rightarrow \Delta p_C = \alpha_I \cdot \Delta p_E$

$I_E = 0 \rightarrow \Delta p_E = \alpha_N \cdot \Delta p_C$

(c) Sketch $\delta p(x_n)$ for each of the cases in part(b).



Prob. 7.13

(a) Show that the definitions of equation 7-40 are correct and tell what q_N represents.

$$\alpha_N = \frac{I_C}{I_E} = \frac{Q_N \cdot (\frac{1}{\tau_{tN}})}{Q_N \cdot (\frac{1}{\tau_{tN}} + \frac{1}{\tau_{pN}})} = \frac{\tau_{pN}}{\tau_{tN} + \tau_{pN}}$$

$I_{EN} = Q_N \cdot (\frac{1}{\tau_{tN}} + \frac{1}{\tau_{pN}}) = I_{ES} \cdot \frac{\Delta p_E}{p_n}$ gives $I_{ES} = Q_N \cdot \frac{p_n}{\Delta p_E} \cdot (\frac{1}{\tau_{tN}} + \frac{1}{\tau_{pN}}) = q_N \cdot (\frac{1}{\tau_{tN}} + \frac{1}{\tau_{pN}})$ with $q_N = Q_N \cdot \frac{p_n}{\Delta p_E}$

and similarly for the inverted mode, $q_N \sim \frac{1}{2} qAW_b p_n$ is the magnitude of the charge stored in the base when the emitter junction is reverse biased and the collector junction is shorted

(b) Show that equation (7-39) corresponds to equation (7-34).

Equation (7-39a) is $I_E = Q_N \cdot \left(\frac{1}{\tau_{tN}} + \frac{1}{\tau_{pN}}\right) - \frac{Q_I}{\tau_{tI}}$

$$Q_N = I_{ES} \cdot \frac{\Delta p_E}{p_n} \cdot \left(\frac{\tau_{tN} \cdot \tau_{pN}}{\tau_{tN} + \tau_{pN}}\right) \text{ and } Q_I = I_{CS} \cdot \frac{\Delta p_C}{p_n} \cdot \left(\frac{\tau_{tI} \cdot \tau_{pI}}{\tau_{tI} + \tau_{pI}}\right)$$

$$I_E = I_{ES} \cdot \frac{\Delta p_E}{p_n} \cdot \left(\frac{\tau_{tN} \cdot \tau_{pN}}{\tau_{tN} + \tau_{pN}}\right) \cdot \left(\frac{1}{\tau_{tN}} + \frac{1}{\tau_{pN}}\right) - I_{CS} \cdot \frac{\Delta p_C}{p_n} \cdot \left(\frac{\tau_{tI} \cdot \tau_{pI}}{\tau_{tI} + \tau_{pI}}\right) \cdot \frac{1}{\tau_{tI}}$$

$$I_E = I_{ES} \cdot \frac{\Delta p_E}{p_n} - I_{CS} \cdot \frac{\Delta p_C}{p_n} \cdot \left(\frac{\tau_{pI}}{\tau_{tI} + \tau_{pI}}\right) = I_{ES} \cdot \frac{\Delta p_E}{p_n} - \alpha_I \cdot I_{CS} \cdot \frac{\Delta p_C}{p_n} = \frac{I_{ES}}{p_n} \cdot (\Delta p_E - \alpha_N \cdot \Delta p_C) = \text{Equation (7-}$$

Equation (7-39b) is $I_C = \frac{Q_N}{\tau_{tN}} - Q_I \left(\frac{1}{\tau_{tI}} + \frac{1}{\tau_{pI}}\right)$

$$Q_N = I_{ES} \cdot \frac{\Delta p_E}{p_n} \cdot \left(\frac{\tau_{tN} \cdot \tau_{pN}}{\tau_{tN} + \tau_{pN}}\right) \text{ and } Q_I = I_{CS} \cdot \frac{\Delta p_C}{p_n} \cdot \left(\frac{\tau_{tI} \cdot \tau_{pI}}{\tau_{tI} + \tau_{pI}}\right)$$

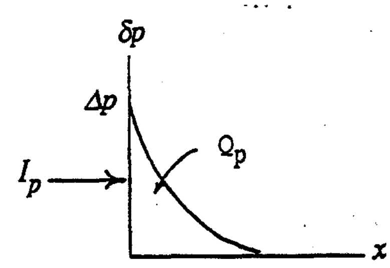
$$I_C = I_{ES} \cdot \frac{\Delta p_E}{p_n} \cdot \left(\frac{\tau_{tN} \cdot \tau_{pN}}{\tau_{tN} + \tau_{pN}}\right) \cdot \frac{1}{\tau_{tN}} - I_{CS} \cdot \frac{\Delta p_C}{p_n} \cdot \left(\frac{\tau_{tI} \cdot \tau_{pI}}{\tau_{tI} + \tau_{pI}}\right) \cdot \left(\frac{1}{\tau_{tI}} + \frac{1}{\tau_{pI}}\right)$$

$$I_E = I_{ES} \cdot \frac{\Delta p_E}{p_n} \cdot \left(\frac{\tau_{pN}}{\tau_{tN} + \tau_{pN}}\right) - I_{CS} \cdot \frac{\Delta p_C}{p_n} = \alpha_N \cdot I_{ES} \cdot \frac{\Delta p_E}{p_n} - I_{CS} \cdot \frac{\Delta p_C}{p_n} = \frac{I_{CS}}{p_n} \cdot (\alpha_I \cdot \Delta p_E - \Delta p_C) = \text{Equation (7-}$$

Prob. 7.14

(a) Explain how the base transit time may be shorter than the hole lifetime.

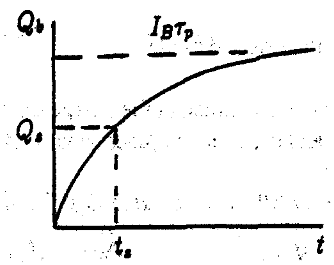
$\delta p(x_n)$ is a steady steady state hole distribution with an average replacement time equal to the hole lifetime. These replaced holes are indistinguishable and transit time is not affected by this replacement; so, the base transit time may be shorter than the hole lifetime.



(b) Explain why the turn-on transient of a BJT is faster when the device is driven into oversaturation.

The turn-on transient is faster in oversaturation because the Q_b=Q_s condition is reached earlier in the exponential rise.

$$Q_b = I_B \cdot \tau_p \cdot \left(1 - e^{-\frac{t}{\tau_p}}\right)$$



$$t_s = \tau_p \cdot \ln\left(\frac{1}{1-\frac{I_C}{\beta \cdot I_B}}\right) \text{ with } \beta \cdot I_B \gg I_C \text{ giving } t_s > \tau_p$$

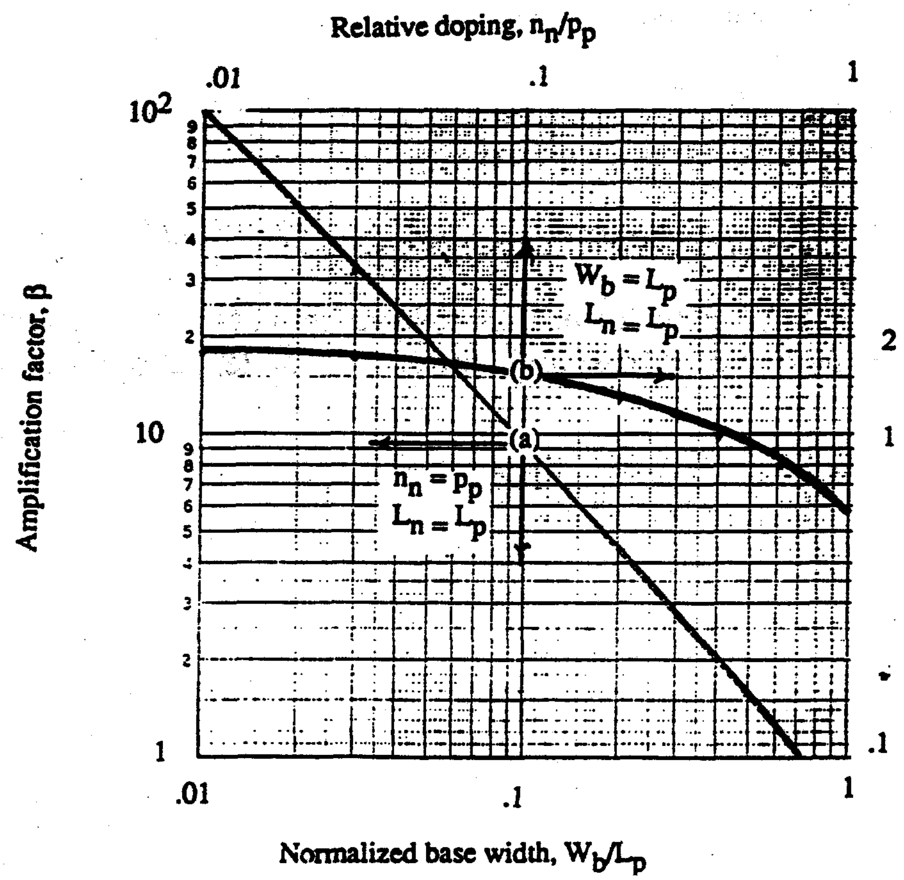
Prob. 7.15

Design an n-p-n HBT with reasonable γ and base resistance.

Since this is an open-ended design problem, there is no unique solution. Students should use the results of Equation 7-81 with the band gap difference of 0.42eV between GaAs and $Al_{0.3}Ga_{0.7}As$, to conclude that the base doping can be considerably higher than the emitter doping while maintaining a good emitter injection efficiency for electrons. It is possible to estimate the base spreading resistance with the higher doping concentration. Note that Appendix III only gives values for light doping. Clearly, much important information will be lost in these estimates because of the sparse information the students have to work with. For example, real HBTs using AlGaAs/GaAs suffer from surface recombination problems, and scaling of these devices to small dimensions is inhibited. Some of the students will be interested enough to read current articles on and will therefore provide comments to this effect. In fact, a good answer to this problem might begin, "I wouldn't use AlGaAs/GAAs. Instead, I would…"

Prob. 7.16

Plot β for the given BJT.



Prob. 7.17

(a) What is Q_b in Figure 7-4 at the d-c bias?

$Q_b = I_B \cdot \tau_p = 10^{-4}A \cdot 10^{-5}s = 10^{-9}C$

or $Q_b = I_C \cdot \tau_t = 10^{-2}A \cdot 10^{-7}s = 10^{-9}C$

(b) Why is B different in the normal and inverted mode of a diffused BJT?

The base transport factor is affected by the built-in field resulting from the doping gradient in the base. This field assists transport in the normal mode, but opposes transport in the inverted mode.

Prob. 7.18

For the given p-n-p transistor, calculate the neutral base width W_b.

The built-in potential at the base-emitter junction can be given by (assuming contribution on heavily doped emitter side is half E_g=0.55 eV):

$$V_{bi_{BE}} \cong 0.55 + \frac{kT}{q}\ln\frac{N_B}{n_i} = 0.898\,\text{V}$$

The built-in potential at the collector-base junction is given by:

$$V_{bi_{BC}} \cong \frac{kT}{q}\left[\ln\frac{N_B}{n_i} + \ln\frac{N_c}{n_i}\right]$$

$$= 0.0259\text{V}\left[\ln\frac{10^{16}}{1.5\cdot10^{10}} + \ln\frac{10^{16}}{1.5\cdot10^{10}}\right] = 0.696\,\text{V}$$

Next calculate the width of the base-emitter and base-collector space charge regions

$$W_{EB} = \sqrt{\frac{2\epsilon_s}{qN_B}\left(V_{bi_{BE}} - V_{EB}\right)}$$

Since N_E>>N_B and the B-E junction is forward biased.

$$W_{EB} = \sqrt{\frac{2\cdot11.8\cdot8.85\cdot10^{-14}\,\frac{F}{cm}}{1.6\cdot10^{-19}C\cdot10^{-16}\,\frac{1}{cm^3}}\left(0.898\text{V} - V_{EB}\right)}$$

for $V_{EB} = 0.2$V $\qquad W_{EB} = 3.02\cdot10^{-5}$ cm

for $V_{EB} = 0.6$V $\qquad W_{EB} = 1.97\cdot10^{-5}$ cm

The width of the collector-base space charge region is given by:

$$W_{CB} = \sqrt{\frac{2\epsilon_s}{q}\frac{N_C + N_B}{N_C N_B}\phi_T}$$

Note: One-sided step junction cannot be assumed since for this problem N_B=N_C

Given: $V_{CB} = 0$

Hence:

$\phi_T = V_{bi_{BC}} = 0.696$V

$W_{CB} = 0.426\cdot10^{-4}$ cm

Calculate width of neutral base region

Given: W = metallurgical base width = 1.5 microns

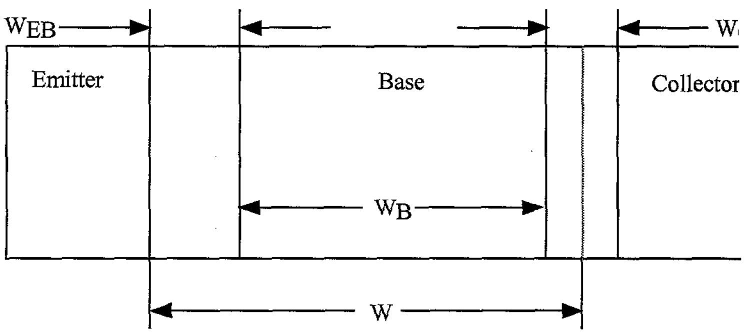
$$W_B = W - W_{EB} - \frac{N_C}{N_B + N_C}\cdot W_{CB}$$

For $V_{EB} = 0.2$V

$$W_B = 1.5 - 0.302 - \frac{0.426}{2} = 0.985\,\text{microns}$$

For $V_{EB} = 0.6$V

$$W_B = 1.5 - 0.197 - \frac{0.426}{2} = 1.09\,\text{microns}$$



Prob. 7.19

For the BJT in Problem 7.18, calculate the base transport factor and the emitter injection efficiency for V_{EB} = 0.2V and 0.6V.

First determine the electron and hole diffusion lengths.

Given $\tau_n = \tau_p = \tau_o = 10^{-7}\text{s}$

$$D_n = D_p = 10\frac{\text{cm}^2}{\text{s}}$$

$$L_n = \sqrt{D_n \tau_n} = \sqrt{10 \cdot 10^{-7}} = 10^{-3}\text{cm}$$

$$L_P = L_n = L = 10\mu\text{m}$$

Calculate the base transport factor, B.

$$\text{For } \frac{W_B}{L_p} << 1,\ B = \frac{1}{\cosh\left(\frac{W_b}{L_p}\right)} \approx 1 - \frac{1}{2}\cdot\left(\frac{W_b}{L_p}\right)^2$$

$$\text{For } V_{EB} = 0.2\text{V} \rightarrow B = 1 - \frac{1}{2}\cdot\left(\frac{0.985\mu\text{m}}{10\mu\text{m}}\right)^2 = 0.995$$

$$\text{For } V_{EB} = 0.6\text{V} \rightarrow B = 1 - \frac{1}{2}\cdot\left(\frac{1.09\mu\text{m}}{10\mu\text{m}}\right)^2 = 0.994$$

Calculate the emitter efficiency γ:

$$\gamma = \frac{I_{E_p}}{I_{E_p} + I_{E_n}}$$

I_{E_p} is the current for holes from the emitter to the base;

I_{E_n} is the current for electrons injected from the base to the emitter.

Calculate I_{E_p} and I_{E_n} as functions of V_{EB}.

I_{E_p} = diffusion current injected across B-E junction by the emitter (holes for p-n-p transistor)

I_{E_n} = diffusion current injected across B-E junction by the base (electrons for p-n-p transistor)

For the given p-n-p:

$$I_{E_p} = A \cdot q \cdot \frac{D_p \cdot n_i^2}{N_B W_B} \cdot e^{\frac{qV_{EB}}{kT}} \text{ (hole current)}$$

$$I_{E_n} = A \cdot q \cdot \frac{D_n \cdot n_i^2}{N_E W_E} \cdot e^{\frac{qV_{EB}}{kT}}$$ (electron current; W_E in the demoniator rather than L_n since $W_E \ll L_n$)

At V_{EB}=0.2V,

$$I_{E_p} = 10^{-5}\text{cm}^2 \cdot 1.609 \cdot 10^{-19}\text{C} \cdot \frac{10\,\frac{\text{cm}^2}{\text{s}} \cdot (1.5 \cdot 10^{10}\,\frac{1}{\text{cm}^3})}{10^{16}\,\frac{1}{\text{cm}^3} \cdot 0.985 \cdot 10^{-4}\text{cm}} \cdot e^{\frac{0.2\text{eV}}{0.0259\text{eV}}} = 8.251 \cdot 10^{-12}\text{A}$$

$$I_{E_n} = 10^{-5}\text{cm}^2 \cdot 1.609 \cdot 10^{-19}\text{C} \cdot \frac{10\,\frac{\text{cm}^2}{\text{s}} \cdot (1.5 \cdot 10^{10}\,\frac{1}{\text{cm}^3})}{10^{19}\,\frac{1}{\text{cm}^3} \cdot 3 \cdot 10^{-4}\text{cm}} \cdot e^{\frac{0.2\text{eV}}{0.0259\text{eV}}} = 2.709 \cdot 10^{-15}\text{A}$$

At V_{EB}=0.6V,

$$I_{E_p} = 10^{-5}\text{cm}^2 \cdot 1.609 \cdot 10^{-19}\text{C} \cdot \frac{10\,\frac{\text{cm}^2}{\text{s}} \cdot (1.5 \cdot 10^{10}\,\frac{1}{\text{cm}^3})}{10^{16}\,\frac{1}{\text{cm}^3} \cdot 0.985 \cdot 10^{-4}\text{cm}} \cdot e^{\frac{0.6\text{eV}}{0.0259\text{eV}}} = 3.8 \cdot 10^{-5}\text{A}$$

$$I_{E_n} = 10^{-5}\text{cm}^2 \cdot 1.609 \cdot 10^{-19}\text{C} \cdot \frac{10\,\frac{\text{cm}^2}{\text{s}} \cdot (1.5 \cdot 10^{10}\,\frac{1}{\text{cm}^3})}{10^{19}\,\frac{1}{\text{cm}^3} \cdot 3 \cdot 10^{-4}\text{cm}} \cdot e^{\frac{0.6\text{eV}}{0.0259\text{eV}}} = 1.38 \cdot 10^{-8}\text{A}$$

From those currents, γ may be calculated.

$$\gamma = \frac{I_{E_p}}{I_{E_p} + I_{E_n}}$$

For V_{EB}=0.2V, $\gamma = \dfrac{8.251 \cdot 10^{-12}\text{A}}{8.251 \cdot 10^{-12}\text{A} + 2.709 \cdot 10^{-15}\text{A}} = 0.9997$

For V_{EB}=0.6V, $\gamma = \dfrac{3.8 \cdot 10^{-5}\text{A}}{3.8 \cdot 10^{-5}\text{A} + 1.38 \cdot 10^{-8}\text{A}} = 0.9996$

Prob. 7.20

For the BJT in Problem 7.18, calculate α, β, I_E, I_B, I_C, and the Gummel number.

To calculate common base current gain alpha

$\alpha = B \cdot \gamma$

V_{EB}=0.2V $\rightarrow$ $\alpha = 0.995 \cdot 0.9997 = 0.9947$

V_{EB}=0.6V $\rightarrow$ $\alpha = 0.994 \cdot 0.9996 = 0.9936$

To calculate beta

$$\beta = \frac{\alpha}{1\text{-}\alpha}$$

$$\text{For } V_{EB} = 0.2 \rightarrow \beta = \frac{0.9947}{1\text{-}0.9947} = 187.7$$

$$\text{For } V_{EB} = 0.6 \rightarrow \beta = \frac{0.9936}{1\text{-}0.9936} = 155.3$$

To calculate currents I_E, I_B and I_C for V_{EB} = 0.2 and 0.6 V the emitter current is given by:

$I_E = I_{E_p} + I_{E_n}$

where I_{E_p} and I_{E_n} are the hole and electron currents, respectively, injected across the base-emitter junction.

For V_{EB} = 0.2V $\rightarrow$ $I_E = 8.251 \cdot 10^{-12}\text{A} + 2.709 \cdot 10^{-15}\text{A} = 8.254 \cdot 10^{-12}\text{A} = 8.254\text{pA}$

For V_{EB}=0.6V $\rightarrow$ $I_E = 3.8 \cdot 10^{-5}\text{A} + 1.38 \cdot 10^{-8}\text{A} = 3.8 \cdot 10^{-5}\text{A} = 38\mu\text{A}$

The collector and base currents can be determined by:

$$I_C = \alpha \cdot I_E \text{ or } I_C = B \cdot I_{E_p} = B \cdot A \cdot q \frac{D_p \cdot n_i^2}{N_B \cdot W_b} \cdot e^{\frac{q \cdot V_{EB}}{kT}}$$

$I_B = (1\text{-}\alpha) \cdot I_E = I_E - I_C$

V_{EB}=0.2V $\rightarrow$ $\alpha = 0.9947$ and $I_E = 8.254\text{pA}$

$I_C = 0.9947 \cdot 8.254\text{pA} = 8.21\text{pA}$

$I_B = 8.254\text{pA} - 8.21\text{pA} = 0.044\text{pA}$

V_{EB}=0.6V $\rightarrow$ α=0.9936 and $I_E = 38\mu\text{A}$

$I_C = 0.9936 \cdot 38\mu\text{A} = 37.8\mu\text{A}$

$I_B = 28\mu\text{A} - 37.8\mu\text{A} = 0.2\mu\text{A}$

Base Gummel Number = $N_B \cdot W_b$

For V_{EB}=0.2V, Gummel number = $10^{16} \frac{1}{\text{cm}^3} \cdot 1.09 \cdot 10^{-4}\text{cm} = 1.09 \cdot 10^{12} \frac{1}{\text{cm}^2}$

For V_{EB}=0.6V, Gummel number = $10^{16} \frac{1}{\text{cm}^3} \cdot 0.985 \cdot 10^{-4}\text{cm} = 9.85 \cdot 10^{11} \frac{1}{\text{cm}^2}$

Prob. 7.21

For the given BJT, calculate β in terms of B and γ and using the charge control model.

In emitter, $L_n^E = \sqrt{\mu_n \cdot \frac{kT}{q} \cdot \tau_n} = \sqrt{150\,\frac{cm^2}{V\cdot s} \cdot 0.0259V \cdot 10^{-10}s} = 1.97 \cdot 10^{-5}cm = 0.197\mu m$

In base,

$$L_p^B = \sqrt{D_p \tau_p} = \sqrt{\mu_p \cdot \frac{kT}{q} \cdot \tau_p} = \sqrt{400\,\frac{cm^2}{V\cdot s} \cdot 0.0259V \cdot 25 \cdot 10^{-10}s} = 1.61 \cdot 10^{-4}cm = 1.61\mu m$$

$$\gamma = \left[1 + \frac{\mu_n^E \cdot N_D^B \cdot W_b}{\mu_p^B \cdot N_A^E \cdot L_n^E}\right]^{-1} = \left[1 + \frac{150\,\frac{cm^2}{V\cdot s} \cdot 10^{16}\,\frac{1}{cm^3} \cdot 0.2\mu m}{400\,\frac{cm^2}{V\cdot s} \cdot 5 \cdot 10^{18}\,\frac{1}{cm^3} \cdot 0.197\mu m}\right]^{-1} = 0.9992$$

$$B = 1 - \frac{W_b^2}{2 \cdot L_p^2} = 1 - \frac{(0.2\mu m)^2}{2 \cdot (1.61\mu m)^2} = .9961$$

$$\beta = \frac{B \cdot \gamma}{1 - B \cdot \gamma} = 213$$

Charge control approach

$$\tau_t = \frac{W_b^2}{2 \cdot D_p} = 0.514 \cdot 10^{-11}s$$

$$\beta = \frac{\tau_p}{\tau_t} = 486$$

These differ because the charge control approach assumes $\gamma = 1$.

Prob. 7.22

For the BJT in 7.21, calculate the charge stored in the base when $V_{CB}=0V$ and $V_{EB}=0.7V$. Find f_T if the base transit time is the dominant delay component.

$$Q_b \approx \frac{1}{2} \cdot q \cdot A \cdot W_b \cdot p_n \cdot e^{\frac{qV_{EB}}{kT}} = \frac{1}{2} \cdot 1.6 \cdot 10^{-19}C \cdot 10^{-4}cm^2 \cdot 0.2 \cdot 10^{-4}cm \cdot \frac{\left(1.5 \cdot 10^{10}\,\frac{1}{cm^3}\right)^2}{10^{16}\,\frac{1}{cm^3}} \cdot e^{\frac{0.7}{.026}} = 1.968 \cdot 10^{-12}C$$

$$\tau_t = \frac{W_b^2}{2 \cdot D_p} = \frac{(0.2 \cdot 10^{-4}cm)^2}{2 \cdot 10.36\,\frac{cm^2}{s}} = 1.93 \cdot 10^{-11}s$$

$$f_T = \frac{1}{2\pi \cdot \tau_t} = 8.24 \cdot 10^9 Hz$$

Prob. 7.23

Find f_T for an n-p-n with τ_T=100ps in the base and C_c=0.1pF and r_c=10Ω in the collector depletion region. The emitter junction charges in 30ps, and electrons drift at v_s through the 1μm depletion region.

The total delay time for the parameters given is

$$\tau_d = 100\text{ps} + \frac{10^{-4}}{10^7}\text{s}\cdot 10^{12}\,\tfrac{\text{ps}}{\text{s}} + 30\text{ps} + 10\Omega\cdot 0.1\text{pF} = 141\text{ps}$$

$$f_T = \frac{1}{2\pi\tau_d} = 1.1\text{GHz}$$

Prob. 7.24

For the given npn Si BJT, find V_{BE} for $\Delta n_E = N_a^B$.

$$\Delta n_E = n_p \cdot e^{\frac{q\cdot V_{BE}}{kT}} = N_a^B \;\rightarrow\; V_{BE} = \frac{kT}{q}\cdot \ln\frac{N_a^B}{n_p} = 0.695\text{V}$$

With $\frac{N_E}{N_B} = 100$, high level injection is not reached until the emitter junction is biased to nearly 0.7V. Since the contact potential, $V_o = \frac{kT}{q}\cdot \ln\frac{N_d^E \cdot N_a^B}{n_i^2} = 0.81\text{V}$, this is a very high bias. Thus γ rolloff due to high injection is not likely in the normal operation range.

Chapter 8 Solutions

Prob. 8.1

For the p-i-n photodiode in figure 8-7,
(a) explain why the photodetector does not have gain.

An electron hole pair created within W by absorption of a photon is collected as the electron is swept to the n side and the hole is swept to the p side. Since only one electron-hole pair is collected per photon, there is no gain.

(b) explain how making the device more sensitive to low light levels degrades speed.

If W is made wider to receive more photons, the transit time to collect the electron hole pairs will be longer; so, response speed will be degraded.

(c) choose a material and substrate to detect light with λ=0.6μm.

In order to detect the light, the band gap must be smaller than the photon energy. A λ=0.6μm photon has an $\text{energy} = \frac{1.24\text{eV}\cdot\mu\text{m}}{0.6\mu\text{m}} = 2.07\text{eV}$. From figure 1-13, $In_{0.5}Ga_{0.5}P$ grown on GaAs or $AlAs_{0.55}Sb_{0.45}$ grown on InP each have a band gap energy slightly below the photon energy (2eV and 1.95eV, respectively).

Prob. 8.2

Find an expression for conductance and the transit time for (a)low and (b)high voltage.

$$\text{conductance} = G = \frac{1}{R} = \frac{A}{L}\cdot q\cdot\mu_n\cdot n$$

$$\text{change in conductance} = \Delta G = \frac{A}{L}\cdot q\cdot\mu_n\cdot\Delta n = \frac{A}{L}\cdot q\cdot\mu_n\cdot g_{op}\cdot\tau_n$$

$$\text{for low voltage, } \tau_t = \frac{L}{v_d} = \frac{L}{\mu_n\cdot\frac{V}{L}} = \frac{L^2}{\mu_n\cdot V}$$

$$\text{for high voltage, } \tau_t = \frac{L}{v_s}$$

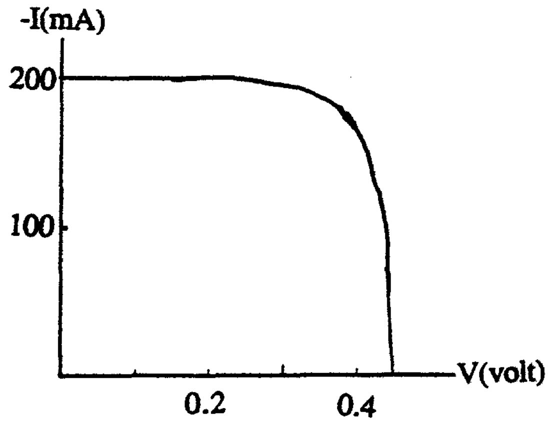
Prob. 8.3

Plot I-V for a Si solar cell with I_{th}=5nA and I_{sc}=200mA.

From Equation 8-2,

$$V = \frac{kT}{q}\cdot \ln\left(\frac{I+I_{sc}}{I_{th}}+1\right) - I_{op} = 0.0259V\cdot \ln\left(\frac{I+0.2A}{5\cdot 10^{-9}A}+1\right)$$

I (mA)	V (volt)
−200	0
−190	0.376
−180	0.39
−160	0.41
−120	0.43
−80	0.44
−40	0.448
0	0.453



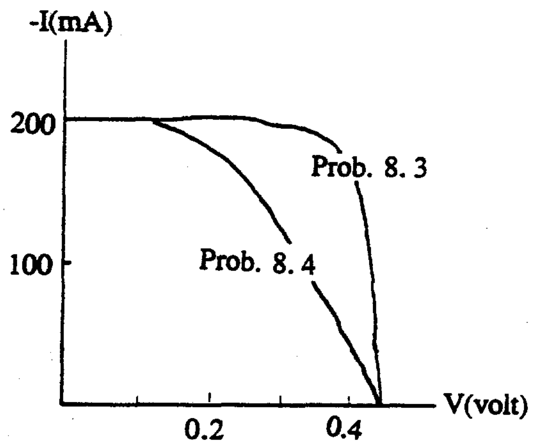
Prob. 8.4

Repeat problem 8.3 with a 1Ω series resistance.

Given current I and terminal voltage $V_a = V+IR$, the voltage across the diode is reduced by IR.

$$I = I_{th}\cdot\left(e^{\frac{q\cdot(V_a - IR)}{kT}} - 1\right) - I_{op} = 5\cdot 10^{-9}A\cdot\left(e^{\frac{(V_a - I\cdot 1\Omega)}{0.0259V}} - 1\right) - 0.2A$$

I	V_a
0	0.453
−0.04	0.408
−0.08	0.36
−0.12	0.31
−0.16	0.25
−0.18	0.21



Prob. 8.5

How can several solar cells be used in a solar cell?

Surface recombination could be reduced by growing a lattice-matched layer with a larger band gap on the surface to keep generated carriers from the surface. For example, AlGaAs (2eV) could be grown on GaAs (1.4eV). Additionally, a secondary cell with a smaller band gap could be placed below the primary cell to absorb light which passes through. For example, Si could be utilized below GaAs.

Prob. 8.6

Find the current density change for 2.5V and 2500V.

for 2.5V, $\mathcal{E} = \dfrac{V}{L} = \dfrac{2.5\text{V}}{5\cdot 10^{-4}\text{cm}} = 5\cdot 10^{3}\,\frac{\text{V}}{\text{cm}}$

electron velocity $= v_{dn} = 1500\,\frac{\text{cm}^2}{\text{V}\cdot\text{s}}\cdot 5\cdot 10^{3}\,\frac{\text{V}}{\text{cm}} = 7.5\cdot 10^{6}\,\frac{\text{cm}}{\text{s}}$

hole velocity $= v_{dp} = 500\,\frac{\text{cm}^2}{\text{V}\cdot\text{s}}\cdot 5\cdot 10^{3}\,\frac{\text{V}}{\text{cm}} = 2.5\cdot 10^{6}\,\frac{\text{cm}}{\text{s}}$

with light $\Delta n = \Delta p = g_{op}\cdot\tau = 10^{20}\,\frac{1}{\text{cm}^3}\cdot 10^{-7}\text{s} = 10^{13}\,\frac{\text{s}}{\text{cm}^3}$

$$\Delta J = q\cdot\Delta n\cdot v_{dn} + q\cdot\Delta p\cdot v_{dp} = 1.6\cdot 10^{-19}\text{C}\cdot 10^{13}\,\frac{\text{s}}{\text{cm}^3}\cdot 1.0\cdot 10^{7}\,\frac{\text{cm}}{\text{s}} = 0.16\,\frac{\text{A}}{\text{cm}^2}$$

for 2500V,

electron velocity $= v_d = v_s = 10^{7}\,\frac{\text{cm}}{\text{s}}$

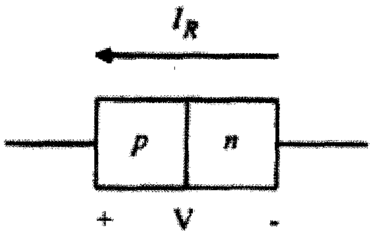
hole velocity $= v_d = v_s = 10^{7}\,\frac{\text{cm}}{\text{s}}$

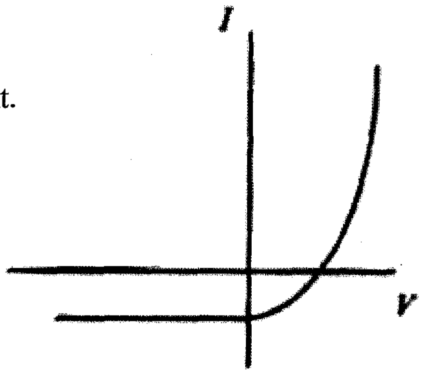
with light $\rightarrow$ $\Delta J = q\cdot\Delta n\cdot v_s + q\cdot\Delta p\cdot v_s = 1.6\cdot 10^{-19}\text{C}\cdot 10^{13}\,\frac{\text{s}}{\text{cm}^3}\cdot 2.0\cdot 10^{7}\,\frac{\text{cm}}{\text{s}} = 0.32\,\frac{\text{A}}{\text{cm}^2}$

Prob. 8.7

(a) Why must a solar cell be operated in the 4^{th} quadrant of the junction I-V characteristics?

Power is only generated in the 4^{th} quadrant (-I,+V). Power is consumed in the 1^{st} (+I,+V) and 3^{rd} (-I,-V) quadrant.





(b) What is the advantage of a quaternary alloy in fabricating LEDs for fiber optics?

A quaternary alloy allows adjustment of both bandgap, and therefore wavelength, and lattice constant for epitaxial growth on convenient substrates.

(c) Why is a reverse-biased GaAs p-n junction not a good photoconductor for light of $\lambda = 1\,\mu m$?

$$E_{photon} = \frac{h \cdot c}{\lambda} = \frac{4.14 \cdot 10^{-15} eV \cdot s \cdot 3 \cdot 10^{10} \frac{cm}{s}}{10^{-4} cm} = 1.24 eV$$

Since the bandgap of GaAs is 1.43eV, the photon is not absorbed; so, GaAs is not a good photoconductor for this light.

Prob. 8.8

Find I_{sc} and V_{oc} for the solar cell.

From Equation 8-1,

$$I_{sc} = I_{op} = q \cdot A \cdot g_{op} \cdot (L_p + L_n + W) = 1.6 \cdot 10^{-19} C \cdot 4cm^2 \cdot 10^{18} \frac{1}{cm^3} \cdot (2\mu m + 2\mu m + 1\mu m) = 0.32mA$$

From Equation 8-3,

$$V_{oc} = \frac{kT}{q} \cdot \ln\left(1 + \frac{I_{op}}{I_{th}}\right) = 0.0259V \cdot \ln\left(1 + \frac{0.32 \cdot 10^{-3} A}{32 \cdot 10^{-9} A}\right) = 0.24V$$

Prob. 8.9

(a) Derive the expression for the voltage at maximum power.

Equation 8-2 can be written as

$$I = I_{th} \cdot \left(e^{\frac{q \cdot V}{kT}} - 1\right) - I_{sc}$$

$$P = I \cdot V = I_{th} \cdot \left(e^{\frac{q \cdot V}{kT}} - 1\right) \cdot V - I_{sc} \cdot V$$

$$\frac{dP}{dV} = I_{th} \cdot \left(e^{\frac{q \cdot V}{kT}} - 1\right) + I_{th} \cdot V \cdot \frac{q}{kT} \cdot e^{\frac{q \cdot V}{kT}} - I_{sc} = 0 \text{ for maximum}$$

$$e^{\frac{q \cdot V_{mp}}{kT}} \cdot \left(V_{mp} \cdot \frac{q}{kT} + 1\right) = 1 + \frac{I_{sc}}{I_{th}}$$

assuming $I_{sc} \gg I_{th}$ and $V_{mp} \gg \frac{kT}{q}$,

$$e^{\frac{q \cdot V_{mp}}{kT}} \cdot V_{mp} \cdot \frac{q}{kT} = \frac{I_{sc}}{I_{th}}$$

(b) Rewrite this equation in the form ln x = C – x.

$$\ln\left(e^{\frac{q\cdot V_{mp}}{kT}}\cdot V_{mp}\cdot\frac{q}{kT}\right)=\ln\left(\frac{I_{sc}}{I_{th}}\right)$$

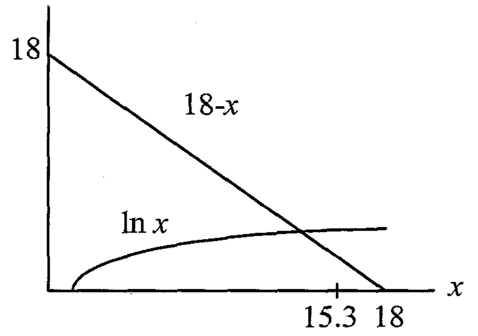
$$\ln\left(V_{mp}\cdot\frac{q}{kT}\right)+\frac{q\cdot V_{mp}}{kT}=\ln\left(\frac{I_{sc}}{I_{th}}\right)$$

$$\ln\left(V_{mp}\cdot\frac{q}{kT}\right)=\ln\left(\frac{I_{sc}}{I_{th}}\right)-\frac{q\cdot V_{mp}}{kT}$$

substitute $x=V_{mp}\cdot\frac{q}{kT}$ and $\ln\left(\frac{I_{sc}}{I_{th}}\right)=\ln\left(\frac{100\cdot 10^{-3}A}{1.5\cdot 10^{-9}A}\right)=18$

$\ln x = 18 - x$

(c) Find a graphical solution for V_{mp} and the maximum power delivered.



solution at x = 15.3

$V_{mp}=15.3\cdot 0.0259V=0.396V$

$I=10^{-9}A\cdot e^{15.3}-10^{-1}A=-96mA$

(d) Find the maximum power at this illumination.

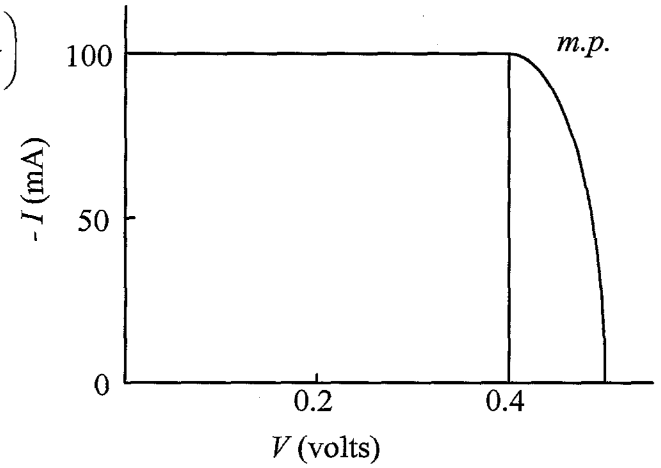
$P_{max}=-I\cdot V=37.9mW$

Prob. 8.10

For the solar cell in 8.9, plot the I-V curve and draw the maximum power rectangle.

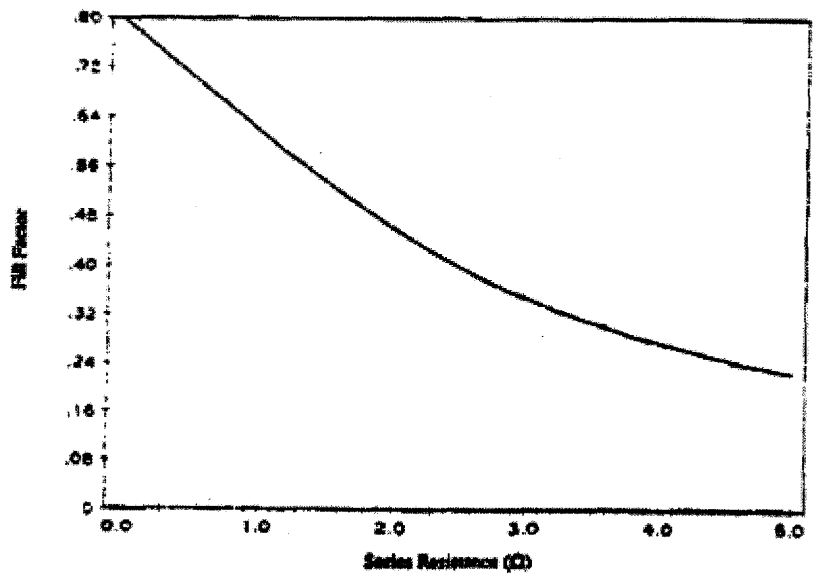
$$V=0.0259eV\cdot\ln\left(1+\frac{0.1A+I}{1.5\cdot 10^{-9}A}\right)$$

I (mA)	V (mV)
0	**467**
-25	**459**
-50	**449**
-75	**431**
-90	**407**
-95	**389**
-98	**365**
-100	**0**



Prob. 8.11

Calculate and plot the fill factor for the solar cell.



Prob. 8.12

Find the frequency and momentum of the emitted photon.

$h \cdot \nu = 1.8\text{eV} \quad \rightarrow \quad \nu = 4.3 \cdot 10^{14}\text{Hz}$

$\lambda = \frac{c}{\nu} = 6.9 \cdot 10^{-7}\text{m}$

$p = \frac{h}{\lambda} = 9.1 \cdot 10^{-28} \frac{\text{kg} \cdot \text{m}}{\text{s}}$

Prob. 8.13

Find the LED emission and tell if can it be used to detect 100nm or 900nm photons.

$$\lambda = \frac{1.24\text{eV} \cdot \mu\text{m}}{E_g} = \frac{1.24\text{eV} \cdot \mu\text{m}}{2.5\text{eV}} = 0.5\mu\text{m} = 500\text{nm}$$

100nm light has $h\nu > E_g$ and will be detected

900nm light has $h\nu < E_g$ and will not be detected

Prob. 8.14

Explain how degeneracy prevents absorption of the emission wavelength.

Since absorption requires promotion of an electron from a filled state in the valence band to an empty state in the conduction band, absorbed photons must have $h\nu > F_n - F_p$. Emitted photons must have $h\nu < F_n - F_p$. This is true only in the inversion region, and absorption becomes important away from the junction.

Prob. 8.15

Show $B_{12}=B_{21}$ at high temperature.

$$B_{12} \cdot n_1 \cdot \rho(\nu_{12}) = A_{21} \cdot n_2 + B_{21} \cdot n_2 \cdot \rho(\nu_{12})$$

$$B_{12} = \left(\frac{A_{21}}{\rho(\nu_{12})} + B_{21}\right) \cdot e^{-\frac{E_2 - E_1}{kT}}$$

$$\text{As } T \to \infty,\ e^{-\frac{E_2 - E_1}{kT}} \to 1 \text{ and } \rho(\nu_{12}) \to \infty$$

$$B_{12} = (0 + B_{21}) \cdot 1 = B_{21}$$

Prob. 8.16

Use Planck's radiation law to find A_{21}/B_{12}.

$$\rho(\nu_{12}) = \frac{A_{21}}{B_{12}} \cdot \frac{n_2}{n_1} + \frac{n_2}{n_1} \cdot \rho(\nu_{12}) = \left(\frac{A_{21}}{B_{12}} + \rho(\nu_{12})\right) \cdot e^{-\frac{h\nu_{12}}{kT}}$$

$$\frac{A_{21}}{B_{12}} = \rho(\nu_{12}) \cdot \left(e^{\frac{h\nu_{12}}{kT}} - 1\right) = \frac{8\pi \cdot h\nu_{12}^3}{c^3}$$

Prob. 8.17

Estimate minimum $n=p$ for population inversion in GaAs.

$$F_n - F_p = E_g = 1.43\text{eV}$$

$$\text{For } n = p,\ F_n - E_i = E_i - F_p = \frac{1.43\text{eV}}{2} = 0.715\text{eV}$$

$$n = p = n_i \cdot e^{\frac{F_n - E_i}{kT}} = 10^6 \frac{1}{\text{cm}^3} \cdot e^{\frac{0.715\text{eV}}{0.0259\text{eV}}} = 10^{18} \frac{1}{\text{cm}^3}$$

Prob. 8.18

For a uniformly illuminate p^+-n diode with g_{op} in steady date, find $\delta_p(x_n)$, $I_p(x_n)$, and $I_p(0)$.

$$\frac{d^2\delta p}{dx_n^2} = \frac{\delta p}{L_p^2} - \frac{g_{op}}{D_p}$$

$$\delta p(x_n) = B \cdot e^{\frac{-x_n}{L_p}} + \frac{g_{op} \cdot L_p^2}{D_p}$$

At x_n=0, $\delta p(0) = \Delta p_n$. Thus $B = \Delta p_n - \frac{g_{op} \cdot L_p^2}{D_p}$

(a) $\delta p(x_n) = \left[p_n \cdot \left(e^{\frac{qV}{kT}} - 1 \right) - \frac{g_{op} \cdot L_p^2}{D_p} \right] \cdot e^{-\frac{x_n}{L_p}} + \frac{g_{op} \cdot L_p^2}{D_p}$

$$\frac{d\delta p}{dx_n} = -\frac{1}{L_p} \left[\Delta p_n - \frac{g_{op} \cdot L_p^2}{D_p} \right] \cdot e^{-\frac{x_n}{L_p}}$$

(b) $I_p(x_n) = -q \cdot A \cdot D_p \cdot \frac{d\delta p}{dx_n} = q \cdot A \cdot \frac{D_p}{L_p} \left[\Delta p_n - \frac{g_{op} \cdot L_p^2}{D_p} \right] \cdot e^{-\frac{x_n}{L_p}}$

$$I_p(0) = q \cdot A \cdot \frac{D_p}{L_p} \cdot p_n \cdot \left(e^{\frac{qV}{kT}} - 1 \right) - q \cdot A \cdot L_p \cdot g_{op}$$

This corresponds to Equation 8-2 for $n_p \ll p_n$ except that the component due to generation on the p side is not included.

Chapter 9 Solutions

Prob. 9.1

Relate the sheet resistance of a diffused layer to $N_a(x)$ and x_j.

$$R_s = \frac{\langle \rho \rangle}{x_j} = \frac{1}{\langle \sigma \rangle \cdot x_j}$$

$$\langle \sigma \rangle = \frac{q}{x_j} \cdot \int_0^{x_j} \mu_p(x) \cdot N_a(x) \cdot dx \quad \text{assuming } N_a(x) \gg N_d \text{ over most of the profile}$$

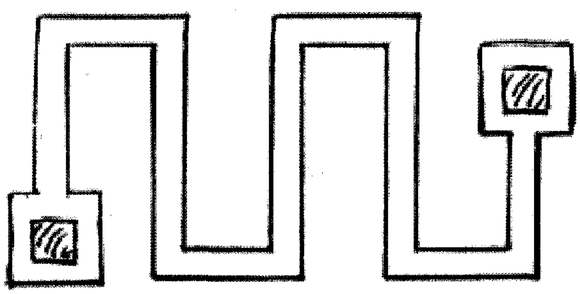
$$R_s = \frac{1}{q \cdot \int_0^{x_j} \mu_p(x) \cdot N_a(x) \cdot dx}$$

Prob. 9.2

Find the aspect ratio and draw a diffusion pattern.

$$R = \frac{R_s \cdot L}{W} \rightarrow \frac{L}{W} = \frac{R}{R_s} = \frac{10^4 \Omega}{200 \Omega} = 50$$

$W = 5\mu m$ and $L = 250\mu m$



Prob. 9.3

(a) Find the required diffusion time.

$$N(x,t) = N_o \cdot \text{erfc}\left(\frac{x}{2\sqrt{D \cdot t}}\right)$$

$$N(3\cdot10^{-4}\text{cm},t) = 10^{20}\,\tfrac{1}{\text{cm}^3}\cdot \text{erfc}\left(\frac{3\cdot10^{-4}\text{cm}}{2\sqrt{2.5\cdot10^{-12}\,\frac{\text{cm}^2}{\text{s}}\cdot t}}\right) = 10^{16}\,\tfrac{1}{\text{cm}^3}$$

$$\text{Using Figure P5-2, } t = \frac{\left(\dfrac{3\cdot10^{4}\text{cm}}{2\cdot \text{erfc}^{-1}\left(\dfrac{10^{16}\,\frac{1}{\text{cm}^3}}{10^{20}\,\frac{1}{\text{cm}^3}}\right)}\right)^2}{2.5\cdot10^{-12}\,\frac{\text{cm}^2}{\text{s}}} = \frac{\left(\dfrac{3\cdot10^{-4}\text{cm}}{2\cdot2.75}\right)^2}{2.5\cdot10^{-12}\,\frac{\text{cm}^2}{\text{s}}} = 1192 \text{ seconds}$$

(b) Find how far would a Sb-doped buried layer move during this diffusion.

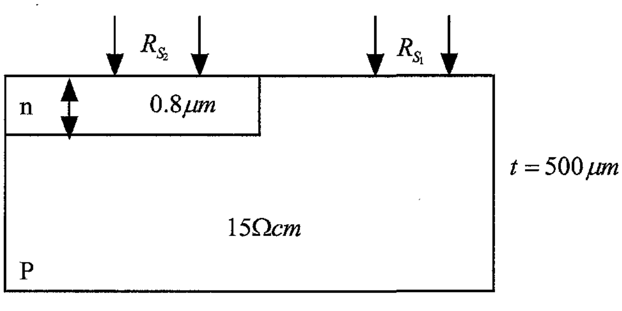
$$N(x,t) = N_o \cdot \text{erfc}\left(\frac{x}{2\sqrt{D \cdot t}}\right)$$

$$N(x,1192\text{s}) = 10^{20}\,\tfrac{1}{\text{cm}^3}\cdot \text{erfc}\left(\frac{x}{2\sqrt{2\cdot10^{-13}\,\frac{\text{cm}^2}{\text{s}}\cdot 1192\text{s}}}\right) = 10^{16}\,\tfrac{1}{\text{cm}^3}$$

$$t = 2\sqrt{2\cdot10^{-13}\,\tfrac{\text{cm}^2}{\text{s}}\cdot1192\text{s}}\cdot \text{erfc}^{-1}\left(\frac{10^{16}\,\frac{1}{\text{cm}^3}}{10^{20}\,\frac{1}{\text{cm}^3}}\right) = 2\sqrt{2\cdot10^{-13}\,\tfrac{\text{cm}^2}{\text{s}}\cdot1192\text{s}}\cdot2.75 = 0.85\mu\text{m}$$

Prob. 9.4

Calculate the average resistivity.



From Irvin curve, $N_{sub} = 10^{15}\,\frac{1}{\text{cm}^3}$

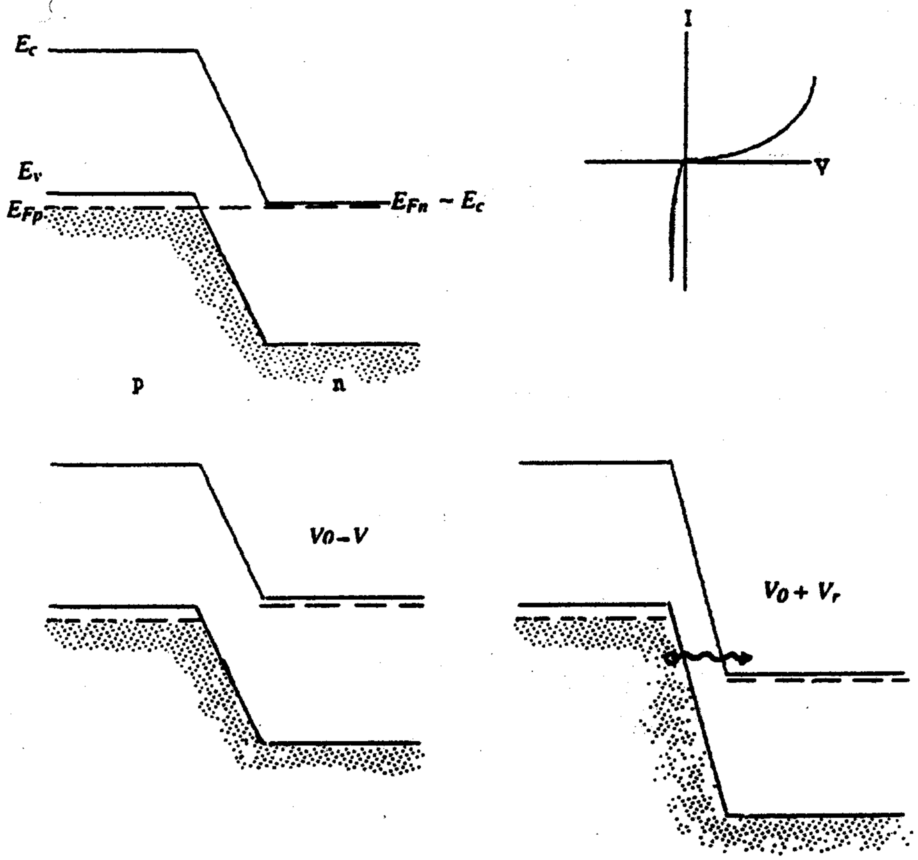
$$R_{S_1} \text{ in undoped } = \frac{P}{t} = \frac{15\Omega\cdot\text{cm}}{500\cdot10^{-4}\text{cm}} = 300\,\tfrac{\Omega}{\text{square}}$$

$$\text{in doped part}, \rho = R_{S_2}\cdot x_j = 90\,\tfrac{\Omega}{\text{square}}\cdot0.8\cdot10^{-4}\text{cm} = 0.0072\Omega\cdot\text{cm} = 72\Omega\cdot\mu\text{m}$$

Chapter 10 Solutions

Prob. 10.1

Sketch the band diagram.

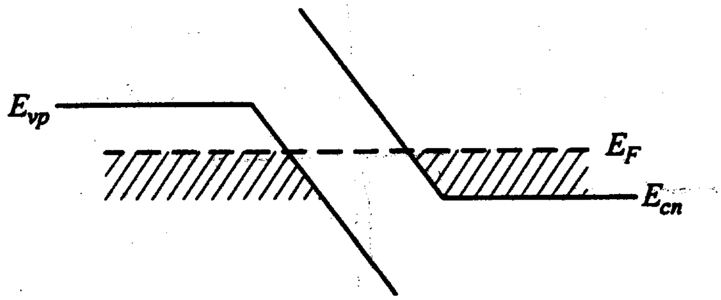


Forward bias **Reverse bias**

This is called a backward diode because it conducts freely in the reverse direction (due to tunneling), but the current remains small for low voltage forward bias.

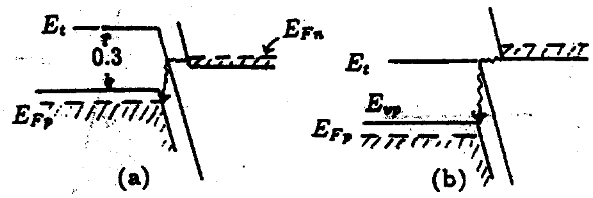
Prob. 10.2

(a) Calculate the minimum forward bias for which tunneling occurs.



The sizes of $E_{vp} - E_F$ and $E_F - E_{cn}$ determine the voltage required to align filled states opposite empty states.

Tunneling begins when $E_{Fn} - E_{Fp} = E_t$ which occurs at a forward bias of 0.3V

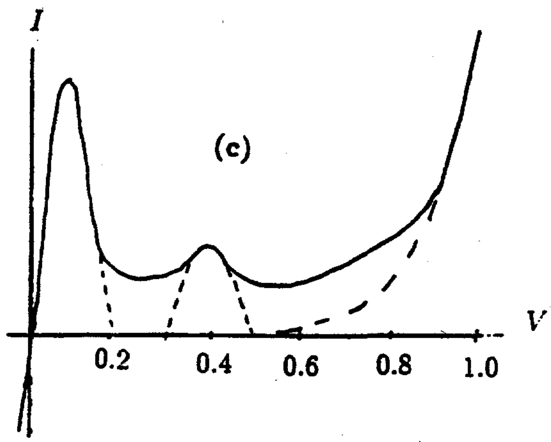


(b) Calculate the maximum forward bias for which tunneling occurs.

Tunneling ends when $E_{cn} - E_{vp} = E_t$ which occurs at a forward bias of
0.3V + 0.1V + 0.1V = 0.5V

(c) Sketch the I-V curve for the diode.

Band to band tunneling is maximized when $E_{Fn} - E_{Fp} = 0.1V$ and is essentially zero when $E_{Fn} - E_{Fp} = 0.2V$.



Prob. 10.3

(a) Relate $\partial\rho/\partial t$ where ρ is the space charge density σ and $\in$ neglecting recombination.

$$J = \sigma \cdot \mathcal{E} = -\sigma \cdot \nabla V$$

$$\nabla \bullet J = -\sigma \cdot \nabla^2 V$$

$$\nabla \bullet J = -\frac{\partial \rho}{\partial t} \cdot \nabla^2 V = -\frac{\rho}{\in}$$

$$\frac{\partial \rho}{\partial t} = -\frac{\sigma \cdot \rho}{\in}$$

(b) Show the space charge $\rho(t)$ decays exponentially with time constant τ_d.

solving differential in (a) gives

$$\rho = \rho_o \cdot e^{-\frac{\sigma}{\in} \cdot t} = \rho_o \cdot e^{-\frac{t}{\tau_d}} \text{ if } \tau_d = \frac{\in}{\sigma}$$

(c) Find the RC time constant of a sample.

$$R = \frac{L}{\sigma \cdot A} \text{ and } C = \frac{\in \cdot A}{L}$$

$$RC = \frac{L}{\sigma \cdot A} \cdot \frac{\in \cdot A}{L} = \frac{\in}{\sigma} = \tau_d$$

Prob. 10.4

Find the criteria for negative conductivity in terms of mobilities and electron concentrations in the Γ and L bands of GaAs.

$$J = \sigma \cdot \mathcal{E} = q \cdot [\mu_\Gamma \cdot n_\Gamma + \mu_L \cdot n_L] \cdot \mathcal{E} = q \cdot [\mu_\Gamma \cdot n_\Gamma + \mu_L \cdot (n_o - n_\Gamma)] \cdot \mathcal{E}$$

$$\frac{dJ}{d\mathcal{E}} = q \cdot [\mu_\Gamma \cdot n_\Gamma + \mu_L \cdot n_L] + q \cdot \mathcal{E} \cdot \left[(\mu_\Gamma - \mu_L) \cdot \frac{dn_\Gamma}{d\mathcal{E}} + n_\Gamma \cdot \frac{d\mu_\Gamma}{d\mathcal{E}} + n_L \cdot \frac{d\mu_L}{d\mathcal{E}} \right] \text{ since } \frac{dn_o}{d\mathcal{E}} = 0$$

$$\frac{dJ}{d\mathcal{E}} < 0 \text{ when } \frac{\mathcal{E} \cdot \left[(\mu_\Gamma - \mu_L) \cdot \frac{dn_\Gamma}{d\mathcal{E}} + n_\Gamma \cdot \frac{d\mu_\Gamma}{d\mathcal{E}} + n_L \cdot \frac{d\mu_L}{d\mathcal{E}} \right]}{\mu_\Gamma \cdot n_\Gamma + \mu_L \cdot n_L} < -1$$

now let $A = \mu_\Gamma \cdot \mathcal{E}$ and $B = \mu_L \cdot \mathcal{E}$ giving $\frac{d\mu_\Gamma}{d\mathcal{E}} = -\frac{A}{\mathcal{E}^2}$ and $\frac{d\mu_L}{d\mathcal{E}} = -\frac{B}{\mathcal{E}^2}$

$$\frac{(A - B) \cdot \frac{dn_\Gamma}{d\mathcal{E}} - \frac{1}{\mathcal{E}} \cdot (n_\Gamma \cdot A + n_L \cdot B)}{\frac{1}{\mathcal{E}} \cdot (n_\Gamma \cdot A + n_L \cdot B)} < -1$$

$$\frac{\mathcal{E} \cdot (A - B) \cdot \frac{dn_\Gamma}{d\mathcal{E}}}{n_\Gamma \cdot A + n_L \cdot B} - 1 < -1$$

$$\frac{\mathcal{E} \cdot (A - B) \cdot \frac{dn_\Gamma}{d\mathcal{E}}}{n_\Gamma \cdot A + n_L \cdot B} < 0$$

B is less than A since $\mu_L \ll \mu_\Gamma$. Thus $\frac{dn_\Gamma}{d\mathcal{E}}$ must be negative. That is the conductivity is negative only while electrons are being transferred from the lower lying Γ valley into the upper L valley.

Prob. 10.5

(a) Find the minimum electron concentration and time between current pulses.

$$n_o \cdot L = 10^{12} \tfrac{1}{cm^2} \rightarrow n_o = \frac{10^{12} \frac{1}{cm^2}}{L} = \frac{10^{12} \frac{1}{cm^2}}{5 \cdot 10^{-4} cm} = 2 \cdot 10^{15} \tfrac{1}{cm^3}$$

$$\tau_t = \frac{L}{v_s} = \frac{5 \cdot 10^{-4} cm}{10^7 \frac{cm}{s}} = 5 \cdot 10^{-11} s$$

(b) Estimate the d-c power dissipated per unit volume.

$$P = I \cdot V = (q \cdot n_0 \cdot v_d \cdot A) \cdot (\mathcal{E} \cdot L)$$

$$\frac{P}{A \cdot L} = q \cdot n_0 \cdot v_d \cdot \mathcal{E} = 1.6 \cdot 10^{-19} C \cdot 2 \cdot 10^{15} \tfrac{1}{cm^3} \cdot 2 \cdot 10^7 \tfrac{cm}{s} \cdot 3 \cdot 10^3 \tfrac{V \cdot s}{cm} = 2 \cdot 10^7 \tfrac{W}{cm}$$

Prob. 10.6

(a) Calculate the ratio of the density of states in the Γ and L conduction bands of GaAs.

$$N_C = 2 \cdot \left(\frac{2\pi \cdot kT \cdot m_n^*}{h^2} \right)^{\frac{3}{2}}$$

$$\text{so } \frac{N_L}{N_\Gamma} = \left(\frac{m_n^*(L)}{m_n^*(\Gamma)} \right)^{\frac{3}{2}} = \left(\frac{0.55}{0.67} \right)^{\frac{3}{2}} = 23.5$$

(b) Assuming a Boltzmann distribution, find the ratio of electron concentrations in these bands at 300K.

$$\frac{n_L}{n_\Gamma} = 23.5 \cdot e^{-\frac{0.30eV}{0.0259eV}} = 2.2 \cdot 10^{-4}$$

The upper L valley is essentially empty at equilibrium at 300K.

(c) Find the equivalent temperature of an electron in the Γ minima.

$$T = \frac{0.0259eV + 0.30eV}{8.62 \cdot 10^{-5} \frac{eV}{K}} = 3782K$$

Prob. 10.7

Explain why two separate BJTs cannot be connected to make a p-n-p-n switch.

The p-n-p-n switching action depends on injection of carriers across both base regions and collection into the base regions of the opposite transistor. For example, transistor action in the p-n-p feeds majority carrier holes to the base of the n-p-n. This cannot occur with separate transistors; so, the p-n-p-n switching effect does not occur.

Prob. 10.8

How does gate bias provide switching in an SCR?

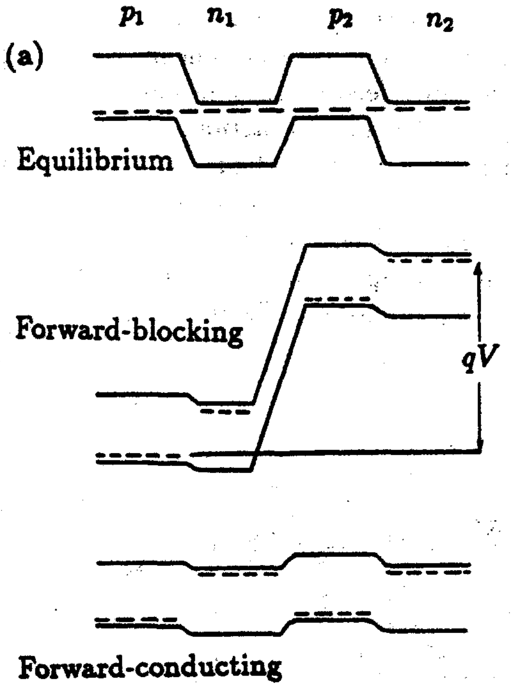
Switching in the SCR of figure 10-13 occurs when holes are supplied to p_2 at a sufficient rate. Although j_3 is forward biased wit i_G=0, transistor action does not begin until hole injection by i_G reaches the critical value for switching.

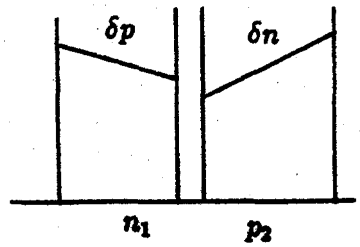
Prob. 10.9

(a) Sketch the equilibrium band diagram in the forward-blocking and forward-conducting states.

(b) Sketch the excess minority carrier distribution in regions n_1 and p_2 in the forward-conducting state.

(a)



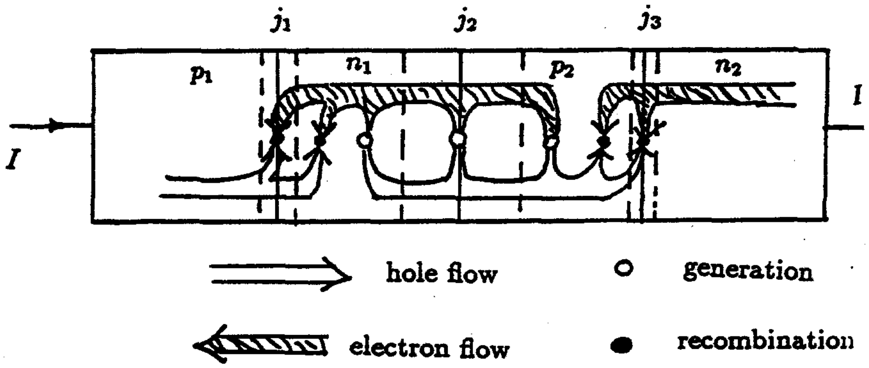


(b) Each equivalent transistor is in saturation. Thus the minority carrier distribution in each base resembles Fig. 7-14b.

Prob. 10.10

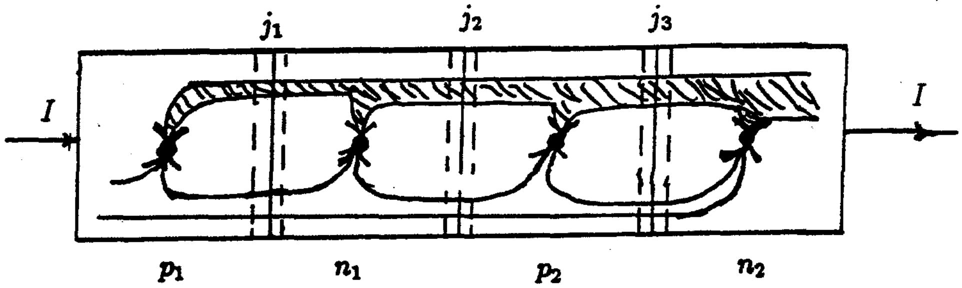
Draw diagrams for the forward-blocking and forward-conducting states of a p-n-p-n.

Forward Blocking State



In the simplified diagram above, we neglect minority carrier transport across each base region. Electrons generated thermally in and about j_2 recombine in n_1 and j_1 with injected holes. Similarly, generated holes feed recombination with injected electrons in p_2 and j_3. In absence of transistor action, current is limited to essentially the reverse saturation current of j_2.
In the figure below, we neglect generation compared with transport due to transistor action. Recombination takes place in n_1 and p_2, but many injected carriers are transported through the device by transistor action. More complete diagrams may be found in the book by *Gentry et al.*, p 72 and 76 (see chapter 10 reading list).

Forward Conducting State



Prob. 10.11

Include avalanche in j_2 in the coupled transistor model.

Referring to figure 10-11,

$$i_{C1} = \alpha_1 \cdot i \cdot M_p + I_{C01} \cdot M_p$$

$$i_{C2} = \alpha_2 \cdot i \cdot M_n + I_{C02} \cdot M_n$$

$$i = i_{C1} + i_{C2} = i \cdot (\alpha_1 \cdot M_p + \alpha_2 \cdot M_n) + I_{C01} \cdot M_p + I_{C02} \cdot M_n$$

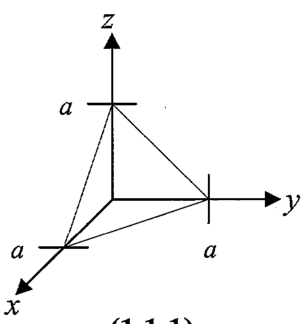
$$i = \frac{I_{C01} \cdot M_p + I_{C02} \cdot M_n}{1 - (\alpha_1 \cdot M_p + \alpha_2 \cdot M_n)}$$

The current becomes large as $\alpha_1 \cdot M_p + \alpha_2 \cdot M_n$ approaches unity.

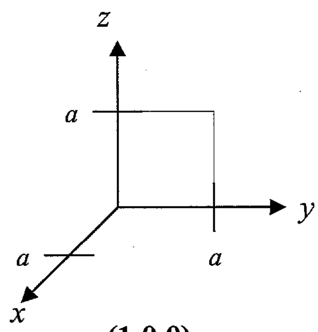
Chapter 1 Self Quiz

Question 1

(a) Label the following planes using the correct notation for a cubic lattice of unit cell edge length *a* (shown within the unit cell).



(1 1 1)



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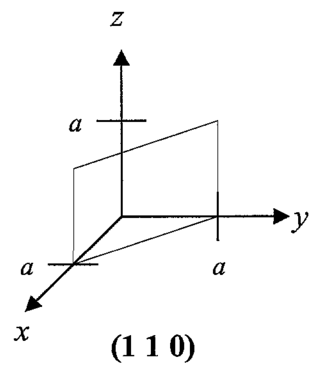


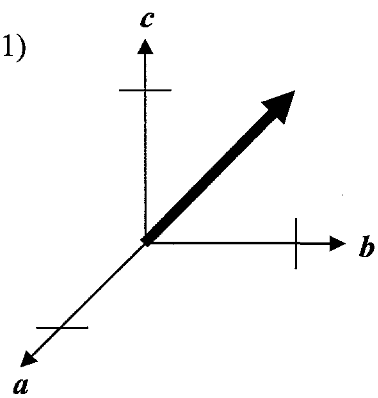
(1 1 0)

(b) Write out all of the equivalent ⟨100⟩ directions using the correct notation.

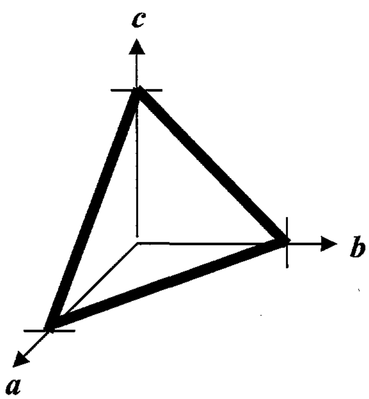
[1 0 0], [0 1 0], [0 0 1], $[\bar{1}\ 0\ 0]$, $[0\ \bar{1}\ 0]$, $[0\ 0\ \bar{1}]$

(c) On the two following sets of axes, (1) sketch the [011] direction and (2) a (111) plane (for a cubic system with primitive vectors ***a***, ***b*** and ***c***.).

(1)



(2)



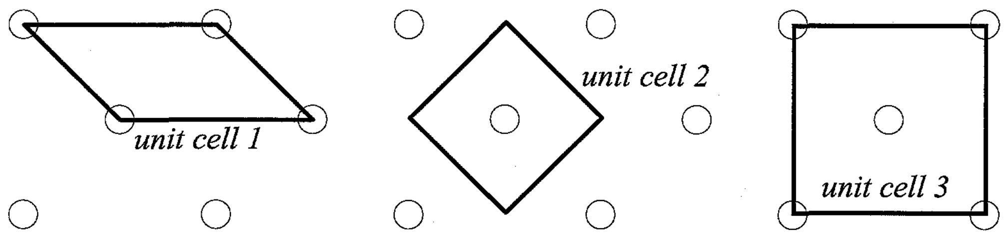
Question 2

(a) Which of the following three unit cells are *primitive cells* for the two-dimensional lattice. Which is the correct combination below.

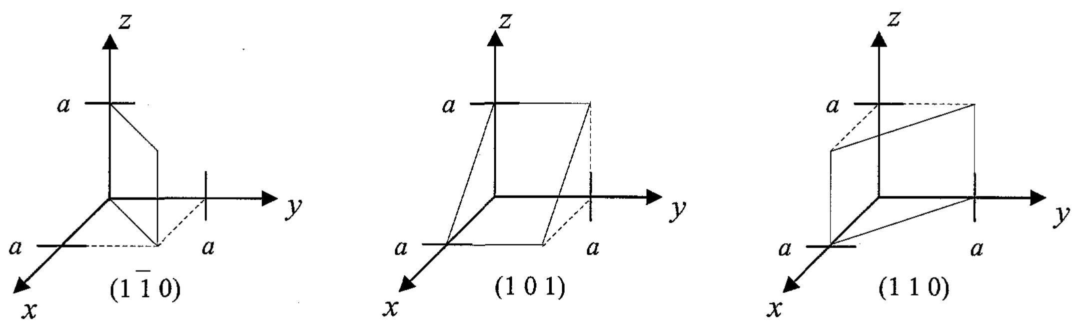
1 / 2 / 3 / 1 and 2 / 1 and 3 / 2 and 3 / 1,2 and 3

1 and 2 each contain a single atom and may be repeated to form the lattice

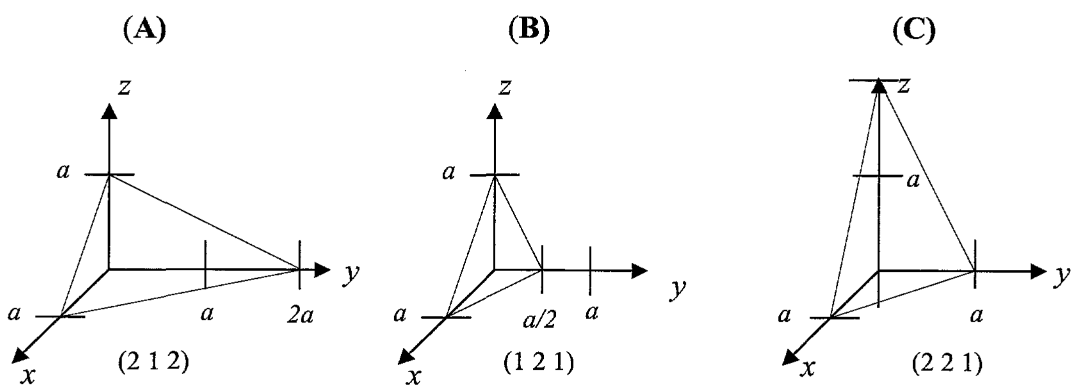
3 may be repeated to form the lattice but contains two atoms



(b) The following planes (shown within the first quadrant for $0 < x,y,z <$ a only, with the dotted lines for reference only) are all from what one set of *equivalent* planes? Use correct notation: ***{1 1 0}***

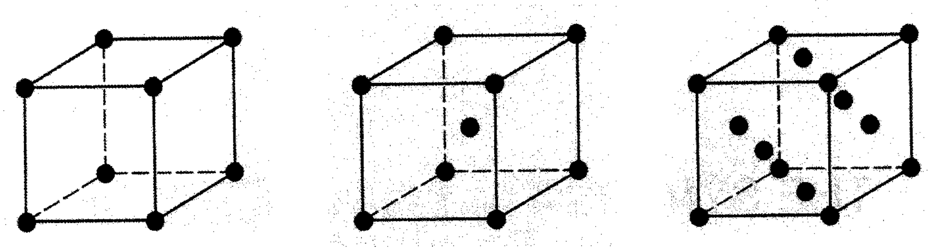


(c) Which of the following three planes (shown within the first quadrant only) is a (121) plane? Which the correct diagram? **B**



Question 3

(a) Diamond and zincblende crystal structures are both composed of a Bravais lattice with a two-atom basis. Which is the correct unit cell for this Bravais lattice. **C**



(A)	(B)	(C)
cubic	body-centered cubic	face-centered cubic

(b) Which statement below is true?

1. GaAs has a ***zincblende*** crystal structure.
2. Si has ***diamond*** crystal structure.

Question 4

Give some examples of 0-dimensional, 1-dimensional, and 2-dimensional defects in a semiconductor.

0D – point defect –impurity atom
1D – line defect – extra plane of atoms between two other planes (dislocation)
2D – area defect – polycrystalline grain boundary, extra plane of atoms not aligned with other planes (stacking fault)

Question 5

(a) What is the difference between a primitive cell and a unit cell? What is the utility of both concepts?

unit cell – volume which is representative of entire lattice and repeated throughout
primitive cell – smallest unit cell which can be repeated to form lattice
use the most convenient unit cell, whether primitive or not, to analyze the full volume often the primitive cell is not most convenient (ex: zincblende)

(b) What is the difference between a lattice and a crystal? How many different 1-D lattices can you have?

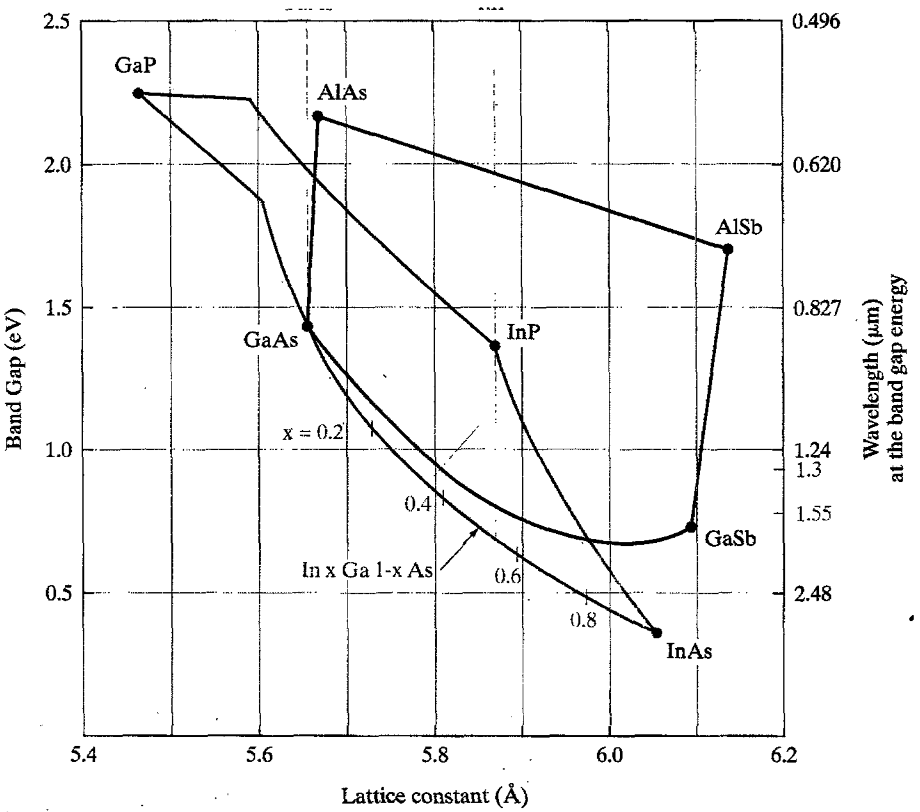
lattice – periodic arrangement of atoms in crystal, mathematical representation
crystal – solid characterized by periodic arrangement of atoms, physical thing
one 1-D lattice

Question 6

Consider growing InAs on the following crystal substrates: InP, AlAs, GaAs, and GaP. For which case would the *critical thickness* of the InAs layer be greatest? You may use Fig. 1-13 from your text.

GaP / GaAs / AlAs / ***InP***

closest lattice constant → least strain → greatest critical thickness



Chapter 2 Self-Quiz

Question 1

Decide whether *each* of the following one-dimensional functions defined between the limits x approaches negative infinity and x approaches positive infinity is an allowed quantum mechanical wavefunction.

a) $\psi(x) = C$ for $-|a| < x < |a|$; $\psi(x) = 0$ otherwise **not allowed**

discontinuous at a

b) $\psi(x) = C(e^{x/a} + e^{-x/a})$ **not allowed**

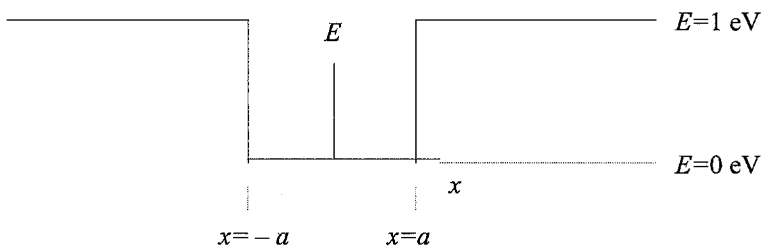
goes to infinity

c) $\psi(x) = C\exp(-x^2/|a|)$ **allowed**

where both C and a are nonzero and finite constants.

Question 2

Consider the finite potential well sketched below.

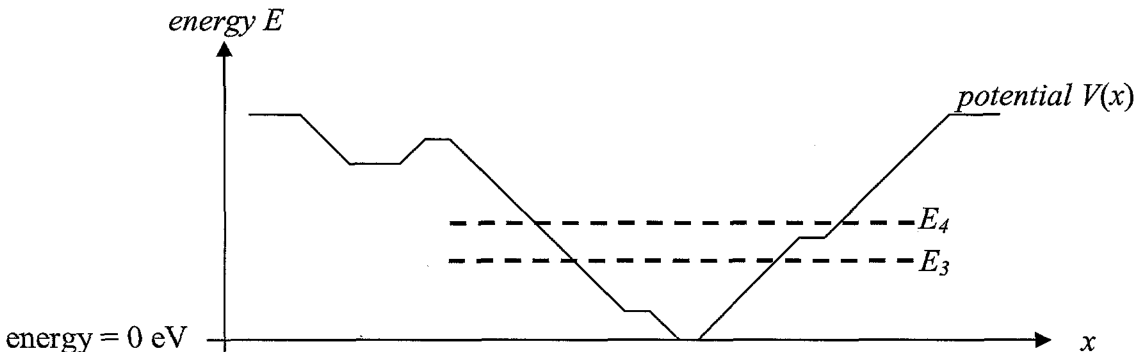


a) Can the measured value of a particle's energy in the well be 0 eV?
No – energy quantization, solution to Schrödinger wave equation

b) If the particle has an energy of $E < 1$ eV, can the measured value of the particle's position be $|x| > a$?
Yes – wavefunction for E < 1eV and |x|>a is not zero because particle may tunnel

Question 3

(a) For a particle in the following potential well of minimum potential energy equal to 0 eV, could the ground state eigenenergy E_1 of the particle be equal to zero? Circle one choice: *yes / no / not enough information provided*

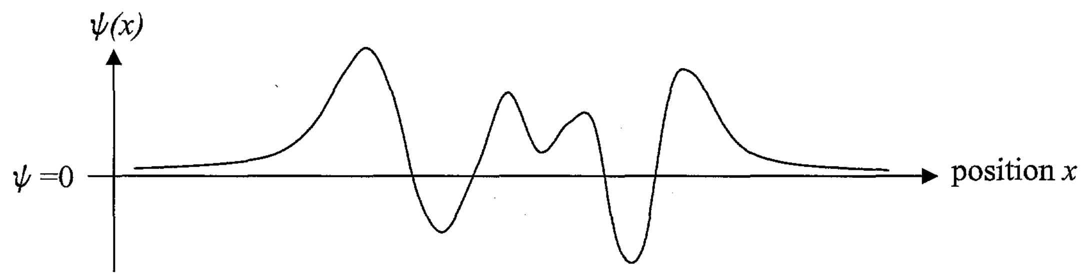


(b) Given the 3[rd] and 4[th] most energetic eigenstates of energies as shown above, is it possible under any circumstances that the expectation value of the particle's energy could be exactly $0.5(E_3 + E_4)$? (Do not assume the particle is in an energy eigenstate.) Circle one choice below.

yes / no / not enough information provided

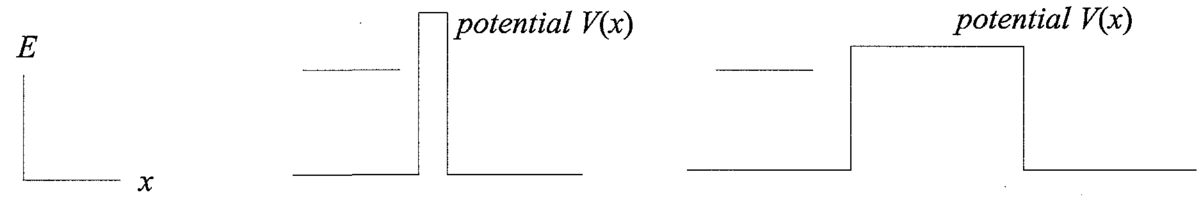
(c) Consider the following *continuous, smooth,* and *normalizable* wavefunction $\psi(x)$. Is this wavefuntion an allowed quantum mechanical wavefunction for a particle (currently) above the potential V(x) of Part (a)? (Circle one.)

yes / no / can't tell



Question 4
Consider quantum mechanical particles incident from the left having well-defined energy as indicated by the vertical positions of the arrows, in the two systems shown below. Will the probability of being reflected for the incident particle in System 1 compared to System 2 be greater, less, same or not enough information is provided?

not enough information provided



System 1
(narrow, tall barrier)

System 2
(wide, low barrier)

Question 5
Suppose five precise measurements were made on a particle in rapid succession, such that the time evolution of the particle wavefunction *between* measurements could be neglected, in the following order: (1) position, (2) momentum, (3) momentum, (4) position, (5) momentum. If the results of the first two measurements were x_o and p_o, respectively, what would be the results of the next three measurements (circle one each)?

Heisenburg uncertainty principle gives the following results:

measurement (3): momentum $\boldsymbol{p_o}$

measurement (4): position **unknown**

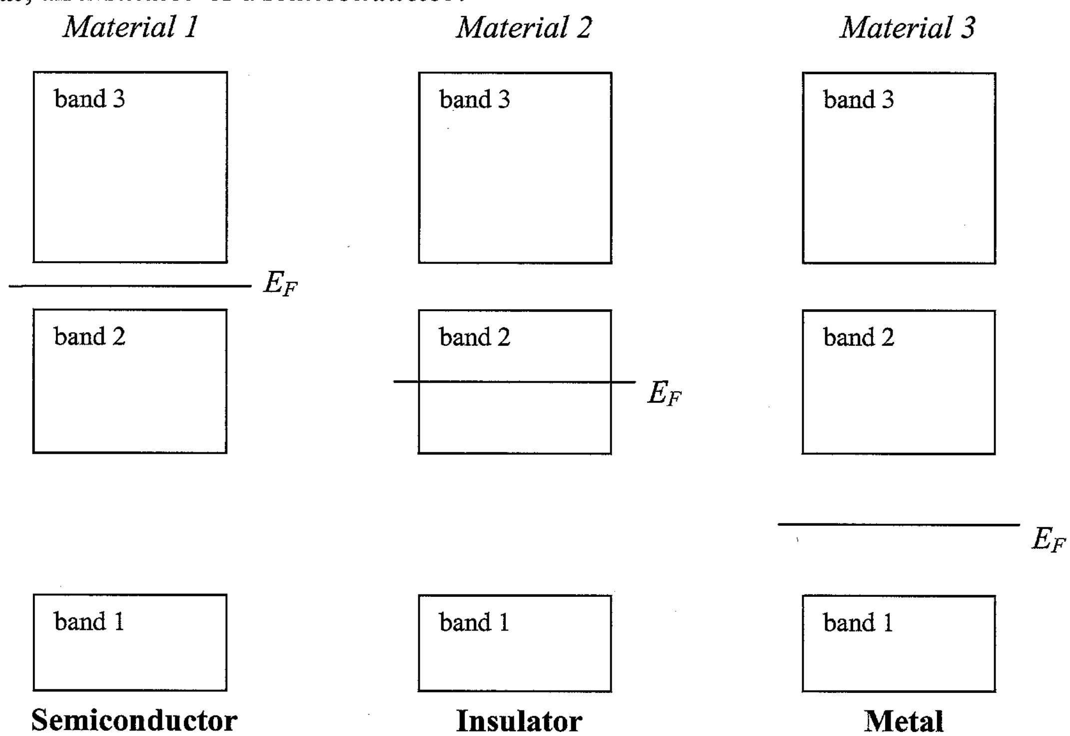
measurement (5): momentum **unknown**

Question 6
(a) If the photoelectric effect were governed by ***classical*** physics rather than quantum mechanics, what would be result of the following experiments?
By increasing the intensity of the incident radiation, what would happen to the energy of ejected electrons?
Energy is increased.
How about increasing the wavelength of the light?
Energy is unchanged.
(b) What would be the quantum mechanical results?
By increasing the intensity of the incident radiation, what would happen to the energy of ejected electrons?
Energy is unchanged.
How about increasing the wavelength of the light?
Energy is decreased since frequency decreases.

Chapter 3 Self- Quiz

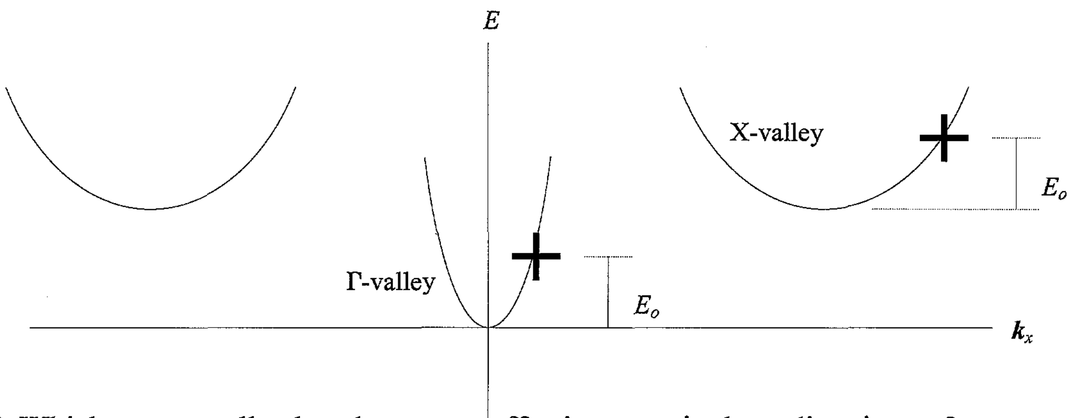
Question 1

(a) The following three diagrams show three different energy bands of some hypothetical crystalline materials (energy varies vertically). The only difference between the three materials is the assumed Fermi level energy E_F. Characterize each material as a ***metal***, an ***insulator*** or a ***semiconductor***.



(b) Assuming you can see through one and only one of the materials of part (a) above, which one would it most likely be? *Material 1 / Material 2 /* ***Material 3***

Question 2
Consider the following conduction band energy E vs. wavevector $\boldsymbol{k_x}$ dispersion relation.



(a) Which energy valley has the greater effective mass in the x-direction m_x?
X-valley

(b) Consider two electrons, one each located at the positions of the heavy crosses. Which has the greater velocity magnitude? **The one in the Γ-valley**

Question 3
These questions refer to the bandstructures of Si and GaAs shown in Fig. 3-10.

(a) Which material appears to have the *lowest* (conduction band) electron effective mass, Si or GaAs? **GaAs**

(b) Which of these would you expect to produce photons (light) more efficiently through electron hole-recombination? **GaAs**

(c) Consistent with your answer to Part (c) and making use of App. III, what would you expect the energy of the emitted photons to be? What would be their wavelength in μm? Would these be visible, infrared or ultraviolet?

E = 1.43eV $\qquad \lambda = \dfrac{1.24\mu\text{m}\cdot\text{eV}}{1.43\text{eV}} = 0.87\mu\text{m}$ $\qquad$ **Infrared**

(d) How many equivalent conduction band minima do we have for Si? GaAs?
6 equivalent minima for Si, 1 equivalent minima for GaAs

Question 4

Refer to Fig. 3-10 which shows the E vs. $\boldsymbol{k}$ dispersion relations for gallium arsenide (GaAs) and for silicon (Si) along the [111] and [100] directions, showing both valence and conduction bands.

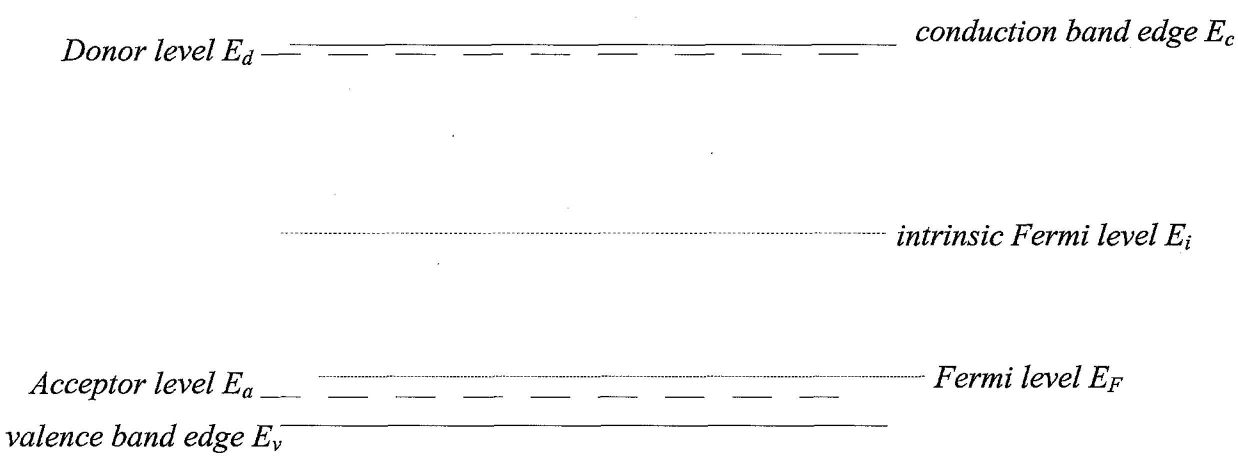
(a) Neglecting differences in electron scattering rates in the two materials, would you expect Si or GaAs to have the greatest electron mobility μ_n? **GaAs**

(b) If a constant force were applied in the [100] direction for a short period of time on an electron initially located at the conduction band minimum of each semiconductor and if scattering were neglected, would the magnitude of change in **k** in Si be greater, equal to or smaller than the magnitude of the change in **k** in GaAs for the same force?

Equal To since $\frac{d}{dt}\hbar \mathrm{k} = \mathrm{F}_{\text{external}}$

Question 5

(a) The equilibrium band diagram for a doped direct gap semiconductor is shown below. Is it n-type, p-type or not enough information provided ? **p-type**



(b) Based on the band diagram above (E_i is exactly in the middle of the gap), would you expect that the conduction band density-of-states effective mass is greater than, equal to or smaller than the valence band effective mass? **equal**

(c) What if any of the following conditions by themselves could lead to the above band diagram? Circle each correct answer.

(a) very high temperature

(b) very high acceptor doping

(c) very low acceptor doping

Question 6

A hypothetical semiconductor has an intrinsic carrier concentration of $1.0 \times 10^{10}/\text{cm}^3$ at 300K, it has conduction and valence band effective densities of states, N_c and N_v, both equal to $10^{19}/\text{cm}^3$.

(a) What is the bandgap, E_g?

$$n_i = \sqrt{N_c N_v} \cdot e^{\frac{-E_g}{2kT}}$$

$$10^{10} \frac{1}{\text{cm}^3} = \sqrt{10^{19} \frac{1}{\text{cm}^3} \cdot 10^{19} \frac{1}{\text{cm}^3}} \cdot e^{\frac{-E_g}{2 \cdot 0.026\text{eV}}}$$

$$E_g = 0.052\text{eV} \cdot \ln \frac{10^{10} \frac{1}{\text{cm}^3}}{10^{19} \frac{1}{\text{cm}^3}} = 1.08\text{eV}$$

(b) If the semiconductor is doped with $N_d = 1 \times 10^{16}$ donors/cm^3, what are the equilibrium electron and hole concentrations at 300K?

$$n_o = 10^{16} \frac{1}{\text{cm}^3} \qquad p_o = \frac{n_i^2}{n_o} = \frac{10^{20} \frac{1}{\text{cm}^6}}{10^{16} \frac{1}{\text{cm}^3}} = 10^4 \frac{1}{\text{cm}^3}$$

(c) If the same piece of semiconductor, already having $N_d = 1 \times 10^{16}$ donors/cm^3, is also doped with $N_a = 2 \times 10^{16}$ acceptors/cm^3, what are the new equilibrium electron and hole concentrations at 300K?

$$p_o = 10^{16} \frac{1}{\text{cm}^3} \qquad n_o = \frac{n_i^2}{p_o} = \frac{10^{20} \frac{1}{\text{cm}^6}}{10^{16} \frac{1}{\text{cm}^3}} = 10^4 \frac{1}{\text{cm}^3}$$

(d) Consistent with your answer to Part (c), what is the Fermi level position with respect to the intrinsic Fermi level, $E_F - E_i$?

$$E_F - E_i = kT \cdot \ln\left(\frac{p_o}{n_i}\right) = 0.026\text{eV} \cdot \ln\left(\frac{10^{16} \frac{1}{\text{cm}^3}}{10^{10} \frac{1}{\text{cm}^3}}\right) = 0.36\text{eV}$$

Question 7

What is the difference between density of states and *effective* density of states, and why is the latter such a useful concept?

Density of states gives available states as a function of energy.
Effective density of states maps to the values at the band edges making calculations of carrier concentrations easy.

Question 8

(a) Does mobility have any meaning at very high field? Why?

No, drift velocity saturates and is no longer linearly dependent on electric field.

(b) How do you measure mobility and carrier concentration?

Hall effect and resistivity measurements.

Chapter 4 Self-Quiz

Question 1
Consider a p-type semiconductor that has a bandgap of 1.0 eV, a minority electron lifetime of 0.1 μs, and is uniformly illuminated by light having photon energy of 2.0 eV.

(a) What rate of uniform excess carrier generation is required to generate a uniform electron concentration of 10^{10}/cm^3?

$$g_{op} \cdot \tau_n = \delta n = \delta p$$

$$g_{op} \cdot 10^{-7} s = 10^{10} \frac{1}{cm^3}$$

$$g_{op} = 10^{17} \frac{1}{eV \cdot cm^3}$$

(b) How much optical power per cm^3 must be absorbed in order to create the excess carrier population of part (a)? (You may leave you answer in units of eV/s-cm^3.)

$$p = h\nu \cdot g_{op} = 2.0 eV \cdot 10^{17} \frac{1}{cm^3 \cdot s} = 2.0 \cdot 10^{17} \frac{eV}{cm^3 \cdot s}$$

(c) If the carriers recombine via photon emission, approximately how much optical power per cm^3 will be generated? (You may leave you answer in units of eV/s-cm^3.)

$$p = E_g \cdot g_{op} = 1.0 eV \cdot 10^{17} \frac{1}{cm^3 \cdot s} = 1.0 \cdot 10^{17} \frac{eV}{cm^3 \cdot s}$$

This is less because carriers go to the band edge before recombining.

Question 2

(a) What do we mean by "deep" versus "shallow" traps? Which are more harmful for semiconductor devices and why? What is an example of a deep trap in Si?

Shallow traps are near the band edge.
Deep traps are near the midgap.
Deep traps are more harmful because they increase the chances of leakage.
Gold (Au) forms deep traps.

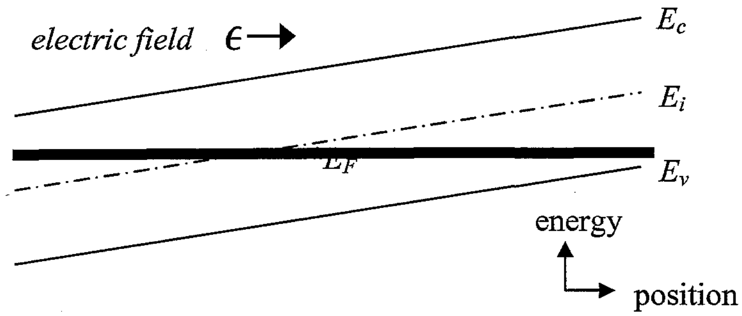
(b) Are absorption lengths of slightly-above-bandgap photons longer in Si or GaAs? Why?

Si; indirect band gap

(c) Do absorption coefficients of photons increase or decrease with photon energy? Why?

The absorption coefficient is very low below the band gap energy, increases abruptly at Eg, and continues to increase slowly at higher energies as more possible transitions become available with higher density of state.

Question 3
Consider the following equilibrium band diagram for a portion of a semiconductor sample with a built-in electric field ϵ:

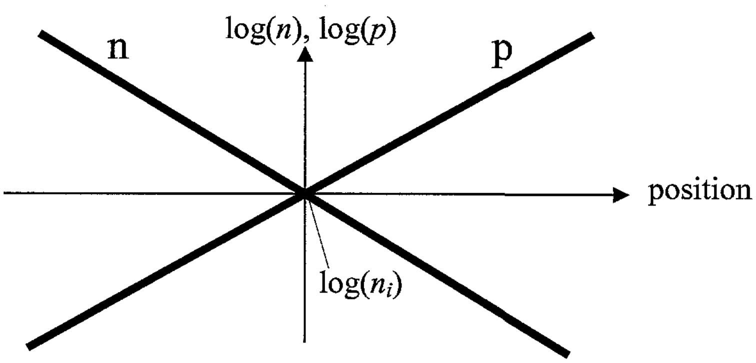


(a) Sketch the Fermi level as a function of position *through* the indicated point, E_F, across the width of the band diagram above. **E_F is flat in equilibrium.**

(b) On the band diagram, sketch the direction of the electric field. Is the field constant or position dependent? **Constant.**

(c) On the following graph, sketch and label both the electron and hole concentrations as a function of position across the full width of the sample. Note that the carrier concentration scale is logarithmic such that exponential variations in the carrier concentration with position appear as straight lines. Note also that the horizontal axis corresponds to the intrinsic carrier concentration of n_i.

$$n = n_i \cdot e^{\frac{E_F - E_i}{kT}} \qquad p = n_i \cdot e^{\frac{E_i - E_F}{kT}}$$



Question 4

(a) Indicate the directions of the hole and electron **flux** densities ϕ due to diffusion and drift under these *equilibrium* conditions corresponding to the previous Question 3. Circulate the appropriate arrow in each case.

$\phi_{p,diffusion}$ $\leftarrow$

$\phi_{p,drift}$ $\rightarrow$

$\phi_{n,diffusion}$ $\rightarrow$

$\phi_{n,drift}$ $\leftarrow$

(b) Indicate the directions of the hole and electron **current** densities j due to diffusion and drift under these *equilibrium* conditions.

$j_{p,diffusion}$ $\leftarrow$

$j_{p,drift}$ $\rightarrow$

$j_{n,diffusion}$ $\leftarrow$

$j_{n,drift}$ $\rightarrow$

Question 5

(a) What are the relevant equations that must be solved in general for a semiconductor device problem?

Drift/Diffusion, Continuity, Poisson

(b) In general how many components of conduction current can you have in a semiconductor device? What are they?

Four – electron drift, hole drift, electron diffusion, and hole diffusion

Question 6

(a) Consider a region in a semiconductor with an electric field directed toward the right (→) and carrier concentrations increasing toward the left (←). Indicate the directions of particle fluxes ϕ (circle one for each) and charge currents j due to drift and diffusion within that region (circle one for each):

ϕ_p(drift) →

ϕ_p(diffusion) →

ϕ_n(drift) ←

ϕ_n(diffusion) →

(b) Based on your answers to part (a), indicate the directions of the charge currents j due to drift and diffusion within that region (again, circle one for each):

j_p(drift) →

j_p(diffusion) →

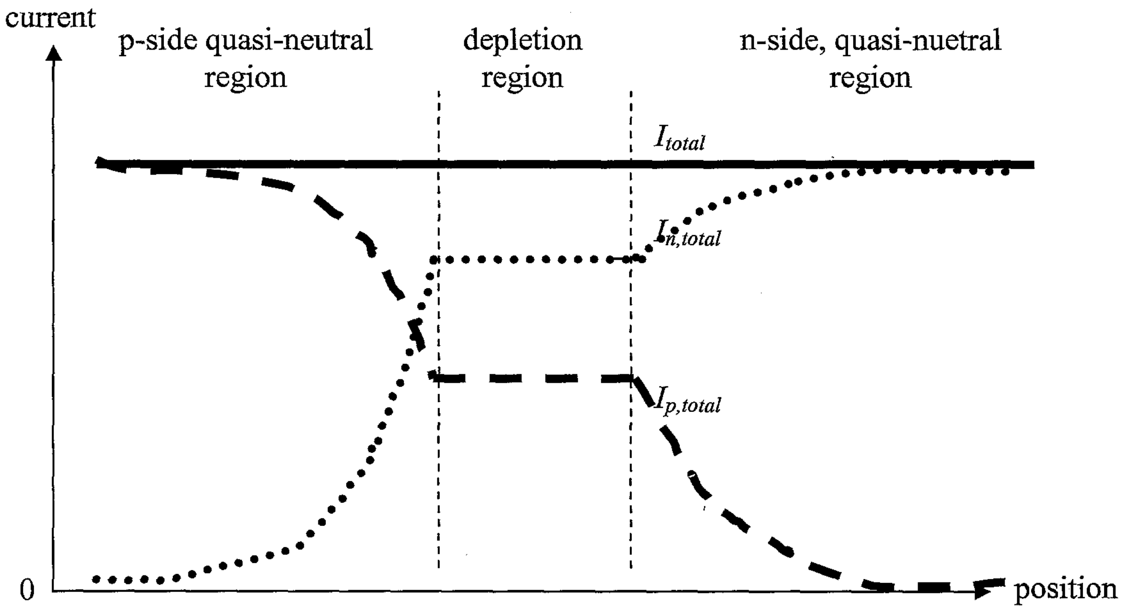
j_n(drift) →

j_n(diffusion) ←

Chapter 5 Self-Quiz

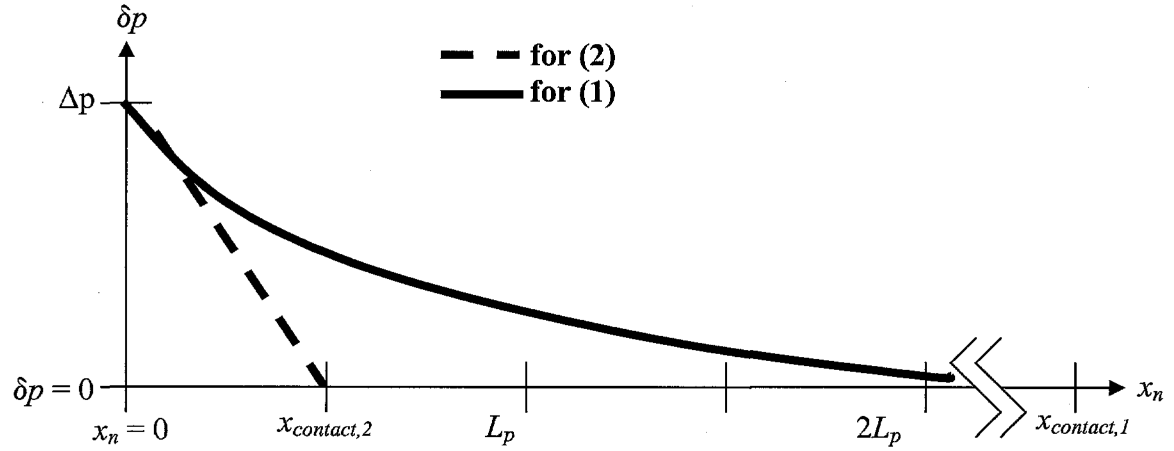
Question 1

Consider a forward biased *ideal* (abrupt junction, no recombination or generation in the depletion region) long p-n junction diode under forward bias. On the following graph, sketch and label the total current I_{total}, the total electron current $I_{n,total}$ and the total hole current $I_{p,total}$ as a function of position throughout the entire device. The value of each has been given at the n-side edge of the depletion region for reference. (Hint: the excess carrier concentrations are essentially pinned to zero at the contacts.)

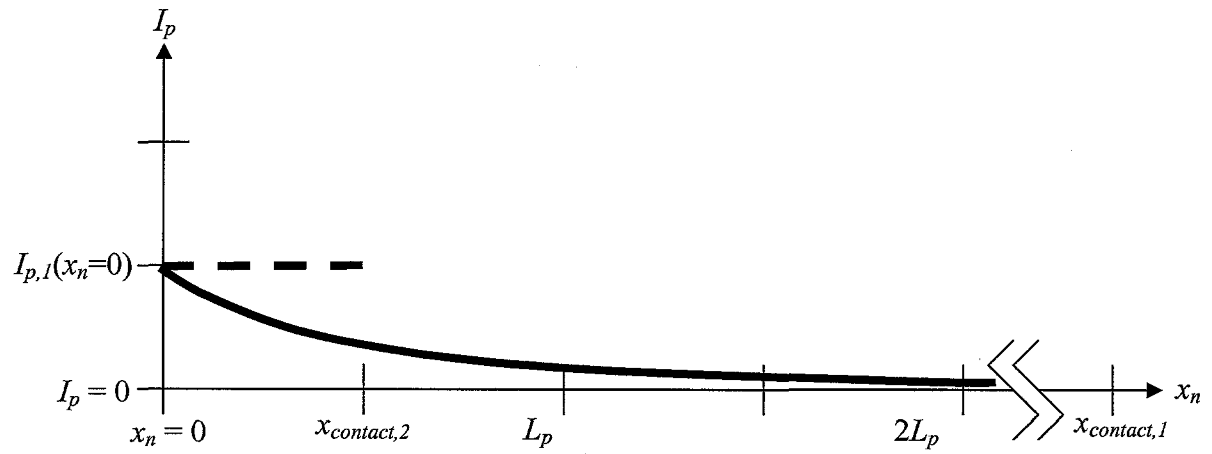


Question 2

(a) Consider (1) a long p-n junction diode with the ohmic contact on the n-side, $x_{contact,1} >> L_p$, and (2) a short p-n junction diode with the ohmic contact on the n-side well within a diffusion length of the depletion region, $x_{contact,2} < L_p$. Given the indicated excess hole (equals electron) concentration at the n-side edge of the depletion region Δp for both diodes, sketch the excess hole concentrations as a function of position $\delta p(x_n)$ within the region for the two cases on the following graph and label them (1) and (2) accordingly.



(b) Consistent with your answers to Part (a), sketch the hole (diffusion) currents within the region for the same two diodes on the following graph, and label them (1) and (2) again. The value of the hole current at edge of the depletion region for the long diode $I_{p,1}(x_n=0)$ is provided for reference.



Question 3

Consider two Si p-n junction diodes, one long and one short (contacts within a diffusion length of the depletion region) but otherwise identical. Under identical forward bias voltage, which diode would have greater current flow?

Short, because it has a steeper concentration profile.

Question 4

If the depletion capacitance of a p-n junction diode is $C_{d,o}$ at equilibrium and the contact potential is 0.5 V, how much reverse bias voltage would have to be applied to reduce the depletion capacitance to $0.5C_{d,o}$?

$$C_j \propto \left(\frac{1}{V_0 - V}\right)^{1/2}$$

$$0.5 \cdot \left(\frac{1}{V_0}\right)^{1/2} = 1 \cdot \left(\frac{1}{V_0 - V}\right)^{1/2}$$

$$V = 3V_0 = 1.5V$$

Question 5

(a) What is the difference between depletion and diffusion capacitance in a diode? Which one dominates in forward bias and why? Reverse bias?

Depletion capacitance is due to stored depletion charge.
Diffusion capacitance is due to stored mobile carriers.
Diffusion capacitance dominates in forward bias.
Depletion capacitance dominates in reverse bias.

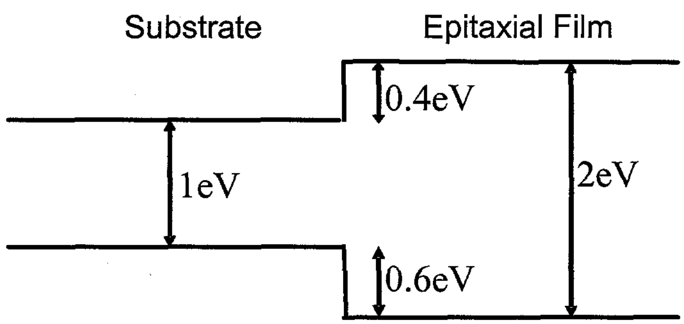
(b) Why is it meaningful to define small signal capacitance and conductance in semiconductor devices such as diodes? How are they defined?

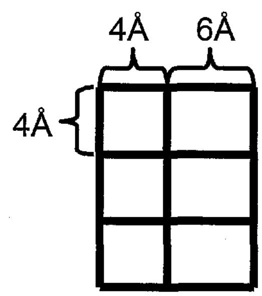
Alternating current signals are typically smaller than direct current bias.

$$C = \frac{dQ}{dV} \qquad G = \frac{dI}{dV}$$

Question 6

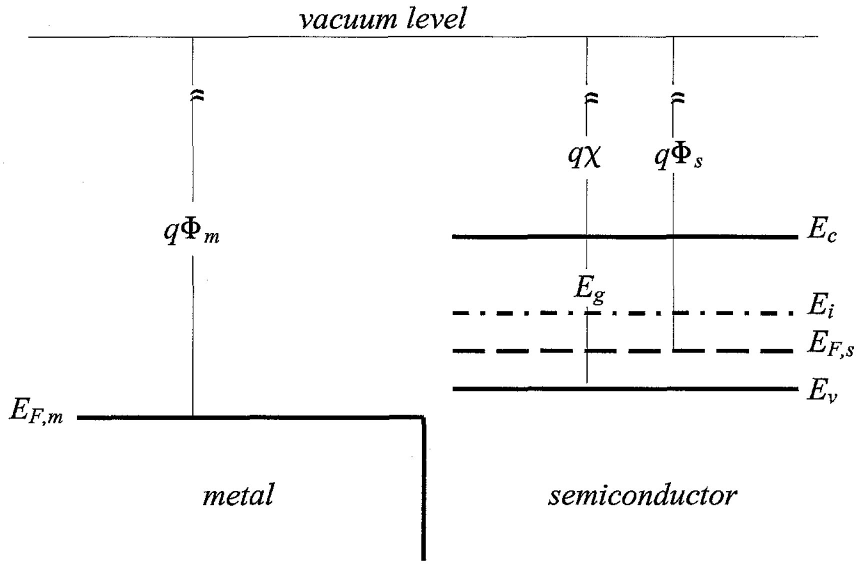
We grow a pseudomorphic heterostructure consisting of an epitaxial film with lattice constant of 6 Å and bandgap of 2 eV on a thick substrate with lattice constant of 4 Å and bandgap of 1 eV. Both the substrate and epitaxial layer have a cubic crystal structure in the unstrained state. If 60% of the band edge discontinuity is in the conduction band, sketch a simplified band diagram of this heterostructure. Also, qualitatively show a 2-D view of the crystal structure in relation to the band diagram.

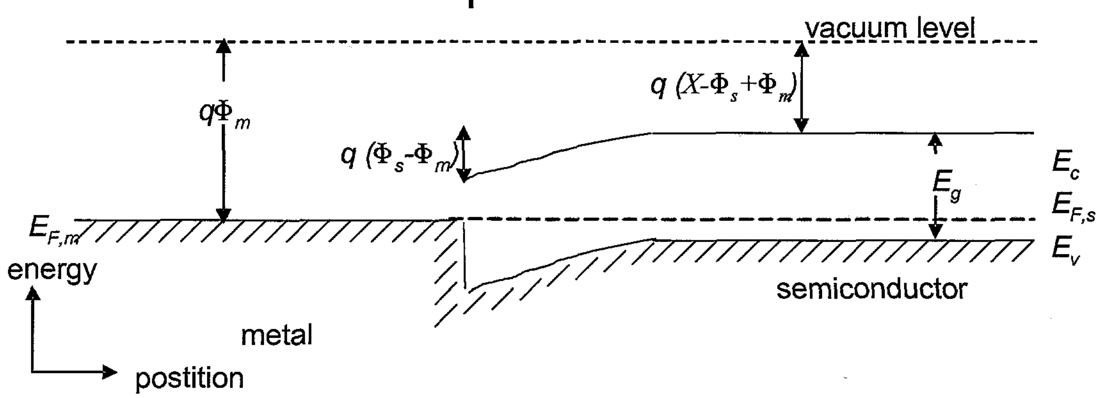




Question 7

(a) In the space below, sketch the equilibrium band diagram resulting from bringing together the illustrated metal and lightly doped semiconductor indicating the Fermi level; the conduction band and valence band offsets from *the* Fermi level at the metal-semiconductor interface in terms of $q\Phi_m$, $q\Phi_s$, $q\chi$ and/or E_g; any band-bending in the semiconductor in terms of $q\Phi_m$, $q\Phi_s$, $q\chi$ and/or E_g; and (qualitatively) any charge depletion or accumulation layer. Assume no interface traps.





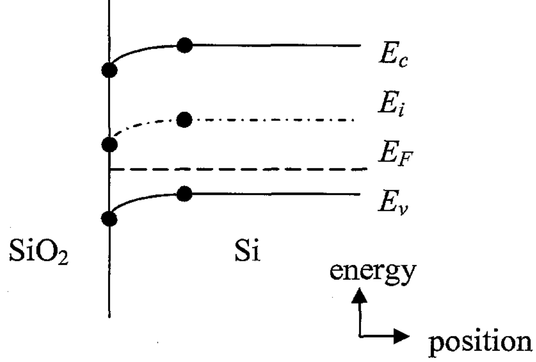
(b) Is this a Schottky contact or an ohmic contact? **Ohmic contact**

Chapter 6 Self-Quiz

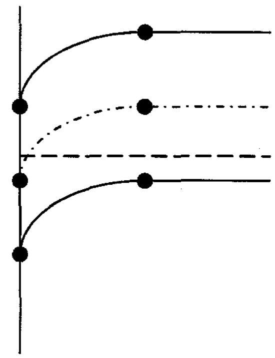
Question 1

Label the following MOS capacitor band diagrams as corresponding to accumulation, weak inversion, depletion, strong inversion, flatband or threshold. Use each possibility only once.

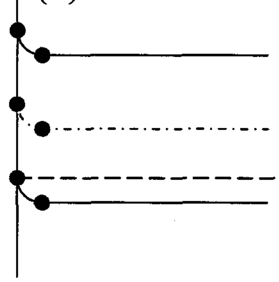
(A) **Depletion**



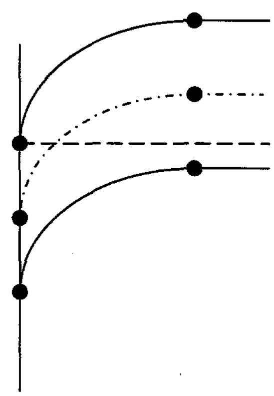
(B) **Weak Inversion**



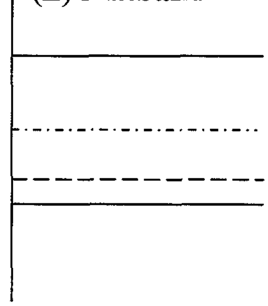
(C) **Accumulation**



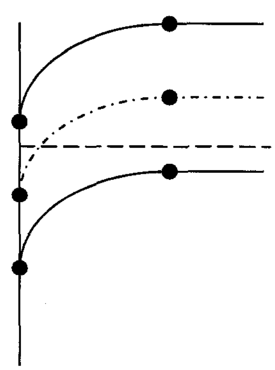
(D) **Strong Inversion**



(E) **Flatband**



(F) **Threshold**



Question 2

(a) What is the main distinction between an active device and a passive device?

An active device gives power gain.

(b) If a device has power gain, where is the higher energy of the a.c. signal at the output coming from?

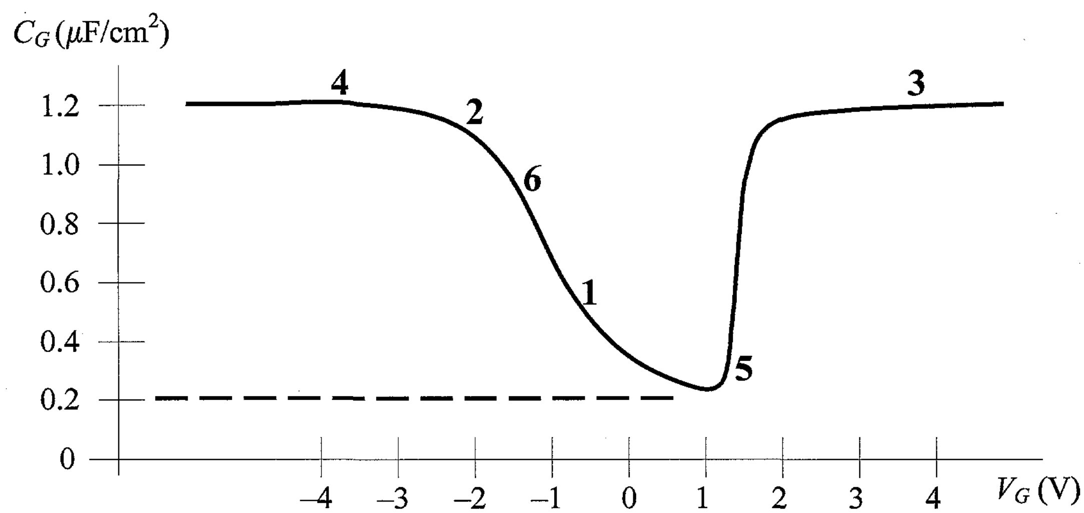
The power comes from the direct current power supply.

(c) What is the distinction between a current controlled versus voltage controlled three-terminal active device? Which is preferable?

A voltage controlled device has much higher input impedence than a current controlled device and is preferable because it consumes less power.

Question 3

For Parts (a) through (c) below, consider the following low-frequency gate capacitance (per unit area) vs. gate voltage characteristic for a metal gate n-channel MOSFET.



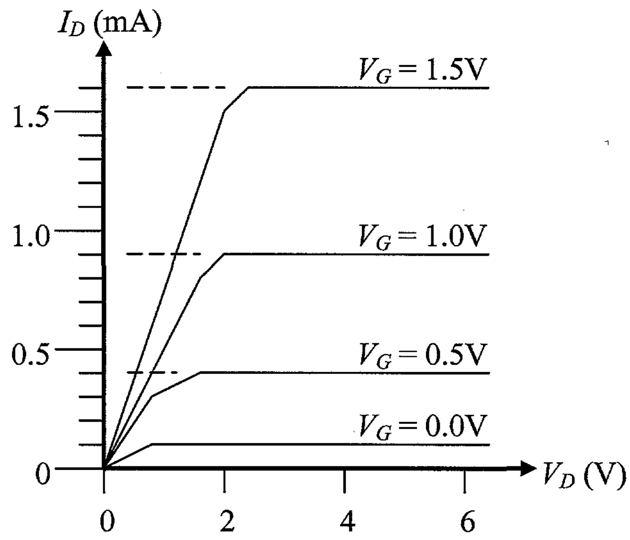
(a) On the above figure label the *approximate* regions or points of

- weak inversion (1)
- flatband (2)
- strong inversion (3)
- accumulation (4)
- threshold (5)
- and depletion (6)

(b) What is the oxide capacitance per unit area? **1.2μF/cm^2**

(c) On the curve above, sketch the high frequency curves for this MOSFET with grounded source/drain. **Same as above.**

Question 4
Consider the following MOSFET characteristic.



(a) Is this an n-channel or p-channel device? **n-channel**

(b) Does this appear to be a long-channel or short-channel device?
Long, because I_D is flat and there is a quadratic dependence $(V_G\text{-}V_T)^2$.

(c) What is the apparent threshold voltage V_T? **-0.5V**

(d) Is this a depletion mode or enhancement mode MOSFET? **depletion mode, $V_T<0$**

Question 5
Is the subthreshold slope of a MOSFET decreased (improved) by:
(a) Increasing or decreasing the oxide thickness? **decreasing**
(b) Increasing or decreasing the substrate doping? **decreasing**
For each of these two methods, name one thing/factor/effect that limits the extent to which each of these methods can be used to reduce subthreshold swing.
(a) oxide leakage and breakdown
(b) punchthrough between source and drain

Question 6
Would increasing the device temperature increase, decrease or leave unchanged (circle the correct answers):
(a) the reverse saturation current of a pn diode ?
increase / decrease / unchanged
(b) the subthreshold source-to-drain leakage current of MOSFET?
increase / decrease / unchanged

Question 7

Assuming no interface charge due to defects and/or traps, would decreasing the oxide thickness/increasing the oxide capacitance of an n-channel MOSFET *increase*, *decrease* or leave essentially *unchanged* the following parameters (circle the correct answers):

(a) the flat band voltage, V_{FB}? $V_{FB} = \phi_{ms} - \frac{Q_{ox}}{C_{ox}}$ is not C_{ox} dependent since $Q_{ox} = 0$

increase / decrease / unchanged

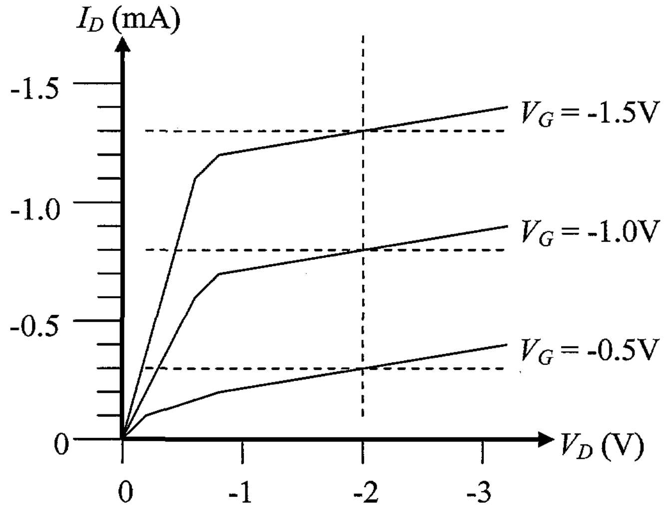
(b) the threshold voltage, V_T? $V_T = V_{FB} + 2\phi_F - \frac{Q_D}{C_{ox}}$ is C_{ox} dependent

increase / **decrease** / unchanged

(c) the subthreshold slope? $S = 60\frac{\text{mV}}{\text{decade}} \cdot \left(1 + \frac{C_D + C_{it}}{C_{ox}}\right)$

increase / **decrease** / unchanged

Question 8
Consider the following MOSFET characteristic.



(a) What is the transconductance g_m at V_D = -1.0V (in units of mhos)?

$$g_m = \frac{\partial I_D}{\partial V_G} = \frac{-0.5\text{mA}}{-0.5\text{V}} = 1\text{m}\mho$$

(b) What is the apparent threshold voltage at V_D = -1.0V?
V_T = -0.2V

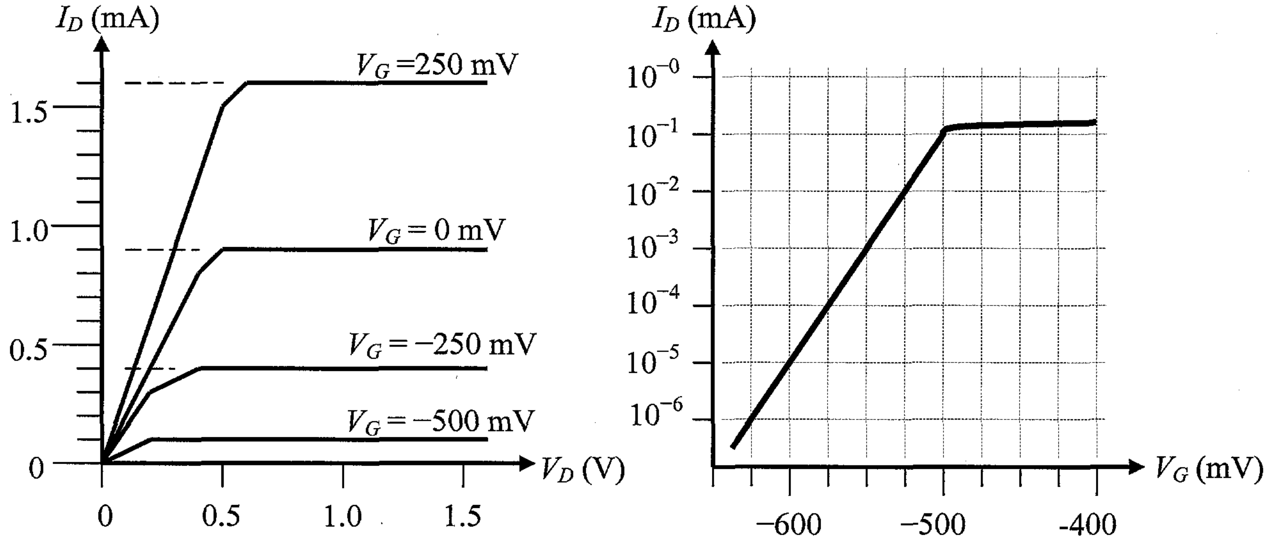
(c) Does this MOSFET appear to be a long-channel or short-channel device?
short channel since $I_D \propto (V_G\text{-}V_T)$

(d) Is this an n-channel or p-channel device?
p-channel

(e) Is this a depletion mode or enhancement mode MOSFET?
enhancement mode because V_T < 0 for pMOSFET

Question 9

A senior in Electrical and Computer Engineering in a device fabrication course presented the following characteristics as those of an n-channel MOSFET that he had fabricated, and characterized at a temperature of 300K.



(a) If we were to believe this student, would this be "*normally on/depletion mode*" or a "*normally off/enhancement mode*" MOSFET? Circle one choice below.

normally off / **normally on**

(b) If we were to believe this student, would this MOSFET appear to be a *long-channel* or *short-channel* device? Circle one.

long channel / short channel

(c) If we were to believe this student, what would be the *subthreshold slope/swing, S*, of this MOSFET?

$$S = \frac{100\text{mV}}{4 \text{ decades}} = 25 \frac{\text{mV}}{\text{decade}}$$

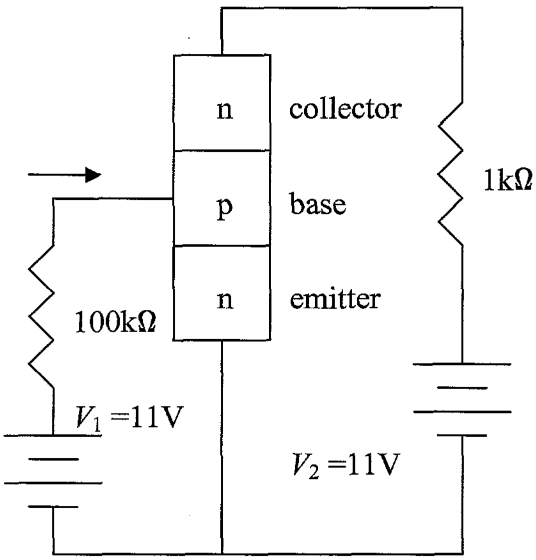
(d) This student, however, was subsequently expelled from the university for falsifying this data leaving him with nothing to show for his years at school but huge college loan debts. What *clearly* physically unrealistic aspect of these MOSFET characteristics (besides less than smooth curves) should have drawn suspicion?

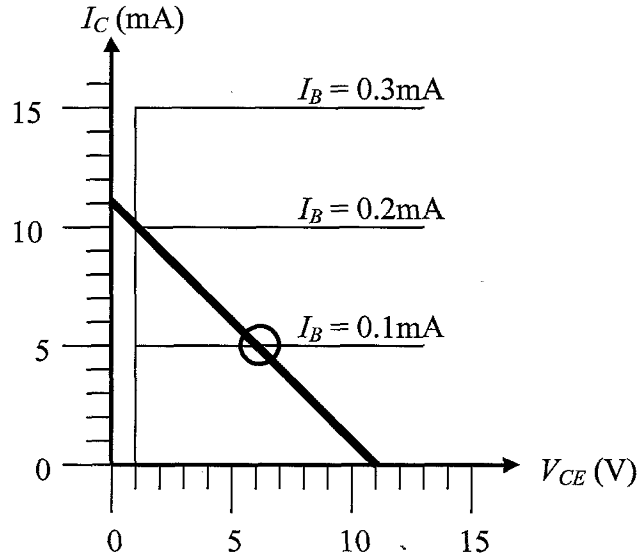
S cannot be less than 60mV/decade.

Chapter 7 Self-Quiz

Question 1

Consider the following bipolar junction transistor (BJT) circuit and somewhat idealized transistor characteristics (where, in particular, the voltage drop across the forward biased base-emitter junction is assumed to be constant and equal to 1V for simplicity.





(a) What is the (common emitter) gain β?

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{5\text{mA}}{0.1\text{mA}} = 50$$

(b) Draw the load line on the transistor characteristics.

V_{CE}=0→ I_C =11mA $\quad$ I_C=0mA→V_{CE}=11V

(c) What is the collector-emitter voltage drop in this circuit within half a volt?

$$I_B = \frac{(11\text{-}1)\text{V}}{100\text{k}\Omega} = 0.1\text{mA} \rightarrow V_{CE} = 6\text{V}$$

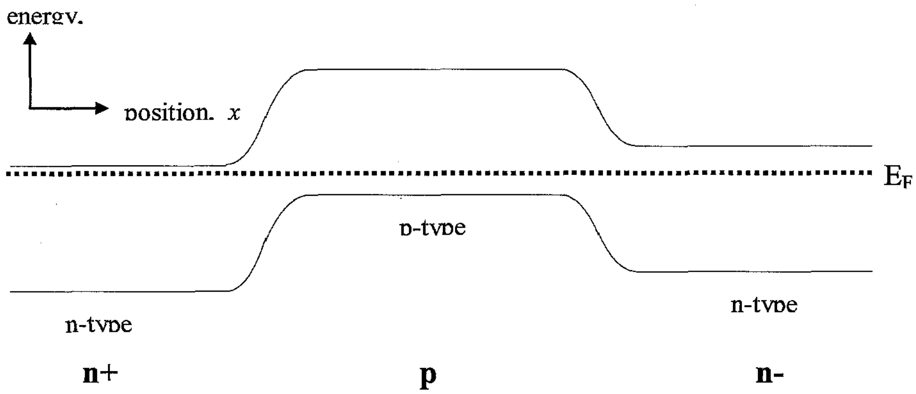
(d) If voltage V_1 could be changed, what value of V_1 would drive the BJT in this circuit to the edge of saturation?

I_B=0.2mA at onset of saturation.

$$\frac{V_1\text{-1V}}{100\text{k}\Omega} = 0.2\text{mA} \rightarrow V_1 = 21\text{V}$$

Question 2

The following is a band diagram within an n-p-n bipolar junction transistor (BJT) at equilibrium. Sketch in the Fermi level as a function of position. Qualitative accuracy is sufficient.



Question 3

Would decreasing the base width of a BJT *increase* or *decrease* or leave *unchanged* the following assuming that the device remained unchanged otherwise?

a) emitter injection efficiency γ? **increase** / unchanged / decrease

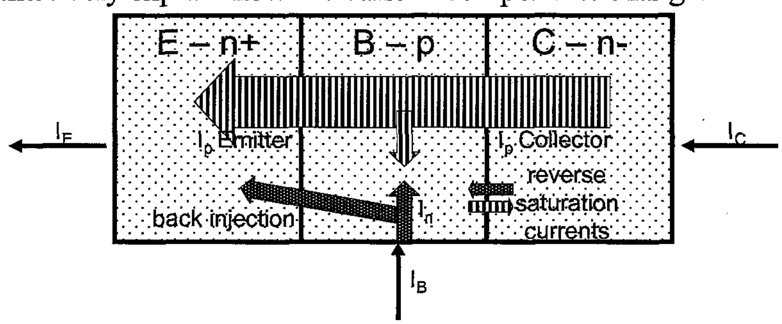
b) base transport factor B? **increase** / unchanged / decrease

c) common emitter gain β? **increase** / unchanged / decrease

d) *magnitude* of the Early voltage V_A? increase / unchanged / **decrease**

Question 4

Sketch the cross section of an n-p-n BJT and point out the dominant current components on it showing the correct directions of the various current vectors. If we increase the base doping, qualitatively explain how the various components change.



If base doping increases, the injected electrons lost to recombination and holes supplied by the base contact for recombination increase, the electrons reaching the reverse-biased collector junction decrease because of lower electron concentration in the base, the thermally generated electrons and holes decrease slightly, and the holes injected across the emitter junction is unchanged.

Question 5
Would decreasing the base doping of the BJT *increase*, *decrease* or leave essentially *unchanged* (circle the correct answers):

(a) emitter injection efficiency γ?

increase / decrease / unchanged

(b) base transport factor B?

increase / decrease / unchanged

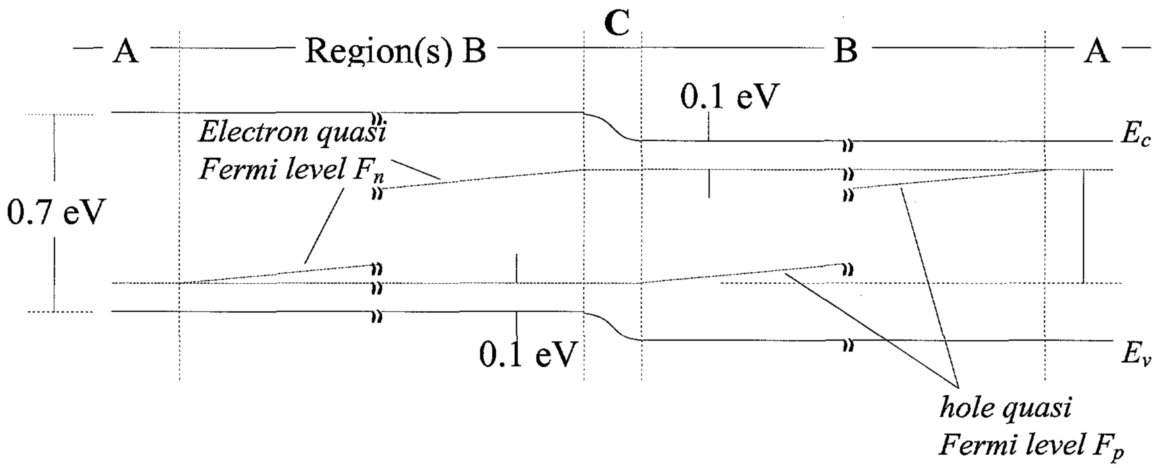
(c) magnitude of the Early voltage V_A?

increase / **decrease** / unchanged

Chapter 8 Self- Quiz

Question 1

Consider the following band diagram of a simple light-emitting diode, subjected to a forward bias of 1.4V. Assume essentially all recombination is direct and results in light emission. The forward bias current consists of holes injected from a contact to the left and electrons injected from a contact to the right.



(a) In which region would you expect the optical recombination rate to be the greatest? Circle one.

Region(s) A / Region(s) B / **Region C**

(b) What is the approximate energy of the emitted photons in eV?
0.7eV

(c) For a steady-state current $I = 10$ mA, assuming all photons escape, what is the optical output power consistent with your answer to Part (b)? **7mW.**
(hint) Watts = Amps·eV/q = Amps·Volts
$\mathbf{10 \cdot 10^{-3} A \cdot 0.7V = 7mW}$

(d) If the voltage drop across the depletion region is 0.4V, what is the separation of the quasi-Fermi levels in Region C? How can this be less than the total forward bias of 1.4V?
0.4V which is less than the bias 1.4V since the resistive drop is in neutral regions

(e) What is the electrical power consumed? **14mW**
$\mathbf{10 \cdot 10^{-3} A \cdot 1.4V = 14mW}$

(f) In terms of the ratio of optical power out to electrical power in, what is the efficiency of this light emitting diode? **7mW/14mW = 50%**

Question 2

A solar cell has a short-circuit current of 50 mA and an open circuit voltage of 0.7 V under full illumination. What is the maximum power delivered by this cell if the fill factor is 0.8?

$$\mathbf{P = 0.8 \cdot (50 \cdot 10^{-3} A \cdot 0.7 V) = 28 mW}$$

Question 3

If one makes an LED in a semiconductor with a direct bandgap of 2.5 eV, what wavelength light will it emit? Can you use it to detect photons of wavelength 0.9 μm? 0.1 μm?

$$\lambda = \frac{1.24 \mu m \cdot eV}{2.5 eV} = 0.5 \mu m$$

0.9μm → No, because $h\nu < E_g$.

0.1μm → Yes

Question 4

What is most attractive about solar cells as a global energy source? Why haven't they been adopted more widely so far?

Solar cells are attractive because the energy is renewable and does not negatively impact the environment.

Solar cells have not been widely adopted because of their high cost.

Chapter 9 Self- Quiz

Question 1

Study the ITRS roadmap chapters on Process, Integration, Devices and Structures (PIDS) and on Front End Processes (FEP) available at http://public.itrs.net/. This has projections about next generation CMOS devices.
Plot some of the projected MOS device parameters from the various tables as a function of time? Do they obey Moore's laws? **Yes**
Based on what you have learned in Chapters 6 and 9, do the required $I_{D(Sat)}$ numbers for NMOSFETs for various technology nodes in Table 47b make sense? How about some of the other MOSFET requirements in other tables that are color coded red? **No**

Question 2
Discuss consequences— one good, one bad —of quantum mechanical tunneling in MOSFETs.

Good: ohmic contacts to source and drain
Bad: gate leakage

Question 3
What is hot electron damage, and is it more or less severe than hot hole damage? Why? How can you minimize hot carrier damage?

Hot electron damage is more severe because the mobility of electrons is higher and barrier injection is lower.

Question 4
Why are MOSFETs manufactured with {100} planes parallel to the $Si\text{-}SiO_2$ interface?

Lowest Q_{ox}

Chapter 10 Self- Quiz

Question 1

Study the ITRS roadmap chapter on Emerging Research Devices available at http://public.itrs.net/.This has projections about next generation CMOS devices as well as novel devices using nanotechnology.
Write a report on which of these devices you think will be used in products in the next 5 years, 10 years, and 20 years.