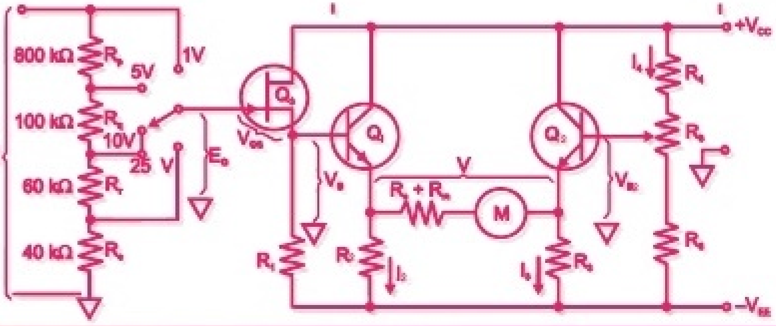
Electronic Instrumentation and Measurement



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Preface

Electronic instrumentation is a science of measurement and their control. The measurement of any quantity, whether it is temperature, voltage, current, pressure, resistance, power, force, velocity, or distance, is determined through an instrument. Some static and dynamic characteristics are to be kept in mind while choosing an instrument for a particular measurement. Measurement is fundamental to all branches of engineering. Keeping this in mind, most universities have incorporated the study of electronic instrumentation along with measurement in BE/BTech in Electronics and Communication Engineering, Electrical and Electronics Engineering, and Electronic Instrumentation Engineering curricula.

The book *Electronic Instrumentation and Measurement* has been written to study the performance, operation and applications of the most important electronic measuring instruments, techniques and instrumentation methods that include both analog and digital instruments. It covers a wide range of topics that deal with the basic measurement theory, measurement techniques, such as analog meter movements, digital instruments, power and energy measurement meters, AC and DC bridges, magnetic measurements, cathode ray oscilloscope, display devices and recorders, and transducers. It also explains generation and analysis of signals along with DC and AC potentiometers, and transformers.

Key Features

- Complete coverage of the subject as per the syllabi of most universities
- Relevant illustrations provide graphical representation for in-depth knowledge
- Numerous mathematical examples for maximum clarity of concepts
- Chapter objectives at the beginning of each chapter for its overview
- Chapter-end summary and exercises for quick review and to test your knowledge
- A comprehensive index at the end of the book for quick access to the topics

Chapter Organization

The book is organized into 15 chapters.

Chapter 1 introduces different units of measurement, dimensional notation of physical quantities, CGS and MKS systems of units. It explains various standards and methods of measurement, and generalized measurement systems.

Chapter 2 focuses on significant figures, static and dynamic performance characteristics for selecting a suitable instrument for measurement. It also discusses different types of errors in measurement, combination of errors, probability of errors and limiting errors. It also describes the statistical analysis on measurement data to analytically determine the uncertainty of final measurement result.

Chapter 3 deals with electromechanical instruments to measure physical quantities, including permanent magnet moving-coil instruments, galvanometers, DC ammeters and voltmeters, series and shunt ohmmeters, and describes the calibration of instruments.

Chapter 4 presents DC and AC measuring instruments. The instruments that can measure both DC and AC quantities include moving-iron type instruments, thermocouples, dynamometers, and transistor voltmeter circuits. The instruments that can measure only AC quantities include rectifier-type instruments which incorporate rectification techniques such as full-wave rectifier voltmeter, half-wave rectifier voltmeter, and half-bridge full-wave rectifier voltmeter.

Chapter 5 throws light on digital instruments, such as digital voltmeter, digital multimeter, and digital frequency meter. It also compares between analog and digital instruments.

Chapter 6 takes into account power and energy measurement instruments, including electrodynamic wattmeter, low power factor wattmeter, induction-type wattmeter, energy meters, single-phase induction-type energy meter. It determines power in three-phase circuits using one-, two- and three-wattmeter methods, and three-phase wattmeter. It also explains maximum demand indicators and trivector meters.

Chapter 7 focuses on DC and AC measurement bridges. It describes Wheatstone bridge and Kelvin bridge as DC bridges, Megger for resistance measurement, fall-of-potential method for measuring earth resistance. It explains different types of AC bridges for measuring unknown capacitance, resistance, frequency, inductance, and mutual inductance, and describes miscellaneous bridges, such as parallel T network, bridge T network, Q meter, and Wagner earthing device.

Chapter 8 explores induction coil, fluxgate, and Hall-effect type magnetometers, and ballistic galvanometer for measuring flux density of a magnetic field. It determines hysteresis loop and B-H curve using different methods. It also discusses Hopkinson, Ewing double bar, Illiovici, and burrow type permeameters along with AC testing of magnetic materials, and measurement and separation of iron losses.

Chapter 9 presents the structure and working of a cathode ray oscilloscope along with its probes. It also discusses multi-input oscilloscopes, special types of oscilloscopes including sampling and storage oscilloscopes, and Lissajous figures.

Chapter 10 familiarizes the reader with various display devices such as decade counting assembly, light emitting diode, liquid crystal display, nixie tube, segmental gas discharge display, and dot matrix display. It also explains the concept of X-Y recorders, magnetic tape recorders, plotters, DC and AC signal conditioning in measurement systems, and data acquisition system.

Chapter 11 presents the classification of transducers as primary and secondary, analog and digital, active and passive, and transducers and inverse transducers. It also throws light on the types of resistive transducers, thermoelectric transducers, inductive transducers, piezoelectric transducers, photoelectric transducers, pressure transducers, and some miscellaneous transducers including load cells, hotwire anemometer, ultrasonic flow meters, seismic transducers, and tachogenerators.

Chapter 12 focuses on the concept of generation of signals, different types of signal generators including pulse and square-wave generators, function generator, arbitrary waveform generator, audio- and radio-frequency signal generators, frequency synthesizer signal generator, sweep frequency generator, and video signal generator.

Chapter 13 presents the need for analysis of waveform, different types of analyzers, including wave analyzer, spectrum analyzer, power analyzer, and harmonic distortion analyzer.

Chapter 14 expounds the basic principle of potentiometers. It also describes different types of DC potentiometers including the basic slide wire, Crompton's, multi-range, precision type, and deflection potentiometers, and AC potentiometers including polar and co-ordinate AC potentiometers. It also discusses the applications of DC and AC potentiometers, and self-balancing potentiometer.

Chapter 15 deals with the basic concepts of instrument transformer and its parameters. It also explains the construction, equivalent circuit, phasor diagrams, characteristics, errors, reduction methods, and testing methods of both current and potential transformers along with their differentiation.

It is hoped readers will enjoy reading the book and it proves to be a good resource for all.

The readers are welcome to send their feedback on the book to *itlesl@rediffmail.com.* All the constructive comments will be highly appreciated.

Rohit Khurana

Acknowledgement

In all my efforts towards making this book a reality, my special thanks go to my technical and editorial teams, without whom this work would not have achieved its desired level of excellence. I sincerely extend my thanks to my research and development team for devoting their time and putting in relentless effort in bringing out this high-quality book. I convey my gratitude to my publisher, Vikas Publishing House Pvt. Ltd, for sharing this dream and giving all the support in realizing it.

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Electronic Measurement

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Define electronic instrumentation and measurements
- Categorize the units of measurement, such as fundamental and derived units
- Comprehend the dimensions of different physical quantities
- Describe the various systems of units like CGS, MKS, Rationalized MKS, and other systems
- Classify standards of measurements into international, primary, secondary, and working
- Explain the standards for mass, length, volume, frequency, time, temperature, luminous intensity, and electrical including resistance, voltage, capacitance, and inductance
- Understand the concept of IEEE standards

1.1 INTRODUCTION

Electronic instrumentation is a science of measurement and its control. It serves all branches of engineering and medicine, and every human purpose. It requires an instrument for measurement. An **instrument** is defined as a device that determines the magnitude or value of a quantity, such as temperature, pressure, level, current, voltage, resistance, power, force, velocity, and distance. Any quantity or parameter can be understood clearly by measuring it.

Measurements are made to monitor a process or operation which is to troubleshoot an existing circuit, characterize and define a circuit, and modify it for improvement. It depends on the measuring equipment or instrument available and how the instrument will function as per the prescribed requirement. The instruments may be simple or complex, such as thermometers, anemometers, barometers which measure environmental conditions, and special monitoring equipments; and electric, gas, and water meters which are used at homes and hospitals.

The quantities measured are expressed in numerical values which possess some units. The standard system of units of measurement was required to be established to accept it throughout the world. This system is called as **international system** and abbreviated as **SI**. In this chapter, we will study units and their types, the system of units, and their measuring standards.

1.2 UNITS OF MEASUREMENT

Any physical quantity which is measured, is expressed in both **kind** and **magnitude**. Each kind of physical quantity is expressed in a standard measure called **unit**. The magnitude of the physical quantity is defined as the product of numerical ratio and the unit. The **numerical ratio**, also called **numerical multiplier** or **number of measures** is defined as the number of times the unit occurs in any given amount of the same quantity. Two kinds of units are used to measure a physical quantity known as *fundamental* and *derived units*.

1.2.1 Fundamental and Derived Units

The measures of length, mass, and time are the fundamental units in mechanics and also fundamental to most other physical quantities, so are called **primary** fundamental units. On the other hand, the measures of certain physical quantities in electrical, thermal, and illumination disciplines are also defined by fundamental units, called **auxiliary** fundamental units.

All other units that can be expressed in terms of fundamental units with the help of physical equations are known as **derived** units. Every derived unit originates from some physical equation or law defining that unit, such as the area (A) of a rectangle is given as the product of length (l) and breadth (b), and expressed as $A = l \times b$. Suppose metre is the unit of length and breadth, then the number of multiplier and unit both are multiplied as $5 \text{ m} \times 6 \text{ m} = 30 \text{ m}^2$.

SI mechanical units

The three fundamental mechanical units in SI system are **metre** (abbreviated as m), **kilogram** (abbreviated as kg), and **second** (abbreviated as s) as the units of length, mass, and time, respectively. The derived mechanical units, such as force, work, power and energy can be obtained from these fundamental units.

Force: A force must be applied to accelerate or decelerate a body proportional to the desired rate of change of velocity, that is, acceleration (or deceleration). Mathematically, it is expressed as:

$$\text{Force} = \text{Mass} \times \text{Acceleration} \quad \Rightarrow \quad F = m \times a \qquad ...(1)$$

When a body is accelerated vertically from earth's surface, the acceleration due to gravity (g) expressed in SI unit is $g = 9.81 \text{ m/s}^2$.

The SI unit of force is newton. It is denoted by N and is defined as the force that accelerates a mass of one kilogram (kg) with one metre per squared second (m/s^2).

Work: The work done in moving a body is the force exerted to overcome the body's resistance to bring it into motion. Mathematically, it is expressed as the product of force and the distance through which the body moves in the direction of force.

$$\text{Work done} = \text{Force} \times \text{Distance} \quad \Rightarrow \quad W = F \times d \qquad ...(2)$$

The SI unit of work is joule. It is denoted by J and is defined as the amount of work done in moving a body through a distance of one metre when a force of one newton acts on it. Joule is thus expressed as newton-metre.

Power: Power is the certain amount of work to be done in a time, that is, time rate of doing work. Mathematically, it is expressed as:

$$\text{Power} = \frac{\text{Work}}{\text{Time}} \Rightarrow P = \frac{W}{t} \qquad ...(3)$$

The SI unit of power is watt. It is denoted by W, and is defined as the power obtained when one joule of work is done in one second.

Energy: Energy is defined as the capacity to do work. It is measured in same units as those of work.

When the power of one watt (1 W) is used for one hour, the work done or energy consumed is one watt-hour (1 Wh) or when the power of 1 W is dissipated for 1 s, the work done or energy consumed is 1 J as power is time rate of doing work. Similarly, when the power of 1 kW is used for one hour, the energy consumed is 1 kWh.

Electrical units

In the SI system, the unit of electric current is considered as the fundamental unit while the units of charge, potential difference, emf, and voltage; resistance and conductance; magnetic flux and flux density; inductance and capacitance are categorized as derived electrical units.

Current and Charge: The electric current is the flow of charge carriers that passes a given point in a conductor in one second. The SI unit of electrical charge or quantity of electricity is coulomb, denoted as C and that of electric current is ampere, denoted as A. Since current is easier to measure accurately than charge, ampere is the fundamental electrical unit and coulomb is the derived unit.

A constant current that flows in each of the two infinitely long parallel conductors 1 metre apart that exerts a force of 2×10^{-7} newton per metre of length on each conductor is called one **ampere**. When a current of one ampere flows, the charge that passes a given point in the conductor per second is called one **coulomb**. Mathematically, ampere is expressed as:

$$\text{Amperes} = \frac{\text{Coulombs}}{\text{Seconds}} \qquad ...(4)$$

Since 6.24×10^{18} electrons or charge carriers carry a total charge equal to 1 coulomb, one electron carries a charge $Q = \dfrac{1}{6.24 \times 10^{18}} = 1.602 \times 10^{-19}\,\text{C}$.

Potential Difference, Emf, and Voltage: The SI unit of potential difference, emf, and voltage is volt. It is denoted as V, and is defined as the potential difference between two points on a conductor that carries a constant current of one ampere when one watt of power is dissipated between these points.

The work done to move 6.24×10^{18} electrons through a potential difference of 1 V is 1 joule. Therefore, one electron volt (eV) of energy is involved to move one electron through a potential difference of 1 V, such that $1\,\text{eV} = \dfrac{1}{6.24 \times 10^{18}}\,\text{J}$.

Resistance and Conductance: The SI unit of resistance is ohm which is symbolized by the Greek capital letter omega Ω. It is defined the resistance that allows the current of 1 ampere to flow when a potential difference of 1 volt is applied to the resistance. The reciprocal of resistance is conductance whose unit is siemen, denoted as *S*.

Magnetic Flux and Flux Density: The SI unit of magnetic flux is weber (Wb) which is defined as the magnetic flux that links a single turn coil producing an emf of 1 V when the flux reduces to zero at a constant rate in 1 second.

The SI unit of magnetic flux density is tesla (T) which is defined as the flux density in a magnetic field when the flux of 1 weber occurs in a plane of 1 square metre. Thus, 1 tesla is also expressed as 1 Wb/m^2.

Inductance and Capacitance: The SI unit of inductance is henry. It is denoted as *H*. When 1 volt of emf is induced by the current that changes at the rate of 1 A/s, the inductance of the circuit is 1 H.

The SI unit of capacitance is farad. It is denoted as *F* and is defined as the capacitance of a capacitor containing a charge of 1 coulomb when the potential difference is 1 volt between its terminals.

Example 1 Determine how long it takes for an engine with 750 W output to raise a 50 kg load vertically through 65 m.

Solution: Given that: m = 50 kg, P = 750 W, d = 65 m, acceleration due to gravity $g = 9.81\ m/s^2$

Force can be computed from Equation (1) as:

$$F = m \times a = 50 \text{ kg} \times 9.81 \text{ m/s}^2 = 490.5 \text{ N}$$

The work done is computed as:

$$W = F \times d = 490.5 \text{ N} \times 65 \text{ m} = 31882.5 \text{ J} \qquad \text{[Refer to Eqn. (2)]}$$

Since power is expressed as: $P = \dfrac{W}{t}$

This implies time taken to raise a load is given as: $t = \dfrac{W}{P} = \dfrac{31882.5}{750} = 42.5 \text{ s}$

1.2.2 Temperature Units

Temperature Scales: The **Celsius scale** and the **Kelvin scale** are the two temperature scales for measuring temperature in SI units. There are 100 equal divisions or degrees between the freezing and boiling temperatures of water. The freezing temperature of water is zero degree Celsius (or 0°C) and boiling temperature is 100°C at normal atmospheric pressure. The Kelvin scale starts at absolute zero of temperature corresponding to –273.15°C which means 0°C corresponds to 273.15 K and 100°C corresponds to 373.15 K. This scale is also known as **absolute scale**. The temperature difference of 1 K and 1°C is the same.

There is also a non-SI temperature scale called **Fahrenheit scale** in which the freezing and boiling temperatures of water are 32°F and 212°F, respectively.

Joules Equivalent: The Joules equivalent, also called **mechanical equivalent of heat,** is used to calculate the energy required to raise a quantity of water through a given temperature change, so that, to raise one litre of water through 1°C, the energy required is 4187 J. The container in which the water is stored must be heated to the same temperature as water so every container must have its own **water equivalent** which is defined as the quantity of water absorbing the same amount of energy as the container through a specified temperature change on being heated.

1.2.3 Scientific Notations and Metric Prefixes

The number multiplied by 10 raised to a power can be easily written to denote very small or very large numbers, such as $0.01 = \frac{1}{10 \times 10} = \frac{1}{10^2} = 1 \times 10^{-2}$, $1000 = 1 \times 10 \times 10 \times 10 = 1 \times 10^3$, $2100 = 2.1 \times 10^3$, and $0.021 = 2.1 \times 10^{-2}$. This form of representation is called **scientific notation**. In SI system of units, instead of commas, spaces are used for writing large numbers, such as ten thousand is written as 10 000 while one thousand as 1000. There is an exception for four numeral numbers.

To simplify the writing of very small and very large quantities, metric prefixes are used, such as 1×10^{-2} A is expressed as 1 centiampere or 1 cA and 1000 Ω as 1 kilo-ohm or 1 kΩ. Table 1.1 shows metric prefixes and symbols for various multiples and submultiples of 10.

Table 1.1 Representation of Numbers, Scientific Notation, and Metric Prefixes

Number	*Scientific Notation*	*Prefix*	*Symbol*
0.1	10^{-1}	deci	d
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1000	10^{3}	kilo	k
100	10^{2}	hecto	h
10	10	deka	da

The representation of quantities in multiples of 10^3 or 10^{-3} as mentioned in Table 1.1 is called **engineering notation**. The quantity such as 55×10^{-7} is more suitably written as 5.5×10^{-6} A or 5.5 μA and 10^7 Ω as 10×10^6 Ω or 10 MΩ. To employ electrical calculations, engineering notation proves to be more convenient than ordinary scientific notation.

1.3 DIMENSIONS

We have already studied different fundamental and derived units of quantities and scientific notation. In this section, we will study the quality of a quantity called **dimension** that distinguishes it from other quantities. Dimensions are expressed in characteristic notations, such as $[M]$ for mass, $[L]$ for length, $[T]$ for time, and so on.

Table 1.2 Representation of SI units, Symbols, and Dimensions

Quantity	*Symbol*	*Units*	*Unit Symbol*	*Dimensions*
Fundamental Units				
Mass	m	kilogram	kg	$[M]$
Length	l	metre	m	$[L]$
Time	t	second	s	$[T]$
Electric current	I	ampere	A	$[I]$
Thermodynamic temperature	T	kelvin	K	$[\Theta]$
Luminous intensity		candela	cd	
Supplementary Units				
Solid angle	Ω	steradian	sr	$[L^2]^\circ$
Plane angle	α, β, γ	radian	rad	$[L]^\circ$
Derived Units				
Area	A	square metre	m^2	$[L^2]$
Volume	V	cubic metre	m^3	$[L^3]$
Frequency	f	hertz	Hz	$[T^{-1}]$
Density	ρ	kilogram per cubic metre	kg/m^3	$[L^{-3}M]$
Velocity	u	metre per second	m/s	$[LT^{-1}]$
Angular velocity	ω	radian pe r second	rad/s	$[L^\circ T]$
Acceleration	a	metre per second squared	m/s^2	$[LT^{-2}]$
Angular acceleration	α	radian per second squared	rad/s^2	$[L^\circ T^{-2}]$
Force	F	newton	$N(kg\,m/s^2)$	$[LMT^{-2}]$
Pressure, Stress	p	newton per square metre	N/m^2	$[L^{-1}MT^{-2}]$
Work, Energy	W	joule	J(N m)	$[L^2MT^{-2}]$
Power	P	watt	W(J/s)	$[L^2MT^{-3}]$
Quantity of electricity	Q	coulomb	C(A s)	$[TI]$
Potential difference, Electromotive force	V	volt	V(W/A)	$[L^2MT^{-3}I^{-1}]$
Electric resistance	R	ohm	Ω(V/A)	$[L^2MT^{-3}I^{-2}]$
Electric capacitance	C	farad	F(As /V)	$[L^{-2}M^{-1}T^4I^2]$
Electric field strength	E, ε	volt per metre	V/m	$[LMT^{-3}I^{-1}]$
Magnetic field strength	H	ampere per metre	A/m	$[L^{-1}I]$
Magnetic flux	Φ	weber	Wb(v s)	$[L^2MT^{-2}I^{-1}]$
Magnetic flux density	B	tesla	$T(Wb/m^2)$	$[MT^{-2}I^{-1}]$
Inductance	L	henry	H(Vs /A)	$[L^2MT^{-2}I^2]$
Magnetomotive force	U	ampere	A	$[I]$
Luminous flux		lumen	lm(cd sr)	
Luminance		candela per square metre	cd/m^2	
Illumination		lux	$lx(lm/m^2)$	

Table 1.2 represents physical quantities, quantity symbols, units and their symbols, and dimensions. The quantities are categorized into fundamental, supplementary, and derived units. A derived unit is always represented by a complete algebraic formula known as dimension. For example, the volume of a cuboid is measured in terms of quantities like length, breadth, and height, all having same unit as metre and dimension as $[L]$. Therefore, the unit of volume is metre cube (m^3) and its dimension is $[V] = [L]\,[L]\,[L] = [L^3]$. Note that the square bracket denotes the dimensional notation only.

The two supplementary units mentioned in the table are: **steradian** for solid angle defined as the angle subtended by the surface of the sphere at its centre whose area is equal to the square of the radius of the sphere and **radian** for plane angle defined as the angle subtended by an arc of a circle whose length is equal to the radius of the circle.

Example 2 Derive the dimensions of the following:
(a) Permeability of a medium
(b) Power using relation $P = I^2 R$ and $P = V^2/R$.

Solution: The dimensions of the following quantities are obtained as:

(a) The permeability of medium μ is expressed as:

$$\mu = \frac{\text{Magnetic flux density}}{\text{Magnetic field strength}} = \frac{B}{H} = \frac{[MT^{-2}I^{-1}]}{[IL^{-1}]} = [MLT^{-2}I^{-2}]$$

(b) The power P is obtained from the relation as:

$$P = I^2R = [I^2][ML^2T^{-3}I^{-2}] = [ML^2T^{-3}]$$

Also, power can be obtained from the relation:

$$P = V^2/R = [ML^2T^{-3}I^{-1}]^2/[ML^2T^{-3}I^{-2}] = [ML^2T^{-3}]$$

1.4 SYSTEM OF UNITS

Scientists of the French Academy of Sciences laid down certain principles to follow a single system of weights and measures as per the French government directive in 1790. The **first principle** stated that the universal system of weights and measures should be based on permanent measures of nature instead of man-made reference standards. According to this principle, any physical quantity can be expressed by three fundamental units, that is, length, mass, and time. The unit of length is chosen as **metre** which is defined in terms of ten-millionth part of distance from equator to pole along the meridian passing through Paris. The unit of mass is chosen as **gram** defined as the mass of one cubic centimetre of distilled water at 4°C. and normal atmospheric pressure (760 mm mercury). The unit of time remains the same traditional **second** defined as 1/86,400 of the mean solar day. The **second principle** states that all other units should be derived from the three fundamental units of length, mass, and time. The third and the **last principle** proposed that the submultiples and multiples of basic units be expressed in decimal system. Also, the system of prefixes was devised by this principle.

By 1875, many countries accepted the proposal of French Academy. Thus, metric system of units became a legal system. In the meantime British Association for the Advancement of Science introduced **CGS** (that is, centimetre-gram-second) **absolute system of units** in which centimetre and gram were the fundamental units of length and mass, and which was used by physicists all over the world. To overcome the complications of CGS system, a new system was introduced known as MKS system of units.

1.4.1 CGS

The systems using centimetre, gram, and second as the fundamental mechanical units for the purpose of science and engineering for many years are called as **CGS absolute systems**. They are named as absolute systems as all physical quantities could be expressed in terms of the three fundamental units. The CGS system was categorized as *electromagnetic units system* (or **emu**) and *electrostatic units system* (or **esu**).

Electromagnetic units system

In this system, units of four physical quantities are involved, such as the permeability of the medium (μ), and the units of length, mass and time. The permeability of free space (μ_0) is taken as unity, that is 1. We may define unit magnetic pole as the pole exerting unit force on a similar pole located at 1 cm distance.

The dimensions of various electromagnetic quantities can be obtained as follows.

- **Pole strength:** The dimension of pole strength m can be obtained by considering the law of force between pole strengths m_1 and m_2 separated by a distance d, given as:

$$F = \frac{m_1 m_2}{\mu d^2}$$

On substituting the dimensions of force and distance from Table 1.2, we obtain the value of pole strength when the strengths of two poles are equal as:

$$[MLT^{-2}] = \frac{[m^2]}{[\mu][L^2]}$$

Therefore, the dimensions of pole strength m are: $[m] = [\mu^{1/2} M^{1/2} L^{3/2} T^{-1}]$ ...(5)

- **Magnetizing force:** It is represented as H and measured as the force exerted on a unit pole given as:

$$[H] = \frac{[F]}{[m]} = \frac{[MLT^{-2}]}{[\mu^{1/2} M^{1/2} L^{3/2} T^{-1}]} = [\mu^{-1/2} M^{1/2} L^{-1/2} T^{-1}] \quad ...(6)$$

- **Current:** It is measured by the relation of magnetizing force H which acts at the centre of a loop of radius r, given as:

$$H = \frac{2\pi I}{r}$$

The dimension of current I can be obtained by substituting the value of H from Equation (6) as:

$$[H] = \frac{[I]}{[L]}$$

Substituting the dimensions, we get:

$$[I] = [H][L] = [\mu^{-1/2} M^{1/2} L^{1/2} T^{-1}] \qquad ...(7)$$

- **Charge:** The charge Q, expressed as the product of current I and time T, is given as:

$$[Q] = [I][T] = [\mu^{-1/2} M^{1/2} L^{1/2} T^{-1}][T] = [\mu^{-1/2} M^{1/2} L^{1/2}] \qquad ...(8)$$

- **Potential difference:** It is represented as V and defined as the work done W per unit charge Q and is given as:

$$[V] = \frac{[W]}{[Q]} = \frac{[ML^2T^{-2}]}{[\mu^{-1/2} M^{1/2} L^{1/2}]} = [\mu^{1/2} M^{1/2} L^{3/2} T^{-2}] \qquad ...(9)$$

- **Resistance:** It is expressed as the potential difference V per unit current I and is given as:

$$[R] = \frac{[V]}{[I]} = \frac{[\mu^{1/2} M^{3/2} L^{1/2} T^{-2}]}{[\mu^{-1/2} M^{1/2} L^{1/2} T^{-1}]} = [\mu L T^{-1}] \qquad ...(10)$$

- **Inductance:** The inductance L is expressed as the potential difference V per unit rate of change of current dI/dt and is given as:

$$[L] = \frac{[V]}{[I]/[T]} = \frac{[V][T]}{[I]} = \frac{[\mu^{1/2} M^{1/2} L^{3/2} T^{-2}][T]}{[\mu^{-1/2} M^{1/2} L^{1/2} T^{-1}]} = [\mu L] \qquad ...(11)$$

- **Capacitance:** The capacitance C is expressed as charge Q per unit potential difference V and is given as:

$$[C] = \frac{[Q]}{[V]} = \frac{[\mu^{-1/2} M^{1/2} L^{1/2}]}{[\mu^{1/2} M^{1/2} L^{3/2} T^{-2}]} = [\mu^{-1} L^{-1} T^{2}] \qquad ...(12)$$

Electrostatic units system

In this system also, units of four physical quantities are involved, such as the permittivity of the medium (ε), and the units of length, mass and time. The permittivity of free space (ε_0) is taken as unity, that is 1.

We may define the unit of electrical charge as the charge exerting unit force on a similar charge located at 1 cm distance.

The dimensions of various electrostatic quantities can be obtained as follows.

- **Charge:** The dimension of charge Q can be obtained by considering Coulomb's law which states that the force exerted between two charges Q_1 and Q_2 separated by a distance d is related as:

$$F = \frac{Q_1 Q_2}{\varepsilon d^2}$$

On substituting the dimensions of force and distance from Table 1.2, we obtain the value of charge, when two charges are equal, as:

$$[MLT^{-2}] = \frac{[Q^2]}{[\varepsilon][L^2]}$$

Therefore, the dimensions of charge Q are: $[Q] = [\varepsilon^{1/2} M^{1/2} L^{3/2} T^{-1}]$ ...(13)

- **Current:** The current is represented by the dimension I and is expressed as charge per unit time. Mathematically, it is given as:

$$[I] = \frac{[Q]}{[T]} = \frac{[\varepsilon^{1/2}\ M^{1/2}\ L^{3/2}\ T^{-1}]}{[T]} = [\varepsilon^{1/2}\ M^{1/2}\ L^{3/2}\ T^{-2}] \qquad \text{...(14)}$$

- **Potential difference, or Emf:** Potential difference represented as V is expressed as work done per unit charge. On substituting the dimension of W from Table 1.2 and Q from Equation (13), we get:

$$[V] = \frac{[W]}{[Q]} = \frac{[ML^2T^{-2}]}{[\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-1}]} = [\varepsilon^{-1/2}M^{1/2}L^{1/2}T^{-1}] \qquad \text{...(15)}$$

- **Resistance:** The resistance R is expressed as potential difference per unit current, and is given as:

$$[R] = \frac{[V]}{[I]} = \frac{[\varepsilon^{-1/2}M^{1/2}L^{1/2}T^{-1}]}{[\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-2}]} = [\varepsilon^{-1}L^{-1}T] \qquad \text{...(16)}$$

- **Inductance:** The inductance L is expressed as potential difference V per unit rate of change of current dI/dt as:

$$[L] = \frac{[V]}{[I]/[T]} = \frac{[V][T]}{[I]} = \frac{[\varepsilon^{-1/2}M^{1/2}L^{1/2}T^{-1}][T]}{[\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-2}]} = [\varepsilon^{-1}L^{-1}T^2] \qquad \text{...(17)}$$

- **Capacitance:** The capacitance C is expressed as charge per unit potential difference given as:

$$[C] = \frac{[Q]}{[V]} = \frac{[\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-1}]}{[\varepsilon^{-1/2}M^{1/2}L^{1/2}T^{-1}]} = [\varepsilon L] \qquad \text{...(18)}$$

Among the CGS unit systems discussed above, electromagnetic units system is more convenient to use than electrostatic units system in terms of electrical measurements. In general, if a quantity is expressed in CGS system without any additional designation of electrostatic or electromagnetic, then electromagnetic units system is preferred.

1.4.2 Practical Units

A system of practical units was used because many CGS units were too small or too large for practical engineering applications. These units are derived from absolute units or by referring to arbitrary standards as they are easy to manage and handle. The electromagnetic units can be made into smaller or larger practical units through an appropriate power of 10 so they can be used conveniently in experimental work. The practical unit of resistance, that is, ohm is taken as 10^9 CGS electromagnetic units by the British Association Committee on electrical units. Also, the practical unit of potential difference, that is, volt is taken as 10^8 CGS electromagnetic units. On fixing these two magnitudes, the practical units for other quantities can also be fixed. For example, the practical unit of current, that is, ampere would be expressed as:

$$\text{Ampere} = \text{Volt/Ohm} = 10^8\ \text{emu}/10^9\ \text{emu} = 10^{-1}\ \text{emu of current}$$

1.4.3 MKS

In the theoretical CGS systems, there occurred a problem when in some equations, a factor of 4π was inappropriately present. Due to this, both emu and esu CGS systems were regarded as irrational. This led to the need of a new system, the MKS system, which was introduced by Professor Giorgi in 1901. The three fundamental mechanical units in this system were taken to be metre, kilogram, and second. Along with three fundamental quantities one more quantity was used to connect electrical and mechanical quantities, that is, permeability of free space taken as $\mu_o = 10^{-7}$. The permeability of any medium (μ) is expressed as $\mu = \mu_o\mu_r$ where μ_o is unity in CGS system and μ_r is the relative permeability. Therefore, MKS unit of permeability = 10^7 × CGS unit of permeability.

1.4.4 Rationalized MKS

The MKS system was rationalized in which a factor 4π was relocated by redefining certain units assuming the value of permeability of free space as $\mu_o = 4\pi \times 10^{-7}$ and permittivity of free space as $\varepsilon_o = 1/(36\pi \times 10^{-9})$. This rationalized system uses four fundamental units, namely, metre, kilogram, second, and ampere. International Electro-technical Commission recommended the use of ampere as the fourth fundamental unit.

1.4.5 Other System of Units

The traditional English language system of measurements, that is, American and Imperial systems uses the fundamental mechanical units for length, mass, and time as foot (ft), pound-mass (lb), and second (s), respectively. One-twelfth of the foot is defined as inch whose value is fixed to be exactly 25.4 mm. Similarly, the value of pound is fixed to be exactly 0.45359237 kg. With the help of these two units all units in the English system can be converted into SI units. Using the above mentioned fundamental units, the derived units are obtained. For example, the unit of acceleration is expressed as ft/s^2 and of density as lb/ft^3. The unit of force is poundal defined as the force required to accelerate 1 pound-mass at the rate of 1 ft/s^2. The unit of work is expressed as foot-poundal.

1.5 STANDARDS OF MEASUREMENT

A physical representation of a unit of measurement is called **standard of measurement**. A piece of equipment that has a known measure of physical quantity is said to be a **standard**.

However, to realize a unit of measurement, a material standard or a natural phenomenon including atomic and physical constants are referred. For example, in the international system (SI), **kilogram** is the fundamental unit of mass which is defined as the mass of a cubic decimetre of water at its temperature of maximum density 4°C. A material standard, (that is International Prototype) for kilogram which consists of a platinum–iridium hollow cylinder preserved at the International Bureau of Weights and Measures at Sevres, near Paris is used to represent a unit of mass of kilogram.

1.5.1 Classification of Standards

The standards of measurement are classified on the basis of their functionality and areas of application into four categories that include *international*, *primary*, *secondary*, and *working standards*.

International standards

The international agreement laid the basis for international standards which are preserved at International Bureau of Weights and Measures. Of these, some units in physical form can be represented to the closest possible accuracy achievable with the scientific and technological methods using these standards. They are monitored regularly by absolute measurements in terms of fundamental units. However, international units have become superfluous due to improvements in absolute measurements and accuracy and are now replaced by absolute units as wire resistance standards, for resistance could be constructed permanently which do not vary with time. For example, absolute ampere took over international ampere in 1948 as absolute unit which was determined by means of current balance that weighs the force between two current-carrying coils. The relation between force and current that produces force can be calculated from the concepts of electromagnetic field theory. Therefore, the fundamental unit of electric current in SI system is absolute ampere.

Primary standards

Primary standards are highly accurate absolute standards, and are used as ultimate reference standards which are maintained by national standards laboratories in various countries of the world, like the National Bureau of Standards in Washington. These standards must be highly stable whose values should vary as small as possible over a long period of time. The fundamental units and some of the derived electrical and mechanical units that are represented as primary standards are calibrated independently by absolute measurements at each national laboratory. The result of these measurements are compared with each other. The main function of the primary standards is to verify and calibrate the secondary standards.

Secondary standards

Secondary standards are used in industrial measurement laboratories as reference standards for calibrating high accuracy components and equipments, and for verifying the accuracy of working standards. The industrial laboratory holds the responsibility to maintain and calibrate these standards. However, the national standard laboratories periodically check their calibration and compare them with primary standards and then send back to the industry with certification of their measured value in terms of primary standards.

Working standards

Working standards are the important tools used in measurement in laboratory which are used to check and calibrate general laboratory instruments for performance and accuracy. The standard resistors, inductors, and capacitors are the working standards used by manufacturer of precision to check his testing equipment. The standard resistances are made up of materials having very low temperature coefficient such as manganin and range from 0.01 Ω to 1 MΩ with an accuracy range of ±0.01% to ±0.1%. The standard inductance value ranges from 100 μH to 10 H with a typical accuracy of ±0.1% while the standard capacitance values range from 0.001 μF to 1 μF with a typical accuracy of ±0.02%.

1.5.2 Mass, Length, and Volume Standards

Mass: The material representation of unit of mass which is preserved at the International Bureau of Weights and Measures near Paris, is the International Prototype kilogram. The National Physical Laboratories of every country keeps the Prototype kilogram as the primary standard of mass with an accuracy of 1 part in 10^8. This Prototype is occasionally compared to the standard kept at the International Bureau. The secondary standards of mass have an accuracy of 1 ppm (that is, part per million) and are kept by industrial laboratories. They are verified against the primary standards. The wide range of working standards of mass are available to suit almost any application having an accuracy of the order of 5 ppm. These standards are checked against the secondary standards.

Length: In 1960, metre was defined as the international unit of length in terms of the optical standard as the orange-red radiation emitted from krypton atom. The wavelength of the orange light emitted from internationally recognized krypton-86 discharge lamp on being excited and observed well under defined conditions constitute the basic unit of length. It has an accuracy of 1ppm. For over 20 years, 1,650,76.73 wavelengths of orange-red light radiated from krypton-86 atom in vacuum was recognized as the international standard metre. The unit, metre was redefined in 1983 because the standard explained before could not prove to be precise so a new standard was adopted which explains metre in terms of time. It states that the length travelled by light in vacuum in time interval of 1/299792458 second is one metre. Hence, the speed of light is given as 299792458 m/s. The most commonly used working standards of length in industry are precision gauge blocks which are made up of steel blocks having two parallel surfaces separated by a specified distance with an accuracy tolerance in the range of 0.5–0.25 micron where 1 micron is equal to one millionth of 1 metre. It is advantageous to use gauge blocks due to their low cost, precision, and accuracy of 1 ppm. Due to the features of this block, it is possible to manufacture interchangeable industrial components in an economical manner.

Volume: The unit of volume is not represented by an international standard, rather is a derived quantity. A number of primary standards have been constructed by National Physical Laboratories for volume to be calibrated in terms of absolute dimensions of mass and length. The secondary derived standards calibrated in terms of primary standards are also available.

1.5.3 Frequency and Time Standards

Frequency and time standards can be understood by the development and refinement of atomic resonators through which frequency of an oscillator can be controlled to a great extent.

Since frequency is inversely proportional to time, atomic clocks can be constructed with great accuracy and precision. When electrons in an atom transit between two energy levels E_1 and E_2, the radiations emitted or absorbed are expressed as:

$$E_2 - E_1 = hf$$

where h is Plank's constant and f is physical constant, that is, frequency.

The physical constant f depends on the internal structure of an atom only, and the energy states should not be affected by magnetic fields or any other external conditions. The International Committee of Weights and Measures defined the unit of time, that is, second, in terms of frequency of cesium transitions that assigns a value of 9,192,631,770 Hz to the hyperfine transition of cesium atom without being affected by external fields. The primary or absolute standards of measurement which do not need to be referred to any other standard for calibration or checking are **hydrogen maser** and **cesium beam** while the secondary standard is **rubidium vapour standard** which need to be recalibrated periodically with reference to primary standards. The **quartz oscillator** is an independent frequency standard that provides feedback frequency to cesium or rubidium atomic standards as they are passive.

Hydrogen maser standard

The hydrogen maser is relatively large in size and is used in limited areas where stability is the top priority and size is not considered. It is the most stable frequency source known today having a frequency of 1420405751.73 ± 0.03 Hz. It is an active device whose signals are directly amplified or used to drive the crystal oscillator as shown in Figure 1.1. An atomic hydrogen beam is fed to a state select magnet that allows higher energy state atoms to pass into a quartz storage bulb. This bulb causes frequency shift as its walls are coated with teflon to minimize perturbations of the atoms and confines the atoms to a uniform magnetic field in a tuned microwave cavity. The hydrogen atom is transited between states $F = 1$ and $F = 0$ in a cavity set at frequency equal to the transition of the hydrogen atom and undergoes many reflections off the walls of the bulb. The atoms relax and give up their energy to the microwave field while interacting inside the tuned cavity. The atoms are stimulated to radiate till a maser operation is achieved.

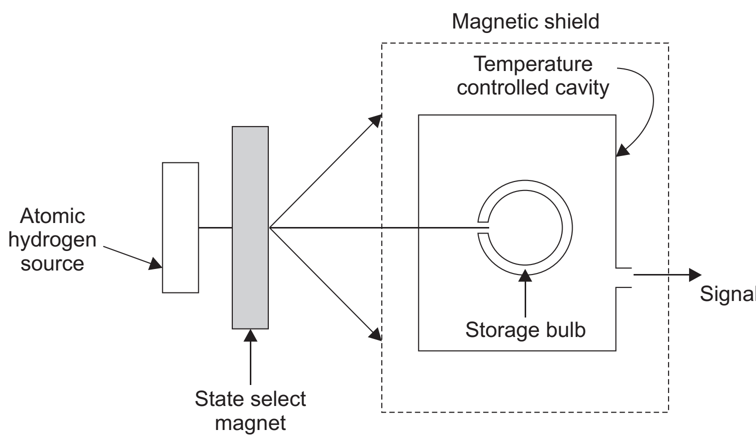


Fig. 1.1 Construction of Hydrogen Maser Resonator

Cesium beam standard

Cesium beam resonator is a primary standard which has good long term and short term stability. Their advantage of having relatively small size and weight has resulted in their acceptance by world standard authorities as frequency reference. Figure 1.2 shows a resonator that contains cesium atoms which are slowly evaporated in an oven and then collimated into a beam. The resonator contains two state select magnets which deflect each atom by some amount depending on the energy state of the atom. The atoms in energy state $F = 4$ only are allowed to form cesium beam in the chamber. The beam passes through a low and uniform magnetic field which is subjected to microwave excitation at frequency 9192.631770 MHz. This energy is sufficient to transit atoms from state $F = 4$ to $F = 3$. The beam then passes through the second state select magnet (similar to the first one) which deflects the atoms that have undergone the required state change onto a hot wire ionizer. The ionized cesium atoms are then converted to electron current by the detector after filtering through a mass spectrometer.

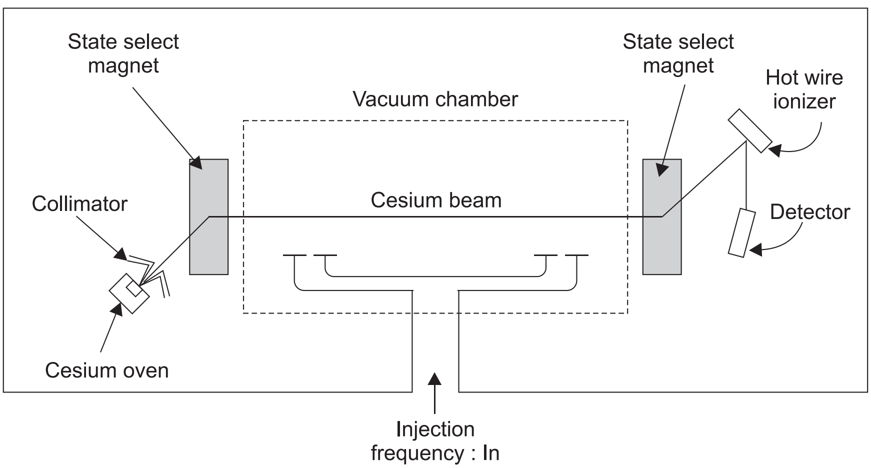


Fig. 1.2 Construction of Cesium Beam Resonator

A control scheme is required for cesium beam resonator which is shown by a block diagram in Figure 1.3 in which a crystal oscillator produces microwave field frequency. The

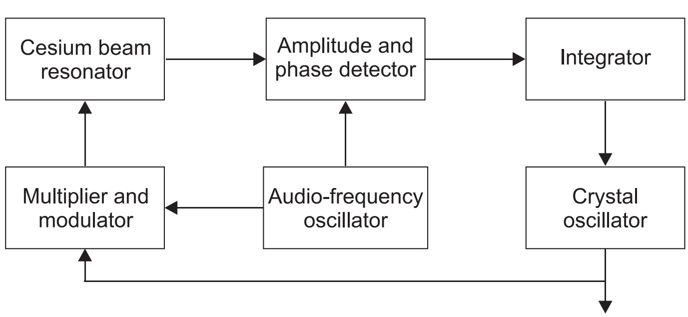


Fig. 1.3 Block Diagram of Cesium Beam Resonator

input is fed to the amplitude and phase detector from the resonator which is then amplified and fed to a voltage controlled crystal oscillator. This detector provides signal magnitude and polarity corresponding to the difference of the oscillator frequency from the desired frequency. The integrator filters the signals to remove high-frequency components and then drives the crystal oscillator to the correct frequency required by the cesium beam resonator. The signal from the crystal oscillator is then frequency multiplied and phase modulated by audio-frequency oscillator.

Rubidium vapour standard

The rubidium vapour standard is a secondary standard based on hyperfine transition between energy states $F = 2$ and $F = 1$ in rubidium 87 gas whose transition frequency is 6834682608 Hz. It is small in size and has good short-term stability so it is a useful portable instrument. The block diagram in Figure 1.4 shows that the light from the spectral lamp is filtered and fed to the cell containing rubidium 87 vapour in microwave cavity. The Doppler broadening effects can be reduced by adding an inert gas. The multiplication of a quartz oscillator operating in a closed loop gives the microwave signal similar to that used for cesium standard. Due to optical pumping, the population of atoms in $F = 2$ state increases and microwave energy causes transition from energy state $F = 2$ to $F = 1$ and more light is absorbed by the cell. The photodetector output reaches a minimum at a frequency 6834682608 Hz.

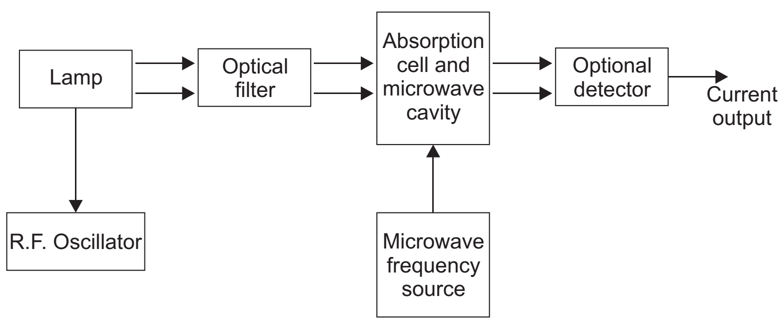


Fig. 1.4 Block Diagram of Rubidium Vapour Resonator

Quartz crystal standard

The frequency standard that is applicable to a wide range of applications is given by quartz oscillator which relies on the piezoelectric properties of quartz. The frequency lies in the range of 1 Hz–750 kHz and 1.5–200 MHz. The ranges between 750 kHz and 1.5 MHz are difficult to cover due to strong resonance. Since the low frequency crystals have less stability and greater physical size so high frequency crystals are preferred. The quartz crystal is aligned by X-rays and cut with a precision along one of the several planes and then grounded and lapped to form exact dimensions. The electrodes are deposited in vacuum along the sides of crystals and leads attached to it. The crystal has an atomic structure that is oriented along x (electrical), or y (mechanical), and z (optical) axes. A different cut may lead to a different change. A cut along x direction, that is X-cut, leads to mechanical change in thickness when electric field is applied in y direction. It has negative temperature coefficient of frequency. A cut along y direction, that is Y-cut leads to mechanical change in length when electric field is applied in x direction. It has positive temperature coefficient of frequency. The positive and negative temperature coefficients can be combined to minimize frequency shift in the crystal by making a cut rotated from x and y axes. Figure 1.5 shows frequency–temperature characteristic of a crystal where the turning point determines the cut of the crystal and could occur in the range of $-50°C$ to $+100°C$. The two most commonly used cuts for the quartz controlled oscillators are AT and BT. The cut that has low variation of frequency with temperature is AT cut and the cut preferred over 10 MHz frequency is BT cut as it is less susceptible to load capacitances and drive levels.

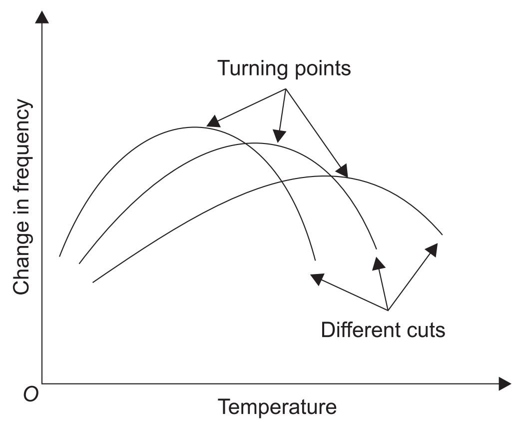


Fig. 1.5 Variation of Frequency with Temperature Showing the Effect of Cut in a Crystal on its Turning Point

1.5.4 Temperature and Luminous Intensity Standards

One of the basic SI temperature quantities is thermodynamic temperature whose unit is Kelvin. It forms the fundamental temperature scale for the reference of all other temperatures. The temperature on this scale is denoted by the symbol T and is designated as K. The thermodynamic temperature of triple point of water at exactly 273.16 K is defined through which the magnitude of Kelvin is fixed. The equilibrium temperature between ice, liquid water, and its vapour is the triple point of water.

Temperature measurements are difficult to be made on thermodynamic scale so the practical scale known as the International Practical Scale of Temperature was adopted in the Seventh General Conference of Weights and Measures. The temperature on this scale is denoted by the symbol t and designated as °C (or degree Celsius). There are two fundamental fixed points on the Celsius scale mentioned at the atmospheric pressure, that is, triple point of water as 0.01°C and boiling point of water as 100°C. There are a number of primary fixed points established above and below the two fundamental points referring to the boiling point of oxygen at $-182.97°C$, boiling point of sulphur at 444.6°C, freezing point of zinc at 419.58°C, freezing point of silver at 960.8°C, and the freezing point of gold at 1,063°C. The relationship for conversion between Kelvin scale and Celsius scale is given as $t(°C) = T(K) - T_o$ where T_o is 273.15 degrees. The platinum resistance thermometer is considered as the primary standard for temperature which is constructed from strain free platinum. Based on the resistance properties of the platinum wire, the values between the fundamental and primary fixed points on the scale can be calculated accurately.

Luminous intensity has primary as well as secondary standards. Its unit is **candela** which is defined as one-sixtieth of luminous intensity per square metre of one full radiator at freezing temperature of platinum under standard atmospheric pressure. The full radiator, that is a black body or Planckian radiator at solidification temperature of platinum at approximately 2,042 K under a pressure of 101.325 kN/m^2 is the primary standard of luminous intensity. The radiator has a cylinder of fused thorium oxide with a length of 45 mm with internal diameter of about 2.5 mm. The powdered fused thorium oxide is packed the bottom of the tube. Some pure platinum is contained in fused thorium oxide crucible in which the cylinder is supported vertically. The cylinder has a lid with small hole of diameter of about 1.5 mm in the centre. It is embedded in powdered fused thorium oxide in a large refractory container having a funnel-shaped opening as shown in Figure 1.6. A high-frequency coil carrying an alternating current of 1.6 MHz surrounds the outer container which induces eddy currents in the container and the platinum melts. The platinum is then allowed to cool slowly and change from liquid state to solid state. During this period of interchanging state, measurements for the luminous intensity of light passing from the aperture of known diameter can be made as the temperature remains constant for a sufficiently long time. The luminous intensity measured from this primary standard is 598,000 international candles/m^2. This unit was used prior to 1948; but now candela is the unit for measurement.

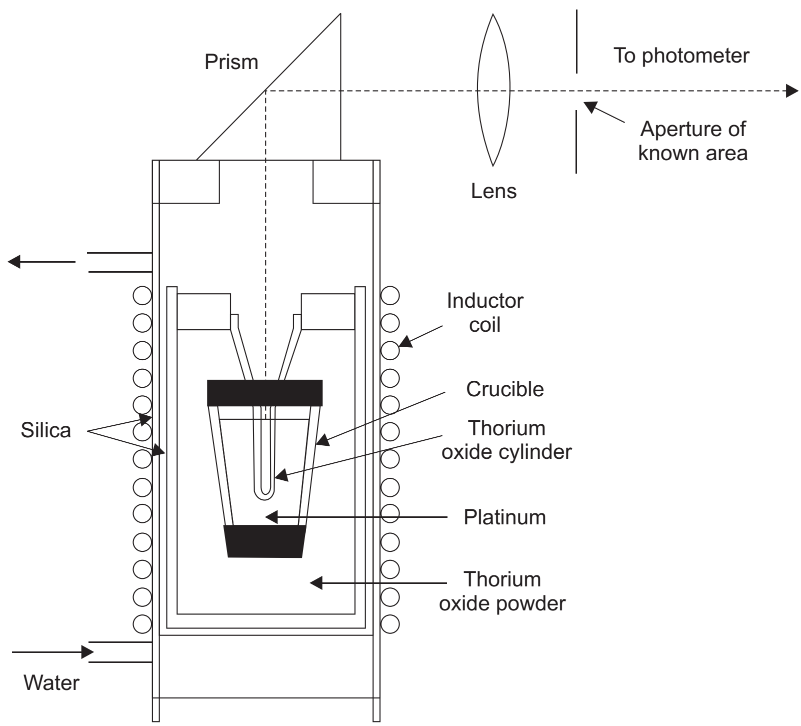


Fig. 1.6 A Radiator to Measure Luminous Intensity as Primary Standard

The special type of tungsten filament lamps operating at a temperature where their spectral power distribution in the visible region matches the basic standard is the secondary standard of luminous intensity.

1.5.5 Electrical Standards

Resistance standards

The SI unit of resistance is the absolute value of ohm which is defined in terms of the fundamental units of length, mass, and time. The International Bureau of Weights and Measures, and national standards laboratories of various countries preserve a group of primary resistance standards and carry out absolute measurements of ohm. These primary standards are checked periodically and verified occasionally by absolute measurements. The value of standard resistor is 1 Ω which is in the form of a coil of wire made from some alloy like manganin having low temperature coefficient of resistance and high electrical resistivity (Figure 1.7). The atmospheric moisture causes changes in resistance, which can be prevented by mounting the resistance coil in a double-walled sealed container.

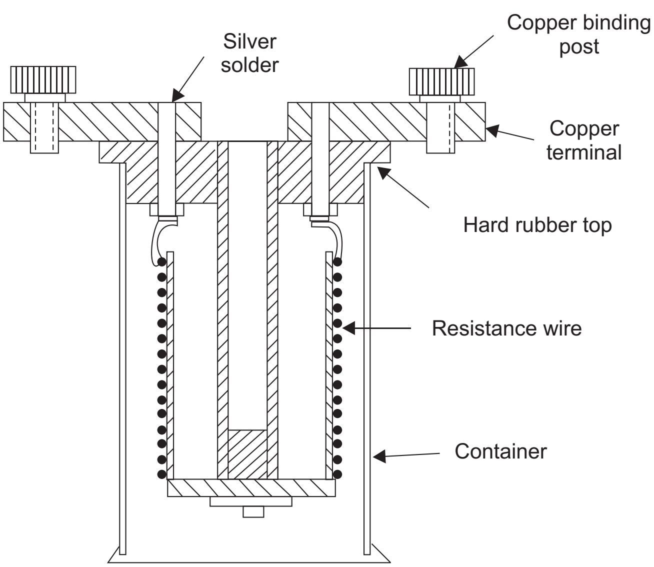


Fig. 1.7 A Double-walled Resistance Standard

Instrument manufacturers provide a wide range of values of resistance, usually of multiples of 10 Ω for secondary and working standards. The secondary standard resistors are sometimes called **transfer resistor** whose resistance coil is immersed in moisture-free oil and is supported between a polyester film to reduce stress on the wire and to improve resistor stability. This arrangement is placed in a sealed can. The coil connections are silver soldered and the terminal hooks are nickel-plated copper free from oxygen. These resistors find application in standards, research, industrial, and calibration laboratories. They are also used for ratio and resistance determinations.

The resistance wire so selected for transfer resistor provides almost constant resistance over a wide range of temperatures whose resistances can be calculated at any temperature given as: $R_t = R_{25°C} + \alpha(t - 25) + \beta(t - 25)^2$, where α, β are temperature coefficients, R_t is resistance at ambient temperature t, $R_{25°C}$ is resistance at 25°C. The value of α is less than 10×10^{-6} and of β lies between -3×10^{-7} and -6×10^{-7}.

Voltage standards

In 1962, British physicist, Brian Josephson introduced a new standard for volt in which a voltage is developed across a thin-film junction which is then cooled to nearly absolute zero and irradiated with microwave energy. This voltage is expressed as:

$$\nu = \frac{hf}{2e} \qquad ...(15)$$

where f is frequency of the microwave irradiation
h is Planck's constant, given as 6.63×10^{-34} Js
e is the charge of an electron, given as 1.602×10^{-19} C

From Equation (15), we can say that this standard volt varies only with irradiating frequency so it is related to the standard of time or frequency. The accuracy of standard volt is one part in 10^8 including all system inaccuracies when the microwave irradiating frequency is locked to an atomic clock.

A primary and secondary standard used for calibration is the standard cell, also called **Weston cell**. This cell has two electrodes, the positive electrode is made of mercury and the negative electrode is made of cadmium amalgam (10% cadmium). A solution of cadmium sulphate is filled as electrolyte and all these components are placed in a H-shaped glass container. The Weston cell is categorized into two types, namely, **saturated cell** and **unsaturated cell**. The saturated cell, also called as **normal Weston cell,** has its electrolyte saturated at all temperatures and electrodes are covered with cadmium sulphate crystals as shown in Figure 1.8. The voltage variation in saturated cell is approximately –40° μV per 1°C rise. The unsaturated cell is one in which the concentration of cadmium sulphate saturates at a particular temperature of 4°C. This cell has negligible temperature coefficient of voltage at normal room temperatures. However, the saturated cell is more stable than unsaturated cell and is maintained as primary standard for voltage. To control its temperature within 0.01°C, these cells are kept in oil bath. The absolute voltage of this cell is 1.01858 V at 20°C and the voltage at all other temperatures is computed as:

$$e_t = e_{20°C} - 0.000046(t-20) - 0.00000095(t-20)^2 + 0.00000001(t-20)^3 \qquad ...(16)$$

The Weston saturated cells are not well suited for general laboratory use as secondary or working standards as they are temperature sensitive. On being carefully treated, they can retain their voltage standards over a period of 10 to 20 years and their drift in voltage is of the order of 1 μV per year

The Weston unsaturated cells are preferred as secondary and working standards for voltage. They do not require exact temperature control and their voltage lies in the range of 1.0180 V to 1.0200 V, varying less than 0.01 per cent from 10°C to 40°C. The internal resistance of these cells varies in the range of 500 Ω to 800 Ω. To keep the nominal voltage

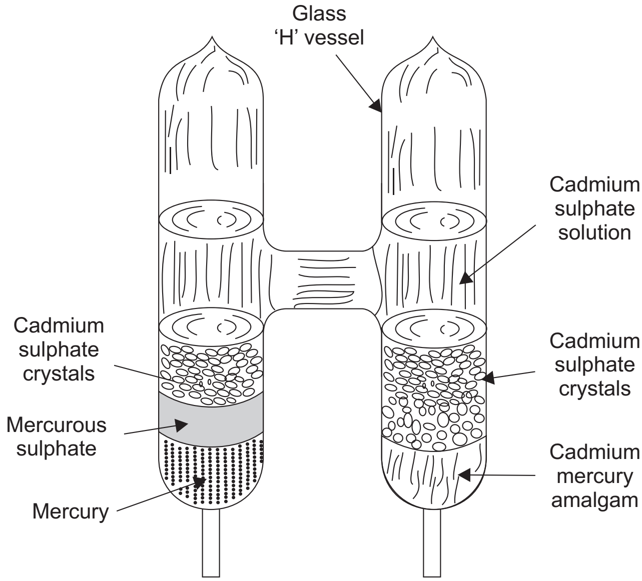


Fig. 1.8 A Configuration of Weston Saturated Cell

unaffected from the internal voltage drop, the current drawn from this cell should not exceed 100 μA.

Earlier, the standard cells were used for laboratory voltage standards, but now they are based on semiconductor devices like zener diode as voltage reference element.

Capacitance standards

The SI unit of capacitance is farad which is measured with Maxwell DC commuted bridge as shown in Figure 1.9. Since frequency and resistance both can be determined accurately, the capacitance can be computed with great accuracy from the frequency of DC commutation and resistive bridge arms of the DC commuted bridge in which the capacitor C is charged and discharged alternately through resistor R and commutating contact. The bridge is balanced by adjusting the value of R_3 and therefore the value of capacitance is determined.

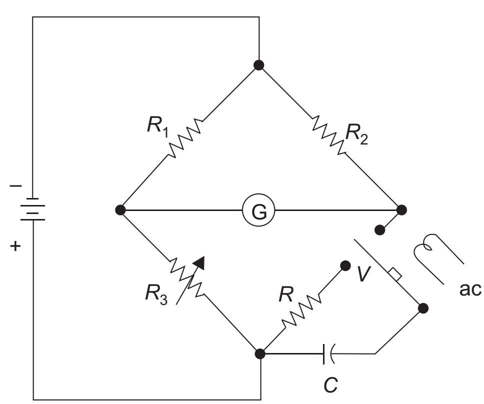


Fig. 1.9 Capacitance Measured from Commuted DC Bridge

The standard capacitors are made up of metal plates with air as the dielectric material. Their capacitance can be determined by knowing the area of the plates and the distance between them. These standard capacitors are used to calibrate secondary and working standards of industrial users and measurement laboratories.

The working standards of capacitance can be obtained in suitable range values. The air capacitors usually have small values and solid dielectric materials capacitor have larger values. The compactness of these standards account for high dielectric constant and very thin dielectric layer. The excellent working standards that are stable, have low temperature coefficient, and dissipation factor with little or no aging effect are silver-mica capacitors.

Inductance standards

The SI unit of inductance is henry, denoted by H and its dimensions in CGS emu system is [μL] which shows that it depends only on length, provided that the permeability is considered to be dimensionless. The standards for both mutual and self-inductance are explained here whose value depends on their physical dimensions and the number of turns of the inductance coil. The inductance standard is so chosen that the materials used to construct the inductor should have dimensions that do not change with time, temperature, and humidity. The inductance should not depend on the value of current passing so the inductors are made of air-cored coils instead of ferromagnetic materials as they depend on the value of permeability and hence inductance depends on flux density which depends on the current passing in the coil.

The primary standard selected for mutual and self-inductance by national standard laboratories is *Campbell standard* which is always a fixed standard, that is, single valued. Figure 1.10 shows Campbell standard consisting of a single-layer primary coil of bare copper wire wound under tension in grooves machined in a cylinder of marble. The marble is used as it is a perfect insulator, non-magnetic, has low temperature coefficient of thermal expansion, is unaffected by atmospheric conditions, cheap, and easy to work with. The primary coil is

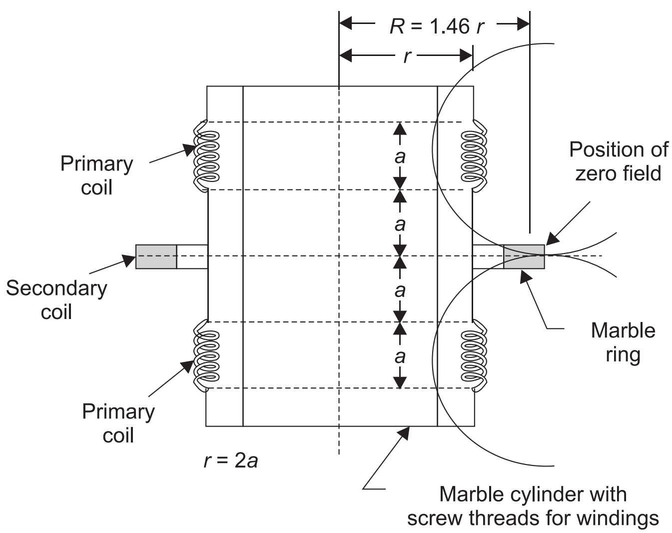


Fig. 1.10 Campbell Primary Standard

divided into two parts which is then connected in series and separated from each other with a distance equal to three times the axial length of each coil. The secondary coil is placed midway between two halves of the primary coil and has a diameter of about one and half times the diameter of the primary coil. This coil has many layers and is placed in a channel cut in the marble cylinder. Hence, the secondary coil lies in a position of almost zero field. Thus, the value of flux linked by secondary winding is independent of the determination of its diameter to great accuracy as it is difficult to evaluate the effective diameter of the secondary winding having many layers. The high value of inductance is obtained by using multilayer secondary winding instead of the single-layer winding.

The secondary and working standards for mutual inductance could be fixed or variable. Their inductance should not depend on supply frequency, and should not vary with temperature and time. The fixed secondary standards consist of two coils wound on a hard paraffined wood former or marble separated by a flange. To reduce the effect of eddy currents in conductors, the wire is made of stranded copper. The coils and former are then immersed in hot paraffin wax after winding. The variable standards are known as **inductometers** which consist of two sets of coil, one fixed and other movable to vary the mutual inductance, and are frequently used in laboratories.

The secondary standards for self-inductance are constructed to possess a nominal value of simple fraction of 1 H. For this, the coils are wound with silk covered stranded copper wire. The marbles are used to make formers. The coils are then immersed in melted paraffin wax after winding.

1.5.6 IEEE Standards

IEEE is the Institute of Electrical and Electronics Engineers which is headquartered in New York city. There are standard procedures, definitions, and nomenclature instead of physical items standards available for comparing secondary standards as we have studied earlier. The IEEE standards control the instrument's front panels, for test and measuring procedures, for electrical hardware and installations in particular situations.

These standards define certain standard test methods for the evaluation of the various electronic components and devices. Some standards are meant to specify test equipment that each manufacturer must adopt, like the arrangement and names of knobs, controls, and functions of common laboratory oscilloscope. If this is not adopted it becomes difficult for the oscilloscope operator to operate different oscilloscopes differently.

There are other standards as well that concern the safety of wiring for ships, industrial buildings, power plants, and many more applications. The standard voltage and current ratings have been specified so that when any component is to be changed, it can be changed without any damage. The standard logic and schematic symbols have been defined for engineering drawing to make understandable by engineers. The most important IEEE standard is the standard hardware for interfacing the laboratory test equipment to computers for monitoring and control purposes.

1.6 METHODS OF MEASUREMENT

We have already studied various units and standards of measurement. In this section, we will discuss the different methods employed to measure a quantity. The measurement of a quantity involves both the magnitude of a value and the unit of measurement. For example, if the voltage is measured as 10 V, then the value '10' represents magnitude and 'V' (volts) represents the unit of measurement. There are two methods of measurement, namely *direct comparison method* and *indirect method*.

1.6.1 Direct Comparison Method

In this method, any physical quantity that is unknown (also called **measurand**) is directly compared with a primary or secondary standard. For example, the length of a bar can be measured with the help of a measuring scale or tape which act as a secondary standard. The result of measurement is then expressed as a numerical value and a unit where metre is the unit of length. If a quantity is directly compared with secondary standard, the primary standard need not to be known as secondary standards are made from them.

The measured value of a quantity remains same whether it is directly compared with primary or secondary standard.

This method of measurement is not always accurate due to the involvement of human factors. Human beings can make comparisons with a precision of about 0.25 mm while measuring a length, thus some errors may occur. Similarly, when some mass is measured by comparing it with some standard, an error may occur as there will always be a difference between the two masses, no matter how small that difference is. The length can be measured to great degree of accuracy as compared to mass as it becomes difficult for human beings to differentiate between wide margins of mass. This method is rarely used due to lack of practicality, feasibility, and possibility.

1.6.2 Indirect Method

To overcome the drawbacks of direct comparison method, indirect method for measurement is preferred. This method measures quantities which cannot be measured directly. This method consists of a system that senses, converts and finally presents an analog output in the form of chart or displacement. A transducing device called **transducer** is used for this purpose which is coupled to a number of connecting apparatus forming a part of the measuring system. The transducer converts the input (a quantity to be measured) into output (some other measurable quantity). Both input and output are proportional to each other. The reading obtained from the transducer is the actual value of the quantity being measured and the output is well calibrated.

1.7 GENERALIZED MEASUREMENT SYSTEMS

The generalized measurement system is employed for indirect method of measurement as it consists of various functional elements. The three main functional elements of the measurement system include *primary sensing element*, *variable conversion and manipulation element*, and *data presentation element*. Each functional element performs the definite and required steps in the measurement which may be a single component or group of components.

1.7.1 Primary Sensing Element

The quantity to be measured (measurand) is detected firstly by the primary sensor as shown in Figure 1.11. This measurand is then converted into analogous electrical signal by a transducer, which is a device that converts one form of energy to another. Specifically in electrical measuring system, the transducer converts a physical quantity into an electrical quantity. The output of the primary sensing element is an electrical signal which may be a voltage or frequency or some other electrical parameter. This is the first stage of measurement and is known as **detector transducer stage**.

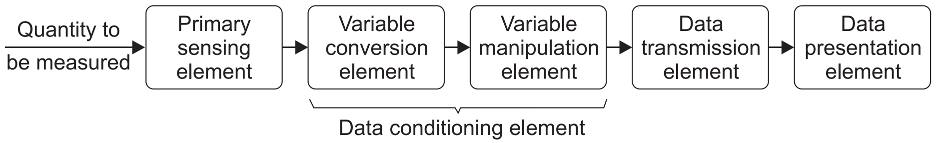


Fig. 1.11 Functional Elements of a Measurement System

1.7.2 Variable Conversion and Manipulation Element

Sometimes the output of the primary sensing element may not be suited to the system so the output is converted to some other suitable form while preserving the information content of the original signal. For example, if the output is in the form of analog and the next stage requires input signal in digital form, then analog-to-digital converter is used. The signal received from the conversion element is then manipulated keeping the original nature of the signal preserved. The manipulation is a change in numerical value of the signal or an increase in amplitude or power of the input signal. The variable manipulation element not necessarily follows variable conversion element, it may precede the conversion element in some cases. For example, if in a case, voltage is too high, attenuators are used to lower it for subsequent stages of the system.

A problem may arise when unwanted signals, like noise, interfere with the original output signal due to extraneous source, or a weak signal may get distorted by processing equipment. Certain operations, like attenuation, amplification, addition, subtraction, integration, differentiation, filtering, modulation, detection, sampling, clipping, and chopping, are performed on the signal to bring it to the desired form for the next stage of measurement system to accept known as **signal conditioning**. Signal conditioning contains many other functions in addition to variable conversion and variable manipulation. The element after the primary sensing element in any measurement system is called **signal conditioning element**. The element that performs transmission of data from one element to another is known as **data transmission element**. The stage which comprises signal conditioning and data transmission is known as **intermediate stage**.

1.7.3 Data Presentation Element

Data presentation element is the **final** or **terminating stage** of measurement. It conveys the information of the quantity being measured to the system or personnel to handle the instrument for monitoring, control, or analysis purposes. The information is presented in an intelligible form. The data is monitored by visual display devices which may be analog or digital indicating instruments like ammeters and voltmeters. However, if data is needed to be recorded, different types of recorders, like magnetic tapes, storage type CRT, high speed camera and TV equipment, printers, microprocessors or analog and digital computers may be used. Control and analysis is done by computers or microprocessors.

Consider, for an example, a bourdon tube pressure gauge as shown in Figure 1.12 is used as a measurement system. The bourdon tube acts as the primary sensing element and a variable conversion element in which pressure is sensed as an input quantity. When the pressure is applied, the closed end of the bourdon tube gets displaced which converts pressure into small displacement. The closed end of the bourdon tube is connected to a gearing arrangement through mechanical linkage. The small displacement is amplified by this gearing arrangement and thus the pointer rotates through a large angle. The mechanical linkage works as a data transmission element while gearing arrangement works as data manipulation element (see Figure 1.13). The pointer and dial arrangement works as a data presentation element which is calibrated with known pressure input to give the indication of the applied pressure signal to the bourdon tube.

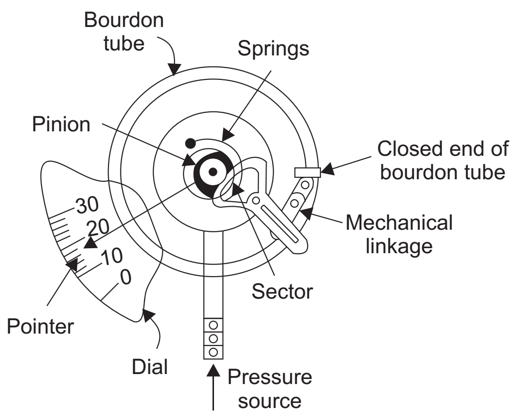


Fig. 1.12 Bourdon Tube

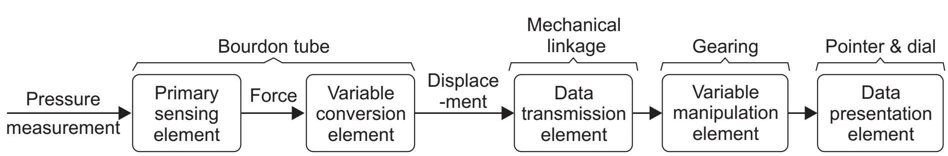


Fig. 1.13 Block Diagram of Bourdon Tube Pressure Gauge

Let us Summarize

1. Electronic instrumentation is a science of measurement and its control. It requires an instrument for measurement.
2. An instrument is defined as a device that determines the magnitude or value of a quantity, such as temperature, pressure, level, current, voltage, resistance, power, force, velocity, and distance.

3. The measure of any physical quantity is expressed in both kind and magnitude. Each kind of physical quantity is expressed in standard measure called unit. The magnitude of a physical quantity is defined as the product of a numerical ratio and the unit.
4. Two kinds of units are used to measure a physical quantity known as fundamental and derived units.
5. The three basic fundamental mechanical units in SI system are metre, kilogram, and second, as the units of length, mass, and time, respectively. The derived mechanical units, such as force, work, power and energy can be obtained from fundamental units.
6. In SI electrical unit system, the unit of electric current is considered as the fundamental unit while the units of charge, potential difference, emf, and voltage; resistance and conductance; magnetic flux and flux density; inductance and capacitance are categorized as derived electrical units.
7. Celsius scale and Kelvin scale are the two SI temperature scales for measuring temperature.
8. The quality of a quantity is called dimension. The dimension of a quantiy distinguishes it from other quantities.
9. The systems using centimetre, gram, and second as the fundamental mechanical units for the purpose of science and engineering for many years are called CGS absolute systems.
10. CGS system was categorized as electromagnetic units system (or emu) and electrostatic units system (or esu).
11. As many CGS units were too small or too large for practical engineering applications, a system of practical units was used. These units are derived from absolute units or by referring to arbitrary standards as they are easy to manage and handle.
12. MKS system was introduced by Professor Giorgi in 1901. The three fundamental mechanical units in this system were taken to be metre, kilogram, and second.
13. Along with three fundamental quantities one more quantity was used to connect electrical and mechanical quantities, that is, permeability of free space taken as $\mu_o = 10^{-7}$.
14. A physical representation of a unit of measurement is called standard of measurement. The standards of measurement are classified on the basis of their functionality and areas of application into four categories that include international, primary, secondary, and working standards.
15. The frequency and time standards can be understood by the development and refinement of atomic resonators through which frequency of an oscillator can be controlled to a great extent.
16. The SI unit of resistance is the absolute value of ohm which is defined in terms of the fundamental units of length, mass, and time. The SI unit of capacitance is farad which is measured with Maxwell DC commuted bridge.
17. The SI unit of inductance is henry, denoted as H and its dimensions in CGS emu system is $[\mu L]$ which shows that it depends only on the length provided that permeability is considered to be dimensionless.
18. IEEE standards are the standards for the controls on instrument's front panels, for test and measuring procedures, for electrical hardware and installations in particular situations.
19. There are two methods of measurement, namely direct comparison method and indirect method.
20. The three main functional elements of the measurement system include primary sensing element, variable conversion and manipulation element, and data presentation element.

EXERCISES

Fill in the Blanks

1. The units whose sizes cannot be chosen independently are _________.
2. The internal resistance of Weston saturated cell ranges from _________ to _________.
3. _________ material is used for making wires for resistance standards.
4. The dimensions of resistance in CGS emu system are _________.
5. The symbol nm stands for _________.

Multiple Choice Questions

1. The voltage standards based on Zener sources
   (a) can be designed to cover wide range of voltages
   (b) are reverse biased
   (c) are immune to short circuit effects
   (d) all the above
2. The fundamental units in SI system are
   (a) metre, kilogram, second, ampere, kelvin
   (b) metre, kilogram, second
   (c) metre, kilogram, second, ampere, kelvin, candela
   (d) metre, kilogram, second, ampere
3. The working standards of capacitance
   (a) are stable
   (b) have low temperature coefficient
   (c) have low dissipation factor
   (d) all of these
4. The dimensions of potential difference in CGS emu system are
   (a) $[\mu^{1/2}M^{1/2}L^{3/2}T^{-2}]$
   (b) $[\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-2}]$
   (c) $[\mu M^{3/2}L^{1/2}T]$
   (d) $[\mu^{-1}M^{1/2}LT^{-2}]$
5. The primary standard for inductance measurement is
   (a) Campbell
   (b) commuted DC bridge
   (c) Weston saturated cell
   (d) none of these

State True or False

1. The dimensions of inductance in CGS esu system are $[\varepsilon^{-1}L^{-1}T^{2}]$.
2. The international standard of length is defined as the wavelength in vacuum of radiation of krypton-86 atom in its two specified transitions.
3. The three main atomic standards for frequency and time are cesium, hydrogen, and rubidium.
4. The derived electrical units in SI system are force, work, energy, and power.
5. The Kelvin scale starts at absolute zero of temperature which corresponds to –373.15°C.

Descriptive/Numerical Questions

1. Define the term unit and its different types.
2. Explain fundamental and derived units in detail.
3. Discuss the different types of standards of measurement.

4. Describe CGS system of units and explain all electromagnetic and electrostatic unit systems.
5. Describe how MKS system of units is better than CGS system of units. Differentiate between rationalized MKS and MKS system of units.
6. Differentiate between international, primary, secondary, and working standards.
7. What are IEEE standards? How do these standards differ from those maintained by national standard laboratories.
8. Describe the international standards of mass, length, and volume.
9. Explain time and frequency standards of measurement.
10. What are the different electrical standards of measurement?
11. Explain the primary standard for luminous intensity.
12. Explain the primary and secondary standards for voltage measurement.
13. Explain the primary standards of measurement for inductance.
14. Describe the various methods of measurements with the help of suitable block diagrams.

2

Measurement Characteristics

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Explain significant figures, and performance characteristics, including static and dynamic characteristics
- Discuss the various errors in measurement, including gross errors, systematic errors, absolute and relative errors, and random errors
- Differentiate among various types of errors and apply measures to reduce them
- Learn the concept of combination of errors and limiting errors
- Analytically reduce errors using different methods, such as statistical analysis, and probability of errors

2.1 INTRODUCTION

In the previous chapter, we studied the measurements of physical quantities, measurement systems, various units associated with measurements, and the standards for measurements. In addition, there are many other characteristics associated with a measurement. An appropriate instrument on the basis of certain characteristics must be chosen to perform a specific measurement. Every measuring instrument possesses some inaccuracies or errors due to defect in the instrument, wrong observance, or environmental factors. These errors further get combined to each other and result in even more errors. Although the errors may cancel out each other completely but we always assume the worst case of combination of errors while performing measurements. Hence, an analysis of these errors is must.

The accuracy and precision of an instrument is defined by the material used for making an instrument, the design selected, and the skills used to construct it. Thus, while choosing an instrument for a particular application it is necessary to know the desired level of accuracy. When a higher degree of accuracy is required for an instrument, expensive materials and highly skilled labour is employed. For example, while choosing a thermometer for room temperature measurement, an accuracy of $\pm 0.5°C$ would not cause any significant problem whereas using the same thermometer for measuring the temperature of a chemical reaction will give erroneous results as a variation of $0.5°C$ can cause significant change in rate of reaction or products of process. Thus, in order to minimize such conditions, manufacturers specify certain specifications along with the instruments. In this chapter, we will discuss the measurement characteristics and distinguish them on the basis of their desirability. Also, we will discuss various types of errors, their causes and methods to remove them with mathematical or statistical analysis.

2.2 SIGNIFICANT FIGURES

The significant figures provide the information about the magnitude and precision of a measured quantity. The number of significant figures in the measurement of a quantity indicates the precision with which it is measured. The more the number of significant figures, the more precise is the quantity. For example, if the value of the current is specified as 345 A, its value must be taken as close as possible to 345 A rather than to 344 A or 346 A. Now, if the value of the current is specified as 345.0 A, then its value must be taken close to 345.0 A than to 345.1 A or 344.9 A. Here, 345 A contains three significant figures, while 345.0 has four significant figures. Thus, having more significant figures implies better precision than the former. This example shows the significance of a zero after a decimal point.

It is to be noted that the number of significant figures in the result of the calculation must be equal to the number of significant figures in the original quantities. For example, consider the calculation involving two original quantities, that is a current I of 2.34 A and a voltage V of 5.42 V. The resistance R is obtained as:

$$R = \frac{V}{I} = \frac{5.42}{2.34} = 2.31623932\ \Omega$$

Here, the resistance is expressed using nine digits. However, this is not the appropriate answer. The correct answer should contain the same number of significant figures as in the original quantities. Hence, the correct value of resistance R in the example should contain three significant figures, that is 2.31 Ω.

Note: *When the number of digits in original quantities is not equal, then the number of digits in the result should be taken equal to the number of digits in the least precise number.*

2.3 PERFORMANCE CHARACTERISTICS

A specific measurement can be done using various measuring instruments. Thus, to select a suitable instrument, the performance characteristics of these instruments must be known and compared with each other. These characteristics are divided into two parts, namely, *static* and *dynamic* characteristics.

2.3.1 Static Characteristics

Performance characteristics which measure slowly varying or unvarying data and thereby indicate the response of the instruments are known as **static characteristics**. Static calibration is used to obtain static characteristics. Static characteristics of an instrument include *accuracy*, *precision*, *resolution*, *repeatability*, *reproducibility*, *static error, sensitivity*, and *drift* as discussed hereafter.

Accuracy

Accuracy is defined as the degree of closeness or exactness of a measured value compared to the expected value (also termed as **true** or **desired value**) of the quantity measured. The manufacturer of the instrument also specifies accuracy as the maximum amount of error which will not be exceeded. Therefore, it can be termed as conformity to truth. For example, if accuracy of ±1% is specified for a 100 V voltmeter, the true value of the voltage lies between 99 V and 101 V with maximum error for any observed value or reading not exceeding ±1V. The accuracy of an instrument can be specified in either of the following ways.

- **Point accuracy:** This type of accuracy does not specify the general accuracy of an instrument, rather it gives the information about the accuracy at only one point on its scale. However, constructing a table of accuracy at a number of points in the range of the instrument may help in calculating the general accuracy of an instrument.
- **Percentage of scale range accuracy:** In this case, the accuracy of a uniform scale instrument is expressed in terms of scale range. This type of accuracy specification can be highly misleading. For example, consider a thermometer having a range and accuracy of 200°C and ±0.5% of scale range, respectively. This implies that for a reading of 200°C, the accuracy is ±0.5%, while for a reading of 40°C, the same accuracy yields a greater error as computed herebelow:

$$\frac{200}{40} \times (\pm 0.5) = 2.5\%$$

- **Percentage of true value accuracy:** In this case, the accuracy is defined in terms of true value of the quantity being measured. Thus, the errors are proportional to the readings, that is, smaller the reading, lesser is the error. This method is considered the best way to specify the accuracy of an instrument.

Precision

Precise means ‘sharply or clearly defined’. It is a measure of the degree up to which repeated readings in a group of measurements are similar, provided they are measured under same conditions. It is to be noted that a precise reading need not be accurate or vice versa. For example, consider a voltmeter having a very high degree of precision on account of its finely divided, clearly legible, incredibly sharp pointer, and mirror-backed distinct scales which remove parallax. Assume that this device can measure voltage to a value of 1/1000th of a volt but the zero adjustment of the voltmeter is not accurate. Thus, the voltmeter yields highly precise but not accurate readings due to inaccurate zero adjustment.

There are two characteristics of precision, namely, *conformity* and *number of significant figures*. For example, consider that the true value of a resistance is given as 2,485,672 Ω, which is indicated as 2.5 MΩ when measured using an ohmmeter. The ohmmeter will consistently indicate this reading and consequently due to scale limitation, the instrument produces a precision error. It is evident from the above example that due to lack of significant figures, the result or measurement is not precise, though its closeness to true value implies its accuracy. Thus, conformity is necessary but not a sufficient condition for precision and vice versa.

Resolution

Resolution, also termed as **discrimination**, is defined as the smallest change in the measured quantity to which an instrument will respond. The needle of the instrument will show no deflection unless a change equal to its resolution is achieved in the input. For example, consider a 200 V voltmeter in which the needle shows a deflection or reading change from zero only when the minimum input is 1 V. This instrument cannot be used to measure 50 mV because its resolution is 1 V. In general, there will be no effect on the instrument for any input or change in input less than 1 V.

Repeatability

Repeatability, also termed as **test-retest reliability**, is the variation of scale reading when the input is applied randomly. It provides the closeness with which we can measure a given input value repeatedly. Sometimes, repeatability is also termed as **inherent precision of the measurement equipment**. If an input of a constant magnitude is applied intermittently, the output reading must be the same, otherwise the instrument is said to have poor repeatability. Figure 2.1 shows the relation between input and output with ± repeatability.

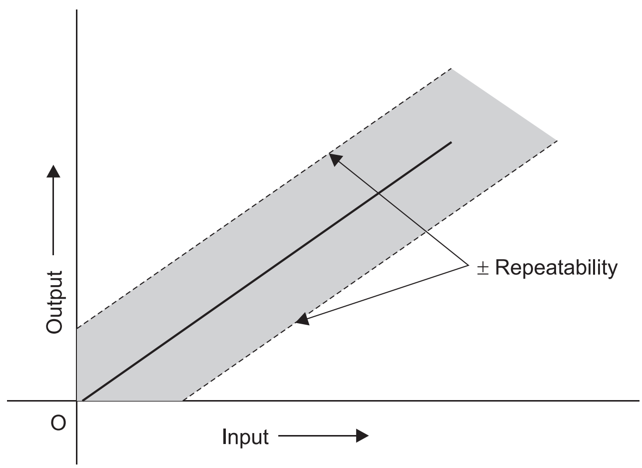


Fig. 2.1 Graph showing Repeatability

Reproducibility

Similar to repeatability, reproducibility is also a measure of the closeness with which we can measure a given input repeatedly when the input is applied constantly. However, it is measured over a period of time. It indicates the steady state response of an instrument. The instrument should have good reproducibility, which is possible only when the output reading of the instrument remains the same when an input with a constant magnitude is applied continuously over a period of time. Otherwise, the instrument is said to have poor reproducibility.

Static error

Error is defined as the difference between the measured value and the true value. If it is not possible to measure true value of a quantity so the best measured value is used. The error may be equal to or less than the value given by accuracy (which indicates the maximum error) for a particular instrument. When this error difference is constant, the error is said to be **static error**.

Sensitivity

Sensitivity is defined as the ratio of change in output with respect to the change in input of the instrument. For example, consider a voltmeter in which the input voltage V_i changes by 0.2 V, then the output reading should also change by 0.2 V. Therefore, the sensitivity is expressed as $\Delta V_o / \Delta V_i$. Thus, it represents how truly an instrument responds to a change in the input.

The graphical representation of input-output relationship of a quantity helps to determine sensitivity as the slope of the calibration curve. The change in input is expressed as ΔQ_i and change in output is expressed as ΔQ_o. When the calibration curve is a straight line, sensitivity $\Delta Q_o / \Delta Q_i$ is constant over the entire range of the instrument as shown in Figure 2.2.

In the other case, when the curve is not a straight line, sensitivity is no more constant and varies with input as shown in the figure.

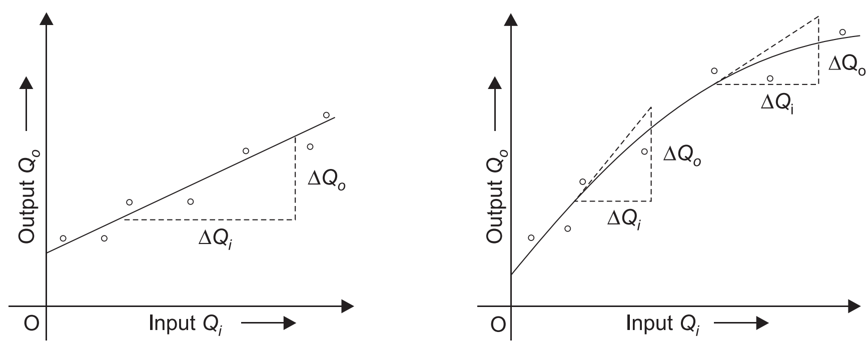


Fig. 2.2 Graphical Representation of Sensitivity

Drift

Drift is defined as the gradual shift in the indication of an instrument over a period of time during which true value of the quantity does not change. It is categorized into three types, namely, *zero drift*, *span drift*, and *zonal drift*.

Zero drift

The same amount of shifting in whole calibration is termed as zero drift, also termed as **calibration drift** as shown in Figure 2.3(a). It can occur due to many reasons including slippage, undue warming up of electronic tube circuits, or if an initial zero adjustment in an instrument is not made.

Span drift

Span drift is defined as the drift which increases gradually with the deflection of the pointer [see Figure 2.3(b)]. It is also termed as **sensitivity drift** and is not constant. The combined zero and span drift is shown in Figure 2.3(c).

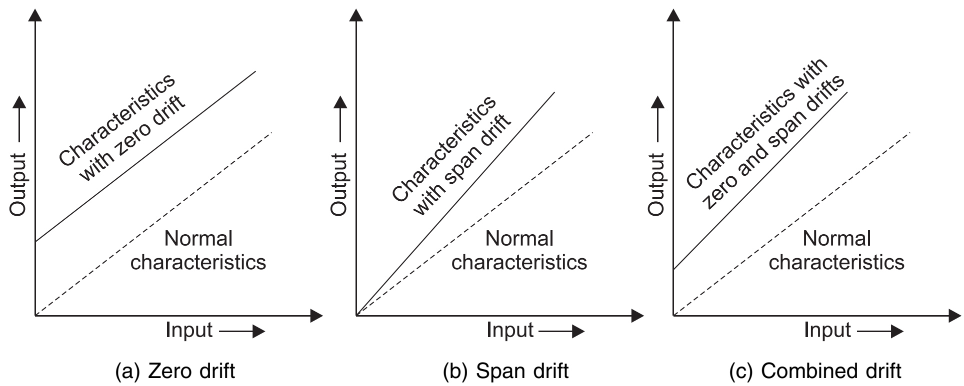


Fig. 2.3 Types of Drift

Zonal drift

If the drift occurs only in a particular zone of an instrument, it is said to be **zonal drift**. It may be caused due to various environmental factors, such as, change in temperature, thermal emfs, mechanical vibrations, wear and tear, stray electric and magnetic fields, and high mechanical stresses developed in some parts of the instruments and systems.

2.3.2 Dynamic Characteristics

The characteristics which indicate the response of instruments that measure time-varying quantities in which the input varies with time and so does the output are known as **dynamic characteristics**. The time-varying input could be a step input, ramp or sinusoidal input.

Various factors, such as mass, fluid capacitance, thermal capacitance or electric capacitance cause slowness in the instruments that are used to measure time-varying quantities. This, consequently, results in delay of the response of instruments with respect to the change in the measured variables. In addition to this, some delay may arise when the instrument is waiting for some other reaction to occur. Therefore, it is important to study the dynamic behavior of the instruments. The dynamic characteristics of an instrument include *measuring lag*, *fidelity*, *speed of response*, and *dynamic error*.

Measuring lag

Any change in the measured quantity causes delay or retardation in the response of an instrument. This delayed response is known as **measuring lag**. It can be categorized into two types, namely *retardation type* and *time delay type*. When a change in the measured quantity occurs and the instrument responds immediately, it is said to be **retardation type**. However, in **time delay type**, the instrument responds after a dead time after the input is applied. Dead time causes dynamic error by simply shifting the system response along the time scale. The time delay type measuring lag is usually very small and can be ignored. However, when the quantity being measured varies quickly, the performance of the system is adversely affected by the dead time.

Fidelity

The quality of reproducing faithful values is known as **fidelity**. It is defined as the ability of an instrument to indicate the changes in measured quantity without the dynamic error.

Speed of response

The quickness with which an instrument responds to changes in the measured quantity is known as **speed of response**.

Dynamic error

Dynamic error, also called **measurement error**, is defined as the difference between the true value of a time-varying quantity and the value indicated by the measurement system in the absence of static error.

2.4 ERRORS IN MEASUREMENT

Various factors affect measurement, some of which are due to the instruments themselves and some others are due to their inappropriate use. Common sources of errors in measurement include improper functioning of hardware, poor design, insufficient knowledge of design conditions and process parameters, poor maintenance, and design limitations. Depending upon these factors, errors can be categorized into different types, such as *gross errors*, *systematic errors*, *absolute and relative errors*, and *random errors*.

2.4.1 Gross Errors

Errors due to human mistakes in using instruments, recording observations, and calculating measurement results are known as gross errors (also termed as **human errors**). Sometimes the measurement may go wrong due to the operator even if the instrument is good. This type of error may occur due to various reasons as follows.

- The observer may misread the scale which may result in wrong reading. For example, 21.3°C may be read as 31.3°C due to an oversight.
- The observer may transpose the readings while recording. For example, 24.9°C may be recorded as 29.4°C.
- The units of the reading may be misunderstood in case of digital devices which do not display units along with readings. For example, 21mV may be recorded as 21V by a beginner.
- A wrong scale may be chosen to record observations in case of analog devices which have multiple scales.

Gross errors will always be present, since a human is always involved in measurements. These cannot be eliminated completely, however, they can be avoided to a certain extent by implementing the following measures:

- Carefully reading and recording observations.
- A large number of readings should be taken at different reading points in order to avoid re-reading with same error and then plotting these readings in graphical form or substituting them in a proper equation assures reduced errors.
- Using proper analog instruments zeroed electrically and mechanically.

2.4.2 Systematic Errors

Measurement of all quantities is affected by systematic errors. They are sometimes referred to as **bias** and defined as a constant uniform deviation of the operation of an instrument. The shortcomings of the instrument for example, worn out or defective parts, environmental effects, ageing effects are all included in these errors. They are broadly classified into three categories namely, *instrumental errors*, *observational errors*, and *environmental errors*.

Instrumental errors

Instrumental errors occur in measurements due to the instrument even when human errors are avoided. They may be divided into three parts depending upon their causes as discussed below.

Misuse of instruments

Sometimes a good instrument may give wrong results due to its inappropriate usage. Misuse of instruments may be due to improper zero adjustment, using connecting wires of very high resistance, and poor initial adjustments. They give erroneous results despite the fact that the instrument used is good in condition. These practices cause errors but may not cause permanent damage to the instrument.

Using the instruments contrary to the manufacturer's specifications and instructions also produces errors and may cause permanent damage to the instruments due to overheating and overloading, and sometimes it may even lead to the system failure.

Inherent shortcomings of instruments

The mechanical structure of the instruments gives rise to such errors. They may occur due to construction, operation, or calibration of the measuring devices or instruments. For example, friction in the bearings of various moving components, stretching of spring, irregular spring tension due to improper handling or overloading of device result in errors. These errors may cause the instrument to read too low or too high along its entire scale.

Loading effects

Indicating instruments, such as voltmeter, ammeter when connected across a circuit change its conditions to some extent. Thus, the measured quantity will depend on the way measuring device is connected across the circuit or the method employed for its measurement. For example, a calibrated voltmeter when connected across high resistance circuit will give a misleading reading whereas the same voltmeter when connected across a low resistance circuit may give a more dependable reading. Thus, the voltmeter is said to have a loading effect on the circuit, resulting in changing the actual circuit conditions by the measurement process.

Methods of prevention

We have studied different types of instrumental errors. Some of them are inherent while some others result due to carelessness of the operator. The methods to eliminate them are listed below.

- Choosing a suitable instrument for a particular application.
- Carefully planning the procedure for measurement.

- The instrument should be carefully calibrated against a standard.
- Precautions should be taken to ensure that the instrument operates properly.
- Continuously monitoring for faults by checking for erratic behaviour, reproducibility and stability of results.
- After determining the amount of instrumental errors, appropriate correction factors should be applied.
- Loading effects should be considered while planning any measurement.

Observational errors

The errors that are introduced by an observer are termed as **observational errors**. For example, while reading a meter scale, estimation error may get incurred or if the pointer of the voltmeter is somewhat above the surface of the scale, error due to parallax may get introduced when the line of vision of the observer is not exactly above the pointer as shown in Figure 2.4. The practice of the observer in holding head too far in the right or left while noting down the observation continues to lessen the accuracy in the measurement and introduces errors.

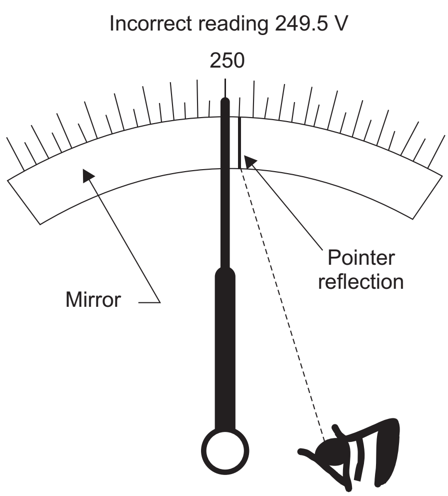


Fig. 2.4 Parallax Error when the Observer's Line of Vision is not Exactly above the Pointer

When two individuals note the observations, they may not be the same due to their different sensing capabilities and therefore, affect the accuracy of the measurement. For example, one observer may note down the reading at a particular time while the other observer may read it too soon causing an error. Also, when light and sound measurements are involved, two different experimenters produce different results as no two observers have the same physical responses.

Digital instruments are used to avoid the above mentioned observational errors in which the observation is displayed in the form of digits and hence, no error can occur on account of sensing and observations. Also, the parallax errors can be minimized by using instrument having highly accurate meters provided with mirrored scales as the pointer's image is hidden by the pointer and observer's eye is directly in line with the pointer [see Figure 2.5]. Parallax errors can be eliminated by using the instruments having same plane for the pointer and scale.

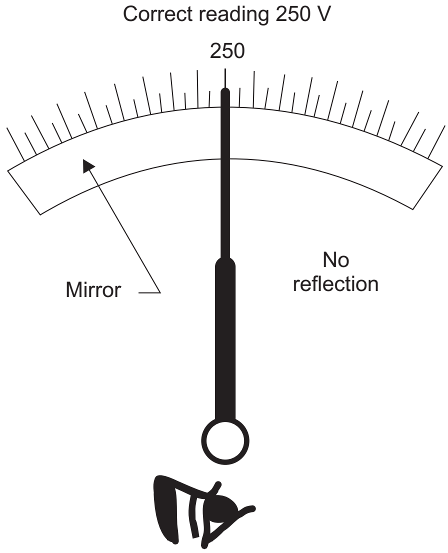


Fig. 2.5 Parallax Errors Minimized by using Mirror Scaled Meters

Environmental errors

The errors which arise on account of environmental factors are known as **environmental errors**. They occur due to the conditions in the surrounding area of the instrument (that is, external to the device), that include, effects of changes in pressure, temperature, dust, humidity, vibrations, and electrostatic or magnetic fields. For example, a change in ambient temperature of a moving coil instrument causes a change in elasticity of its spring and thus, a change in the reading results. Various corrective measures that can be taken to reduce these errors are as follows.

- Precautions should be taken to ensure that the instrument operates in constant conditions. For example, a temperature controlled enclosure can be used to keep the temperature constant around the instrument.
- Electrostatic or magnetic shield may be used to protect the instrument from electrostatic or magnetic fields.
- Hermetically sealing the instrument to eliminate the effects of dust and humidity.
- Equipments which are immune to environmental changes can be used. For example, resistance materials with a very low resistance temperature co-efficient can be used to minimize the variations in resistance with temperature.

2.4.3 Absolute and Relative Errors

The **absolute error**, denoted as Δe, is defined as the amount of physical error in any measurement. This physical error is expressed in the same units of the quantity being measured.

For example, a 300 Ω resistance is said to have a possible error of ±20 Ω. Here, ±20 Ω represents an absolute error in terms of the same unit, that is, ohm (Ω). Mathematically, the absolute error is expressed as:

$$\Delta e = A_m - A_t \qquad ...(1)$$

where A_m is the measured value, and A_t is the true or expected value.

Relative error, also known as **fractional error,** denoted as e_r, is defined as the ratio of the absolute error to the true value of the quantity being measured. It is usually expressed in percentage, ratio, parts per thousand, or parts per million relative to the total quantity. For example, a 300 Ω resistance is said to have a possible error of ±8 %. It gives an indication of the quality of the measurement relative to the size of the measured quantity. Mathematically, relative error is expressed as:

$$\text{Relative error} = \frac{\text{Absolute error}}{\text{True value of the quantity}}$$

Or, it can be expressed as:

$$e_r = \frac{\Delta e}{A_t} \qquad ...(2)$$

The percentage relative error $\%\ e_r$ is expressed as:

$$\%\ e_r = e_r \times 100 = \frac{\Delta e}{A_t} \times 100 \qquad ...(3)$$

where e_r denotes the relative error.

Also, the true value A_t can be expressed by rearranging and combining Equations (1) and (2) as:

$$A_t = \frac{A_m}{1 + e_r}$$

Example 1 An ammeter reads 6.7 A and the true value of the current is 6.54 A. Determine the absolute error and the correction for this instrument.

Solution: Given that: measured value A_m = 6.7 A, and true value A_t = 6.54 A

The absolute error Δe is given as:

$$\Delta e = A_m - A_t \qquad \text{[Refer to Eqn. (1)]}$$

Substituting the given values, we get:

$$\Delta e = 6.7 - 6.54 = 0.16 \text{ A}$$

The correction for this ammeter is obtained as:

$$-\Delta e = -0.16 \text{ A}$$

Example 2 The current through a resistor is 2.5 A, but the measurement yields a value of 2.45 A. Calculate the percentage error of measurement.

Solution: Given that: measured value $A_m = 2.45$ A and true value $A_t = 2.5$ A

The absolute error Δe is given as:

$$\Delta e = A_m - A_t \qquad \text{[Refer to Eqn. (1)]}$$

Substituting the given values, we get:

$$\Delta e = 2.45 - 2.5 = -0.05 \text{ A}$$

Now, the relative error e_r is given by the relation:

$$e_r = \frac{\Delta e}{A_t}$$

$$\Rightarrow \qquad e_r = \frac{-0.05}{2.5} = -0.02$$

The percentage error $\%\ e_r$ is obtained as:

$$\%\ e_r = e_r \times 100 \qquad \text{[Refer to Eqn. (3)]}$$

$$\Rightarrow \qquad \%\ e_r = e_r \times 100 = -0.02 \times 100 = -2\%$$

2.4.4 Random Errors

Errors which occur due to unknown factors are termed as **random errors**. These errors persist even after all the substantial removal or correction of the systematic errors and hence, referred to as **residual errors**. These errors are normally small but of great concern when a high degree of accuracy is required. They generally occur due to a large number of small factors which cause fluctuation from one measurement to another. Hence, a variation in successive readings occurs even when the measurement is done in an ideal environment and the instrument is calibrated accurately before the measurement. For example, if the value of a current is to be monitored using an ammeter after equal time intervals of 20 minutes, a slight variation in the observations can be seen.

These random errors cannot be controlled by known control methods and can only be analyzed statistically. The best approximation of the true value can be obtained by increasing the number of readings and using statistical means.

2.5 COMBINATION OF ERRORS

When a quantity is calculated by a combination of two or more measurements made on different instruments, the errors due to inaccuracies of these instruments get combined in a way to increase the amount of the error. This will further depend upon the type of combination, that is, whether the quantities are combined as *sum*, *difference*, *product*, or *quotient*.

2.5.1 Sum of Quantities

The total error is the sum of the absolute error in each measurement, when the quantity determined is the sum of the two measurements. Let A and B be the two different measurements with ΔA and ΔB as their measured absolute errors, respectively. The sum of these measurements can be represented as:

$$S = (A \pm \Delta A) + (B \pm \Delta B)$$

Or, it can be written as:

$$S = (A + B) \pm (\Delta A + \Delta B)$$

Thus, the total possible error is the sum of the absolute values of individual errors.

2.5.2 Difference of Quantities

Similar to the sum of quantities, here also, the total error is the sum of the absolute errors in each measurement, when the quantity determined is the difference of two measurements. Let A and B be two different measurements with ΔA and ΔB as their respective measured absolute errors. The difference of these measurements can be represented as:

$$D = (A \pm \Delta A) - (B \pm \Delta B)$$

Or, it can be written as:

$$D = (A - B) \pm (\Delta A + \Delta B)$$

Thus, the total possible error is the sum of the absolute values of individual errors.

Note: *When the errors are given in percentage, they are converted to their absolute values before taking sum or difference of quantities.*

2.5.3 Product of Quantities

When the resultant quantity is the product of two or more quantities, the sum of the percentage errors in each quantity gives the percentage error of the resultant quantity. Let A and B be two different measurements and ΔA and ΔB be their respective measured absolute errors. The product of these measurements can be represented as:

$$P = AB$$

Or, it can be written as:

$$P = (A \pm \Delta A)(B \pm \Delta B)$$

$$\Rightarrow \quad P = AB \pm A\Delta B \pm B\Delta A \pm \Delta A\Delta B$$

Now, since $\Delta A\Delta B$ is a very small quantity, neglecting this term, we get:

$$P \simeq AB \pm (A\Delta B + B\Delta A)$$

And, the percentage error in P is calculated as:

$$= \frac{A\Delta B + B\Delta A}{AB} \times 100\%$$

$$= \left(\frac{A\Delta B}{AB} + \frac{B\Delta A}{AB}\right) \times 100\%$$

It can be written as:

$$= \left(\frac{\Delta B}{B} + \frac{\Delta A}{A}\right) \times 100\%$$

Thus, we get:

$$\% \text{ error in } P = (\% \text{ error in } A) + (\% \text{ error in } B)$$

2.5.4 Quotient of Quantities

Similar to the product of quantities, here also the sum of percentage error of each quantity gives the percentage error of the quotient of the quantities. Mathematically, it can be represented as:

$$\% \text{ error in } \frac{A}{B} = (\% \text{ error in } A) + (\% \text{ error in } B)$$

where A and B are two quantities.

2.5.5 Quantity Raised to a Power

The percentage error in a quantity A raised to the power of another quantity B, that is, A^B can be given as:

$$\% \text{ error in } A^B = B(\% \text{ error in } A)$$

Example 3 Calculate the maximum percentage error in the sum and difference of two measured voltages when $V_1 = 100 \text{ V} \pm 1\%$ and $V_2 = 80 \text{ V} \pm 5\%$.

Solution: Given that: $V_1 = 100 \text{ V} \pm 1\%$ and $V_2 = 80 \text{ V} \pm 5\%$

Expressing the voltages in absolute value of errors, we get:

$$V_1 = 100 \text{ V} \pm 1 \text{ V} \quad \text{and} \quad V_2 = 80 \text{ V} \pm 4 \text{ V}$$

Sum of the quantities S is expressed as:

$$S = V_1 + V_2 = (100 \text{ V} \pm 1 \text{ V}) + (80 \text{ V} \pm 4 \text{ V}) \quad \text{[Refer to Section 2.5.1]}$$

$$= 180 \text{ V} \pm (1 \text{ V} + 4 \text{ V})$$

$$= 180 \text{ V} \pm 5 \text{ V}$$

Thus, the sum of voltages can be expressed in terms of percentage error as $180 \text{ V} \pm 2.8\%$.

Now, the difference of the quantities D can be written as:

$$D = V_1 - V_2 = (100 \text{ V} \pm 1 \text{ V}) - (80 \text{ V} \pm 4 \text{ V}) = 20 \text{ V} \pm 5 \text{ V} \qquad \text{[Refer to Section 2.5.2]}$$

In terms of percentage error, it can be written as:

$$D = 20 \text{ V} \pm 25\%$$

Example 4 Two capacitors of $100 \pm 1.4\ \mu\text{F}$ and $80 \pm 1.5\ \mu\text{F}$ are connected in parallel. Determine the error of the resultant capacitance in μF and in percentage.

Solution: Given that: $C_1 = 100 \pm 1.4\ \mu\text{F}$ and $C_2 = 80 \pm 1.5\ \mu\text{F}$

Since the capacitors are connected in parallel, the resultant capacitance C is given as:

$$C = C_1 + C_2 = (100 \pm 1.4) + (80 \pm 1.5)\ \mu\text{F}$$

$$C = 180 \pm 2.9\ \mu F$$

Thus, the value of resultant capacitance in absolute error is 180 ± 2.9 μF.

Percentage error is computed as:

$$= \pm \frac{2.9}{180} \times 100 = \pm 1.62\%$$

Thus, the value of resultant capacitance in percentage error is 180 ± 1.62%.

2.6 STATISTICAL ANALYSIS

Statistical analysis is performed on measurement data to analytically determine the uncertainty of final measurement result. This method is employed when the deviation of measurement from its true value is to be determined and the reason for specific error is unpredictable. A large number of readings are taken to analyze the data statistically, thereby providing information about the correctness of the observations taken. However, it cannot remove a fixed bias which is present in all readings, hence, while performing statistical analysis of data, we must ensure that systematic errors are small as compared to random errors.

2.6.1 Arithmetic Mean

A large number of readings should be taken to find the best approximation. The method employed to find the most probable value of a measured variable on taking a number of readings of the same quantity which are not exactly equal to each other is **arithmetic mean**. The arithmetic mean for n readings $x_1, x_2, x_3, \ldots x_n$ is given as:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\sum_{n=1}^{n} x_n}{n} \qquad \ldots(4)$$

where $\bar{x}$ is the arithmetic mean. Sometimes, while calculating the arithmetic mean or average value of a number of readings, it is found that the difference between some measurements and the mean is much larger as compared to the difference between other measurements and the mean. If the number of such readings is small, we can reject the readings with larger difference and determine the average from other readings. However, if large number of measurements differ greatly from the mean, they all should be rejected and measurements should be repeated again. The effect of random errors can be significantly minimized by arithmetic mean.

2.6.2 Deviation

The departure of a measured value from the arithmetic mean of a group of measurements is known as **deviation**. It is expressed as:

$$d_1 = x_1 - \bar{x}$$

$$d_2 = x_2 - \bar{x}$$

$$d_n = x_n - \bar{x}$$

where d_1 is the deviation of first reading and so on. The deviations can be positive or negative. The algebraic sum of all deviations is given as:

$$\begin{aligned}\text{Algebraic sum} &= d_1 + d_2 + d_3 + \ldots + d_n \\ &= (x_1 - \bar{x}) + (x_2 - \bar{x}) + (x_3 + \bar{x}) + \ldots + (x_n - \bar{x}) \\ &= (x_1 + x_2 + x_3 + \ldots + x_n) - (n\bar{x}) \qquad \ldots(5)\end{aligned}$$

Now, from Equation (4), we have:

$$(x_1 + x_2 + x_3 + \ldots + x_n) = n\bar{x} \qquad \ldots(6)$$

Thus, from Equations (5) and (6), we notice that the algebraic sum of all deviations is zero, that is:

$$d_1 + d_2 + d_3 + \ldots + d_n = 0$$

Average deviation

The **average deviation** is defined as the average of the absolute values of deviations. The absolute value implies that the signs of the deviations are neglected. Mathematically it is expressed as:

$$D = \frac{|d_1| + |d_2| + |d_3| + \ldots + |d_n|}{n} = \frac{\sum |d_n|}{n} \qquad \ldots(7)$$

It indicates the precision of the instruments used in measurement. A low average deviation between the readings implies a highly precise instrument and vice versa.

2.6.3 Standard Deviation

The standard deviation or the **root mean square deviation** of an infinite number of measurements is defined as the square root of the sum of all individual deviations squared and divided by the number of readings. It is denoted by σ and is expressed as:

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n}} = \sqrt{\frac{\sum d_n^2}{n}} \qquad \ldots(8)$$

However, practically the number of readings cannot be infinite. Thus, the standard deviation for a finite number of readings is expressed as:

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n-1}} = \sqrt{\frac{\sum d_n^2}{n-1}} \qquad \ldots(9)$$

For a good measurement, the value of standard deviation for random errors is low.

2.6.4 Variance

The square of the standard deviation is known as **variance** or **mean square deviation**. It is denoted by σ^2 and is expressed as:

$$\text{Variance} = \sigma^2 \qquad \text{...(10)}$$

Thus, from Equation (8), we get:

$$\sigma^2 = \frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n} = \frac{\sum d_n^2}{n} \qquad \text{...(11)}$$

For finite number of readings, we have:

$$\sigma^2 = \frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n-1} = \frac{\sum d_n^2}{n-1} \qquad \text{...(12)}$$

Example 5 A set of current measurements were taken and readings were recorded as 11.3 mA, 11.6 mA, 10.9 mA, 12.1 mA, 11.0 mA, 12.5 mA, and 11.9 mA. Calculate:

(a) arithmetic mean

(b) deviations from the mean

(c) average deviation

Solution: (a) The arithmetic mean $\bar{x}$ is computed from Equation (4) as:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\sum_{n=1}^{n} x_n}{n}$$

Here, n = 7. Thus, the mean is calculated to be:

$$\bar{x} = \frac{11.3 + 11.6 + 10.9 + 12.1 + 11.0 + 12.5 + 11.9}{7} = \frac{81.3}{7}$$

$$\Rightarrow \qquad \bar{x} = 11.6$$

(b) The deviation d from the mean of each measurement is given as:

$$d_n = x_n - \bar{x} \qquad \text{[Refer to Section 2.6.2]}$$

Substituting the value for each reading, we get the corresponding deviation from mean as:

$$d_1 = x_1 - \bar{x} = 11.3 - 11.6 = -0.3$$

$$d_2 = x_2 - \bar{x} = 11.6 - 11.6 = 0$$

$$d_3 = x_3 - \bar{x} = 10.9 - 11.6 = -0.7$$

$$d_4 = x_4 - \bar{x} = 12.1 - 11.6 = 0.5$$

$$d_5 = x_5 - \bar{x} = 11.0 - 11.6 = -0.6$$

$$d_6 = x_6 - \bar{x} = 12.5 - 11.6 = 0.9$$

$$d_7 = x_7 - \bar{x} = 11.9 - 11.6 = 0.3$$

(c) The average deviation D is obtained as:

$$D = \frac{|d_1| + |d_2| + |d_3| + ... + |d_n|}{n} = \frac{\sum |d_n|}{n} \qquad \text{[Refer to Eqn. (7)]}$$

$$\Rightarrow \qquad D = \frac{0.3 + 0 + 0.7 + 0.5 + 0.6 + 0.9 + 0.3}{7}$$

$$D = \frac{3.3}{7} = 0.47$$

Example 6 A current passing through a resistor is recorded by 10 different observers and the readings obtained are as 100.1 A, 101.7 A, 100.9 A, 102.1 A, 101.5 A, 101.0 A, 100.0 A, 102.1 A, 102.3 A, and 101.3 A. Calculate:

(a) arithmetic mean
(b) standard deviation
(c) variance

Solution: Let us represent this data in tabular form to avoid any confusion.

Given readings (in amperes) x	*Deviation from mean,* $d_n = x_n - \bar{x}$	*Square of deviation,* d_n^2
100.1	−1.2	1.44
101.7	0.4	0.16
100.9	−0.4	0.16
102.1	0.8	0.64
101.5	0.2	0.04
101.0	−0.3	0.09
100.0	−1.3	1.69
102.1	0.8	0.64
102.3	1.0	1.0
101.3	0	0
$\Sigma x_n = 1013.0$	$\Sigma \lvert d_n \rvert = 6.4$	$\Sigma d_n^2 = 5.86$

(a) Arithmetic mean $\bar{x}$ is obtained as:

$$\bar{x} = \frac{\sum x_n}{n} = \frac{1013.0}{10} = 101.3 \text{ A} \qquad \text{[Refer to Eqn. (4)]}$$

(b) Standard deviation σ for a finite number of observations is given by Equation (9) as:

$$\sigma = \sqrt{\frac{\sum d_n^2}{n-1}} = \sqrt{\frac{5.86}{9}} = 0.806$$

(c) Variance σ^2 is calculated as:

$$\text{Variance} = \sigma^2 = (0.806)^2 = 0.649 \text{ A}^2 \qquad \text{[Refer to Eqn. (10)]}$$

2.7 PROBABILITY OF ERRORS

The effects of errors on the quantity being measured can be positive or negative. For most of the measurements, positive effects are nearly equal to the negative effects resulting in small net error. By making a large number of measurements, we can obtain a central value if all positive and negative effects are equal and cancel out each other. Since this condition occurs frequently, we can apply probability or mathematical laws to study about random errors.

2.7.1 Error Distribution

When a number of measurements of the same quantity are taken, the result is a scatter of data around the most probable value known as **central value**. This scattering of measurement can be represented graphically in the form of block diagram or **histogram** (also called **frequency distribution curve**). Table 2.1 shows 56 readings taken by an ammeter at small time intervals. These readings are recorded to the nearest 0.1 A and the nominal value of the measured current is 200 A.

Table 2.1 Table showing Current Readings

Current reading (amperes)	*Number of readings*
199.7	2
199.8	5
199.9	13
200.0	20
200.1	11
200.2	4
200.3	1

These readings are plotted as shown in Figure 2.6. A graph is plotted between the number of observations and the corresponding readings. As can be seen from the figure the maximum number of readings occur at the central value of 200.0 A while the other readings are placed more or less in a symmetrical way on either side of it.

As more readings are taken at relatively smaller increments, the curve of the histogram becomes smoother. This bell-shaped curve is known as **Gaussian curve** and is shown by dotted line in Figure 2.6. This curve is symmetrical with respect to the central value or mean reading. Central value of the curve is the most probable value of the true reading. The narrower and sharper the curve, the more definitely a central value can be stated.

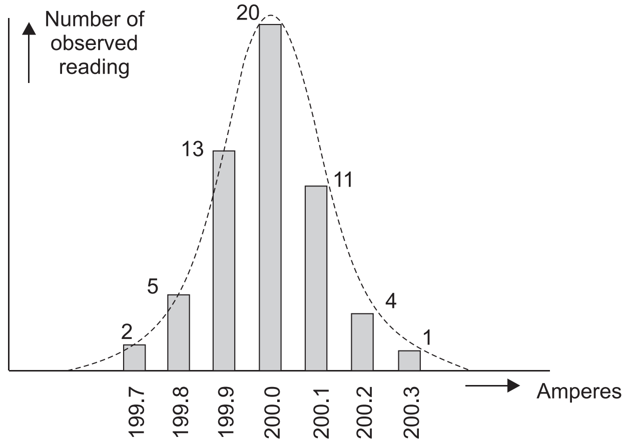


Fig. 2.6 Frequency Distribution Curve

The Gaussian law of error, also known as **normal law**, is the basis for the analytical study of random effects and is mathematically expressed as:

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2\omega^2} \qquad \text{...(13)}$$

where y is the probability of occurrence of deviation ω, which is the magnitude of deviation from mean value, and h is a constant equal to $\frac{1}{\sigma\sqrt{2}}$. Here, σ is the standard deviation. Now, a graph can be plotted between y and ω for the calculation of standard deviation. The result of this calculation can also be represented in the form of a graph as shown in Figure 2.7. It is evident from the graph that the average value is itself the true value if there occurs a peak at zero deviation due to its maximum probability of occurrence.

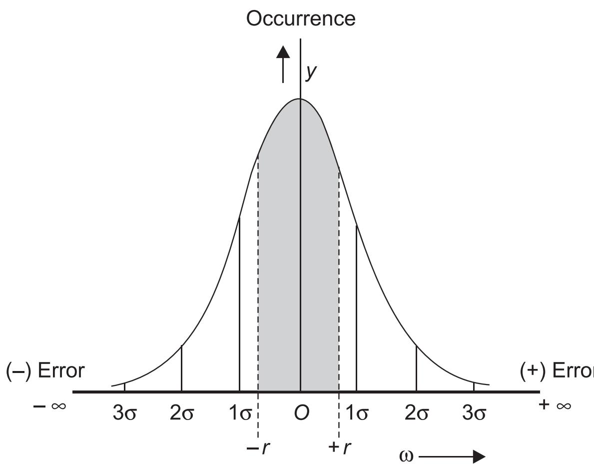


Fig. 2.7 Depiction of Normal Law

2.7.2 Probable Error

The total number of observations is represented by the area under the Gaussian probability curve in the range of $+\infty$ to $-\infty$ as shown in Figure 2.7. Among the observations that differ from the mean by a value less than standard deviation are represented by the area under the same curve between the range $+\sigma$ to $-\sigma$. Integrating this area, we may get the total number of cases that fall under this area. Generally, 68% of all the cases lie in this range for normally distributed data when the Gaussian distribution is considered.

A quantity known as **probable error** or simply **PE** measures the precision of the quantities. In Figure 2.7, the points $+r$ and $-r$ marked in Gaussian probability curve cover half of the total area under the curve. Hence, half of the total deviation lies in this range. The value of r can be found using Equation (13). Integrating the equation from $-r$ to $+r$ and equating it to 1/2 as:

$$\frac{h}{\sqrt{\pi}}\int_{-r}^{+r} e^{-h^2\omega^2}\,d\omega = \frac{1}{2}$$

we get the value of r as:

$$r = \frac{0.4769}{h} \qquad ...(14)$$

Also, we have:

$$h = \frac{1}{\sigma\sqrt{2}}$$

Substituting this in the above relation, we get:

$$\text{Probable error, } r = \pm 0.6745\,\sigma \qquad ...(15)$$

After calculating the value of r for a certain number of measurements if further measurements are made, then there are equal chances for the new value to lie in between $+r$ and $-r$.

Example 7 Calculate the probable error for a number of measurements if the standard deviation is given as 0.0025.

Solution: Given that: standard deviation σ = 0.0025

The relation between standard deviation σ and probable error r is given as:

$$\text{Probable error, } r = \pm 0.6745\,\sigma \qquad \text{[Refer to Eqn. (15)]}$$

On substituting the values, we obtain:

$$r = \pm(0.0025)\,(0.6745)$$

$$\Rightarrow \qquad r = \pm 0.00168$$

2.8 LIMITING ERRORS

The manufacturer guarantees accuracy within a certain percentage of full scale reading for most of the circuit components of the instrument so that the purchaser can choose the one suitable for his requirement. The limits specified by the manufacturers up to which a quantity can deviate from its specified value are known as **limiting errors**, also referred to as **guarantee errors**. They provide guarantee that the error in the instrument is not beyond the specified limit. For example, the resistance of a resistor with its value 200 Ω ± 5% lies in the range of 190 Ω to 210 Ω. Here, it is to be noted that the manufacturer does not specify the probable error or standard deviation, instead only ensures that the errors would not be higher than the set limits.

Since the limiting errors have fixed magnitudes based on full scale reading of the meter, an increase in the per cent limiting error occurs when the smaller readings are taken. Thus, the measurements should be done as close to the full scale as possible.

Example 8 A 0–400 V voltmeter is specified to be accurate within 1.2% full scale deflection. Calculate the limiting error when the instrument measures a voltage of 175 V.

Solution: Given that: Accuracy at FSD is 1.2%.

Thus, the magnitude of limiting error can be calculated as:

Limiting error = 0.012 × 400 V = 4.8 V

Now, the reading indicated by the meter is 175 V. The limiting error at this reading can be obtained as:

$$\text{Limiting error at 175 V} = \frac{4.8}{175} = 0.027.$$

And, the per cent limiting error is given as:

$$0.027 \times 100 = 2.7\%$$

Let us Summarize

1. The accuracy and precision of an instrument is defined by the material used for making an instrument, the design selected, and the skills used to construct it.
2. Significant figures provide the information about the magnitude and precision of a measured quantity. The number of significant figures in the measurement of a quantity indicates the precision with which it is measured.
3. The performance characteristics of an instrument are divided into two parts, namely static and dynamic characteristics.
4. Performance characteristics which measure the slowly varying or unvarying data and thereby indicate the response of the instruments are known as static characteristics. Static characteristics of an instrument include accuracy, precision, resolution, repeatability, reproducibility, static error, sensitivity, and drift.
5. The characteristics that indicate the response of instruments measuring time-varying quantities in which the input varies with time and so does the output are known as dynamic characteristics. The dynamic characteristics of an instrument include measuring lag, fidelity, speed of response, and dynamic error.
6. Various factors affect the measurement, some of which are due to the instruments themselves and some others are due to their inappropriate use. Depending upon these factors, errors can be categorized into different types, such as gross errors, systematic errors, absolute and relative errors, and random errors.

7. When a quantity is calculated by the combination of two or more measurements made on different instruments, the errors due to inaccuracies of these instruments get combined in a way to increase the amount of the error. This will further depend upon the type of combination, that is, whether the quantities are combined as sum, difference, product, or quotient.
8. Statistical analysis is performed on measurement data to analytically determine the uncertainty of the final measurement result. The methods used for statistical analysis include arithmetic mean, deviation, standard deviation, and variance.
9. We can apply probability or mathematical laws to study random errors. A quantity known as probable error or simply PE measures the precision of the quantities.
10. The limits specified by the manufacturers up to which a quantity can deviate from its specified value are known as limiting errors, also referred to as guarantee errors.

EXERCISES

Fill in the Blanks

1. The average deviation D is mathematically expressed as __________.
2. The closeness of the measured value to the true value is known as __________.
3. The expression for probable error is given as __________.
4. Incorrect setting of range of an ammeter results in __________ error.
5. Regulation in ambient temperature results in the reduction of __________ error.

Multiple Choice Questions

1. An 0–10 A ammeter has an accuracy of 1% of full scale deflection. The limiting error corresponding to a reading of 2.5 A is:
   (a) 3% (b) 4%
   (c) 1% (d) None of these
2. The difference between the expected value and measured value of a variable is known as:
   (a) gross error (b) instrumental error
   (c) absolute error (d) random error
3. The resultant resistance for a series combination of two resistances 100 Ω ± 5 Ω and 150 Ω ± 15 Ω is expressed as:
   (a) 250 Ω ± 10 Ω (b) 250 Ω ± 12.6 Ω
   (c) 250 Ω ± 20 Ω (d) 250 Ω ± 15.8 Ω
4. The mathematical expression for Gaussian distribution can be given as:
   (a) $y = \dfrac{h}{\sqrt{\pi}} e^{-h^2\omega^2}$ (b) $y = \dfrac{h}{2\sqrt{\pi}} e^{-h^2\omega^2}$
   (c) $y = \dfrac{h}{\sqrt{\pi}} e^{-h^3\omega^2}$ (d) $y = \dfrac{h}{\pi} e^{-h^2\omega^2}$
5. Gross errors are caused due to:
   (a) instrumental error (b) human error
   (c) random error (d) both (a) and (b)

State True or False

1. Manufacturer's specifications of accuracy are termed as limiting errors.
2. A set of readings having a wide range is said to have a low precision.
3. Random errors may occur even when the gross and systematic errors are accounted for.
4. A temperature having the value 24.60°C has three significant figures.
5. Measuring lag is the delay in the response of an instrument.

Descriptive/Numerical Questions

1. Define the following terms as applied to an electronic instrument:
   (a) Accuracy (b) Precision (c) Resolution
2. A circuit was tuned for resonance by 8 different students and the value of their resonant frequency in kHz were recorded as 532, 548, 546, 531, 543, and 536. Calculate:
   (a) Arithmetic mean (b) Average deviation
   (c) Deviation from mean (d) Variance
3. Write short notes on:
   (a) Systematic error (b) Random error
   (c) Gross error
4. Define static characteristics of an instrument.
5. What is meant by absolute error of measurement?
6. Describe the dynamic characteristics of a measuring instrument in brief.
7. How the performance characteristics of an instrument are classified?
8. A 600 V voltmeter is specified to be accurate within $\pm 2.5\%$ at full scale deflection. Calculate the limiting error when the instrument is used to measure a voltage of 400 V.
9. Explain the various methods that are available to minimize and eliminate errors.
10. Define speed of response, significant figures, and reproducibility.
11. A true value of voltage across a resistor is 50 V, the instrument reads 49 V. Calculate the absolute error and percentage error.
12. Determine the limiting error in the computed value of power dissipation in a resistor of $200 \pm 0.5\ \Omega$, if a current of 1.98 ± 0.04 A is passing through it.
13. The following eight observations were recorded when measuring a voltage as 21.7, 22.0, 21.8, 22.1, 23.2, 22.6, 21.9, and 22.2 volts. Find the probable error.
14. A 240 Ω resistance has an accuracy of $\pm 15\%$ and carries a current of 4 mA. Calculate the power dissipated in the resistance and the accuracy of the result if the current is measured by a 50 mA range ammeter having an accuracy of $\pm 5\%$ of full scale.

3

Electromechanical Instruments

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Understand the concept of electromechanical instruments, including PMMC, galvanometer, DC ammeter, DC voltmeter, and ohmmeter
- Classify the instruments on the basis of direct measuring and comparison instruments, absolute and secondary instruments
- Explain the working, construction, and the forces acting in PMMC instrument
- Explain the construction, working principle of different types of galvanometer, including d'Arsonval galvanometer and ballistic galvanometer
- Discuss the working of DC ammeters and DC voltmeters and their further modification to multi-range ammeters and voltmeters
- Appreciably understand series and shunt ohmmeters
- Know the fundamentals of instrument calibration

3.1 INTRODUCTION

In the previous chapters, we have studied measurement units and systems, standards of measurement, measurement characteristics and various errors with their statistical analyses. In this chapter, we will deal with **electromechanical instruments** which are electrically operated mechanical instruments used to measure physical quantities and how to keep these instruments in proper running condition. A permanent magnet moving coil is the basic building block of all electromechanical instruments, (i) galvanometers (to measure small currents and voltages), (ii) DC ammeters and voltmeters (to measure direct current and voltage), and (iii) series and shunt ohmmeters (to measure resistance). All measuring instruments should fulfil the essential conditions that when they are placed in the circuit for measurement, they should not alter the circuit conditions and should consume only a small amount of power.

The permanent magnet moving coil, abbreviated as **PMMC** is an instrument that consists of a light weight coil of copper wire suspended in the field of permanent magnet. The current flowing in the wire produces a magnetic field by the coil which interacts with the field from the magnet thereby resulting in partial rotation of the coil. The current flowing in the wire is indicated by the deflection on a calibrated scale through a pointer connected to the coil.

The PMMC instrument acts as a basic building of other electromechanical instruments, such as it can function as DC ammeter or DC voltmeter by connecting resistors in parallel with the coil to measure a wide range of direct current levels or by connecting appropriate value resistors in series to measure voltage, respectively. The ohmmeters are also made from PMMC instruments, precision resistors, and batteries.

This chapter gives a brief insight of constructional details, working principle, and functioning of all these electromechanical instruments which will be very helpful in their practical handling. Further, the calibration of instruments is covered as an integral part which holds the importance to keep track of the system's accuracy.

3.2 CLASSIFICATION OF INSTRUMENTS

A number of electrical measuring instruments, namely ammeter, voltmeter, ohmmeter, wattmeter are classified according to their function. They can be categorized as analog and digital. Analog instruments, also termed as **deflection instruments** use scale and pointer to indicate the quantity being measured, whereas digital instruments use numerical (or digital) format to display the measurement. Digital instruments are discussed in later chapters. Analog instruments are further classified as passive and active instruments. In **passive instruments**, the pointer moves over the scale in accordance with the quantity being measured, whereas in **active instruments**, an external power supply and an amplifier are used to monitor the electromechanical movement of the pointer. The additional electronic circuits increase the sensitivity of the active instruments. Both active analog and digital instruments are termed as **electronic instruments**. Analog instruments are broadly classified as follows.

Direct measuring and comparison instruments

In *direct measuring* instruments, the value of the unknown quantity is measured, recorded or displayed directly. These instruments are actuated by the energy obtained by converting the measurand energy directly. For example, ammeters, voltmeters, wattmeters, and energy meters are all direct measuring instruments. In *comparison* instruments (also termed as *null-type instruments*), the value of the unknown quantity is measured by comparing it with a standard value, that is a precisely known quantity. For example, AC and DC bridges are comparison type instruments.

The most simple and inexpensive instruments commonly used in engineering practice are direct measuring, since the measurements are made in short span of time. On the contrary, comparison type instruments are used where high accuracy is required.

Absolute and secondary instruments

Absolute instruments, also known as **primary instruments**, measure the quantity which is expressed in terms of fundamental units of length, mass, and time. These instruments are used only in a standards laboratory. For example, in Rayleigh current balance instrument, the magnetic effect of a current in the coil is weighed in a balance. The mass and acceleration due to gravity of the balancing weight determines the force exerted by it. Therefore, the current is precisely obtained from the force and coil dimensions.

Secondary instruments are those which display the quantity being measured with a scale and pointer. Hence, virtually all the instruments in use are secondary instruments without the concern of accuracy.

3.3 PERMANENT MAGNET MOVING-COIL INSTRUMENT

The permanent magnet moving-coil (PMMC) instrument is the most accurate instrument used for DC measurements. It is considered to be the basic building block of all other DC measuring instruments. The PMMC and instruments based on it are deflection instruments that use a pointer to move on a calibrated scale to indicate the measurement. There are three operating forces, namely *deflecting force*, *controlling force*, and *damping force* which are required for their operation.

Principle

PMMC works on the principle of Fleming's left hand rule which states that a force is experienced by a current-carrying conductor (or coil) when placed in a magnetic field. This force tends to move the coil in the direction as shown in Figure 3.1 where the thumb, the first finger, and the middle finger are all perpendicular to each other. The thumb represents the direction of the force on the conductor while the first and middle finger represent the direction of magnetic field and the direction of the current in the conductor, respectively.

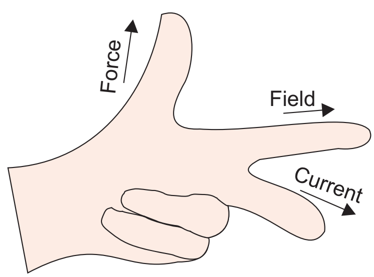


Fig. 3.1 Fleming's Left Hand Rule

Constructional details

A PMMC instrument consists of a U-shaped permanent magnet made up of magnetic materials like alnico or alcomax and two soft iron pole shoes. A cylindrical soft iron core is located between these pole shoes so that the air gaps between the core and the shoes must be very narrow to have a strong magnetic flux across the gap. A coil of thin wire with a number of turns is mounted on a rectangular aluminium frame or former positioned between the poles of the permanent magnet that moves in this narrow gap as shown in Figure 3.2.

There are two non-magnetic spiral springs, preferably made of phosphor bronze whose one end is fastened to the coil and the other end is connected to an adjustable zero position control for controlling purposes. The zero-position control can be adjusted with the help of

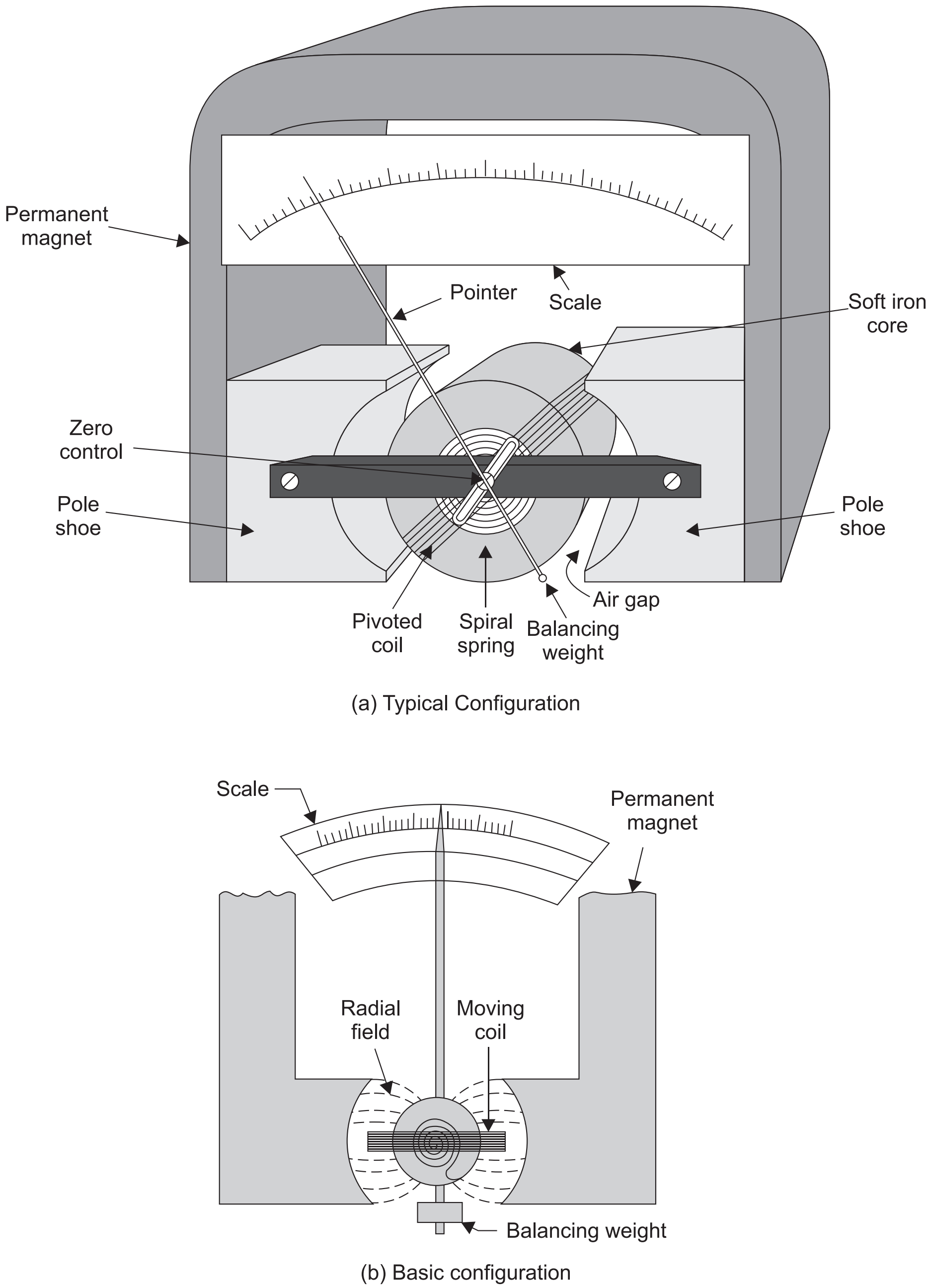


Fig. 3.2 Construction of a PMMC Instrument

a screw on the instrument cover to move the end of the spring. These low resistance springs keep the coil and the pointer at their respective zero positions in the absence of coil current.

The balancing weights or counter-weights provide accurate mechanical balance to the moving system in order to avoid the gravitational effect on the accuracy of the instrument. The moving system of the deflecting instrument is supported by the jewelled-bearing suspension where the pivots or pointed ends of the shafts are inserted in cone-shaped cuts in jewel (sapphire or glass) bearing. This allows the coil to move freely in the field of the permanent magnet with minimum friction [see Figure 3.3].

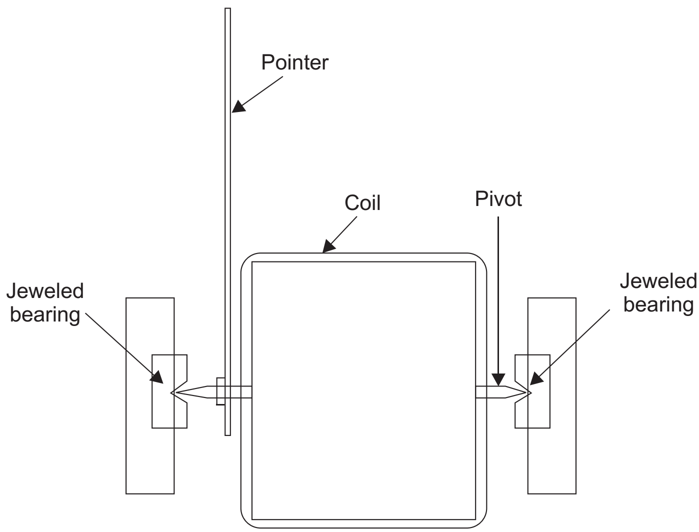


Fig. 3.3 Jewelled-bearing Suspension

Sometimes, springs are also mounted in the jewel bearings at both ends so as to lessen the damage to the instrument due to shocks as shown in Figure 3.4.

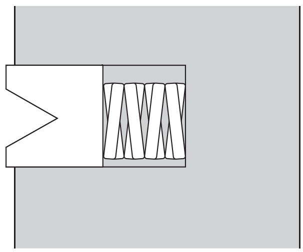


Fig. 3.4 Jewel Bearing Supported with Spring

Working

The operating current flows through the coil when PMMC instrument is connected in the circuit to measure electrical quantity, such as voltage or current. This coil current sets up a magnetic field which interacts with the field of the permanent magnet thereby producing a force known as **deflecting force** that causes the pointer (which is fixed to the coil) to move over the calibrated scale in clockwise (positive) direction from the zero position to indicate the value of electrical quantity being measured. This movement of a pointer over calibrated scale is called **d'Arsonval movement**. When the current in the coil is reversed, the coil rotates in opposite direction due to the interaction of magnetic flux from the coil with the field of permanent magnet and pointer moves in anti-clockwise direction, that is, to the left of zero position [see Figure 3.5]. Therefore, PMMC instrument is said to be polarized with a positive and negative terminals indicating the correct polarity for connection. Hence, it is purely direct current measuring instrument and it can measure alternating current only with the use of rectifiers.

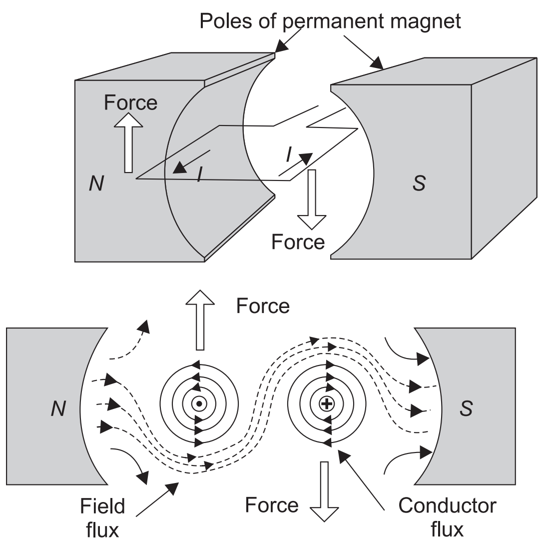


Fig. 3.5 Interaction of Magnetic Flux of Coil with the Field of Permanent Magnet

Spiral springs in the PMMC instrument provide the force to control the movement of the pointer. The coil and the pointer retain their zero position through springs with no current flow. When the coil rotates due to current flow, the springs wind up and the deflecting force exerted on the coil by them increases as the deflection of coil increases. When the controlling force and the deflecting force become equal, the coil and the pointer stop rotating to attain final position as shown in Figure 3.6. Note that the pointer and the coil oscillate for sometime before settling down at their final positions. For this, the damping force is required to settle or damp the oscillations. This damping force is produced by the movement of the coil which induces eddy currents in the aluminium frame (non-magnetic conductor) of the coil setting up a magnetic flux to oppose the coil motion to damp the oscillations. The pointer settles rapidly to its final position with critical damping and it overshoots once and then settles with less than critical damping. In the worse case, it settles very slowly to its final position with large amount of damping [see Figure 3.7].

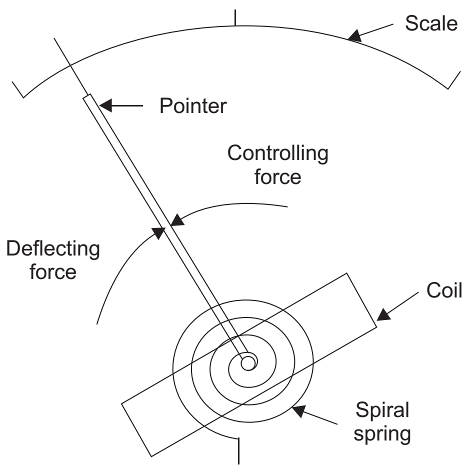


Fig. 3.6 Stationary Pointer when Controlling and Deflecting Forces are Equal

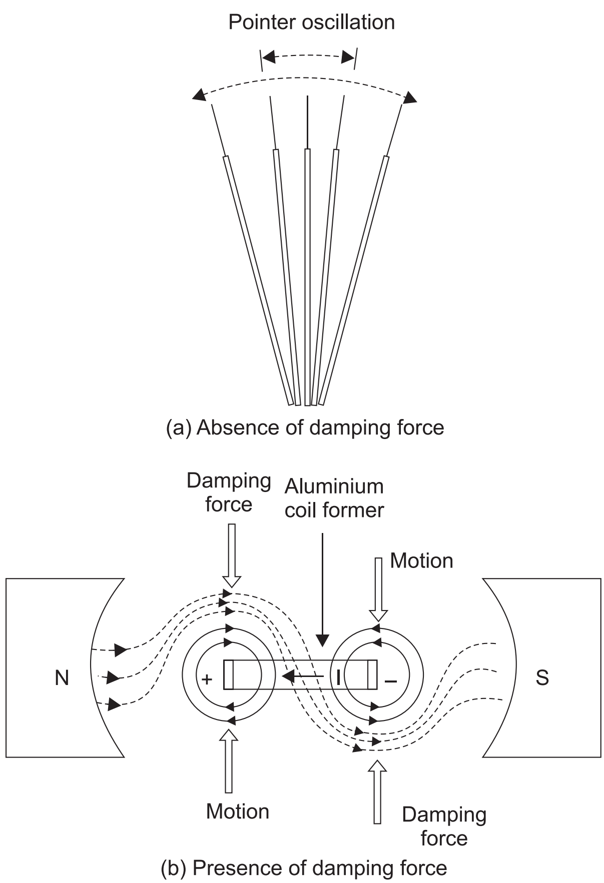


Fig. 3.7 Effect on Pointer in Absence and Presence of Damping Force

Deflecting torque equation

Due to soft iron core in PMMC instruments, the magnetic field in the air gap is radial. This makes the coil conductors to move at right angles to the field. When a current is passed through the coil, the forces act on both sides of the coil resulting in deflecting torque as shown in Figure 3.8. The strong magnetic field increases the deflecting torque which in turn results in increased sensitivity of the instrument. The deflecting torque, denoted by T_D can be expressed mathematically as:

$$T_D = \text{Force} \times \text{Perpendicular distance between the coil and the field} \quad ...(1)$$

If a current of I amperes is passed through the coil, then the force acts on each side of the coil which is given as:

$$F = BINl \sin \alpha \text{ newtons} \quad ...(2)$$

where B is the magnetic flux density in the air gap, Wb/m^2
I is the coil current, A
N is the number of turns of the coil
l is the length of the coil, m
α is the angle between the direction of magnetic field and the conductor

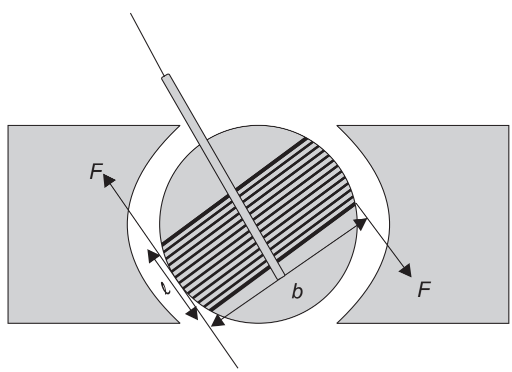


Fig. 3.8 Force Acting on Each Side of Coil

Since the magnetic field is radial, the angle α becomes equal to 90°. Hence, Equation (2) becomes:

$$F = BINl \text{ newtons} \quad ...(3)$$

Now, let the perpendicular distance between the coil and the magnetic field be represented by b, the breadth of the coil. Thus, substituting Equation (2) in (1), we may write:

$$T_D = BINl \times b$$

Or,
$$T_D = BINA \quad ...(4)$$

where A (= $l \times b$) is the area of the coil. The deflecting torque T_D is measured in newton-metres (Nm). Since all the quantities except current are constant for any given instrument, the deflection torque in Equation (4) is proportional to the coil current expressed as:

$$T_D \propto I$$

The instrument is controlled by the spiral springs, so the resultant torque is said to be the **controlling** or **restoring torque** denoted as T_C. This controlling torque is in proportion to the actual angle of deflection of the pointer θ, therefore it can be written as:

$$T_C \propto \theta \quad \text{or} \quad T_C = K\theta \qquad \text{...(5)}$$

where K is the spring or control constant of the instrument measured in newton-metres per radian (Nm/rad).

When the controlling torque and the deflecting torque are equal to each other, the pointer attains a final steady deflection. Thus, equating Equations (4) and (5), we get:

$$T_C = T_D = K\theta = BINA \qquad \text{...(6)}$$

Also, at the final steady deflection, the essential condition is expressed as:

$$\theta \propto I \qquad \text{...(7)}$$

From the above equation, we can say that the deflection in the pointer is always proportional to the coil current. Therefore, it is a requisite for the instrument to have a uniformly or linearly divided scale, illustrating that 1 mA current produces a movement of 1 cm on the pointer from zero. The full scale deflection (FSD) current level for typical moving coil in PMMC instrument is 50 µA to 1 mA.

Example 1 The magnetic flux density in the air gap of a PMMC instrument is 0.5 T. The length and width of the coil are given as 1.20 cm and 2 cm, respectively. Calculate the number of turns required to deflect a torque of 4.2 µNm if the coil current is given as 100 µA.

Solution: Given that: Magnetic flux density B = 0.5 T

Length of the coil l = 1.2 cm

Width of the coil b = 2 cm

Deflection torque T_D = 4.2 µN-m

Coil current I = 100 µA

The relation for the deflection torque is given from Equation (4) as:

$$T_D = BINlb$$

Thus, the number of turns N is calculated as:

$$N = \frac{T_D}{BIlb}$$

Substituting the given values, we get:

$$N = \frac{4.2 \times 10^{-6}}{0.5 \times 100 \times 10^{-6} \times 1.2 \times 10^{-2} \times 2 \times 10^{-2}}$$

$$\Rightarrow \qquad N = 350$$

Example 2 The coil of a moving coil meter has 200 turns wound on a non-inductive former, its dimensions are 2.5 cm × 2 cm. It works in a constant magnetic flux density of 0.15 T in the air gap. The control spring produces a torque of 100×10^{-7} Nm. Calculate the current in the coil to produce a deflection of 110°.

Solution: Given that: number of turns $N = 200$

Length of the coil $l = 2.5$ cm

Breadth of the coil $b = 2$ cm

Magnetic flux density $B = 0.15$ T

Controlling torque $T_C = 100 \times 10^{-7}$ Nm

Deflection $\theta = 110°$

The relation for the deflection torque T_D is given as:

$$T_D = BINlb \qquad \text{[Refer to Eqn. (4)]} \quad ...(2a)$$

We need to calculate the control constant K from the controlling torque T_C as:

$$T_C = K\theta \qquad \text{[Refer to Eqn. (5)]} \quad ...(2b)$$

$$100 \times 10^{-7} = 110° \text{ K}$$

$$\Rightarrow \qquad K = 0.909 \times 10^{-7} \text{ Nm/rad}$$

The pointer reaches final steady deflection when the deflecting torque T_D and controlling torque T_C are equal to each other. Therefore, Equating (2a) and (2b) as:

$$T_D = T_C = K\theta = BINlb$$

Therefore, the current I in the coil which produces a deflection of 110° can be obtained as:

$$I = \frac{K\theta}{BNlb}$$

Substituting the given values, we get:

$$I = \frac{0.909 \times 10^{-7} \times 110°}{0.15 \times 200 \times 2 \times 10^{-2} \times 2.5 \times 10^{-2}}$$

$$I = 0.66 \text{ mA}$$

3.3.1 Advantages and Disadvantages of PMMC Instruments

The PMMC instruments have several advantages as follows.

- The scale of PMMC instruments is linear and uniform, that is, evenly divided.
- The efficiency of such instruments is very high.
- These instruments require very low power for their operation, in the range of 25 μW to 200 μW.
- The eddy current damping is very effective in these instruments.

- The magnetic field in these instruments is constant, which ensures no hysteresis loss. Also, as the magnetic field is very strong and so are the operating forces, the errors due to external stray magnetic fields have nominal effect on the reading which in turn provides accurate and reliable readings.
- A single PMMC instrument may be used to measure several ranges of voltage and current by using different values for shunts and multipliers.

The PMMC instruments also have some disadvantages as below.

- The PMMC instruments are expensive because of their accurate design.
- These instruments are used for DC measurements. This is because when the current in the coil reverses, the torque also reverses. Thus, if AC measurement has to be taken, the pointer would not be able to follow frequent reversals of the current; therefore the deflection would correspond to the mean torque, that is, zero.
- Due to ageing or temperature changes, variations either in strength of control springs or in the permanent magnet cause some errors.
- Some friction gets introduced due to jewel bearing suspension.

3.4 GALVANOMETER

A galvanometer is a PMMC instrument designed to detect and measure the magnitudes of small currents and voltages in a circuit. This instrument is very sensitive to enormously low levels of electrical quantities. It has a deflection system in which the pointer can deflect on either side of the zero on a centre-zero scale in accordance with the direction of the coil current. There are two types of galvanometer, namely, *d'Arsonval galvanometer* and *ballistic galvanometer*.

3.4.1 D'Arsonval Galvanometer

The d'Arsonval galvanometer is a PMMC instrument based on d'Arsonval movement whose basic construction and working operation along with its taut-band suspension, damping, moving system, and sensitivity are discussed in this section.

Construction and operation

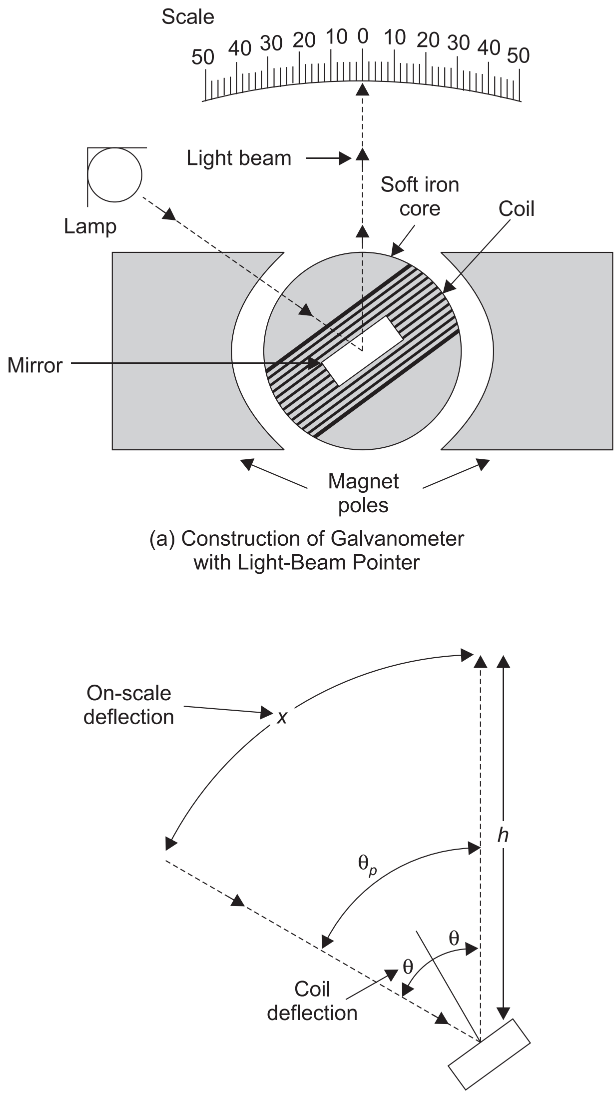
Highest sensitivity of a galvanometer can be achieved when the weight of the moving coil is reduced to the minimum possible and a small mirror is mounted on the coil in place of a pointer [see Figure 3.9(a)]. The centre-zero scale is set at 1 m distance from the mirror on which a beam of light reflected by the mirror is focused. A very small coil current deflects the light beam which behaves as a very long weightless pointer. Hence, in comparison to pointer instruments, light-beam galvanometers are sensitive to much lower current levels. The deflection of light-beam pointer is twice that of mirror and the coil as shown in Figure 3.9(b). The deflection angle of light-beam pointer θ_P is given as:

$$\theta_P = \frac{x}{h}$$

where x represents the on-scale deflection and h is the length of the light beam. Thus, the deflection angle θ of the mirror and the coil is given as:

$$\theta = \frac{x}{2h}$$

Here θ_P and θ are expressed in radians. The scale of the galvanometer may be calibrated in microamperes (μA) or simply on a millimetre (mm) scale.



(a) Construction of Galvanometer with Light-Beam Pointer

(b) Deflection of Light-Beam Pointer with Respect to Deflection of Mirror and Coil

Fig. 3.9 D'Arsonval Galvanometer

Taut-band suspension

Extremely sensitive galvanometers use taut-band suspension, also known as **ribbon suspension**, to support the moving coil system as shown in Figure 3.10. It is seen from the figure that the springs hold two flat metal ribbons (made of platinum alloy or phosphor bronze) under tension to support the coil. The ribbons make electrical connections by carrying current to the coil and are held under tension which behaves like rubber and make the instrument extremely rugged. On twisting the ribbons, they exert a controlling torque. Taut-band suspension offers less friction as compared to jewelled-bearing suspension with full scale deflection (FSD) happening by as little as 2 μA of coil current.

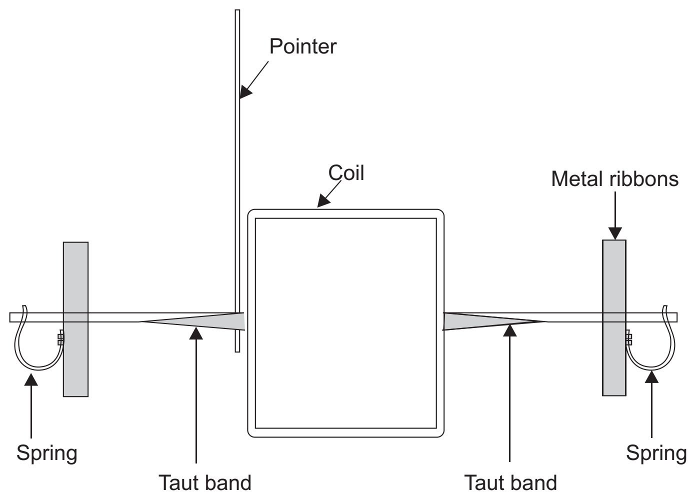


Fig. 3.10 Taut-band Suspension

The instruments with taut-band suspension have much higher sensitivity than the instruments using pivots and jewels. Moreover, such instruments are comparatively insensitive to shock and temperature. The taut-band suspension instruments are capable of handling greater overloads.

Damping

The damping eddy currents are produced by the moving coil wound on a non-conducting and non-magnetic former. The damping resistor, denoted as R_d is connected in series with the coil resistance, denoted as R_C and the source of the coil current to control the current levels. We assume the source resistance much smaller than the coil resistance and the total damping resistance is the sum of damping resistance R_d and coil resistance R_C. The damping torque T_d is present which depends on the resistance and critical damping, obtained by adjusting the value of resistance. The **critical damping** or **dead beat** allows the pointer to move rapidly and settle to its final steady state smoothly with no overshoots. The damping with a little less than critical stage causes the pointer to settle only after one small overshoot, similar to that in PMMC instrument. **Underdamping** causes the pointer to oscillate about final steady state with decreasing amplitude and comes to rest after some time. **Overdamping** causes the pointer to move slowly to its final steady state as the damping torque is more than that required for critical damping. All the three types of damping are shown in Figure 3.11 which illustrates that the underdamping results in great waste of time, overdamping results in loss of time and a faster response is obtained with critical damping and with damping little less than critical damping.

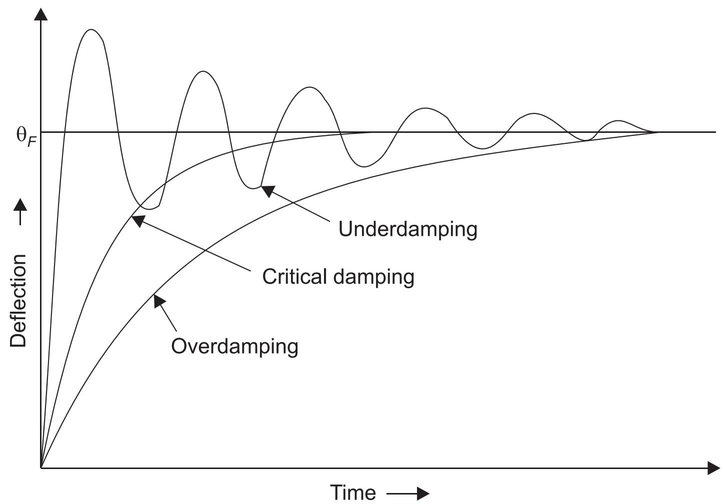


Fig. 3.11 Damping in Galvanometer

Moving coil system

The moving coil could be either circular or rectangular in shape consisting of a number of turns of wire suspended to move freely about its vertical axis of symmetry. The soft iron core is spherical in shape for a circular coil and is cylindrical for a rectangular coil. The core produces a strong magnetic field for the coil to move and provides magnetic flux path that increases the deflecting torque and also the sensitivity of the galvanometer. The four types of torques act on the moving system, namely, deflecting torque T_D (that accelerates the system), inertia torque T_j, damping torque T_d, and controlling torque T_C (which retards the system). The **inertia torque** T_j is the retarding torque produced due to inertia of the moving system and depends on the moment of inertia J (measured in kg-m^2) of the moving system and the angular acceleration $d^2\theta/dt^2$, where θ is the deflection at any time t. The **damping torque** T_d depends on the damping constant D (measured in Nm/rad/s) and velocity of the moving system $d\theta/dt$. The friction due to movement of the coil in air and induced electrical effects in a closed circuit provides damping. The **controlling torque** T_C tries to restore the moving system back to its original position and depends on the control constant K (measured in Nm/rad or Nm/degree) and deflection θ (measured in radians). At any time instant t and for any deflection angle θ, the sum of inertia torque T_j, damping torque T_d, and controlling torque T_C is equal to the deflecting torque T_D, such that:

$$T_j + T_d + T_C = T_D$$

Or $$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta = GI \qquad ...(8)$$

Here, I is the instantaneous current measured in amperes (A) and G can be evaluated from deflecting torque T_D as studied in Equation (4). Since the deflecting and controlling torque are equal at final steady deflection, such that:

$$T_C = T_D = K\theta = BINA \qquad \text{[Refer to Eqn. (6)]}$$

Therefore, $$K\theta = GI \qquad ...(9)$$

where $G = BNA$ known as the **galvanometer displacement constant** measured in Nm/A.

At steady state condition, $d^2\theta/dt^2 = 0$, $d\theta/dt = 0$, and $\theta = \theta_F$, where θ_F is final steady state deflection.

Substituting the above values in Equation (8), we get:

$$\theta_F = GI/K \qquad ...(10)$$

Equation (8) can be written in auxiliary form as $Jm^2 + Dm + K = 0$. The solution of this equation is given as:

$$\theta = A \exp(m_1 t) + B \exp(m_2 t)$$

where A and B are the constants and m_1 and m_2 are the roots of the equation given as:

$$m_1 = \frac{-D + \sqrt{D^2 - 4\,\mathrm{KJ}}}{2\,\mathrm{J}} \quad \text{and} \quad m_2 = \frac{-D - \sqrt{D^2 - 4\,\mathrm{KJ}}}{2\,\mathrm{J}}$$

Using Equation (10), the complete solution of the differential equation (8) at steady state deflection is given as:

$$\theta = A \exp(m_1 t) + B \exp(m_2 t) + \theta_F \qquad ...(11)$$

where $A \exp(m_1 t) + B \exp(m_2 t)$ represents motion which could be oscillatory or not. The four possible cases of damping are explained which gives the solution of Equation (11).

Case I: Critical damping results when $D^2 = 4$ kJ, where m_1 and m_2 are equal and real roots of Equation (11). This results in non-oscillatory motion. Here, $m_1 = m_2 = -D/2\,J$. The solution of the equation is given as:

$$\theta = [A + B] \exp(-Dt/2\,\mathrm{J}) + \theta_F \qquad ...(12)$$

where A and B are constants whose values can be found by differentiating Equation (12) as:

$$\frac{d\theta}{dt} = \frac{-D}{2\,\mathrm{J}} \exp(-Dt/2\,\mathrm{J})\,(A + Bt) + B\exp(-Dt/2\,\mathrm{J}) \qquad ...(13)$$

Putting time $t = 0$, deflection $\theta = 0$ and velocity of the moving system $d\theta/dt = 0$ in Equation (12) and (13), we get:

$$0 = \theta_F + A \quad \text{and} \quad 0 = -(D/2\,\mathrm{J})\,A + B$$

Therefore, $A = -\theta_F$ and $B = -(D/2\,\mathrm{J})\,\theta_F$

Substituting the above values in Equation (12), we get:

$$\theta = \theta_F[1 - \exp(-Dt/2\,\mathrm{J})\,(1 + Dt/2\,\mathrm{J})]$$

The damping constant for critical damping is given as $D = D_C = 2\sqrt{\mathrm{KJ}}$.

Here $D/2\,\mathrm{J} = 2\sqrt{\mathrm{KJ}/2\,\mathrm{J}} = \sqrt{\mathrm{K}/\mathrm{J}} = \omega_n$, where ω_n is angular frequency.

Therefore, for critical damping galvanometer, we have:

$$\theta = \theta_F\,[1 - \exp(-\omega_n t)\,(1 + \omega_n t)]$$

Case II: Underdamping results when $D^2 < 4$ kJ where m_1 and m_2 are imaginary roots of the Equation (11). This results in oscillatory motion. The roots m_1 and m_2 will be given as:

$$m_1 = \frac{-D + \sqrt{D^2 - 4\,\mathrm{KJ}}}{2\,\mathrm{J}} = \frac{-D}{2\,\mathrm{J}} + j\frac{\sqrt{4\,\mathrm{KJ} - D^2}}{2\,\mathrm{J}} = -\alpha + j\omega$$

$$m_2 = \frac{-D - \sqrt{D^2 - 4\,\mathrm{KJ}}}{2\,\mathrm{J}} = \frac{-D}{2\,\mathrm{J}} - j\frac{\sqrt{4\,\mathrm{KJ} - D^2}}{2\,\mathrm{J}} = -\alpha - j\omega$$

Substituting the above values in Equation (11), we get:

$$\theta = A\,[\exp(-\alpha + j\omega)\,t] + B\,[\exp(-\alpha - j\omega)\,t] + \theta_F$$

$$\Rightarrow \qquad \theta = \exp(-\alpha t)\,[A\exp(j\omega t) + B\exp(-j\omega t)] + \theta_F$$

Since θ is real and $e^{\pm j\omega t}$ is complex, A and B must be complex. Let $A = a + jb$ and $B = c + jd$, then:

$$\theta = e^{-\alpha t}\,[(a + jb)\exp(j\omega t) + (c + jd)\exp(-j\omega t)] + \theta_F$$

$$= e^{-\alpha t}\,[(a + jb)(\cos\omega t + j\sin\omega t) + (c + jd)(\cos\omega t - j\sin\omega t)] + \theta_F$$

$$= e^{-\alpha t}\,[(a + c)\cos\omega t - (b + d)\sin\omega t + j\,(b + d)\cos\omega t + j(a - c)\sin\omega t)] + \theta_F \qquad ...(14)$$

For all values of t, the imaginary term of the above equation must be zero. Hence,

$$(b + d)\cos\omega t + (a - c)\sin\omega t = 0$$

At $\omega t = 0$, $b + d = 0$ or $b = -d$

At $\omega t = \pi/2$, $a - c = 0$ or $a = c$

Therefore, $A = a + jb$ and $B = a - jb$ which concludes that A and B are complex conjugate pair. Equation (14) can be written as:

$$\theta = 2\,e^{-\alpha t}\,[a\cos\omega t + d\sin\omega t] + \theta_F \qquad ...(15)$$

Now let us assume that $a = (F/2)\sin\phi$ and $d = (F/2)\cos\phi$, thereby $F = 2\sqrt{(a^2 + d^2)}$ and $\phi = \tan^{-1}\phi$

Equation (15) can be written as:

$$\theta = F\,e^{-\alpha t}\,[\sin\phi\cos\omega t + \cos\phi\sin\omega t] + \theta_F = F\,e^{-\alpha t}\sin(\omega t + \phi) + \theta_F$$

$$\Rightarrow \qquad \theta = F\exp(-Dt/2\,\mathrm{J})\,[\sin(\omega_d t + \phi)] + \theta_F \qquad ...(16)$$

where ω_d is angular frequency of damped oscillation expressed as:

$$\omega_d = \frac{\sqrt{4\,\mathrm{KJ} - D^2}}{2\,\mathrm{J}}\ \mathrm{rad/s} \qquad ...(17)$$

Putting time $t = 0$, deflection $\theta = 0$ in Equation (16), we get:

$$0 = F\sin\phi + \theta_F \quad \text{or} \quad \sin\phi = -(\theta_F/F)$$

Here, ϕ and θ_F are positive quantities and F is a negative quantity. On differentiating Equation (16) we get:

$$\frac{d\theta}{dt} = \exp\left(\frac{-D}{2\,\mathrm{J}}\right) F\exp(-Dt/2\,\mathrm{J})\left[\sin(\omega_d t + \phi)\right] + F\exp(-Dt/2\,\mathrm{J})\,[\cos(\omega_d t + \phi)]\,\omega_d$$

Substituting $d\theta/dt = 0$ at $t = 0$ in the above equation, we get:

$$0 = \frac{-D}{2\,\mathrm{J}}F\sin\phi + F\omega_d\cos\phi$$

Substituting the value of $\sin\phi$ in above relation, we get:

$$0 = \frac{-D}{2\,\mathrm{J}}F\left(-\frac{\theta_F}{F}\right) + F\omega_d\cos\phi \;\Rightarrow\; \cos\phi = -\frac{D}{2J\omega_d}\cdot\frac{\theta_F}{F}$$

The final solution for underdamped motion of galvanometer is obtained as:

$$\theta = \theta_F\left[1 - \frac{2\sqrt{\mathrm{KJ}}}{\sqrt{2\,\mathrm{KJ} - D^2}}\exp(-Dt/2\,\mathrm{J})\sin\left(\frac{\sqrt{4\,\mathrm{KJ} - D^2}}{2\,\mathrm{J}}t + \tan^{-1}\frac{\sqrt{4\,\mathrm{KJ} - D^2}}{D}\right)\right]$$

The above equation states that the oscillation will be an attenuated sinusoidal motion and the moving system starting from zero current position will oscillate about its final steady state position θ_F.

Case III: Undamped motion of galvanometer is not a possible case under practical working conditions as states a condition with no damping forces, such that $D = 0$. However, its properties are used to express motion under actual operating conditions. On putting $D = 0$ in Equation (17), we obtain angular frequency as $\omega_n = \sqrt{K/J}$. The undamped frequency, also called natural or free oscillation is expressed as:

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{J}}$$

The free time period of oscillation T_o is given as:

$$T_o = \frac{1}{f_n} = 2\pi\sqrt{\frac{J}{K}}$$

$$\phi_o = \tan^{-1}\infty = 90°$$

Substituting the above values in Equation (16), we get:

$$\theta = F\,[\,\sin(\omega_n t + 90°)] + \theta_F \qquad ...(18)$$

The above equation represents undamped galvanometer oscillation around constant amplitude θ_F and frequency f_n.

Case IV: Overdamping results when $D^2 > 4$ KJ where m_1 and m_2 are real and unequal roots of Equation (11). This results in non-oscillatory motion and the galvanometer reaches final steady position in lethargic manner. The complete solution of Equation (11) will be:

$$\theta = \theta_F\left[1 + \frac{\xi + \sqrt{\xi^2 - 1}}{2\sqrt{\xi^2 - 1}}\exp\{-\omega_n t(\xi - \sqrt{\xi^2 - 1})\} - \frac{\xi - \sqrt{\xi^2 - 1}}{2\sqrt{\xi^2 - 1}}\exp\{-\omega_n(\xi + \sqrt{\xi^2 - 1})\}\right]$$

where $\xi = \frac{D}{D_C}$. The above equation signifies a decaying motion without overshoot or oscillations. Hence, this motion being very slow is not desirable for the indicating instruments.

Sensitivity

Sensitivity of galvanometer can be categorized as current, voltage, and megaohm sensitivities. The **current sensitivity** of the instrument, usually measured in millimetres per microampere (mm/μA) determines the current level producing a measured deflection. The **voltage sensitivity** measures the deflection in scale divisions per unit voltage and is usually expressed in millimetres per microvolt (mm/μV) for a series connected resistance of a given value. Sometimes, **megaohm sensitivity** is also specified for galvanometers as the value of the resistance connected in series with the instrument which restricts the deflection to one scale deflection for a difference of 1 V applied across its terminals.

3.4.2 Ballistic Galvanometer

A ballistic galvanometer is a sensitive instrument, used to measure the quantity of charge passing through it due to transient current. It has a copper wire coil wound on a non-conducting frame. This frame is suspended by the phosphor bronze strip between the poles of the permanent magnet. The magnetic flux in the air gap is increased by placing a soft iron core within the coil as shown in Figure 3.12. The magnetic field is perpendicular to the coil surface and radial in the narrow annular gap as the magnet poles are cylindrical concave in shape. The apparatus is enclosed in a shell to avoid outer disturbances.

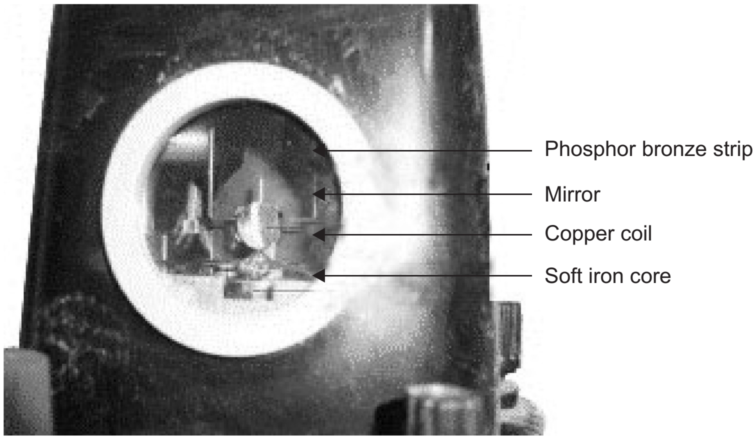


Fig. 3.12 Ballistic Galvanometer

Similar to d'Arsonval galvanometer, the ballistic galvanometer incorporates a moving coil which is meant to rotate between the magnets and deflects its indicating needle or mirror in proportion either to the total charge passing through it or to a short duration voltage pulse. However, this coil has a large moment of inertia that permits the quantity of charge to pass prior to significant movement of the coil. Unlike d'Arsonval galvanometer, the damping force is extremely small or zero in this galvanometer and does not show a steady deflection due to transitory nature of the current passing through it. When the charge passes, it produces an impulse or a momentary torque so that the coil oscillates with decreasing amplitude and slowly swings to some maximum position.

Moving coil system

The moving coil system shows no significant deflection till the time charge passes through the galvanometer and it deflects only on the complete passage of the charge. The moving system deflects to dissipate the energy received to it by the charge in the form of friction and damping. The time interval between which the charge passes, that is, from $t = 0$ to $t = t_1$, the coil of the galvanometer does not move and the equation of motion is expressed by Equation (8) as:

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta = GI$$

Or
$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta = G\frac{dQ}{dt}$$

Or
$$\frac{d^2\theta}{dt^2} + \frac{D}{J}\frac{d\theta}{dt} + \frac{K}{J}\theta = \frac{G}{J}\frac{dQ}{dt}$$

Integrating the above equation from $t = 0$ to $t = t_1$, we get:

$$\left.\frac{d\theta}{dt}\right|_{t=0}^{t=t_1} + \frac{D}{J}\left.\theta\right|_{t=0}^{t=t_1} + \frac{K}{J}\int_{t=0}^{t_1}\theta dt = \frac{G}{J}Q$$

During the time interval, the deflection θ remains zero, then $d\theta/dt = (G/J)\,Q$.

When the charge has passed completely, that is after time t_1, the current flowing through the coil is zero. Hence, the deflecting torque $T_D = GI = 0$. Therefore, the above equation becomes:

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta = 0 \qquad \text{...(19)}$$

The motion of the galvanometer is undamped given from Equation (16) as:

$$\theta = F\exp(-Dt/2\text{ J})\,[\sin(\omega_d t + \phi)] + \theta_F$$

Since the galvanometer shows no steady state deflection, therefore $\theta_F = 0$, the above equation becomes:

$$\theta = F\exp(-Dt/2\text{ J})\,[\sin(\omega_d t + \phi)]$$

Also the damping is very small, so $\omega_d \approx \omega_n$.

Hence $$\theta = F\exp(-Dt/2\text{ J})\,[\sin(\omega_n t + \phi)] \qquad \text{...(20)}$$

Differentiating the above equation, we get:

$$\frac{d\theta}{dt} = F\omega_n\exp(-Dt/2\text{ J})\,[\cos(\omega_n t + \phi)] - F(D/2\text{ J})\exp(-D/2\text{ J})\,[\sin(\omega_n t + \phi)] \qquad \text{...(21)}$$

The initial conditions at $t = 0$ are given as $\theta = 0$ and $d\theta/dt = (G/J)\,Q$.

Substituting the above values in Equation (20) and (21), we get:

$$F\sin\phi = 0 \quad \text{or} \quad \phi = 0 \quad \text{and} \quad F = GQ/J\omega_n$$

Therefore, Equation (20) can be written as:

$$\theta = (GQ/J\omega_n)\exp(-Dt/\text{J})\sin\omega_n t$$

$$\Rightarrow \qquad \theta = (GQ/J)\cdot\sqrt{\text{J/K}}\,\exp(-Dt/2\text{ J})\sin\sqrt{\text{K/J}}\,t \qquad \left[\text{Since } \omega_n = \sqrt{\text{K/J}}\right] \qquad \text{...(22)}$$

From Equation (22), we can say that the charge Q is proportional to the deflection θ at any time instant t which implies that the motion of the galvanometer is oscillatory with decreasing amplitude.

Example 3 A galvanometer has the following parameters given as magnetic flux density $B = 8 \times 10^{-3}$ Wb/m^2, number of turns $N = 300$, length and width of coil as 15 mm and 30 mm and control constant is 2.5×10^{-9} Nm/rad. Calculate the final steady state deflection of the galvanometer when a current of 1 μA flows through it.

Solution: Given that: Magnetic flux density $B = 8 \times 10^{-3}$ Wb/m^2

Number of turns $N = 300$

Length of coil $l = 15$ mm

Width of coil $b = 30$ mm, and

Current $I = 1 \times 10^{-6}$ A

Control constant $K = 2.5 \times 10^{-9}$ Nm/rad

The deflection torque T_D of the galvanometer is given by Equations (4) and (9) as:

$$T_D = BINA = GI$$

where $A = l \times b$ is the area of the coil, and G is the galvanometer displacement constant obtained as:

$$G = BNA = 8 \times 10^{-3} \times 300 \times 15 \times 10^{-3} \times 30 \times 10^{-3} = 1.08 \times 10^{-3}$$

The final steady state deflection θ_F is given as:

$$\theta_F = \frac{GI}{K} = \frac{1.08 \times 10^{-3} \times 1 \times 10^{-6}}{2.5 \times 10^{-9}} = 0.432 \text{ rad}$$

3.5 DC AMMETER

A DC ammeter is the instrument used for measuring the direct current (DC) flowing through an electrical circuit and is connected in series to the circuit under consideration. The instrument has an internal resistance which should be much smaller in comparison to the resistance of the circuit. The basic movement of a DC ammeter is constituted by a PMMC d'Arsonval galvanometer. Thus, the basic construction of the DC ammeter remains the same as galvanometer. The deflection of the pointer is in proportion to the current flowing in the moving coil. Here, the maximum deflection of the pointer is produced by a very small current as the coil winding is very light and thin. The large currents may destroy the coil winding. Therefore, to measure such large currents, the ammeter is provided with a resistor of very low resistance, generally less than 1 Ω, and referred to as **shunt** connected in parallel to the instrument coil or meter movement. The shunt may be external or internal to the instrument. The external shunt is made up of manganin or constantan with the low resistance, whereas the internal shunt used with the basic movement is in the form of a constant temperature resistance wire encased within the instrument. However, more often external shunts are used to measure large currents. When a large amount of current is to be measured, the major part of it must be bypassed via the shunt so that the ammeter does not get damaged. The basic configuration of an ammeter circuit is shown in Figure 3.13.

Here, R_{sh} represents shunt resistance and R_m is meter resistance (also known as **coil circuit resistance** or **internal resistance of the movement**). From Figure 3.13, we can say that the total current I is bifurcated into the meter current I_m (also termed as **full scale deflection current of the movement I_{FSD}**) and shunt current I_{sh}, such that, on summing the total current I is $I = I_m + I_{sh}$. It necessitates that the voltage drop across the movement and the shunt must be the same. Thus, the required shunt resistance R_{sh} can be calculated.

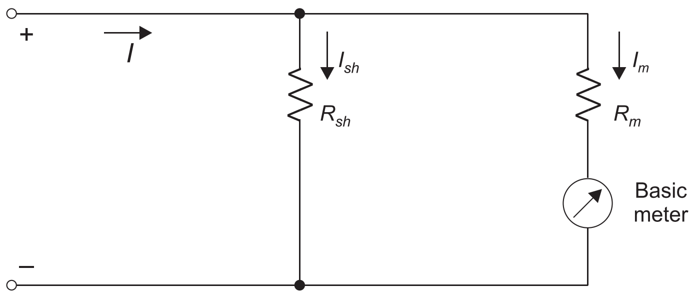


Fig. 3.13 Basic DC Ammeter Circuit

Since $$V_{sh} = V_m$$

Or $$I_{sh}R_{sh} = I_m R_m$$

$\Rightarrow$ $$R_{sh} = \frac{I_m R_m}{I_{sh}} \quad ...(23)$$

Here, we know that:

$$I_{sh} = I - I_m \quad ...(24)$$

Substituting the value of I_{sh} from Equation (24) into Equation (23), we get:

$$R_{sh} = \frac{I_m R_m}{I - I_m} \quad ...(25)$$

Given the values of total current I, meter current I_m, and meter resistance R_m, the value of shunt resistance R_{sh} can be calculated using the relation in Equation (25). Further, the above relation can be modified as:

$$\frac{R_{sh}}{R_m} = \frac{I_m}{I - I_m} \quad \text{or} \quad \frac{R_m}{R_{sh}} = \frac{I - I_m}{I_m}$$

$\Rightarrow$ $$\frac{R_m}{R_{sh}} = \frac{I}{I_m} - 1 \quad \text{or} \quad \frac{I}{I_m} = \frac{R_m}{R_{sh}} + 1 \quad ...(26)$$

Here, term $\frac{I}{I_m}$ represents the multiplying power of the shunt and is denoted by m. Thus, Equation (26) can be written as:

$$m = \frac{R_m}{R_{sh}} + 1$$

Or $$R_{sh} = \frac{R_m}{m - 1} \quad ...(27)$$

Equation (27) is used to determine the shunt resistance of the circuit when the multiplying power of the shunt is known. Note that if shunt resistance R_{sh} is 1 Ω and meter resistance R_m is exactly 99 Ω and the meter shows full scale deflection for a coil current $I_m = 0.1$ mA. Then, the scale should be calibrated as 100 × 0.1 mA = 10 mA to read at full scale.

Temperature compensation

The moving coil in a PMMC instrument is wound with thin copper wire whose resistance changes with change in temperature. The errors are introduced in current measurements due to heating effect of the coil current which produces resistance change. A **swamping resistance** made of manganin or constantan with negligible temperature coefficient having resistance 20 to 30 times the coil resistance is connected in series with the coil and shunt resistance made of manganin is connected in parallel to the combination in order to avoid resistance changes with temperature [see Figure 3.14]. Therefore, the current *I* flowing through the circuit divides in proportion between the meter and the shunt which does not change appreciably with change in temperature.

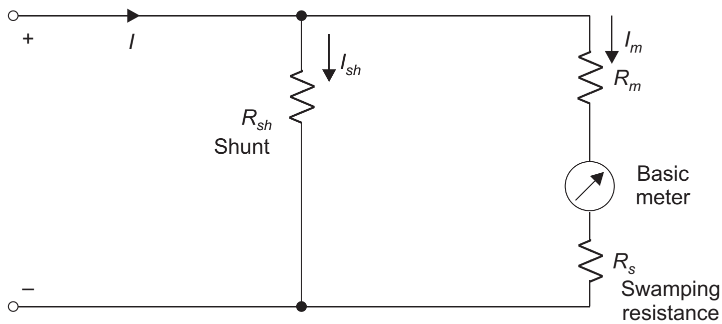


Fig. 3.14 DC Ammeter Circuit with Swamping Resistance

Example 4 A moving coil meter has a resistance of 5 Ω and gives a full scale deflection current with 5 mA. Show that it can be used to measure current up to 10 A and find its shunt resistance.

Solution: Given that: coil resistance $R_m = 5\ \Omega$

Full scale deflection current of the meter $I_m = I_{\text{FSD}} = 5$ mA

Required full scale deflection current $I = 10$ A

We know that the meter voltage is given as:

$$V_m = I_m R_m$$

Substituting the given values in the above relation, we get:

$$V_m = 5 \times 5 \times 10^{-3} = 25 \text{ mV}$$

Also, from Equation (24), the shunt current I_{sh} is given as:

$$I_{sh} = I - I_m$$

$$I_{sh} = 10 - 5 \times 10^{-3} = 9.995 \text{ A}$$

Hence, it can be seen that when I_{sh} is 9.995 A and I_m is 0.005 A, the total current an ammeter can measure is 10 A.

The shunt resistance R_{sh} can be obtained as:

$$R_{sh} = \frac{I_m R_m}{I_{sh}} = \frac{V_m}{I_{sh}} \qquad \text{[Refer to Eqn. (23)]}$$

Substituting the given values, we get:

$$R_{sh} = \frac{25 \times 10^{-3}}{9.995} = 2.501 \text{ m}\Omega$$

Example 5 A PMMC instrument with an FSD of 30 ΩA and a coil resistance of 1.2 kΩ is to be used as DC ammeter. A resistance of 133.3 Ω is connected in shunt with the instrument. Determine the measured current at:

(a) FSD
(b) 0.5 FSD
(c) 0.33 FSD.

Solution: Given that: PMMC instrument FSD I_m = 30 μA
Coil resistance R_m = 1.2 kΩ
Shunt resistance R_{sh} = 133.3 Ω

(a) The meter voltage V_m is given as:

$$V_m = I_m R_m$$

Substituting the values, we get:

$$V_m = 30 \times 10^{-6} \times 1.2 \times 10^3$$

$$V_m = 36 \text{ mV}$$

Now, the shunt current I_{sh} is computed as:

$$I_{sh} = \frac{V_m}{R_{sh}} = \frac{36 \times 10^{-3}}{133.3} = 0.27 \text{ mA} \qquad \text{[Refer to Eqn. (23)]}$$

Thus, the total current I is given as:

$$I = I_m + I_{sh}$$

$$I = 30 \times 10^{-6} + 270 \times 10^{-6}$$

$$I = 300 \text{ μA}$$

(b) At 0.5 FSD, the meter current I_m can be calculated as:

$$I_m = 0.5 \times 30 \times 10^{-6} = 15 \text{ μA}$$

The meter voltage V_m is given as:

$$V_m = I_m R_m$$

Substituting the values, we get:

$$V_m = 15 \times 10^{-6} \times 1.2 \times 10^3$$

$$V_m = 18 \text{ mV}$$

Now, we may calculate the shunt current I_{sh} as:

$$I_{sh} = \frac{V_m}{R_{sh}} = \frac{18 \times 10^{-3}}{133.3} = 0.135 \text{ mA} \qquad \text{[Refer to Eqn. (23)]}$$

The total current I can be given as:

$$I = I_m + I_{sh}$$

or

$$I = 15 \times 10^{-6} + 135 \times 10^{-6}$$

$$I = 150 \ \mu\text{A}$$

(c) At 0.33 FSD, the meter current I_m can be calculated as:

$$I_m = 0.33 \times 30 \times 10^{-6} = 9.9 \ \mu\text{A}$$

And, the meter voltage V_m is given as:

$$V_m = I_m R_m$$

Substituting the values, we get:

$$V_m = 9.9 \times 10^{-6} \times 1.2 \times 10^3$$

$$V_m = 11.88 \text{ mV}$$

Now, the shunt current I_{sh} can be calculated as:

$$I_{sh} = \frac{V_m}{R_{sh}} = \frac{11.88 \times 10^{-3}}{133.3} = 0.089 \text{ mA} \qquad \text{[Refer to Eqn. (23)]}$$

The total current I can be given as:

$$I = I_m + I_{sh}$$

$$I = 9.9 \times 10^{-6} + 89 \times 10^{-6}$$

$$I = 98.9 \ \mu\text{A}$$

3.5.1 Multi-Range Ammeter

Earlier, we have studied the DC ammeter as a basic current measuring instrument which is able to measure the current up to a specific range. However, this current range can be extended further by using a number of shunts with different resistance values which can be selected by range switch. Such an ammeter is capable of measuring DC current in several ranges and is called a **multi-range ammeter** as shown in Figure 3.15.

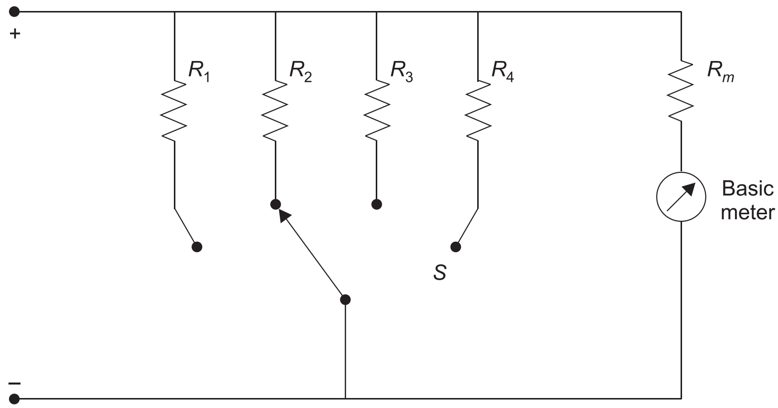


Fig. 3.15 Multi-Range Ammeter

The circuit provides four different current ranges since four shunts R_1, R_2, R_3, and R_4 are connected in parallel to the meter movement. The switch S shown in the figure is a rotary type *make-before-break* switch used to select the ranges. It is connected in series with the low resistance shunts so as to provide low contact resistance and high current carrying capacity. It is customary to use such a switch to ensure that the instrument should not be left without a shunt even for a small fraction of time. This switch also avoids the possibility of high resistance of the instrument to affect the circuit current as a large current could flow through its moving coil, thereby destroying the PMMC meter. The moving contact of the switch makes contact with the next shunt terminal before breaking its contact with the previous shunt terminal. Thus, there are always two shunts in parallel to the meter during switching.

The range of a multi-range ammeter goes up to 50 A. While using a multi-range ammeter, always start with the highest current range and then switch to some lower range until a satisfactory upscale reading is attained. The overall cost of this meter is comparatively higher due to high precision resistors.

Ayrton shunt

The Ayrton, or **universal,** shunt has its own significance in an ammeter circuit. It eliminates the likelihood of having the meter without a shunt in the circuit. The configuration of an Ayrton shunt ammeter is illustrated in Figure 3.16. The overall resistance gets slightly increased using this shunt.

The switch changes its position offering different current ranges. In the circuit shown in the figure, the switch S can be positioned to any three positions marked as 1, 2, and 3. If the switch S is in position 1, resistance R_1 becomes parallel with the meter resistance R_m and the series combination of R_2 and R_3 resistors (thus, the total resistance in parallel is $R_m + R_2 + R_3$). This implies that the current flowing through the meter is much less than the current through the shunt which protects the meter movement and reduces its sensitivity.

If the switch S is connected to position 2, resistance R_1 together with resistance R_2 (that is, $R_1 + R_2$) becomes parallel to the serially connected meter resistance R_m and resistance R_3 (that is, $R_m + R_3$). In this case, the current through the meter becomes more than the current flowing through the shunt resistance.

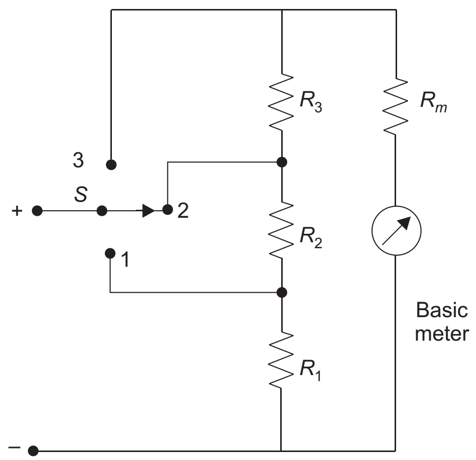


Fig. 3.16 Ayrton Shunt Ammeter

If the switch S is connected to position 3, all three resistances R_1, R_2, and R_3 (that is, $R_1 + R_2 + R_3$) are connected in parallel to the meter resistance R_m. This incurs that the current through the meter is the maximum in this case and very small current flows through the shunt. Hence, the sensitivity of the meter gets increased.

Example 6 A d'Arsonval movement with coil resistance 200 Ω and full scale deflection current 5 mA is to be converted to a multi-range ammeter with ranges:

(a) 0–1 A
(b) 0–5 A
(c) 0–10 A

Design the required multi-range ammeter.

Solution: Given that: full scale deflection current I_m = 5 mA

Coil resistance R_m = 200 Ω

The relation for the shunt resistance of the ammeter is given as:

$$R_{sh} = \frac{I_m R_m}{I - I_m} \quad \text{[Refer to Eqn. (25)]} \quad ...(6a)$$

(a) For the range 0–1 A, that is, 1000 mA, the shunt resistance R_{sh} can be obtained by substituting the given values in Equation (6a), we get:

$$R_{sh} = \frac{5 \times 200}{1000 - 5} = \frac{1000}{995} = 1.005\ \Omega$$

(b) Similarly, for the range 0–5 A, that is, 5000 mA, the shunt resistance R_{sh} comes out to be:

$$R_{sh} = \frac{5 \times 200}{5000 - 5} = 0.200\ \Omega$$

(c) Again, for the range 0–10 A, that is, 10,000 mA, the shunt resistance R_{sh} comes out to be:

$$R_{sh} = \frac{5 \times 200}{10000 - 5} = 0.100\ \Omega$$

3.6 DC VOLTMETER

A DC voltmeter is the instrument used for measuring the potential difference between two points or terminals in a DC circuit or a circuit component. The basic movement of this voltmeter is constituted by PMMC d'Arsonval movement. The PMMC instrument deflects in proportion to the coil current while the coil current is directly proportional to the voltage across the coil. This implies that the PMMC instrument can be used as a DC voltmeter to measure the voltage. It must be connected to the circuit under consideration with proper polarity. The coil resistance R_m is usually very small and the coil voltage also results in small voltage measurements. Therefore, to increase the range of the instrument, a resistor is connected in series with the meter, known as **multiplier** or **multiplier resistor,** denoted as R_s as shown in Figure 3.17. The combination of multiplier and meter resistance (that is, total resistance) is put across the circuit to measure the voltage. The multiplier limits the current flowing through the meter so that it may not surpass the full scale deflection value. This prevents the movement from being damaged.

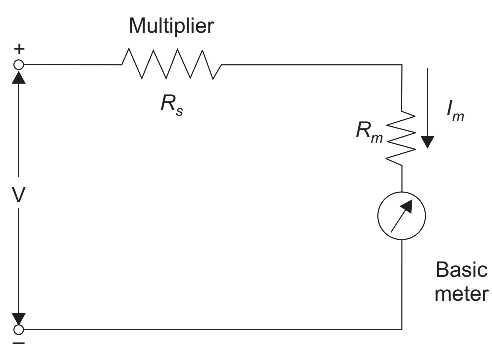


Fig. 3.17 DC Voltmeter Circuit

The required value of the multiplier R_s can be calculated as:

$$V = I_m(R_s + R_m)$$

$$\Rightarrow \quad V = I_m R_s + I_m R_m \qquad \text{...(28)}$$

$$R_s = \frac{V - I_m R_m}{I_m} \quad \text{or} \quad R_s = \frac{V}{I_m} - R_m \qquad \text{...(29)}$$

Equation (29) is used to calculate the resistance of the multiplier provided the values of potential difference V, meter current or full scale deflection current I_m, and meter resistance or internal resistance of meter movement R_m are known.

It can also be expressed in terms of the multiplying factor m as:

$$m = \frac{V}{\upsilon} \quad ...(30)$$

where υ represents the voltage across the meter movement, that is, $\upsilon = I_m R_m$. The above expression shows that the potential difference or full range voltage of instrument V is m times the voltage across the meter movement. Thus, it yields:

$$m = \frac{I_m R_m + I_m R_s}{I_m R_m} = 1 + \frac{R_s}{R_m}$$

Or $$R_s = (m - 1)\, R_m \quad ...(31)$$

From Equations (30) and (31), we can say that the multiplier must be $(m - 1)$ times the meter resistance. Thus, the multiplier increases the voltage range by a factor of 10 as multiplier resistance R_s is nine times the meter resistance R_m.

Loading effect and sensitivity

The voltmeter is always connected in parallel with the points in a circuit at which the voltage is to be measured. It must possess a very high value of resistance because a low resistance may result in changing of voltage of the circuit under test. This is known as **loading effect** of the voltmeter.

When selecting a meter for certain voltage measurement, the **sensitivity** of DC voltmeter is an important factor. It is defined as the total resistance of the instrument per unit volt. The total resistance of the voltmeter is the sum of multiplier resistance R_s and coil resistance R_m, that is, $R_s + R_s$. The voltmeter sensitivity is specified by manufacturer as printed on the scale of the instrument. For a given sensitivity, the total resistance of the voltmeter can be easily obtained as the product of the meter range and sensitivity (meter range × sensitivity). On the other hand, for a known value of the full scale current of the meter, the sensitivity of the meter can be found as the reciprocal of the full scale current.

Effect of temperature changes

Similar to DC ammeter, some errors creep in DC voltmeter when the coil resistance R_m changes with the change in temperature. The multiplier R_s is made of manganin with negligible temperature coefficient connected in series with coil resistance R_m. The variation in coil resistance due to temperature changes are swamped by multiplier R_s as the series resistance of multiplier R_s is much greater than the coil resistance R_m which is made of copper.

Example 7 A PMMC instrument with FSD of 100 μA and a coil resistance of 1 kΩ is to be converted into a voltmeter. Determine the required multiplier resistance if the voltmeter is to measure 50 V at full scale. Also, calculate the applied voltage when the instrument indicates:

(a) 0.8 FSD

(b) 0.5 FSD

(c) 0.2 FSD

Solution: Given that: Full scale deflection current of meter $I_{FSD} = I_m = 100\ \mu A$

Coil resistance $R_m = 1\ k\Omega$

Full range voltage of instrument $V = 50$ V

The multiplier resistance R_s is given as:

$$R_s = \frac{V}{I_m} - R_m \qquad \text{[Refer to Eqn. (29)]}$$

On substituting the given values, we get:

$$R_s = \frac{50}{100 \times 10^{-6}} - 1 \times 10^3$$

$$= 499\ k\Omega$$

The applied voltage across the circuit is given as:

$$V = I_m(R_s + R_m) \qquad \text{...(7a)}$$

(a) For 0.8 FSD, meter current $I_m = 0.8 \times 100 = 80\ \mu A$. Hence, the applied voltage across the instrument is obtained by substituting the values in Equation (7a) as:

$$V = 80 \times 10^{-6}(499 \times 10^3 + 1 \times 10^3) = 40 \text{ V}$$

(b) Similarly, for 0.5 FSD, $I_m = 50\ \mu A$ and thus, the applied voltage can be obtained by substituting the values in Equation (7a) as:

$$V = 50 \times 10^{-6}(499 \times 10^3 + 1 \times 10^3) = 25 \text{ V}$$

(c) For 0.2 FSD, $I_m = 20\ \mu A$, the applied voltage can be obtained by substituting the values in Equation (7a) as:

$$V = 20 \times 10^{-6}(400 \times 10^3 + 1 \times 10^3) = 10 \text{ V}$$

3.6.1 Multi-Range Voltmeter

Similar to the multi-range ammeters, voltmeters can also be made multi-ranged by connecting several multipliers of different values to the meter to measure workable ranges of voltages. Thus, a large range of voltages can be measured with a multi-range voltmeter. A *make-before-break* rotary switch is incorporated to switch between the multipliers R_1, R_2, R_3, and R_4 in accordance with the desired voltage range V_1, V_2, V_3, and V_4 [see Figure 3.18].

The circuit shown in Figure 3.18 has four different ranges of voltages as four multipliers of different values have been used. The switch S establishes the contact with one of the multipliers to measure a specific range of voltage. The moving contact of the switch S makes the contact with the next multiplier before breaking its contact with the previous multiplier. Thus, during switching there are always two multipliers connected to the meter.

The circuit in Figure 3.18 can be modified to a new circuit as shown in Figure 3.19 in which the multipliers are connected in a series string to the meter. This configuration is preferable over the previous one as multipliers R_1, R_2, and R_3 are standard valued and therefore easily available in precision tolerances. However, the low range multiplier R_4 is the only special (non-standard) resistor which is to be manufactured specifically as per the circuit requirements.

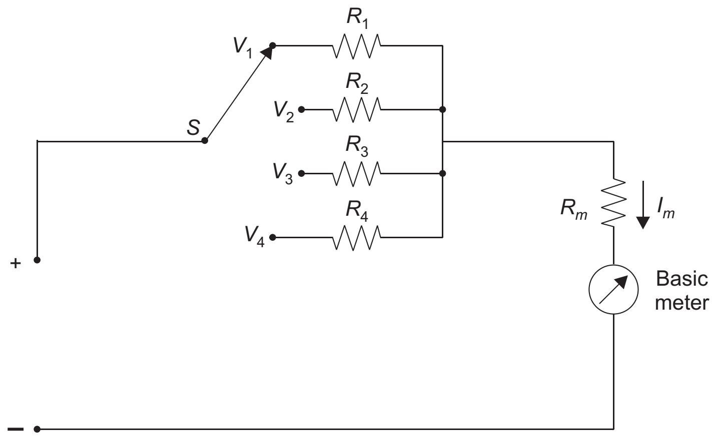


Fig. 3.18 Multi-Range Voltmeter

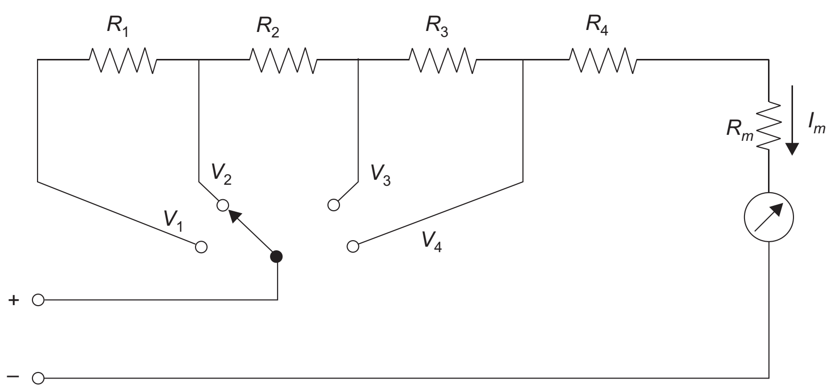


Fig. 3.19 Modified Configuration of Multi-Range Voltmeter

Example 8 Convert a basic d'Arsonval movement with internal resistance 50 Ω, and full scale current 2 mA into a multi-range DC voltmeter with voltage ranges of 0–10 V, 0–50 V, 0–100 V, and 0–250 V.

Solution: Given that: Internal resistance of the meter $R_m = 50\ \Omega$

Full scale deflection of current $I_{FSD} = I_m = 2$ mA

Here, we know that the lowest range of the voltmeter is 0–10 V. Therefore, this range must correspond to the resistance nearest to the meter, that is, at V_4 position as seen in Figure 3.18.

The total resistance R_T for this voltage range can be obtained as:

$$R_T = \frac{V}{I_{FSD}} = \frac{10}{2 \times 10^{-3}} = 5\ \text{k}\Omega$$

Thus, resistance R_4 comes out to be:

$$R_4 = R_T - R_m = 5000\ \Omega - 50\ \Omega = 4950\ \Omega$$

Similarly, for 0–50 V range, the total resistance R_T becomes:

$$R_T = \frac{50}{2 \times 10^{-3}} = 25\ \text{k}\Omega$$

Resistance R_3 is obtained as:

$$R_3 = R_T - (R_4 + R_m) = 25\ \text{k}\Omega - (4950 + 50)\ \Omega = 20\ \text{k}\Omega$$

For 0–100 V range, the total resistance R_T becomes:

$$R_T = \frac{100}{2 \times 10^{-3}} = 50\ \text{k}\Omega$$

Resistance R_2 is obtained as:

$$R_2 = R_T - (R_3 + R_4 + R_m) = 50\ \text{k}\Omega - (20\ \text{k}\Omega + 4950\ \Omega + 50\ \Omega) = 25\ \text{k}\Omega$$

For 0–250 V range, the total resistance R_T becomes:

$$R_T = \frac{250}{2 \times 10^{-3}} = 125\ \text{k}\Omega$$

Resistance R_1 is obtained as:

$$R_1 = R_T - (R_2 + R_3 + R_4 + R_m) = 125\ \text{k}\Omega - (25\ \text{k}\Omega + 20\ \text{k}\Omega + 4950\ \Omega + 50\ \Omega) = 75\ \text{k}\Omega$$

3.7 OHMMETER

Ohmmeter is an electrical instrument which provides direct reading of the resistance measurement conveniently. The resistance is measured in ohms (denoted as Ω). Ohmmeter is usually a low accuracy instrument consisting of a d'Arsonval movement. It is used for checking the continuity of the circuits, measure the resistance of the circuit components such as machine field coils and heater elements, and for checking the semiconductor diodes. The ohmmeter cannot exist as an individual instrument; instead, it can be configured as *series ohmmeter* and *shunt ohmmeter* in its simplest form.

3.7.1 Series Ohmmeter

The series ohmmeter is based on d'Arsonval PMMC movement. The circuit consists of a battery voltage E_b connected in series with a pair of terminals, A and B (where unknown resistance R_x to be measured is connected across it), standard resistance R_1, and a low current PMMC instrument as shown in Figure 3.20(a). The calibrated scale of the ohmmeter is shown in Figure 3.20(b).

The current flowing through the meter, that is I_m for a total series resistance is obtained as:

$$I_m = \frac{\text{Battery voltage}}{\text{Total series resistance}} = \frac{E_b}{R_x + R_1 + R_m} \qquad \text{...(32)}$$

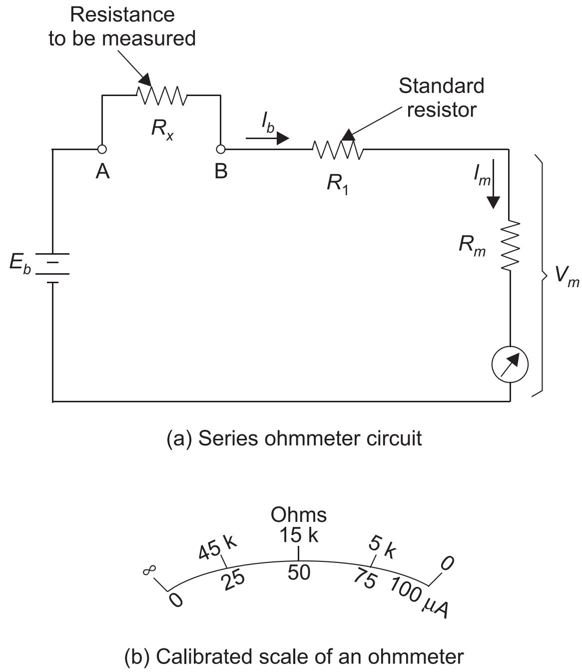


Fig. 3.20 Series Ohmmeter

Or $$R_x = \frac{E_b}{I_m} - (R_1 + R_m)$$

When the terminals A and B are short-circuited, the unknown (external) resistance is zero (that is, $R_x = 0$), and resistors R_1 and R_m give full scale deflection current I_{FSD} as:

$$I_m = I_{FSD} = \frac{E_b}{R_1 + R_m} \qquad ...(33)$$

This condition of full scale deflection when the maximum current flows is marked as zero ohms ("0") at the extreme right on the calibrated scale. On the other hand, when the terminals A and B are open-circuited, that is, unknown resistance R_x is removed from the circuit ($R_x = \infty$), no current flows through the meter and the pointer indicates this zero current position as infinity (∞) at the extreme left on the calibrated scale. When the resistance R_x having a value between zero and infinity is connected across terminals A and B, the meter current I_m lies between zero and FSD. The pointer indication on the scale would be with respect to the relation between R_x and $R_1 + R_m$.

The series ohmmeter circuit discussed in Figure 3.19(a) operates precisely till the battery voltage is exactly at 1.5 V. With the use of battery, its voltage drops and the instrument scale no longer gives correct reading. Even when the terminals A and B are short-circuited to adjust the value of resistance R_1 to give FSD, the scale still gives an error as now the midscale represents the resistance equal to the new value of $R_1 + R_m$. Therefore, an adjustable resistor R_2 is connected in parallel with the meter to adjust the falling battery voltage as shown in

Figure 3.21. The figure shows that the current of the battery I_b is bifurcated into the meter current I_m and the resistor current I_2.

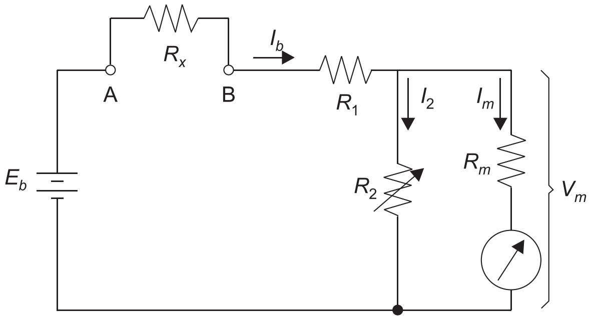


Fig. 3.21 Modified Series Ohmmeter With Adjustable Resistor

Let the terminals A and B be short-circuited, that is, $R_x = 0$, the total circuit resistance then becomes $R_1 + (R_2 \parallel R_m)$. Resistor R_1 is always larger than the parallel combination of R_2 and R_m to a great extent. Therefore, R_1 is considered to be the total resistance of the circuit. Now, if a resistor R_x equal in value to that of R_1 (that is, $R_x = R_1$) is connected across the circuit terminals, the total resistance of the circuit gets doubled making the total circuit current to be halved and so the currents I_m and I_2 are reduced to half of their previous values. This ensures that the midscale measured value of the resistance is equal to the internal resistance of the ohmmeter, that is, R_1. The battery current I_b may be expressed as:

$$I_b = \frac{E_b}{R_x + R_1 + R_2 \parallel R_m} \quad \text{...(34)}$$

Since $R_1 >> R_2 \parallel R_m$, the above relation can be written as:

$$I_b \simeq \frac{E_b}{R_x + R_1} \quad \text{...(35)}$$

From the circuit, the voltage across the meter V_m is represented as:

$$V_m = I_b(R_2 \parallel R_m)$$

Similarly, the current through the meter I_m can be related to the battery current I_b as:

$$I_m = \frac{I_b(R_2 \parallel R_m)}{R_m} \quad \text{...(36)}$$

It should be noted that while using ohmmeter, it should always be short-circuited first and then variable resistor R_2 is adjusted to attain the full scale deflection I_{FSD} to indicate zero ohm on the scale. This procedure ensures that the calibration of the meter prevents incorrect readings of the scale even if the battery voltage falls below its initial value.

Example 9 A series ohmmeter circuit has the battery voltage $E_b = 1.5$ V, $R_1 = 10$ kΩ, $R_m = 60$ Ω, $R_2 = 60$ Ω, and $I_{FSD} = 60$ µA. Calculate: (a) reading of the ohmmeter at 0.5 FSD, (b) value of R_2 when E_b falls to 1.3 V at full scale deflection current, (c) value of unknown resistance for $E_b = 1.3$ V at 0.5 I_{FSD}.

Solution: Given that: Battery voltage $E_b = 1.5$ V

Standard resistance $R_1 = 10$ kΩ

Meter resistance $R_m = 60$ Ω

Variable resistance $R_2 = 60$ Ω, and

Full scale deflection current of the meter $I_{\text{FSD}} = 60$ μA

(a) The meter voltage V_m is given as:

$$V_m = I_m R_m$$

And, at 0.5 FSD, it becomes:

$$V_m = \frac{I_{FSD}}{2} R_m$$

Substituting the given values, we get:

$$V_m = 30 \times 10^{-6} \times 60$$

$$\Rightarrow \qquad V_m = 1.8 \text{ mV}$$

Now, the current flowing through the resistor R_2 can be obtained as:

$$I_2 = \frac{V_m}{R_2} = \frac{1.8 \times 10^{-3}}{60}$$

$$\Rightarrow \qquad I_2 = 30 \text{ μA}$$

The battery current I_b can be calculated as:

$$I_b = I_2 + I_m$$

$$I_b = 30 \times 10^{-6} + 30 \times 10^{-6}$$

$$\Rightarrow \qquad I_b = 60 \text{ μV}$$

From Equation (35), we have:

$$I_b \simeq \frac{E_b}{R_1 + R_x} \quad \Rightarrow \quad R_1 + R_x \simeq \frac{E_b}{I_b}$$

On substituting the values, it gives:

$$10 \times 10^3 + R_x \simeq \frac{1.5}{60 \times 10^{-6}}$$

Or

$$R_x \simeq \frac{1.5}{60 \times 10^{-6}} - 10 \times 10^3$$

$$R_x = 15 \text{ kΩ}$$

Thus, the reading of ohmmeter at 0.5 FSD is 15 kΩ.

(b) The adjusted value of variable resistance R_2 for a new value of battery voltage E_b can be determined

when $R_x = 0$ and $E_b = 1.3$ V.

The battery current I_b is given as:

$$I_b \simeq \frac{E_b}{R_x + R_1} \qquad \text{[Refer to Eqn. (35)]}$$

Substituting the given values, we get:

$$I_b \simeq \frac{1.3}{10 \times 10^3 + 0} = 130 \ \mu\text{A}$$

Also, $$I_b = I_2 + I_{m(\text{FSD})}$$

$$\Rightarrow \quad I_2 = I_b - I_{m(\text{FSD})} = 130 \times 10^{-6} - 60 \times 10^{-6} = 70 \ \mu\text{A}$$

Now, the battery voltage V_m can be obtained as:

$$V_m = I_m R_m = 60 \times 10^{-6} \times 60 = 3.6 \text{ mV}$$

The resistance R_2 can be found as:

$$R_2 \simeq \frac{V_m}{I_2} = \frac{3.6 \times 10^{-3}}{70 \times 10^{-6}} = 51.42 \ \Omega$$

(c) The value of the unknown resistance R_x for $E_b = 1.3$ V and 0.5 I_{FSD} can be obtained as follows.

The meter voltage is found to be $V_m = 1.8$ mV as in part (a). Now, the current through the variable resistor R_2 can be found as:

$$I_2 \simeq \frac{V_m}{R_2} = \frac{1.8 \times 10^{-3}}{51.42} = 35 \ \mu\text{A}$$

Thus, the battery current I_b can be obtained as:

$$I_b = I_2 + I_m = 35 \times 10^{-6} + 30 \times 10^{-6} = 65 \ \mu\text{A}$$

Now the unknown resistance R_x can be obtained from Equation (35) as:

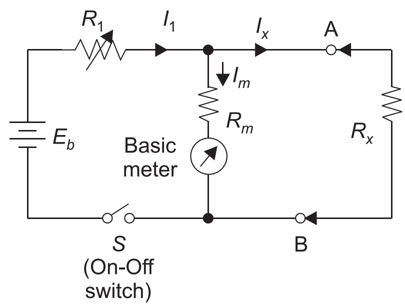
$$I_b \simeq \frac{E_b}{R_1 + R_x} \quad \Rightarrow \quad R_1 + R_x \simeq \frac{E_b}{I_b}$$

$$\Rightarrow \quad 10 \times 10^3 + R_x \simeq \frac{1.3}{65 \times 10^{-6}}$$

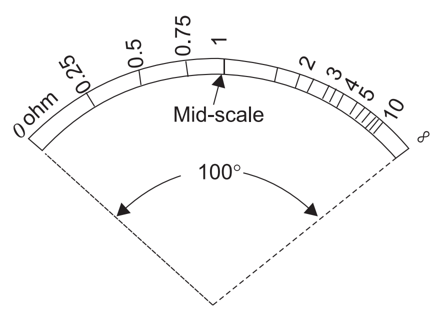
$$\Rightarrow \quad R_x = 10 \text{ k}\Omega$$

3.7.2 Shunt Ohmmeter

The ohmmeter can be configured as a shunt ohmmeter as shown in Figure 3.22(a) consisting of a voltage source (battery voltage E_b) connected in series with a variable (adjustable) resistor R_1 and d'Arsonval movement (meter). The resistor R_x whose value is to be measured is connected in parallel to the meter across the terminals A and B. It is customary to include an on-off switch in the circuit to disconnect the battery whenever the instrument is not in use.



(a) Shunt ohmmeter circuit



(b) Calibrated scale of a shunt ohmmeter

Fig. 3.22 Shunt Ohmmeter

Initially, let the terminals A and B are shorted, that is, $R_x = 0\ \Omega$. The meter reads the minimum current at this point, that is $I_m = 0$ A. Now, if the terminals A and B are open-circuited, that is, $R_x = \infty$, the full scale deflection may be read by the pointer. This is due to the fact that the current flows through the meter of the circuit. Therefore, by selecting an appropriate value of resistor R_1, full scale deflection current I_{FSD} can be measured. The scale of a shunt ohmmeter is different from that of a series ohmmeter. It is marked zero at its left hand side which yields zero meter current while it is marked infinity at its right hand side implying a full scale deflection current as illustrated in Figure 3.22(b). The shunt ohmmeter is used in laboratories for measurements of low resistance values. The shunt ohmmeter can be analyzed similar to that of series ohmmeter. For $R_x = \infty$, the full scale current of the meter I_{FSD} is calculated as:

$$I_{\text{FSD}} = \frac{E_b}{R_1 + R_m} \qquad ...(37)$$

Or
$$R_1 = \frac{E_b}{I_{\text{FSD}}} - R_m$$

When unknown resistance R_x is connected across terminals A and B, the meter current I_m is obtained as:

$$I_m = \left(\frac{E_b}{R_1 + \dfrac{R_m R_x}{R_m + R_x}} \right) \left(\frac{R_x}{R_m + R_x} \right) = \frac{E_b R_x}{R_1 R_m + R_x (R_1 + R_m)} \qquad ...(38)$$

The meter can be calibrated by calculating the fraction of full scale s for a given resistance R_x which can be obtained by dividing Equation (38) by (37) as:

$$s = \frac{I_m}{I_{\text{FSD}}} = \frac{R_x (R_1 + R_m)}{R_1 R_m + R_x (R_1 + R_m)} \qquad ...(39)$$

Since R_1 is very large as compared to the meter resistance R_m. Hence, Equation (40) becomes:

$$s = \frac{I_m}{I_{\text{FSD}}} = \frac{R_x}{R_m + R_x} \qquad ...(40)$$

The above value of s is useful for calibrating the meter in terms of R_x and R_m.

At 0.5 FSD, that is, at half scale reading of the meter $I_m = 0.5\ I_{\text{FSD}}$ and $R_x = R_h$

$$0.5\ I_{\text{FSD}} = \frac{E_b R_h}{R_1 R_m + R_h (R_1 + R_m)} \qquad ...(41)$$

The half scale reading can be obtained by dividing Equation (37) by (40) and solving R_h for a given value of R_1 as:

$$R_h = \frac{R_1 R_m}{R_1 + R_m}$$

Note: *For highest accuracy, the range of ohmmeter must be so selected as to indicate nearly half of the full scale deflection.*

Example 10 A d'Arsonval movement shunt ohmmeter has an internal resistance of 7 Ω, full scale deflection current of 12 mA, and battery voltage of 5 V. The circuit of Figure 3.21(a) is modified by adding appropriate resistor R_{sh} across the movement. Calculate the value of the shunt resistance R_{sh} such that the instrument indicates 0.7 Ω at 0.5 FSD.

Solution: Given that: Internal resistance of the meter = 7 Ω

Full scale deflection current I_{FSD} = 12 mA

Battery voltage E_b = 5 V

For half scale deflection, the current through the movement (meter) I_m is:

$$I_m = 0.5\ I_{\text{FSD}} = 0.5 \times 12 = 6 \text{ mA} \qquad \text{...(10a)}$$

The voltage across the movement (meter) can be calculated as:

$$V_m = I_m R_m$$

At 0.5 FSD, it becomes:

$$V_m = \frac{I_{\text{FSD}}}{2} R_m$$

Substituting the given values, we get:

$$V_m = 6 \times 10^{-3} \times 7 = 42 \text{ mV}$$

Now, the current through the unknown resistance R_x at 0.5 FSD for R_m = 0.7 Ω is determined as:

$$I_x \simeq \frac{V_m}{R_m} = \frac{42 \times 10^{-3}}{0.7} = 60 \text{ mA} \qquad \text{...(10b)}$$

Also, from Figure 3.21(a) we can say, that I_x is the sum of the currents through the shunt I_{sh} and through meter I_m and is obtained as:

$$I_x = I_{sh} + I_m$$

Thus, the current through the shunt resistance I_{sh} can be obtained as:

$$I_{sh} = I_x - I_m$$

Substituting the values of current in the above relation from Equation (10a) and (10b), we get:

$$I_{sh} = 60 \times 10^{-3} - 6 \times 10^{-3} = 54 \text{ mA}$$

Hence, the shunt resistance is computed as:

$$R_{sh} \simeq \frac{V_m}{I_{sh}} = \frac{42 \times 10^{-3}}{54 \times 10^{-3}} = 0.77\ \Omega$$

3.8 INSTRUMENT CALIBRATION

All the DC instruments discussed in this chapter need to be checked for their desired accuracy and smooth functioning. This is done by comparing a given instrument with a standard one which is more accurate. The comparison methods are carried out by highly accurate voltage and current sources, called as **calibrators** and the procedure is known as **instrument calibration**. Let us discuss the calibration technique for voltmeters, ammeters, and ohmmeters individually.

3.8.1 DC Ammeter Calibration

The calibration of a DC ammeter can be done by employing the circuit shown in Figure 3.23. The circuit consists of a constant current source, an ammeter under test to be calibrated, a potentiometer (which measures potential difference), a rheostat R, and a standard resistor R_1. The potentiometer measures the voltage across the standard resistor R_1 by voltmeter method and then the corresponding current is obtained using Ohm's law. This value is then compared with the actual reading of the ammeter under calibration and inserted in the circuit.

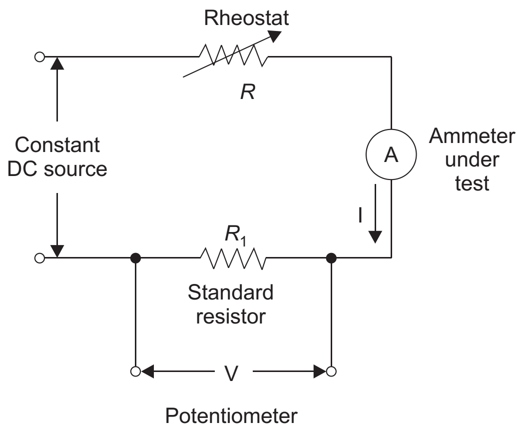


Fig. 3.23 Calibration of a DC Ammeter

The rheostat R is connected in order to control the current level to any desired value. It ensures the calibration of different points on the ammeter scale. The current source must be constant and thus, storage cells or precision power supply can be used.

3.8.2 DC Voltmeter Calibration

The calibration of a DC voltmeter with the help of a potentiometer as a standard instrument is shown in Figure 3.24. The circuit consists of a potentiometer which accurately measures the voltage drop across resistor R_1. The voltmeter under test to be calibrated is connected across the same points as the potentiometer and should therefore indicate the same voltage.

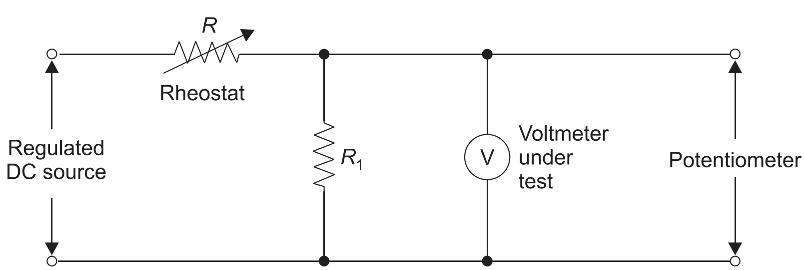


Fig. 3.24 Calibration of a DC Voltmeter

The rheostat R controls the amount of current and thereby the voltage drop across resistor R_1. This ensures a calibration at a number of points on the voltmeter scale. The voltmeters calibrated by this technique are capable of providing an accuracy of ± 0.01 per cent.

3.8.3 Ohmmeter Calibration

An ohmmeter is an instrument which provides moderate accuracy and low precision. Thus, the calibration of an ohmmeter can be done in a rough manner. A standard resistance is measured and the corresponding reading of the ohmmeter is noted. This procedure is repeated a number of times for different points on the ohmmeter scale and for different ranges to obtain an idea about the accuracy of the instrument. Thus the instrument is well calibrated.

Let us Summarize

1. Electromechanical instruments are the electrically operated mechanical instruments used to measure physical quantities and keep these instruments in proper running condition.
2. The electromechanical instruments include permanent magnet moving coil instruments, like galvanometers, DC ammeters and voltmeters, and series and shunt ohmmeters.
3. The permanent magnet moving coil, abbreviated as PMMC is an instrument that consists of light weight coil of copper wire suspended in the field of permanent magnet. The PMMC instrument acts as a basic building block for other electromechanical instruments.
4. The electrical measuring instruments are categorized as analog and digital according to their function.
5. Analog instruments, also termed as deflection instruments use scale and pointer to indicate the quantity being measured, whereas digital instruments use numerical (or digital) format to display the measurement.
6. Analog instruments are further classified as passive and active instruments. Both active analog and digital instruments are termed as electronic instruments.
7. Analog instruments are broadly classified as direct measuring and comparison instruments, and absolute and secondary instruments.
8. PMMC and the instruments based on it are deflection instruments that use a pointer to move on a calibrated scale to indicate a measured quantity. There are three operating forces, namely, deflecting force, controlling force, and damping force required for their operation.
9. A galvanometer is a PMMC instrument designed to detect and measure the magnitudes of small currents and voltages in a circuit. There are two types of galvanometer, namely, d'Arsonval galvanometer and ballistic galvanometer.
10. A DC ammeter is the instrument used for measuring the direct current (dc) flowing through an electrical circuit and is connected in series to the circuit under consideration.
11. An ammeter which is capable of measuring DC current in several ranges is called a multi-range ammeter.
12. A DC voltmeter is the instrument used for measuring the potential difference between two points or terminals in a DC circuit or a circuit component.
13. Similar to the multi-range ammeters, voltmeters can also be made multi-ranged by connecting several multipliers of different values to the meter to measure workable ranges of voltages.
14. Ohmmeter is an electrical instrument which provides direct reading of the resistance value conveniently. The ohmmeters cannot exist as individual instruments; instead, they can be configured as series ohmmeter and shunt ohmmeter in their simplest form.

15. All the DC instruments need to be checked for their desired accuracy and smooth functioning. This is done by comparing a given instrument with a standard one which is more accurate. The comparison methods are carried out by highly accurate voltage and current sources, called calibrators and the procedure is known as instrument calibration.

EXERCISES

Fill in the Blanks

1. The PMMC instruments possess _________, _________, and _________ operating forces.
2. In DC voltmeters, the instrument resistance per unit volt is defined as _________.
3. To attain highest accuracy, the ohmmeter must indicate _________.
4. Calibration of instruments ensures _________.
5. Using Ayrton shunt, the overall resistance of the ammeter gets _________.

Multiple Choice Questions

1. The loading effect in a voltmeter can be avoided by using
   (a) high voltage range (b) low sensitivity meter
   (c) high sensitivity meter (d) accurate and precise meter
2. In shunt ammeters, swamping resistance is connected in series with the meter resistance to
   (a) reduce friction errors (b) reduce temperature errors
   (c) both (a) and (b) (d) none of these
3. To convert a 1mA ammeter having a resistance of 100 Ω into a 1A ammeter, the required shunt resistance is
   (a) 100000 Ω (b) 100 Ω
   (c) 0.001 Ω (d) 0.1001 Ω
4. The instrument used to measure the charge passing through it due to transient current is
   (a) D'Arsonval galvanometer (b) Ballistic galvanometer
   (c) Ammeter (d) Ohmmeter
5. The shunt resistance of an ammeter is given by
   (a) $R_{sh} = \dfrac{R_m}{m-1}$ (b) $R_{sh} = \dfrac{R_m}{m+1}$
   (c) $R_{sh} = \dfrac{m-1}{R_m}$ (d) $R_{sh} = \dfrac{m+1}{R_m}$

State True or False

1. The jeweled bearing suspension provides more stiffness to the moving coil arrangement than the taut band suspension.
2. The increasing deflecting torque increases the sensitivity of the instrument.
3. The multiplying power of the shunt in an ammeter is represented as $\dfrac{R_m}{R_{sh}} + 1$.
4. To achieve maximum accuracy, the range of an ohmmeter must be so selected to indicate 0.5 FSD.
5. In overdamped galvanometers, the moving system moves slowly about its final steady state position.

Descriptive/Numerical Questions

1. Describe the principle of operation, advantages, disadvantages, and applications of PMMC instruments.
2. Derive the steady state deflection of a d'Arsonval galvanometer. What are the intrinsic constants of a galvanometer?
3. Develop the torque equation of moving coil instruments.
4. How is a basic d'Arsonval movement converted into multi-range voltmeter? Explain it using a neat diagram.
5. A basic d'Arsonval movement with an internal resistance R_m = 50 W and full scale deflection current I_{FSD} = 0.5 mA is to be connected to a multi-range DC voltmeter with voltage ranges of 0 –10 V, 0 – 50 V, 0 – 250 V, and 0 – 500 V. Show the arrangement with the help of a neat diagram with values of resistances used.
6. Explain the term loading in voltmeters and give the method to remove the adverse effects of loading.
7. A moving coil instrument gives a full scale deflection for a current of 20 mA with a potential difference of 200 mV across it. Calculate:
   (a) The shunt required to use it as an ammeter to get a range of 0–200 A.
   (b) The multiplier required to use it as a voltmeter of range 0–500 V.
8. Convert a basic d'Arsonval movement with internal resistance R_m = 500 Ω, and full scale current I_{FSD} = 55 μA into a multi-range DC voltmeter with voltage ranges of 0 – 10 V, 0 – 20 V, 0 – 50 V, and 0 – 150 V.
9. A moving coil ammeter has a fixed shunt of 0.02 Ω with a coil resistance of 1000 Ω and a potential difference of 500 mV across it. Full scale deflection is obtained:
   (a) To what shunt current does it correspond?
   (b) Calculate the value of resistance to give full scale deflection current when the current across the shunt resistance is: (i) 20 A (ii) 60 A.
10. Design an Ayrton shunt ammeter with current ranges of 1 A, 5 A, and 10 A using d'Arsonval movement having internal resistance R_m = 25 Ω and full scale deflection current of 5 mA.
11. A series ohmmeter has a battery voltage of 1.5 V, full scale deflection current of 150 μA. The limiting resistance R_1 is such that $R_1 + R_m$ = 10 kΩ. Determine the instrument indication when the circuit is shorted. Also find the resistance at 0.25 FSD, 0.5 FSD, and 0.75 FSD.
12. Design a series ohmmeter with internal resistance of 70 Ω, full scale deflection of 3 mA, and battery voltage of 5 V. The indication at 0.5 FSD must be 1000 Ω. Determine the values of R_1 and R_2. Also, calculate R_2 if the battery voltage varies from 3 V to 5 V.
13. A d'Arsonval movement shunt ohmmeter has an internal resistance of 10 Ω and battery voltage of 3 V. It provides the full scale deflection current (I_{FSD}) of 15 mA. Determine the value of the shunt resistance of the instrument so as to indicate 0.7 Ω at 0.5 FSD. Also find the value of the current limiting resistance.
14. Sketch circuits to show how DC voltmeters and ammeters should be calibrated using standard instruments. Explain.
15. How can a potentiometer be used:
    (a) for the calibration of a voltmeter?
    (b) for the calibration of an ammeter?

4

DC and AC Measuring Instruments

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Explain the moving iron type instruments–repulsion type and attraction type
- Discuss the principle, construction, and working of thermocouple instruments along with its various types
- Discuss the principle, construction, and working of dynamometer instruments along with different types of errors occurring in them
- Explain the different types of transistor voltmeter circuits
- Describe the concepts of analog ammeters and voltmeters–rectifier based instruments
- Describe the electronic multimeter with its functioning and different types of multimeter probes

4.1 INTRODUCTION

Earlier we have studied about the instruments which are used to measure direct current or average values of the quantities under measurement. However, these instruments cannot be used for alternating current (AC) measurements. If an alternating current is applied to the moving coil of such an instrument, the generated torque will alternate its direction in accordance with the alternating half-cycles of the current. Furthermore, when the frequency of current is low, the pointer of the instrument moves back and forth about the zero of the scale. On the contrary, for a high frequency current, the immense inertia of the coil does not let the pointer move in accordance with swift transitions of the signal. Thus, the pointer moves around the zero and keeps on vibrating slightly.

Therefore, there are some other instruments to measure AC quantities. Some of these instruments can measure both AC and DC quantities. In this chapter, instruments like moving iron type instruments, dynamometers, thermocouples, and electronic multimeter will be discussed which are used to measure both AC and DC quantities. Such instruments produce deflection regardless of the direction of the signal and thus, are capable of measuring AC as well as DC quantities. In addition, some other instruments like rectifier type instruments which measure only AC quantities will also be explained in detail.

4.2 MOVING IRON TYPE INSTRUMENTS

Generally, the alternating currents and voltages are measured by moving iron type instruments. The most common ammeters and voltmeters used for power frequency applications are moving iron type instruments. These instruments are rugged, inexpensive, and accurate and therefore are used invariably.

Such instruments are suitable for both AC and DC measurements, since the deflection in such instruments is always irrespective of the direction of the passing current. However, for DC measurements, these instruments are subjected to minor frequency errors.

These instruments incorporate a moving element which is generally a vane or plate of soft iron or high permeability steel. The quantity to be measured (that is, voltage or current) is fed into a stationary coil so as to excite it. Upon excitement, the coil starts behaving as an electromagnet. The iron vane is placed in such a way that it moves in the magnetic field produced by the coil and increases the flux produced by electromagnet. The vane endeavors to get the minimum reluctance position. This results in a force with such a direction that the coil inductance gets increased, since inductance and reluctance of the coil are in inverse proportion to each other.

Based on the number of iron vanes used and the principle of operation, the moving iron type instruments are classified into the following as:

- Repulsion type instruments
- Attraction type instruments

We will discuss here both types of moving iron instruments with their overview, construction and functional details.

4.2.1 Repulsion Type Instruments

In repulsion type instruments, there are two cylindrical soft iron vanes inside the coil. One of these vanes is fixed while the other one is free to move. When an operating current flows through the coil, there occurs a repulsion between the two iron vanes due to which the movable vane moves.

Principle

These instruments operate on the principle that two soft iron vanes are magnetized with the same polarity irrespective of the instantaneous direction of current, and hence, repulsion takes place. The repulsive force causes the movable vane to rotate away from the fixed vane. A pointer, attached to the spindle follows this rotation and deflects in accordance over the scale of the instrument.

Constructional details

The movement of moving iron type instruments consists of a stationary coil with a number of turns. The coil is meant to carry the operating current to be measured. There are two cylindrical iron vanes placed inside the coil, one of which is firmly attached to the frame of coil and is fixed in its position while the other one is attached to the pointer shaft of the instrument and is free to rotate. The displacement of moving vane indicates the magnitude of current in the coil. The load current is only carried by the fixed vane which is designed so as to withstand high current transients. Figure 4.1 shows the basic structure of repulsion type moving iron instruments in which control spring produces a controlling or restoring torque.

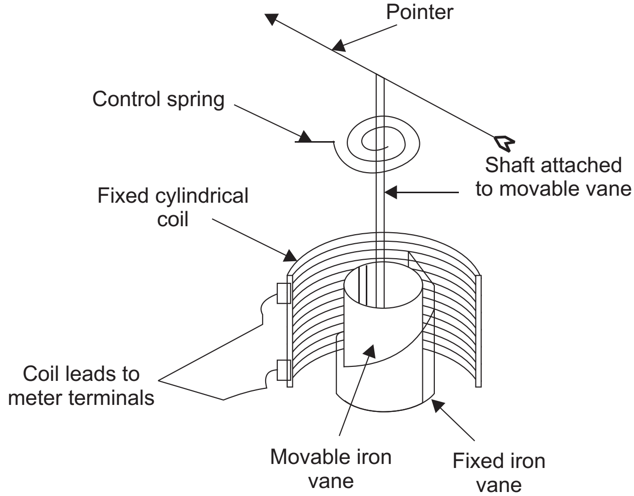


Fig. 4.1 Repulsion Type Instrument

The scale of the moving iron instruments is non-linear and is crowded in the lower range of measurement. The non-linearity of the scale can be eradicated by accurate positioning and shaping of the iron vanes in accordance with the fixed coil. The scale may be directly calibrated in amperes or volts.

Working

When the operating current is applied to the fixed coil assembly, both the fixed and the movable vanes get magnetized with the same polarity irrespective of the current direction. Due to this, a repelling force is experienced by the vanes, which in turn produces a proportional rotation of the movable vane. The deflecting torque thus generated is in proportion to the square of the coil current which makes the instrument to read the true rms (root mean square) value of the quantity being measured. The movement of the rotating vane causes the pointer of the instrument to deflect corresponding to its movement, that is, proportional to the operating current. The restoring torque produced by the control spring opposes the rotation of the moving vane.

4.2.2 Attraction Type Instruments

In attraction type moving iron instruments, only one iron vane is present. The soft iron vane in these instruments is movable and is mounted on the spindle. It is attracted towards the fixed coil on applying operating current.

Principle

When an unmagnetized soft iron is brought near a current carrying coil, it gets attracted into the coil in the similar way as it would be attracted by a bar of magnet. A disc of soft iron is pivoted onto the spindle of the instrument near the coil. On applying operating current through the coil, the disc gets attracted to it, that is, it moves from weaker field outside the coil to the stronger field inside it. The iron disc is always magnetized in such a way that it is attracted towards the coil, irrespective of the direction of the coil current. The coil current causes the pointer to deflect in the same proportion.

Constructional details

In attraction type instruments, there is a fixed flat coil having a number of turns and a narrow slot like opening. The fixed coil is fed with an operating current. An oval-shaped soft iron disc is mounted near the coil on the spindle between bearings of the instrument. The disc is pivoted in such a way so as the major bulk of the iron moves into the center of the coil. A pointer is attached to the spindle and deflects in proportion to the operating current. Figure 4.2 illustrates the configuration of an attraction type instrument.

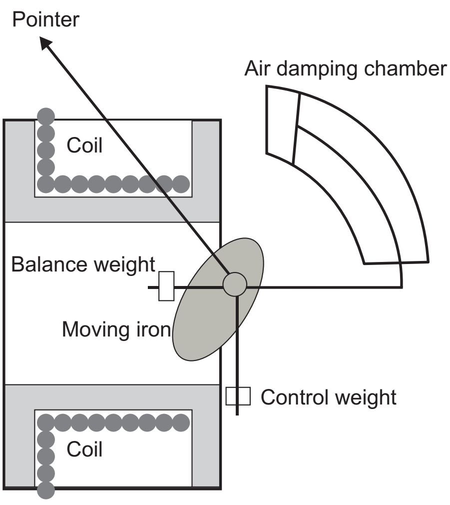


Fig. 4.2 Attraction Type Instrument

The hair or **control spring** produces a controlling torque to oppose the rotation of the moving iron. The balance weight or **control weight** is used to calibrate the instrument. Thus, the null deflection of the scale can be achieved by adjusting the balance weight. The air damping chamber shown in the figure consists of a light movable aluminum piston which is attached to the moving assembly and provides the damping by air friction.

Working

On applying an operating current through the coil, the magnetic field is produced and iron disc gets attracted towards the fixed coil. The magnetic field strength is highest at the center of the coil. This movement of the disc causes the pointer to move and indicate the measurement on the calibrated scale. The pointer moves in proportion to the movement of the disc and the amount of the coil current. Also, the movement of the disc is always in the same direction regardless to the direction of the coil current. This makes the moving iron instruments suitable for both DC as well as AC measurements.

4.2.3 Deflecting Torque Equation

In the moving iron type instruments, when there is small amount of increment in the operating current, the torque generated can be expressed by considering the energy relations. The increased current dI causes the inductance of the coil to be increased by dL and a small deflection $d\theta$ occurs; where I, L, and θ are initial current, inductance, and deflection of the instrument, respectively. Let us proceed to derive the required expression for deflecting torque T_D of the instrument in terms of inductance L.

The increment in the current occurs due to increment in the applied voltage or energy V, given as:

$$V = \frac{d}{dt}(LI) = L\frac{dI}{dt} + I\frac{dL}{dt}$$

The electric energy supplied to the coil in differential time dt is given as:

$$VIdt = I^2 dL + ILdI$$

The change in stored energy in the magnetic field can be given as:

$$= \frac{(I + dI)^2\ (L + dL)}{2} - \frac{I^2 L}{2}$$

If the second and higher order terms are neglected, the above relation reduces to:

$$\simeq ILdI + \frac{I^2 dL}{2}$$

According to the principle of energy conservation, it is stated that:

Electrical energy obtained from supply = Increment in stored energy + Mechanical work done

This gives,

$$I^2 dL + ILdI = ILdI + \frac{I^2 dL}{2} + T_D d\theta$$

$$\Rightarrow \qquad T_D d\theta = \frac{I^2 dL}{2} \qquad \text{...(1)}$$

where T_D is the value of deflecting torque and is measured in newton-metre (Nm) and corresponds to the deflection θ in degrees. Therefore, the amount of additional energy stored due to change in the deflection $d\theta$ is represented by $T_D\, d\theta$. The above equation can be written as:

$$T_D = \frac{I^2 dL}{2d\theta} \qquad \text{...(2)}$$

The controlling torque T_C provided by control spring balances the deflecting torque T_D. As studied in *Section 3.3*, controlling torque $T_C = K\theta$ where K is the spring constant and θ is deflection.

At equilibrium state:

$$T_C = T_D$$

Or,

$$K\theta = \frac{I^2 dL}{2d\theta}$$

$$\Rightarrow \qquad \theta = \frac{I^2 dL}{2Kd\theta} \qquad ...(3)$$

From Equation (3), we can say that deflection is proportional to the square of rms value of the operating current. The current could be of any polarity but the deflecting torque is always unidirectional, that is, it acts in the same direction.

4.2.4 Advantages and Disadvantages of Moving Iron Type Instruments

The advantages of moving iron type instruments can be discussed as follows.

- The moving iron instruments can be used for both AC and DC applications. In conjunction with the AC measurements, these instruments are also used as DC current indicators to measure and indicate the charging and discharging of current in automobile applications.
- The fixed current carrying parts make the construction of these instruments very simple and robust.
- The errors due to friction are very small in these instruments. This is because the current carrying coil of the instrument (that is, heavy part) is not moving and the moving arrangement of the instrument is quite light in weight. This leads to a high torque-weight ratio.
- Within the limits of industrial grades and precision, these instruments provide a high level of accuracy. At frequencies between 25 Hz and 125 Hz, high grade instruments show initial accuracy of 0.75%. If the instrument is designed carefully, the accuracy may be within 0.2% to 0.3% at 50 Hz frequency.
- Such instruments are available with 240° circular scale. This extended range can be considered as an advantage to the instrument.

The moving iron type instruments also have some disadvantages which are as follows.

- When moving iron type instruments are used for DC measurements, some error gets induced due to hysteresis effect. However, for AC measurements, the errors due to hysteresis are in the order of 5% to 10% which is acceptable. They measure AC voltage and current in the industry.
- Due to temperature variations, some errors may get introduced in the instruments.
- Due to non-linear characteristics of the iron, deflecting torque is not exactly proportional to the square of the operating current.
- Variation of frequency also causes some errors. These errors change the reactance of the coil and also change the eddy currents induced in neighboring metal.

- The magnetizing force produced by the fixed coil gets decreased due to stray magnetic fields.
- The scale of these instruments is not uniformly divided. It is crowded at the lower end, so the accurate readings are not possible.

Example 1 In a moving iron instrument, when the pointer deflects from 0° to 100° its self inductance increases by 5.33 mH. Determine the coil current if the controlling spring constant is given as 25 μNm/degree.

Solution: Given that:

Controlling spring constant K = 25 μNm/degree

Deflection θ = 100°

Increase in self inductance dL = 5.33 mH

At equilibrium, the deflecting and controlling torque are equal and are computed as:

$$T_C = T_D = K\theta$$

Substituting the values, we get:

$$T_C = T_D = 100° \times 25\ \mu\text{Nm} = 2.5\ \text{mNm}$$

From Equation (1), we have:

$$T_D d\theta = \frac{I^2 dL}{2}$$

$$\Rightarrow \qquad I = \sqrt{\frac{2T_D d\theta}{dL}}$$

On substituting the required values, we get:

$$I = \sqrt{\frac{2 \times 2.5 \times 10^{-3} \times (100° \times \pi/180°)}{5.33 \times 10^{-3}}} = 1.279\ \text{A} \quad \text{(Since } d\theta \text{ is in radians)}$$

Example 2 The controlling spring constant of a moving iron instrument is 5 μNm/degree constant. The instrument requires a 200 mA current to deflect 120° on scale. Calculate the rate of change of self inductance per degree of deflection.

Solution: Given that:

Controlling spring constant K= 5 μNm/degree

Deflection on scale θ = 120°

Current I = 200 mA

At equilibrium, the deflecting and controlling torque are equal and computed as:

$$T_C = T_D = K\theta$$

Substituting the values, it gives:

$$T_C = T_D = 600\ \mu\text{Nm}$$

From Equation (1), we have:

$$T_D d\theta = \frac{I^2 dL}{2}$$

$$\Rightarrow \qquad \frac{dL}{d\theta} = \frac{2T_D}{I^2}$$

Substituting the required values, we get:

$$\frac{dL}{d\theta} = \frac{2 \times 600 \times 10^{-6}}{(200 \times 10^{-3})^2}$$

On solving, we get:

$$\frac{dL}{d\theta} = 30 \text{ mH/rad}$$

Or, $30 \text{ mH} \times \pi/180° = 0.523 \text{ mH/degree}$ (Since $d\theta$ is in radians)

4.3 THERMOCOUPLE INSTRUMENTS

A thermocouple consists of two dissimilar metals or wires forming a junction. It is a temperature measuring device that produces voltage when its junction is heated by operating current. The voltage produced is in proportion to the temperature of the junction and output of the thermocouple is delivered to a sensitive DC PMMC microammeter. They are most commonly used as temperature sensors for control and measurement. Thermocouples also convert temperature gradient or heat energy into electrical energy.

A thermocouple instrument is a combination of a thermocouple element and a PMMC instrument. Since the operation of the instrument is based on the thermocouple element, it is named so. It can be used for DC as well as AC applications and can measure high frequency voltages and currents very accurately.

Thermocouple instruments are sometimes referred to as RF instruments since the signals with frequencies up to 50 MHz can be measured with an accuracy of about 1%. Above these frequencies, skin effect causes the current to flow through the outer surface of the conductor. This results in increase in effective resistance of the heater element and reduced accuracy of the instrument. A solid and very thin heater element or wire is used for small current (up to 3 A) applications while for currents above 3 A, the heating element is made from tubing. This reduces the errors due to skin effect.

Principle

The thermocouple works upon the principle that when a junction between two different metals is heated, different work functions of the metals produce different temperatures at their ends. This in turn yields an electric quantity, that is, voltage in proportion to the junction temperature. This effect is referred to as **Seebeck effect** and forms the basis of the thermocouple instruments.

Constructional details

The two different wires when joined together, form a thermocouple. Different combinations of alloys can be used to form the wires of thermocouples, such as copper–constantan, platinum–rhodium, iron–constantan, and chromel–alumel. The junction of thermocouple is either welded directly to the heater or thermally connected to it using a ceramic bead as shown in Figure 4.3.

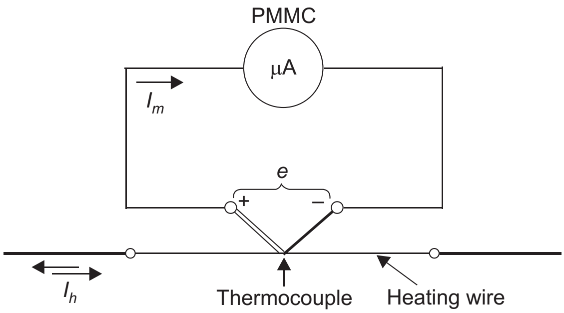


Fig. 4.3 Basic Thermocouple Instrument

The connected end of the wires is termed as **hot junction** or **sensing junction** while the other end is termed as **cold junction**. The thermocouple instrument is connected to an electric circuit to perform measurement. The circuit current (to be measured) provides the required heat at the junction.

One leg of the thermocouple which is positive is connected to the positive port of the PMMC instrument while the negative leg is connected to the negative port of the PMMC instrument. The temperature at the cold junction is referred to as **reference temperature** and must be known for precise measurements. The difference in the temperature of these two junctions leads to the generation of an electromotive force (emf). The more the temperature difference, more is the induced emf. This emf is used to indicate the temperature measurement.

Working

When a thermocouple is connected to an electric circuit, it feeds the current to the thermocouple in order to heat up the thermocouple junction. The cold junction is kept at a reference temperature, say 0°C while the temperature at the hot junction is to be measured. Due to difference in the temperature of both junctions, an electromotive force (emf) is produced gets induced in the thermocouple. The temperature difference between the hot and cold junctions determines the amount of voltage or emf generated at the output of the thermocouple. This voltage is very low, in the range of milli-volts. It can be regarded as a measure of root mean square (rms) value of the operating current since the current heats up the heater element in proportion to its rms value. This voltage is used to indicate the amount of temperature generated at the junction. The scale is essentially a milli-volt scale on which the standard value of reference temperature at the cold junction is marked as 32° F, that is, 0°C.

Note: *To ensure proper functioning of the thermocouple, there must not be any other metal between the two junctions of the instrument.*

4.3.1 Types of Thermo-Elements

There are various types of thermo-elements available with their specialties and specifications. They can be used in accordance with the requirement of the application and can be classified with respect to their measurement ranges, temperature and frequency withstanding capability. The position and placement of the heater element is significant in the configurations of different thermo-elements. The various types of thermo-elements include *contact type*, *non-contact type*, *vacuum type* and *bridge type thermo-elements*.

Contact type thermo-element

This type of thermo-element consists of a separate heater wire through which the current to be measured is passed. The hot junction of the thermocouple is in contact with the heater wire. The basic configuration of an instrument consisting of contact type thermo-element is shown in Figure 4.4.

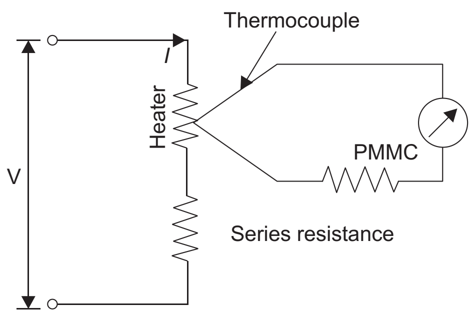


Fig. 4.4 Contact Type Thermo-element

Non-contact type thermo-element

In a non-contact type thermo-element, there is an electrical insulation between the heater wire and the thermocouple, thereby, no direct electric contact exists between the two. This insulation is necessary to measure current in high potential circuits. The thermal energy or heat is transferred from the heater to the junction across the insulation. As a result, the instrument consisting of this type of element becomes slothful and less sensitive as compared to the contact type element instruments. It is also referred to as **separate heater type thermo-element**.

Vacuum thermo-element

In vacuum elements, the thermocouple and heater element are placed inside a glass tube to increase the sensitivity and efficiency of the element. The heater does not cool down due to convection air currents which results in increased efficiency of the instrument. Thus, the heater element of this instrument can be subjected to much higher temperature. Also, there is an insulating bead in between the heater and the thermocouple to provide an electrical insulation (see Figure 4.5). Due to this insulation, the capacitance effects are reduced at high frequency operations between the thermocouple and the rest of the circuit. The capacitive currents are reduced as this insulation creates a small capacitance between the isolated circuits and is in series with the capacitance from instrument to other parts of the circuit or to the ground. This current would otherwise get diverted through parts of the heater element and causes errors in indication.

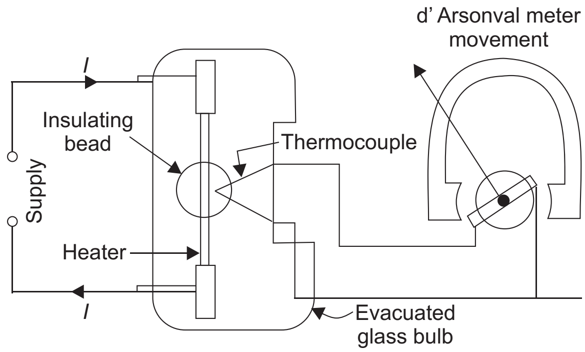


Fig. 4.5 Vacuum Thermo-element Instrument

Up to 100 mA current measurements, this instrument gives the same response as that of a contact type element instrument. It is capable of measuring the current in the range of 2 mA to 500 mA at frequencies up to 100 MHz or more depending upon the resistivity and size of the heater wires. The voltage measurements at frequencies up to 10 kHz can be done using series resistance and up to 100 MHz using special multipliers.

Bridge type thermo-element

The bridge type thermo-element is different from all other types discussed. No heater is used in this type of element. In this configuration, thermocouples are connected as shown in Figure 4.6 and the current to be measured is passed through them directly to increase their temperature. The temperature increment is in proportion to I^2R.

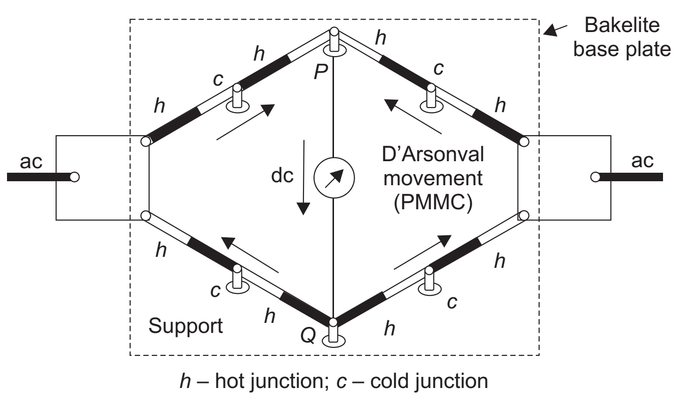


Fig. 4.6 Bridge Type Thermo-element Instrument

From Figure 4.6, it can be seen that there are six pins embedded in bakelite base plate and marked as cold junctions. The hot junctions are placed midway between the pins. The resistance in each of the bridge arms is equal and the bridge is balanced. Therefore, no AC current flows through the instrument. A DC potential difference is generated by the resultant thermal voltage. This arrangement is capable of providing a much higher output as compared to single thermocouple in vacuum type. Also, the sensitivity of the instrument gets increased to a great extent using several thermocouples in series. The voltage available from these elements is as much as 25 mV and made with AC ratings from 100 mA to 1 A. This instrument follows the square law response given as $\theta = KI^2$, where θ is the deflection in PMMC meter and K is the spring constant.

4.3.2 Advantages and Disadvantages of Thermocouple Instruments

Thermocouple instruments are used to measure DC as well as AC quantities and are preferred because of the following advantages.

- Thermocouple instruments are rugged, reliable, and inexpensive.
- The stray magnetic fields do not affect the thermocouple instruments.
- The high sensitivity of the instruments is a significant merit.
- The instruments can provide an accuracy of about 1% for frequencies up to 50 MHz in indicating a measurement.
- Thermocouple instruments can measure current in the range of 0.5 to 20 A. The heater element is placed outside the instrument for higher current ranges while for lower current ranges, the bridge type instrument is suitable.
- The instrument can be transformed into a voltmeter to measure voltages up to 500 V by using a multiplier resistance in series with a low current thermocouple.
- Frequency errors do not exit in such instruments. Thus, the measurements can be done over a wide frequency range.
- A wide temperature range can be covered using these instruments.

Inspite of all these advantages, thermocouple instruments have some disadvantages too.

- When the normal rated current is passed through the heater, it gets heated with a temperature of 300°C. As per the square law, if a current with a value equal to twice its rated value is to be measured it would heat the heater with a temperature of 1200°C. This temperature is enough to destroy the heater wire. Thus, some measures must be taken in order to protect the heater from such overloads as fuses do not provide any protection.
- Thermocouple instruments have very low overload capacity in comparison to other measuring instruments and is about 150% of full scale current.

Example 3 A thermocouple ammeter has a perfect square law response. Full-scale deflection can be obtained by this ammeter for a current of 20 A. Determine the current required for half-scale deflection.

Solution: The deflection θ in the thermocouple ammeter is given as:

$$\theta = KI^2 \qquad \text{[Refer to } \textit{Section 4.3.1}\text{]}$$

Thus, for full-scale deflection, it becomes:

$$\theta_F = KI^2$$

Substituting the value of full-scale current, we get:

$$\theta_F = K(20)^2 \Rightarrow \frac{\theta_F}{K} = 400 \qquad \text{...(3a)}$$

And the current required for half-scale deflection can be obtained as:

$$\frac{\theta_F}{2\,K} = I^2$$

Substituting the value of $\frac{\theta_F}{K}$ from Equation (3a), we get:

$$\frac{400}{2} = I^2 \Rightarrow I = 14.14 \text{ A}$$

4.4. DYNAMOMETER INSTRUMENTS

The fundamental standards (Weston cell and precision resistors) to which all the measuring instruments should be calibrated are DC standards, but the moving-iron type instruments as well as some other AC instruments are difficult to be calibrated with these standards. In order to correctly calibrate such instruments, a **transfer instrument** is required that can measure AC quantities with no modification after being calibrated with a DC source. The accuracy of a transfer instrument is same for both AC and DC measurements. The dynamometer instrument, also referred to as **electrodynamic** or **electrodynamometer instrument** is a transfer instrument which is capable of fulfilling these requisites. The transfer instrument is first calibrated on DC. This calibration is then transferred to an AC instrument which is being operated in its normal ratings. Due to its high accuracy, it is used as a calibration standard for many other instruments. In lower audio and power frequency ranges, it can be used as an AC ammeter and voltmeter.

Principle

The electrodynamometer instruments work on the principle that if the permanent magnet in a PMMC instrument is replaced by two fixed coils connected serially, the torque produced is then proportional to the product of currents in moving and fixed coils. The application of a reverse current will reverse the polarity of the field and the moving coil; however, the direction of the turning force remains the same. Therefore, the dynamometer instruments are suitable for both AC and DC measurements as turning force does not reverse its direction with reverse current.

Constructional details

The electrodynamometer instrument consists of a fixed (or field) coil cut into two sections to allow the passage of the instrument shaft and to provide a more uniform field near the center. When a current is passed through the fixed field coils, it produces a magnetic field in which a moving coil rotates. This moving coil is wound on a non-magnetic former or as a self-sustaining coil. If a magnetic former is used, the alternating current would introduce eddy currents. Therefore, the air acts as a core for both fixed and moving coils [see Figure 4.7].

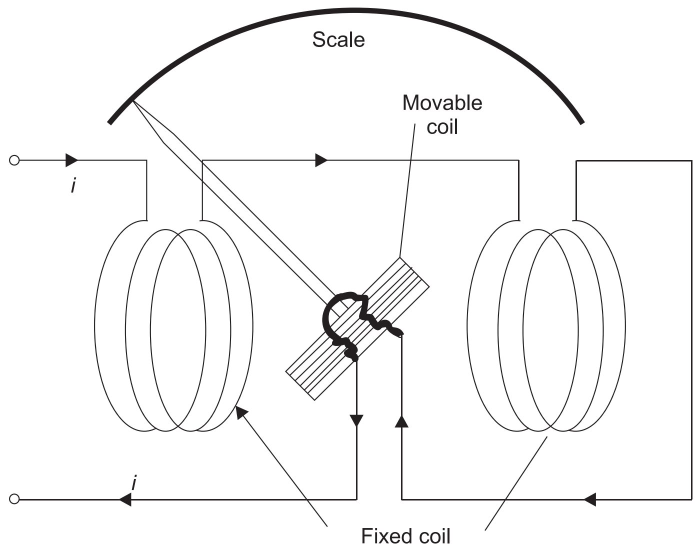


Fig. 4.7 Electrodynamometer Instrument

There are two control springs which provide controlling torque. The counter-weights and the pointer are carried by the moving system in which the moving coil is mounted on an aluminium spindle. A pair of aluminium vanes moves in the sector shaped chamber and is attached at the bottom of the spindle to provide air damping. The magnetic field of a dynamometer instrument is weaker due to air cored coils of the instrument. Thus, eddy current damping cannot be employed, otherwise the magnetic field may get distorted due to the permanent magnet, it requires. The instrument is shielded by enclosing it in a laminated hollow cylinder of iron with closed ends, in order to eradicate the effects of external magnetic fields since the field of the instrument is weaker than that of other instruments. The laboratory standard instruments are encased in polished wooden cases so that they remain stable over a long time. The adjustable leveling screws support the case. Using the machine sub-dividing equipment, the scales are hand drawn.

Working

When the operating current is applied to the air cored coils of the instrument, a proportional magnetic field is produced in which the movable coil is pivoted. A current starts flowing through the movable coil that yields a flux. This flux interacts with the flux produced by the fixed coils. Due to this interaction, the movable coil starts rotating on its axis, resulting in a deflecting torque proportional to the product of the currents in the field coils and the movable coil. The control springs provide controlling torque to oppose this movement, and hence the pointer deflects in proportion to the operating current.

4.4.1 Deflecting Torque Equation

In dynamometers, the flux path is entirely an air path since no iron core is used. The deflecting torque of this instrument depends on coil current, coil dimensions, coil turns, and field flux. The flux of both the coils (that is, fixed and moving coils) is directly proportional to their respective coil currents. The torque can be computed in terms of inductance of the instrument.

Let the fixed and moving coils have their respective self inductances as L_{FC} and L_{MC} and their mutual inductance be represented as M. Then, the total energy stored in the magnetic fields of the coils can be given as:

$$W_T = \frac{1}{2} i_{FC}^2 L_{FC} + \frac{1}{2} i_{MC}^2 L_{MC} + M i_{FC} i_{MC} \quad \text{...(4)}$$

where i_{FC} and i_{MC} are the coil currents through field coil and moving coil, respectively. It is important to note here that the inductances of the individual coils L_{FC} and L_{MC} are not the functions of the deflection θ, whereas their mutual inductance M depends upon the relative position of the movable coil and hence is a function of its deflection θ. Therefore, from Equation (4), the expression for the energy input in the form of deflecting torque T_D, that produces coil movement in the instrument, can be obtained as:

$$T_D = \frac{dW}{d\theta} = i_{FC}\, i_{MC} \frac{dM}{d\theta} \quad \text{...(5)}$$

Now, the equivalent inductance of both the coils can be expressed as:

$$L_{\text{eq}} = L_{FC} + L_{MC} + 2M$$

It gives the mutual inductance as:

$$M = \frac{1}{2}[L_{\text{eq}} - (L_{FC} + L_{MC})] \quad \text{...(6)}$$

When both the fixed and the moving coils are aligned with each other, that is, at the deflection of 180°, the maximum flux linkage between the coils is obtained, while at 0° deflection the mutual inductance becomes negative of its maximum value. Thus, we may write as:

At 180° deflection, $M = M_{\max}$

And, at 0° deflection, $M = -M_{\max}$

For a more general approach, if the plane of the moving coil makes an angle θ with the flux density B of the fixed coil, the mutual inductance can be expressed as:

$$M = -M_{\max} \cos\theta \quad \text{...(7)}$$

Since the dynamometer instruments are capable of measuring DC as well as AC, the deflection torque equation of such instruments can be obtained individually for both operations.

DC Operation

The expression for the deflecting torque given in Equation (5) can be transformed by assuming that a DC current is flowing through the coils. Thus, the expression can be rewritten as:

$$T_D = I_{FC}\, I_{MC} \frac{dM}{d\theta} \qquad ...(8)$$

where I_{FC} and I_{MC} denote the DC components of AC currents i_{FC} and i_{MC}, respectively.

The control is provided by the hair springs or the control springs, thus the controlling torque is in proportion to the deflection angle θ. Thus, we can write it as:

$$T_C \propto \theta$$

Or

$$T_C = K\theta \qquad ...(9)$$

where K is the control or spring constant.

Now, at the steady state deflection condition, we know that both the deflecting and controlling torques are equal. Therefore, we may write as:

$$T_C = T_D$$

Substituting the relations of controlling and deflecting torque from Equations (8) and (9), we get:

$$K\theta = I_{FC}\, I_{MC} \frac{dM}{d\theta}$$

$\Rightarrow$

$$\theta = \frac{I_{FC}\, I_{MC}}{K} \frac{dM}{d\theta} \qquad ...(10)$$

The above equation gives the expression for the deflection angle under DC operation.

AC operation

When the dynamometer instrument is used to measure AC quantities, the inertia of its moving coil influences it to move to a position where the average deflecting torque is balanced by the controlling torque of the control spring for a complete cycle of the supply. Since the deflecting torque is proportional to the mean value of the square of the operating current or voltage, the instrument scale can be calibrated to read the rms value of the quantity being measured.

The average deflecting torque can be expressed when the AC current is flowing through the coils as:

$$T_{D,av} = \frac{1}{T}\int_0^T i_{FC}(t)\, i_{MC}(t) \frac{dM}{d\theta}\, dt \qquad ...(11)$$

Let the coil currents be expressed as:

$$i_{FC}(t) = I_{FC,\,\max} \sin \omega t$$

and,

$$i_{MC}(t) = I_{MC,\max} \sin(\omega t - \alpha)$$

where α represents the phase difference between the currents flowing through the fixed coil and moving coil.

Now, Equation (11) can be written as:

$$T_{D,av} = \frac{1}{T}\int_0^T I_{FC,\max} \sin \omega t \cdot I_{MC,\max} \sin(\omega t - \alpha)\frac{dM}{d\theta}\,dt$$

$$\Rightarrow \qquad T_{D,av} = I_{FC,\text{rms}}\, I_{MC,\text{rms}} \cos\alpha \frac{dM}{d\theta} \qquad ...(12)$$

The controlling torque in AC operations is also produced by the hair springs, thus, it is expressed as:

$$T_C = K\theta$$

In the steady state condition, the average deflecting torque must be equal to the controlling torque. Thus, we may write it as:

$$T_{D,av} = T_C$$

On substituting the respective relations, we get:

$$I_{FC,\text{rms}} I_{MC,\text{rms}} \cos\alpha \frac{dM}{d\theta} = K\theta$$

It gives the deflection angle θ as:

$$\theta = \frac{I_{FC,\text{rms}}\, I_{MC,\text{rms}} \cos\alpha}{K}\frac{dM}{d\theta} \qquad ...(13)$$

In ammeters and voltmeters, both the coils are connected in series which ensures that an identical current flows through them, hence, Equation (13) can be written as:

$$\theta = \frac{I_{\text{rms}}^2}{K}\frac{dM}{d\theta} \quad (\text{since } \alpha = 0°) \qquad ...(14)$$

Equation (11) gives the required relation for the deflection angle θ under AC operations.

4.4.2 Dynamometer Ammeter

The dynamometer instruments can be used as an ammeter. The basic configuration is shown in Figure 4.8 in which the moving coil is constructed in such a way that the current through it does not exceed a certain upper limit, that is 100 mA. It is connected in series with the fixed coils, and hence the current is identical through both the coils.

The flux of both fixed and moving coils repel each other since the similar poles of the coils are adjacent to each other. The resulting deflecting torque which is expressed by Equation (14) moves the pointer in a clockwise direction from its zero position to a steady deflection in accordance with the magnitude of the current flowing through the coils. Now, if the direction of the current is reversed, the pointer still moves in the same direction. Thus, irrespective of the current direction, the deflecting torque influences the pointer to move in

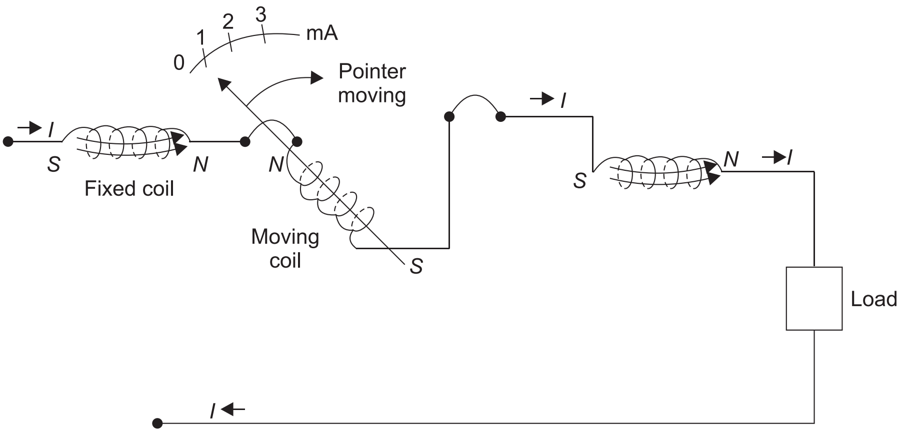


Fig. 4.8 Dynamometer Ammeter

the same direction. This implies that the dynamometer instruments can be used for DC as well as AC measurements.

When larger currents are to be measured, the range of the ammeter has to be increased to ensure its smooth functioning. This is done by modifying the circuit as shown in Figure 4.9.

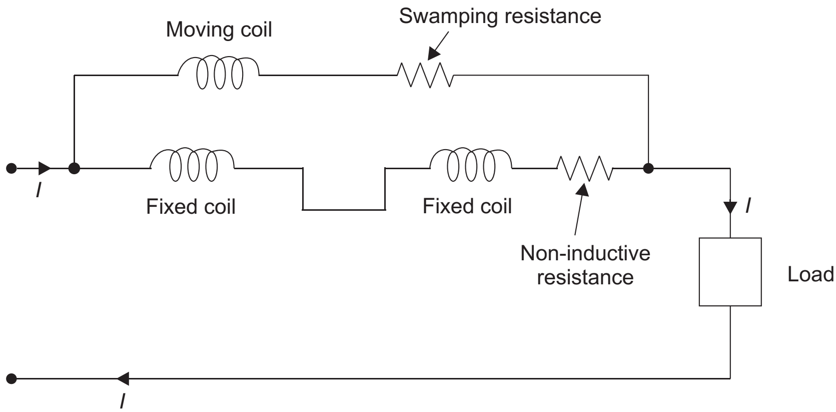


Fig. 4.9 Modified Dynamometer Ammeter

It can be seen that the moving coil is connected in parallel with the fixed coils, and hence the current through the fixed and the moving coils is not the same. It is because the control springs are not suitable for high current ranges. The circuit is used for measuring currents above 250 mA. The moving coil is subjected to a current within 200 mA while the rest of the current is passed through the fixed coils. The moving coil current is a small fraction of the measured current. For the circuit to be highly accurate, the time constant of both fixed and the moving coils must be the same in addition with the same percentage change in the resistance with respect to temperature in the two branches.

4.4.3 Dynamometer Voltmeter

The dynamometer instruments can be used as a highly accurate voltmeter when the fixed and moving coils are connected serially with a highly non-inductive resistance. Hence, the same current flows through both fixed and movable coils. The basic configuration of a dynamometer voltmeter is illustrated in Figure 4.10.

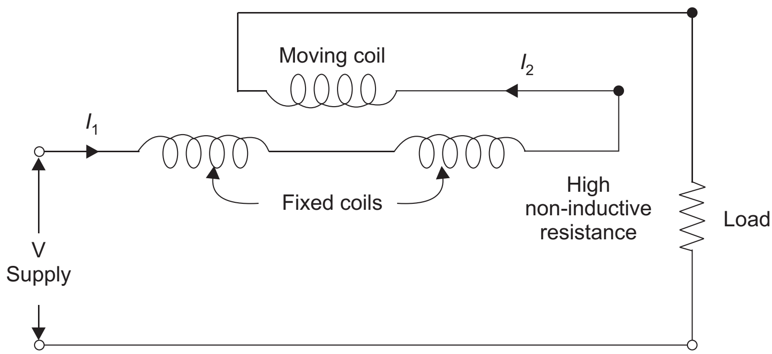


Fig. 4.10 Dynamometer Voltmeter

The circuit shown is connected across a load whose voltage is to be measured. The deflecting torque equation for such voltmeters can be obtained by using Equation (8) as:

$$T_D = I_{FC}\, I_{MC} \frac{dM}{d\theta}$$

Since $I_{FC} = I_{MC} = \left(\frac{V}{Z}\right)$, the above equation becomes:

$$T_D = \left(\frac{V}{Z}\right)\left(\frac{V}{Z}\right)\frac{dM}{d\theta}$$

where V is the voltage across the instrument and Z is the impedance of the circuit.

The controlling torque is given as: $T_C = K\theta$

At steady state deflection, $T_D = T_C$

This implies, $$\left(\frac{V^2}{Z^2}\right)\frac{dM}{d\theta} = K\theta$$

Or $$\theta = \left(\frac{V^2}{Z^2 K}\right)\frac{dM}{d\theta} \qquad ...(15)$$

Equation (15) gives the expression for the deflection θ. It can be seen that the deflecting torque is proportional to the square of the voltage V. If the deflection angle is varied in the range of 45° to 135°, then the term $\frac{dM}{d\theta}$ is kept almost constant.

4.4.4 Errors in Dynamometer Instruments

The *low torque/weight ratio*, *strong external magnetic field*, *eddy currents*, *changes in temperature*, and *operating frequency* are all the factors that cause errors in dynamometer instruments. Let us discuss each of these factors in detail.

Low torque/weight ratio

The operating magnetic field generated by the air cored coils is very small. As a consequence, very few flux linkages per ampere are produced in the moving coil which in turn yields a low deflecting torque. In order to increase this deflecting torque, the magneto-motive force (mmf) of the coil must be increased by increasing the number of turns or by passing a large amount of current through the coil. However, the current increased beyond 200 mA would result in heating up of springs. Also, more the number of turns, more will be the weight of the moving assembly of the instrument yielding a low torque/weight ratio.

The low torque/weight ratio signifies heavy weight of the moving system due to which the instrument incurs frictional losses. As compared to other types of instruments, frictional errors in dynamometer instruments are much higher on account of low torque/weight ratio.

Strong external magnetic fields

We know that dynamometer instruments possess a weak magnetic field. Thus, such instruments must be protected from strong external fields which may influence the instrument and induce errors in them. For this, metal shields are used in portable instruments which effectively isolate the coils from these external fields. However, in precision type instruments, metal shielding is not provided as eddy currents get induced in metal shields that may result in errors in the instrument. To protect such instruments against strong external magnetic fields, an **astatic system** is used. An astatic system is basically a dynamometer in which two identical pairs of fixed and moving coils are connected on a single shaft. The arrangement of the two fixed coils is such that the polarity of their magnetic fields is opposite, thereby producing opposite currents through them. This makes the applied deflection torque to act in the same direction for both the coils. A uniform external field would reduce the field of one of the coils, thereby increasing its deflecting torque and increasing the field of another coil by the same amount (as both the fields are in opposition). This in turn reduces the deflecting torque of that coil. This implies that the net amount of deflecting torque in the two coils becomes zero due to external magnetic field.

Eddy currents

When there is a coupling between the moving coil and its adjacent metal parts, a deflecting torque gets produced by the eddy currents, thereby producing frequency errors in the instrument. Thus, to avoid the effect of eddy currents, the metal in supports and other parts should be used as minimally as possible. Also, it must have high resistivity.

Changes in temperature

When an operating current is fed into the coils of the instrument, the coils get heated up. Due to this heat produced, some errors occur in the coils due to self-heating. Therefore, some technique must be employed to avoid these errors. Temperature compensation is such a technique that neutralizes the effects of changes in the temperature of the instrument. It is achieved by using temperature compensating resistors.

Operating frequency

Self-reactance of the coils in dynamometer instruments vary in accordance with the operating frequency causing frequency errors. The deflection angle θ of the dynamometer voltmeter is given by the relation:

$$\theta = \left(\frac{V^2}{Z^2 K}\right)\frac{dM}{d\theta} \qquad \text{[Refer to Eqn. (15)]}$$

where impedance $Z = \sqrt{R^2 + (2\pi fL)^2}$ [Refer to Figure (4.10)]

The above relation is evident that the impedance Z of the voltmeter changes correspondingly with a change in its operating frequency. This makes voltmeter to read high when the operating frequency goes low and vice-versa. These frequency errors can be reduced if the coil winding of the circuit is made very small, providing the inductive reactance of the circuit as a small fraction of the total impedance, that is, $(2\pi fL)^2 << R$. This implies that $Z \approx R$. Hence, the calibration of the instrument does not get affected by the frequency.

Similar to the dynamometer voltmeters, frequency errors also affect the dynamometer ammeters. To reduce them, the current ratio of moving coil and the fixed coil in the ammeters must be made frequency independent. This can be accomplished if the time constants of both the coils are kept the same, that is,

$$\left(\frac{L}{R}\right)_{FC} = \left(\frac{L}{R}\right)_{MC}$$

Since

$$\frac{I_{MC}}{I_{FC}} = \frac{Z_{FC}}{Z_{MC}} = \frac{\sqrt{R_{FC}^2 + (2\pi fL_{FC})^2}}{\sqrt{R_{MC}^2 + (2\pi fL_{MC})^2}} \cdot \frac{R_{FC}}{R_{MC}} = \frac{\sqrt{1 + (2\pi f)^2\,(L_{FC}/R_{FC})^2}}{\sqrt{1 + (2\pi f)^2\,(L_{MC}/R_{MC})^2}}$$

where I_{FC}, R_{FC}, Z_{FC}, and L_{FC} are the current, resistance, impedance, and inductance of the fixed coil; and I_{MC}, R_{MC}, Z_{MC}, and L_{MC} are the current, resistance, impedance, and inductance of the moving coil.

When the time constants are equal, the above equation becomes:

$$\frac{I_{MC}}{I_{FC}} = \frac{R_{FC}}{R_{MC}}$$

This equation of ratio of currents holds for both DC and AC for same time constants of both coils. Also, both the currents have 0° phase difference between them.

4.4.5 Advantages and Disadvantages of Dynamometer Instruments

The dynamometer instruments have many advantages, some of which are as follows.

- Dynamometer instruments may be used for both DC and AC applications as they have precision grade accuracy of about 0.5 % of full-scale deflection.
- These instruments can be used as voltmeters, ammeters, and wattmeters.
- Dynamometer instruments can be used as a transfer instrument, that is, they can be calibrated on DC and then operated on ac.
- These instruments do not have hysteresis and eddy current errors since they are air cored.
- The frequency range for lower grade instruments is 15 Hz to 1 kHz. The precision grade accuracies can be obtained in the range of 40 to 500 Hz, and the instruments incorporating astatic movement can be operated for the frequencies up to 10 kHz.
- Dynamometer voltmeters are used invariably for the applications which require accurate rms voltages irrespective of the waveforms.

Despite the above advantages, dynamometer instruments have several disadvantages which are described as follows.

- These instruments have non-uniform scales.
- The low torque/weight ratio of the instruments yields more frictional errors which in turn lessens the accuracy.
- The instruments require larger operating current which causes the sensitivity of the instrument to decrease as they have weak magnetic field.
- The power consumption of such instruments is higher than that of the PMMC instruments but lower than moving iron type instruments.
- These instruments are more expensive than PMMC instruments as well as moving iron type instruments.
- Dynamometer instruments have limited range of AC frequencies.
- These instruments need to be handled with great care as they are sensitive to mechanical impacts and overloads.

Example 5 When the pointer of an electrodynamic instrument deflects from 0° to 45°, the mutual inductance between its series-connected fixed and moving coils changes by 20 mH. Determine the coil current if the spring controlling constant is given as 1.5 μNm/degree.

Solution: Given that:

Spring controlling constant $K = 1.5$ μNm/degree

Maximum deflection $\theta = 45°$

Therefore, we may find the controlling torque T_C as:

$$TC = K\theta = 1.5 \times 45° = 67.5 \ \mu\text{Nm}$$

Also, from Equation (14), we have:

$$\theta = \frac{I_{\text{rms}}^2}{K}\frac{dM}{d\theta}$$

$$\Rightarrow \qquad I_{\text{rms}}^2 = K\theta\frac{d\theta}{dM}$$

Substituting the values, we get:

$$I_{\text{rms}}^2 = 67.5\times10^{-6}\frac{45°\times\pi}{20\times10^{-3}\times180°} = 2.65 \text{ mA} \quad \text{(since } d\theta \text{ is in radians)}$$

Thus, $I_{\text{rms}} = 1.63$ mA

Example 6 If the currents flowing through fixed coil and moving coil of an electrodynamometer instrument are given as $I_{FC} = I_{MC} = 120$ mA, it gives a deflection of 100° on its scale. Obtain the increment in the mutual inductance starting from zero value of current. The spring controlling torque is given to be 0.055 μNm/degree.

Solution: The controlling torque T_C is given as 0.055 μNm/degree

At steady state deflection, controlling torque is equal to deflecting torque. Thus, we get:

$$T_C = T_D = K\theta = 100° \times 0.055 = 5.5 \ \mu\text{Nm}$$

From Equation (14), we have:

$$\theta = \frac{I_{\text{rms}}^2}{K}\frac{dM}{d\theta} \Rightarrow dM = K\theta\frac{d\theta}{I_{\text{rms}}^2}$$

Substituting the required values in the above equation, we get:

$$dM = 5.5\times10^{-6}\frac{100°\times\pi}{180°\times(120\times10^{-3})^2}$$

On solving, we get: $dM = 666.6$ μH

4.5 TRANSISTOR VOLTMETER CIRCUITS

In earlier chapter we have studied the voltmeters made of series combination of multiplier resistors and moving coil instruments with some limitations as having low resistance due to which they possess loading effect and cannot measure very low voltages. These low voltages need to be amplified to measurable levels and electronic circuits are required to offer high input resistance. The instruments possessing such electronic circuits are called **electronic voltmeters** which may be analog or digital instruments. These electronic circuits are transistors, operational amplifiers (or op-amp) offering advantages in measurement of resistance, current, and voltages. In this section, we will study the different types of transistor voltmeter circuits, such as *emitter-follower voltmeter*, *FET-input voltmeter*, and *difference amplifier voltmeter*.

4.5.1 Emitter-Follower Voltmeter

An emitter-follower provides a high input resistance to the voltages under measurement and a low resistance to the deflection meter at the output in order to drive the current through the coil of the instrument. A BJT emitter follower can be used for this purpose. The basic configuration of an emitter-follower voltmeter circuit is shown in Figure 4.11 with the required voltage connections.

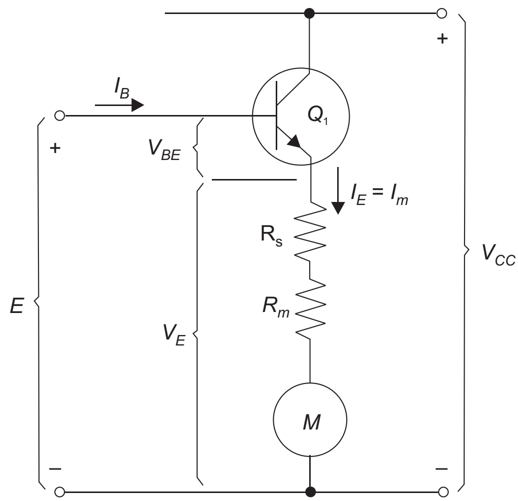


Fig. 4.11 Emitter-Follower Voltmeter Circuit

In Figure 4.11, a PMMC instrument and multiplier resistance R_s are connected in series with the emitter of a bipolar junction transistor (abbreviated as BJT). The negative terminal of the DC supply, that is V_{CC}, is connected to the PMMC meter while the positive terminal is connected to the collector of the transistor. The voltage to be measured, that is, E, is connected to the base of the transistor with its positive terminal while its negative terminal is connected to the meter.

The base current of the transistor can be calculated as:

$$I_B \approx \frac{I_m}{h_{fe}} \quad ...(16)$$

where h_{fe} represents the current gain of the transistor. The base current I_B is much lower than the current I_m flowing through meter. The input resistance R_i of the voltmeter can be given as:

$$R_i \approx \frac{E}{I_B} \quad ...(17)$$

Here the input resistance of the circuit is much larger than the series combination of the meter resistance and multiplier resistance, that is, $R_m + R_s$.

The voltage drop V_{BE} across the base-emitter junction causes an error during measurements. This error can be eradicated by introducing one more emitter-follower and a voltage divider in the circuit. The modified circuit is shown in Figure 4.12 to which a ± 12 V dual polarity supply is connected.

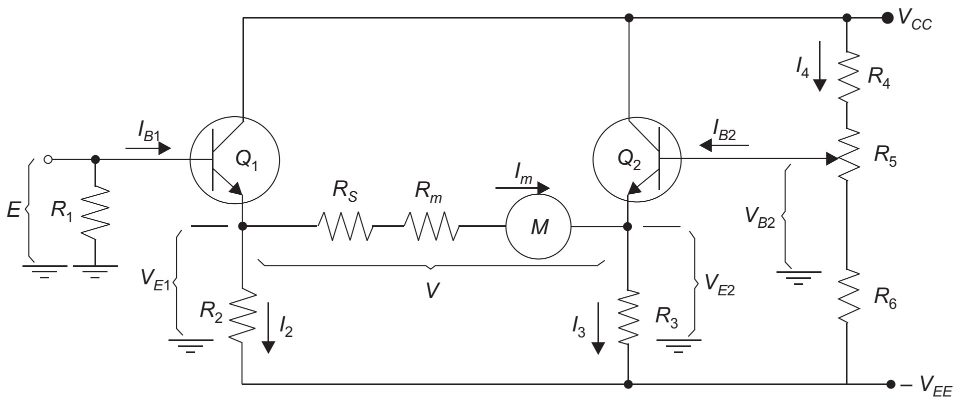


Fig. 4.12 Modified Circuit of Emitter Follower Voltmeter

It can be seen from the above figure that the base of transistor Q_1 is biased to ground through resistor R_1 and the transistor Q_2 is fed with an adjustable bias voltage V_{B2} at its base terminal via a voltage divider, made by resistors R_4, R_5, and R_6. Resistors R_2 and R_3 connect the emitters of both the transistors to the negative supply voltage $-V_{EE}$. The parallel combination of the input resistance of transistor Q_1 and resistor R_1 is the total input resistance of the circuit. The PMMC meter is connected between the emitters of transistors.

In the absence of the input voltage E, the base voltage of transistor Q_2 is adjusted in such a way that the meter indicates zero current. Therefore, we may write as:

$$V_{B2} = 0 \text{ V}$$

While the emitter voltage of both the transistors is:

$$V_{E1} = V_{E2} = -0.7 \text{ V}$$

Thus, the meter voltage $V = 0$ V

Now, if $E = 3$ V is applied to the base of transistor Q_1, the voltage across the meter comes out as:

$$\begin{aligned} V &= V_{E1} - V_{E2} \\ &= (E - V_{BE1}) \\ &= (3 - 0.7) \text{ V} - (-0.7 \text{ V}) = 3 \text{ V} \end{aligned}$$

This shows that when the circuit is modified, all the voltage to be measured appears across the meter since there is no base-emitter voltage drop in BJT.

Example 7 Specifications of a simple emitter-follower voltmeter are: V_{CC} = 12 V, R_m = 2 kΩ, 1 mA FSD meter current, and current gain h_{fe} = 50. Determine the following such that the voltmeter can give FSD when an input voltage of 5 V is applied to it.

(a) Appropriate multiplier resistance

(b) Input resistance

Solution: From the simple emitter-follower voltmeter shown in Figure 4.11, the emitter voltage V_E can be determined as:

$$V_E = E - V_B = 5 - 0.7 = 4.3 \text{ V}$$

(a) The multiplier resistance R_s can be found as:

$$R_s = \frac{V_E}{I_m} - R_m \qquad \text{[Refer to Figure 4.11]}$$

On substituting the values, it becomes:

$$R_s = \frac{4.3}{1 \times 10^{-3}} - 2 \times 10^3 = 2.3 \text{ k}\Omega$$

(b) The base current I_B can be calculated from Equation (16) as:

$$I_B \simeq \frac{I_m}{h_{fe}} = \frac{1 \times 10^{-3}}{50} = 20 \ \mu\text{A}$$

Now, the input resistance R_i can be obtained from Equation (17) as:

$$R_i = \frac{E}{I_B} = \frac{5}{20 \times 10^{-6}} = 250 \text{ k}\Omega$$

4.5.2 Field Effect Transistor (FET)-input Voltmeter

Field Effect Transistors (FETs) have inherent property of providing extremely high input resistance so they can be used in transistor voltmeters. A Junction Field Effect Transistor (JFET) source follower is used for this purpose since its source terminal is capable of supplying the required base current to transistor Q_1 while its gate terminal has an input resistance in excess of 1 MΩ (see Figure 4.13).

Resistances R_p, R_q, R_r, and R_s, connected at the input of the circuit allow a large amount of voltage to be measured by accurately dividing it before applying it to the input transistor. This section of the circuit is called as **input attenuator**. The input voltage (or **gate voltage**, E_G) of transistor Q_3 always remains at 1 V provided that maximum input is applied, regardless of the range. For instance, let the input voltage be 5 V. Now, the input voltage E_G at gate terminal of transistor Q_3 is calculated as:

$$E_G = E \times \frac{R_q + R_r + R_s}{R_p + R_q + R_r + R_s}$$

$$= 5 \text{ V} \times \frac{100 \text{ k}\Omega + 60 \text{ k}\Omega + 40 \text{ k}\Omega}{800 \text{ k}\Omega + 100 \text{ k}\Omega + 60 \text{ k}\Omega + 40 \text{ k}\Omega} = 1$$

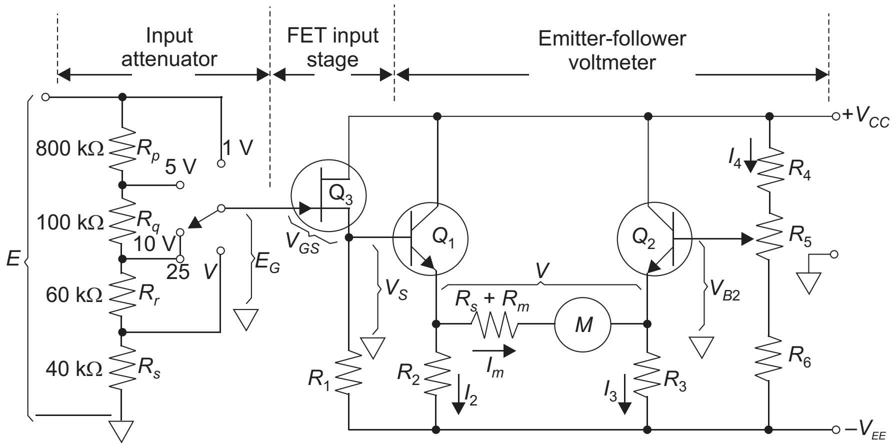


Fig. 4.13 Emitter-follower with FET and Input Attenuator Circuit

Let initially the input voltage $E = 0$ which ensures zero voltage at the gate terminal of the n-channel FET. Now if a gate to source voltage V_{GS} of -5 V is applied to the Q_3, a voltage of $+5$ V appears at its source terminal. This is due to the prerequisite of n-channel FETs that the source terminal must be positive with respect to its gate terminal. Now, the voltage at the base of transistor Q_1 is $+5$ V and the base voltage of transistor Q_2 is also $+5$ V, since there must be equal potential at the base of two transistors. Resistance R_5 is adjustable so that the meter voltage indicates zero whenever the input voltage is zero.

Now let the voltage to be measured be applied to the circuit input. As we know that E_G can attain a maximum value of 1 V, the FET source terminal voltage V_S increases until V_{GS} gets up to -5 V again. This implies that V_S is increased from $+5$ V to $+6$ V in order to keep V_{GS} equal to -5 V. This causes the base voltage of transistor Q_1 to also increase by 1 V which is then indicated by the meter.

The total input resistance of the above circuit is 1 MΩ, provided by the attenuator section. This can be further increased to 10 MΩ by connecting a 9 MΩ resistance in series with the input terminal. As a consequence of this, the input voltage will be further divided by a factor of 10 before being applied to the input terminal of transistor Q_3.

Example 8 The full-scale meter current of an FET input voltmeter is given to be 60 μA and a 20 V supply voltage is fed to it. The BJTs employed in the circuit have a current gain of 100 while the source-gain voltage of the FET is -5 V. Use the following resistance values for calculation.

$R_1 = 5.6$ kΩ, $R_2 = R_3 = 4.5$ kΩ, $R_4 = 2.5$ kΩ, $R_5 = 600$ Ω, $R_6 = 3.5$ kΩ, $R_s + R_m = 30$ Ω,
At $E = 0$, calculate:

(a) V_p (b) I_s (c) I_2 (d) I_3 (e) I_4 (f) adjustment range of V_p.

Solution: Given that:

Gate-source voltage $V_{GS} = -5$ V

Source terminal voltage $V_S = 5$ V

Positive supply voltage $V_{CC} = 20$ V

Negative supply voltage $V_{EE} = -20$ V

(a) When $E = 0$, $V_p = V_s = E_G - V_{GS}$

Substituting the values, we get:

$$V_p = V_s = 0 - (-5)\ V = 5\ \text{V}$$

(b) The current through source terminal I_s can be found as:

$$I_s = (V_s - V_{EE})/R_1 \qquad \text{[Refer to Figure 4.13]}$$

Substituting the values, we obtain:

$$I_s = \frac{5-(-20)}{5.6\times 10^3} = 4.464\ \text{mA}$$

(c) Current through resistor R_2 can be calculated as:

$$I_2 = (V_s - V_{EE} - V_{BE})/R_2 \qquad \text{[Refer to Figure 4.13]}$$

On substituting the given value, it gives:

$$I_2 = \frac{5-(-20)-0.7}{4.5\times 10^3} = 5.4\ \text{mA}$$

(d) Current through resistor R_3 will be equal to that through resistor R_2. Thus,

$$I_3 = I_2 = 5.4\ \text{mA}$$

(e) Current I_4 can be obtained as:

$$I_4 = \frac{V_{CC} - V_{EE}}{R_4 + R_5 + R_6}$$

Substituting the given values, it yields:

$$I_4 = \frac{20-(-20)}{2.5\times 10^3 + 600 + 3.5\times 10^3} = 6.6\ \text{mA}$$

(f) Now, the range of adjustment of V_p can be determined as:

$$V_{p(\max)} = V_{CC} - I_4 R_4 = 20 - (6.6 \times 10^{-3} \times 2.5 \times 10^3) = 3.5\ \text{kV}$$

$$V_{p(\min)} = V_{CC} - I_4 (R_4 + R_5) = 20 - [6.6 \times 10^{-3} (2.5 \times 10^3 + 600)] = -0.46\ \text{V}$$

4.5.3 Difference Amplifier Voltmeter

A maximum voltage of 25 V can be measured using the circuits discussed so far. However, this range can be increased by modifying the input attenuator section of the previous circuit. In addition, the minimum (full-scale) measurable voltage, that is, 1 V can also be changed to a much lower level, say 100 mV by using a meter that indicates FSD when 100 mV appears across $R_m + R_s$. However, it becomes elusive to measure such small voltages and thus, should be amplified before being applied to the meter. To accomplish this, consider the following circuit showing a suitable amplifier.

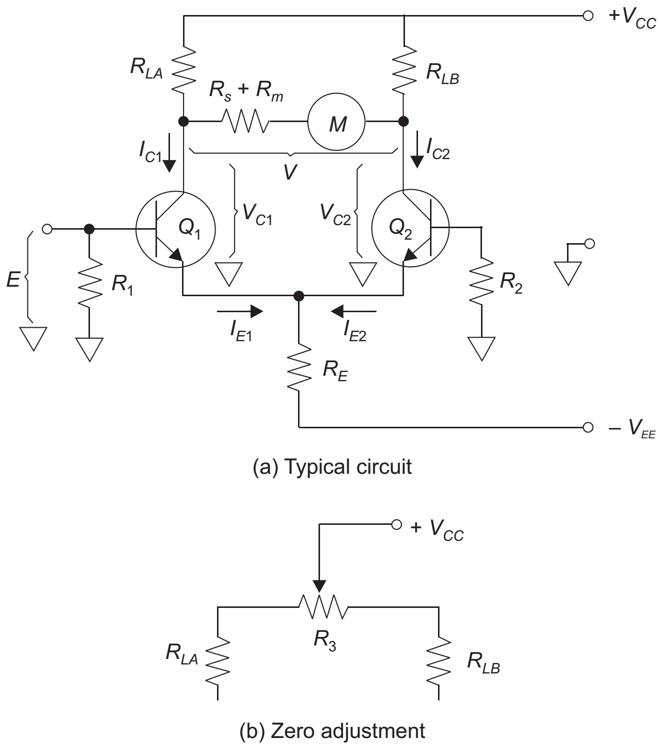


Fig. 4.14 Difference Amplifier Voltmeter

The circuit in Figure 4.14(a) amplifies the difference between the base voltages of transistors Q_1 and Q_2 whenever $V_{B2} = 0$ and input voltage is applied at the base of transistor Q_1, such that, $V_{B1} = E$. Therefore, this circuit is called **difference amplifier voltmeter**. The amplified difference is then applied to the meter circuit. Transistor Q_1 and Q_2 collectively with resistances R_{LA}, R_{LB}, and R_E configure a *differential amplifier* (or *emitter-coupled amplifier*).

When the base of transistor Q_1 is fed with a small positive voltage, it increases the current through Q_1 and decreases the current through Q_2. The increased collector current I_{C1} of transistor Q_1 leads to the increment of $I_{C1}R_{LA}$ which in turn causes V_{C1} to decrease. Similarly, the decreased collector current I_{C2} of transistor Q_2 decreases the $I_{C2}R_{LB}$, thereby increasing $V_{C2.}$ Therefore, the voltage across meter circuit is positive on the right-hand side of the scale and negative on its left-hand side. The voltage V of the meter is in proportion to the applied input voltage E.

Zero adjustment can be provided to the circuit by incorporating an adjustable resistance or potentiometer R_3 as shown in Figure 4.14(b). If the rotating contact of this potentiometer is moved to the righthand side, its proportion added to R_{LA} gets increased while that added to R_{LB} gets decreased and it happens vice-versa on rotating a contact to the left-hand side. In this way, the potentiometer R_3 differentially adjusts V_{C1} and V_{C2}, it can also adjust the meter voltage to zero.

Example 9 The difference amplifier voltmeter has the following components: $V_{CC} = \pm 12$ V, $R_1 = R_2 = 20$ kΩ, $R_{LA} = R_{LB} = 5$ kΩ, $R_E = 3.5$ kΩ, $R_S = 35$ kΩ, and $R_m = 500$ Ω. The meter full-scale current is given to be 40 μA. At the input voltage $E = 0$ V, determine the transistor voltage levels.

Solution: Given that at input voltage $E = 0$ V, the base currents of both transistors will be zero as well.

Thus, $V_{B1} = V_{B2} = 0$ V

And, $V_{E1} = V_{E2} = V_B - V_{BE} = 0 - 0.7 = -0.7$ V

The sum of emitter currents of both transistors can be given as:

$$I_{E1} + I_{E2} = \frac{I_E}{2} = \frac{V_E - V_{EE}}{R_E}$$

Substituting the given values, we get:

$$I_{E1} + I_{E2} = \frac{-0.7 - (-12)}{3.5 \times 10^3} = 3.23 \text{ mA}$$

Therefore,
$$I_{E1} = I_{E2} = \frac{I_E}{2} = \frac{3.23}{2} = 1.62 \text{ mA}$$

Now, the collector voltage level of both transistors can be calculated as:

$$V_{C1} = V_{C2} = V_{CC} - I_{E1} R_{LA}$$

Substituting the required values, we obtain:

$$V_{C1} = V_{C2} = 12 - (1.62 \times 10^{-3} \times 5 \times 10^3) = 3.9 \text{ V}$$

4.6 MEASUREMENT OF AC—RECTIFIER TYPE INSTRUMENTS

So far the instruments we have read in this chapter could measure DC as well as AC quantities. Now we will discuss the instruments which measure AC only. These instruments incorporate rectification technique and therefore are called **rectifier type instruments**. A rectifier element, preferably silicon diode (due to its high forward current and low reverse current ratings) is employed in such instruments which convert conventional AC signal to DC which is then indicated by a DC meter (PMMC meter). In this way, the DC meter shows the amount of rectified ac.

Rectification is essential since the DC meter cannot follow the frequent transitions of AC signal due to its damping mechanism and inertia. An AC signal with a substantially low frequency, not more than 0.1 Hz, can cause the pointer of the meter to follow the instantaneous changing levels, but for a normal (60 Hz) or high frequency AC signal, the meter settles at the average value of the signal, that is, it indicates zero.

4.6.1 AC Voltmeter

The AC voltmeters can be categorized on the basis of the rectification technique employed as *full-wave rectifier voltmeter*, *half-wave rectifier voltmeter*, and *half-bridge full-wave rectifier voltmeter*.

Full-wave Rectifier Voltmeter

A full-wave rectifier voltmeter incorporates a full-wave rectifier which consists of diodes that convert the AC signal into a series of uni-directional current pulses that pass through the PMMC instrument to cause positive deflection. The circuit of a full-wave rectifier voltmeter is shown in Figure 4.15.

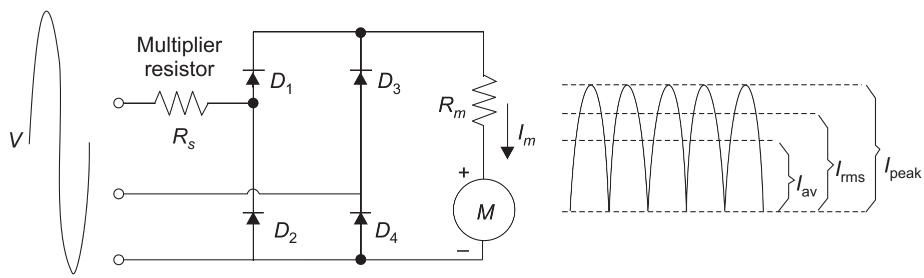


Fig. 4.15 Full-wave Rectifier Voltmeter Circuit

When a sinusoidal input signal is applied to the circuit, the positive half-cycles of the signal are passed by the circuit while the negative half-cycles are inverted. Diodes D_1 and D_4 conduct whenever the positive half-cycles appear at the input. As a result, the current flows through the PMMC meter from top to bottom. On the contrary, the negative half-cycles at the input cause diodes D_2 and D_3 to conduct causing the current to flow again through the meter in the same direction. Thus, the output current of the circuit is a series of positive half-cycles with no intervening space between them (see Figure 4.15).

The multiplier resistance R_m is connected to limit the current through the meter in the same way as in the case of DC voltmeter. The meter must indicate the rms value, (that is, 0.707 × peak value) of the quantity (that is, current or voltage) being measured while this meter deflects in proportion to the average value of the current (0.637 × peak current). This value of the measured current can be expressed in terms of rms values, average values, and peak values as they are directly related to each other. The rms value of a quantity is 1.11 times its average value. Therefore, the linear scale of the meter can be calibrated accordingly.

The low input voltage causes the rectifier current to be low, which in turn causes errors due to variations in voltage drop of diodes. To eliminate this effect, half-wave rectifier voltmeters having a shunt resistance across the meter are used.

Note: *The full-wave rectifier voltmeter deflects the rms value of the quantity only when the pure sinusoidal signal is applied to its input.*

Half-wave Rectifier Voltmeter

The circuit of a half-wave rectifier voltmeter is illustrated in Figure 4.16. It consists of two diodes (D_1 and D_2), a multiplier resistance R_s, a PMMC meter, and a shunt resistance R_{sh}.

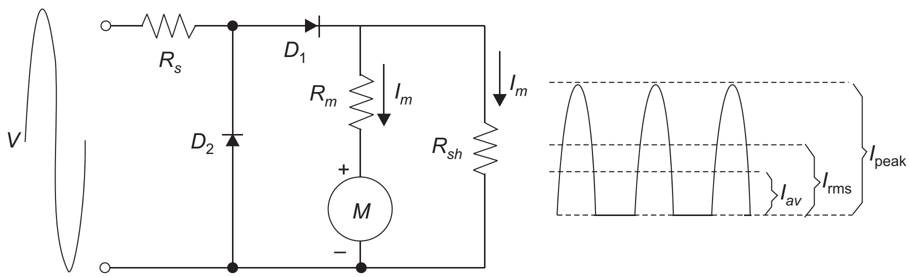


Fig. 4.16 Half-wave Rectifier Voltmeter Circuit

The shunt resistance R_{sh} is included in the circuit so that a sufficiently large current (larger that meter current I_m) can flow through diode D_1 when a positive half-wave appears at the input, that is, when diode D_1 is forward biased. For this, the diode is biased above the knee voltage and well in its linear range of characteristics. Now, when a negative half-wave appears at the input, diode D_2 starts conducting. Some voltage drop V_F occurs across the meter and diode D_1 due to D_2 during negative half-cycle. This eliminates the possibility of any reverse leakage current flowing through the meter via diode D_1. The meter is also protected by diode D_2 against any reverse voltages.

It can be seen from the figure that the output current waveform developed across the meter and shunt resistance R_{sh} consists of intervening spacing in a series positive half-cycles. The meter of the half-wave rectifier voltmeter indicates the average value of the current under measurement as $I_{av} = 0.5 \times (0.637\ I_m)$ which can be converted to its rms equivalent by calibrating the meter scale.

Half-bridge full-wave rectifier voltmeter

The circuit of a half-bridge full-wave rectifier voltmeter is shown in Figure 4.17 which consists of two diodes (D_1 and D_2), two resistances (R_1 and R_2), and a PMMC meter.

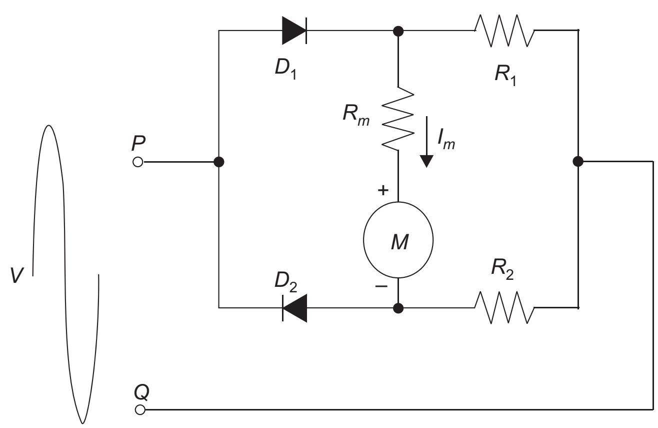


Fig. 4.17 Half-bridge Full-wave Rectifier Voltmeter

The full-wave bridge rectifier consists of four diodes while this voltmeter circuit consists of only two diodes, hence it is called **half-bridge full-wave rectifier voltmeter**. The full-wave rectified current of this circuit passes through the meter, whereas only some part of the current is bypassed through the meter in case of half-wave rectifier voltmeter.

When the positive half-cycle appears at the input, the current starts flowing through terminal P making diode D_1 forward biased and D_2 reverse biased. It then passes through the meter and resistance R_2, and finally reaches terminal Q. Since the series combination of resistance R_2 and the meter is parallel to resistance R_1, a small portion of the current flows through the meter and R_2 while the major part of the current flowing in D_1 passes through R_1.

Similarly, when a negative half-cycle appears at the input, the current flows through terminal Q making diode D_2 forward biased and D_1 reverse biased. The current in this case flows through R_1, meter, diode D_2 and thus, reaches terminal P. Now, the series combination of resistance R_1 and meter is parallel to resistance R_2, as a the major part of current flows through R_2 and only a small portion of the current flows through the series combination.

In this voltmeter circuit, the diodes are forced to operate above the knee voltage of their characteristics and the differences in the characteristics of both diodes (if any) are compensated.

Example 10 A PMMC instrument with meter resistance 1 kΩ gives a full-scale deflection of 80 μA. It is to be used as a full-wave rectifier voltmeter to give FSD of 80 V (rms). Determine the required value of multiplier resistance if silicon diodes are used in the circuit.

Solution: Given that:

Meter resistance R_m = 1 kΩ

The rms voltage of voltmeter V_{rms} = 80 V

At full-scale deflection, the average current flowing through the PMMC meter is:

$$I_{av} = 80\ \mu\text{A}$$

The peak value of current I_m can be obtained as:

$$I_m = \frac{I_{av}}{0.637}$$

Thus, we get:

$$I_m = \frac{80 \times 10^{-6}}{0.637} = 125.6\ \mu\text{A}$$

Now, since the voltage drop for a full-wave rectifier voltmeter is 2 V_F and silicon diodes are used in the circuit as per the given data, the voltage comes out as:

$$\text{Voltage drop} = 2 \times 0.7\ \text{V} = 1.4\ \text{V}$$

We know that the amount of peak voltage V_{peak} applied to the voltmeter is obtained as:

$$V_{\text{peak}} = 1.414 \times \text{rms value of voltage} = 1.414 \times V_{\text{rms}} = 1.414 \times 80\ \text{V} = 113.12\ \text{V}$$

Now, as the total circuit resistance is given as:

$$R_T = R_s + R_m$$

The multiplier resistance R_s can be determined using the relation for meter current I_m as:

$$I_m = \frac{V_{\text{peak}} - 2V_F}{R_S + R_m} \qquad \text{[Refer to Figure 4.15]}$$

$$\Rightarrow \qquad R_s = \frac{V_{\text{peak}} - 2V_F}{I_m} - R_m$$

On substituting the values, we get:

$$R_s = \frac{113.12 - 1.4}{125.6 \times 10^{-6}} - 1 \times 10^3 = 888.5 \ \text{k}\Omega$$

Example 11 A bridge rectifier AC voltmeter employing silicon diodes gives a full-scale deflection of 100 μA. Calculate the value of applied rms voltage when the meter indicates FSD, if meter coil resistance is given to be 1 kΩ and multiplier resistance is 800 kΩ.

Solution: Given that:

Meter resistance R_m = 1 kΩ

Multiplier resistance R_s = 800 kΩ

At FSD, an average current of 100 μA flows through the meter. Thus, the peak current through meter I_m can be obtained as:

$$I_m = \frac{I_{av}}{0.637} = \frac{100 \times 10^{-6}}{0.637} = 156.98 \ \mu\text{A}$$

Now, the peak voltage V_{peak} applied to the voltmeter can be determined as:

$$V_{\text{peak}} = I_m \, (R_s + R_m) + 2V_F$$

Substituting the values, we get:

$$V_{\text{peak}} = 156.98 \times 10^{-6} \, (800 + 1) \times 10^3 + 2 \times 0.7$$

$$V_{\text{peak}} = 127.14 \text{ V}$$

Having obtained the value of V_{peak}, we can now find V_{rms} as:

$$V_{\text{rms}} = 0.707 \times V_{\text{peak}}$$

Substituting the values, we get:

$$V_{\text{rms}} = 89.89 \text{ V}$$

4.6.2 AC Ammeter

Rectifier instruments cannot be used directly as AC ammeters. The prerequisite for an AC ammeter is to have very low resistance which implies that a very low (typically less than 100 mV) voltage drop should occur across the ammeter as it is connected in series with the circuit in which the current is to be measured. Typically, there is a voltage drop of 0.3 V to 0.7 V across the diodes, and for a bridge rectifier, the total voltage drop goes high to 0.6 V to 1.4 V. Therefore, a rectifier instrument cannot be used to measure AC current directly.

However, using a current transformer with a rectifier instrument provides a very low terminal resistance and low voltage drop. If the secondary turns of the transformer are more than its primary turns, the input voltage gets stepped up so that sufficient voltage can be provided for rectifier operation. At the same instant, the current through the primary winding is stepped down by the transformer so that the PMMC meter can measure efficiently. Hence, the current transformation ratio, $\frac{N_S}{N_P} = \frac{I_P}{I_S}$ of the transformer in the ammeter circuit holds utmost importance, where N represents the number of turns in the transformer winding and I is the current flowing through them. The subscripts S and P represent secondary and primary winding of the transformer, respectively. The circuit of a rectifier ammeter is shown in Figure 4.18.

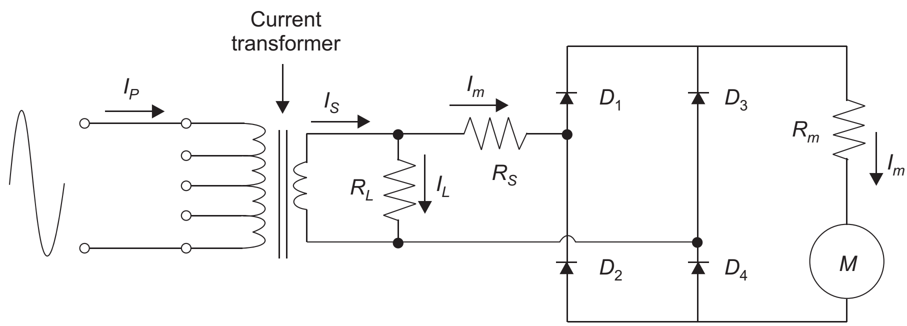


Fig. 4.18 Rectifier Ammeter Circuit (bell 4.10)

The load resistance R_L, connected in parallel to secondary winding, is so selected that only the required amount of current passes through the meter. It can be switched to different values to alter the range of the ammeter. However, the method of changing the number of primary turns by using additional terminals on the primary winding can also be used to change the range of the instrument.

Example 12 A rectifier circuit is to be used in an AC ammeter which deflects 2 A rms current at FSD while the PMMC meter of the instrument deflects 300 μA at FSD. The meter resistance is given to be 1 kΩ and the multiplier resistance is 120 kΩ. If the turn ratio of the transformer is $\frac{N_S}{N_P} = \frac{5000}{10}$, find the required value of the secondary shunt resistance. Assume that silicon diodes are used in the circuit.

Solution: Given that:

Average current flowing through meter at FSD, I_{av} = 300 μA

Multiplier resistance R_s = 120 kΩ

Meter resistance R_m = 1 kΩ

Transformer turn ratio = $\frac{N_S}{N_P} = \frac{5000}{10}$

The peak meter current I_m comes out to be:

$$I_m = \frac{I_{av}}{0.637} = \frac{300 \times 10^{-6}}{0.637} = 470.96 \ \mu\text{A} \qquad \text{[Refer to } \textit{Section 4.6.1}\text{]}$$

The transformer peak secondary voltage V_{peak} can be computed as:

$$V_{\text{peak}} = I_m \ (R_s + R_m) + 2V_F \qquad \text{[Refer to Figure 4.18]}$$

Substituting the values, we get:

$$V_{\text{peak}} = 470.96 \times 10^{-6} \ (120 \times 10^3 + 1 \times 10^3) + 1.4$$

$$= 58.38 \text{ V}$$

Now, the secondary rms voltage $V_{S,\text{rms}}$ can be obtained as:

$$V_{S,\text{rms}} = 0.707 \times V_{\text{peak}} = 41.27 \text{ V} \qquad \text{[Refer to } \textit{Section 4.6.1}\text{]}$$

The rms current through the meter $I_{m,\text{rms}}$ can be determined as:

$$I_{m,\text{rms}} = 1.11 \times I_{av} = 1.11 \times 300 \times 10^{-6} = 333 \ \mu\text{A}$$

Now, using the current transformation ratio, the rms current through secondary winding comes out as:

$$\frac{N_S}{N_P} = \frac{I_P}{I_S} \Rightarrow \frac{5000}{10} = \frac{2}{I_S} \qquad \text{(since } I_P \text{ is given to be 2A)}$$

Thus, $$I_S = \frac{20}{5000} = 0.004 \text{ A}$$

And, the current through the load is obtained as:

$$I_L = I_S - I_{m,\text{rms}} = 0.004 - 333 \times 10^{-6} = 3.67 \text{ mA}$$

Finally, the shunt resistance of the secondary winding can be found as:

$$R_L = \frac{V_{S,\text{rms}}}{I_L} = \frac{41.27}{3.67 \times 10^{-3}} = 11.24 \ \text{k}\Omega$$

4.7 ELECTRONIC MULTIMETER

Earlier we have studied instruments like ammeters, voltmeters, and ohmmeters, all based on different mechanisms such as PMMC meter movement and moving iron mechanism, regardless of the quantities they measure. The only difference that lies in them is their configuration. All these instruments can be configured into a single unit which is capable of measuring different quantities, that is, current, voltage, and resistance of an electronic circuit. The resultant single unit circuit is known as an **electronic multimeter**. The front panel of a multimeter is shown in Figure 4.19.

An electronic multimeter can be used to directly measure DC voltages, AC voltages, and resistances with multiple ranges. The FUNCTION and RANGE switches shown in the

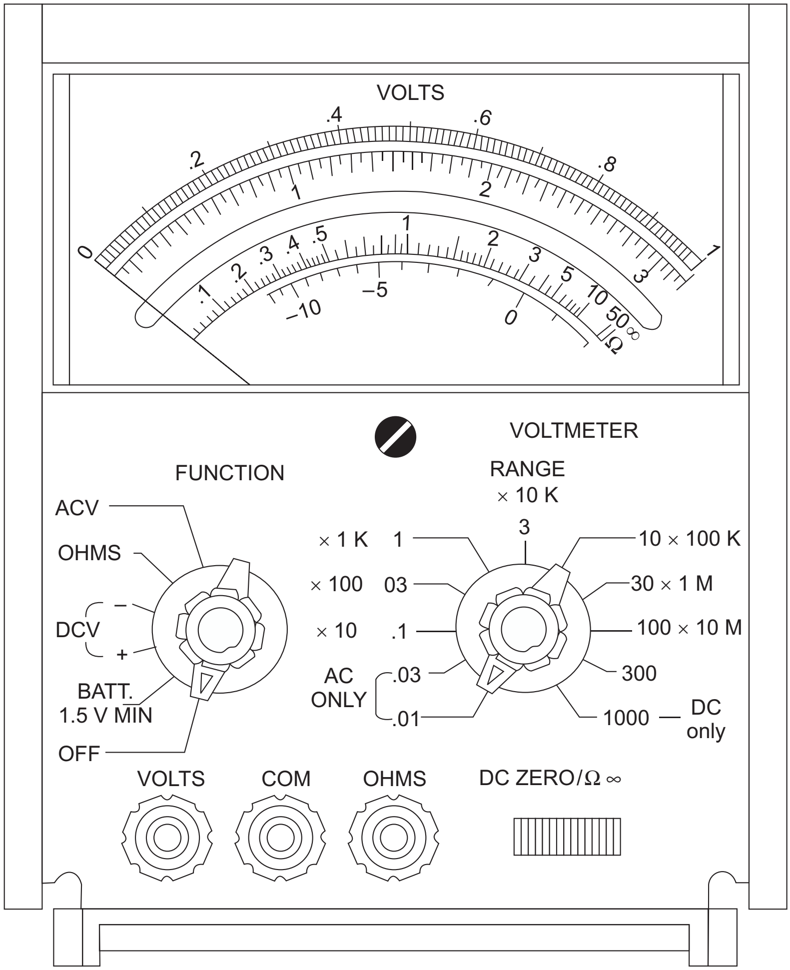


Fig. 4.19 Front Panel of an Electronic Multimeter

figure are used for selection purpose. Essentially, the multimeter must be accompanied with a function switch, which on rotating connects the meter movement to the desired circuit. There exist several ranges of DC and AC voltages. For the multimeter shown, nine ranges are available for DC voltage measurements which vary from 0.1 V (full scale) to 1000 V with an accuracy of ±2 % full scale; the input resistance is 10 MΩ for all ranges. For AC voltage measurements, there are ten ranges starting from 10 mV to 300 V with a frequency range of 10 Hz to 1 MHz. The range switch or an input attenuator is used to set the required range of input voltage. However, the high frequency probes are designed to extend the range for use with voltages ranging from 0.25 V to 30 V, the frequency range can be further extended to 500 MHz. In AC measurements, the instrument gives an accuracy of about ±2% of full scale and input impedance is 10 MΩ shunted by 40 pF capacitance on ranges up to and including 1 V. For voltage ranges 3 V and greater, it is 10 MΩ shunted by 20 pF.

When the FUNCTION knob is set to position ±DCV, voltages with positive or negative polarities can be measured. At the bottom of the front panel, can be seen three terminals, namely, VOLTS, COM, and OHMS. For measuring voltages, VOLTS and COM terminals are used while for measuring resistance, OHMS and COM terminals are used. The DC ZERO/Ω ∞ is meant for electrical and mechanical zero control for the pointer.

The 1 position at the scale centre is the ohmmeter scale as shown in Figure 4.19. For the precise reading of two voltage scales; 0 to 1 and 0 to 3 are provided along with knife-edge pointer and mirror.

4.7.1 Probes Used in Multimeters

Multimeters are sometimes used to measure some other quantities, like temperature. A number of probes and adapters are available for this purpose. Also, the range of measurements of a multimeter can be extended using such probes, namely, *high-current probe*, *high-voltage probe*, *radio-frequency probe*, and *hall-effect probe*.

High-current probe

The high-current probe is used with a multimeter in order to extend its current range. The AC current probe is based on the principle of a current transformer which reduces the high levels of alternating current. As shown in Figure 4.20, the current to be measured is carried by a conductor.

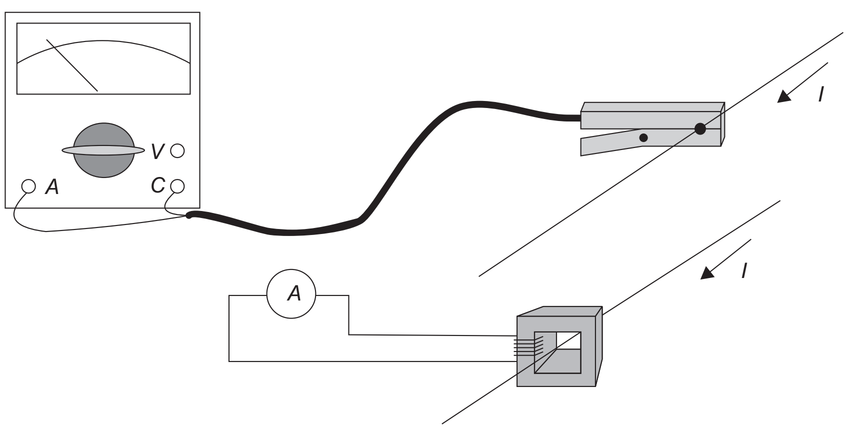


Fig. 4.20 High-current Probe

The core of the transformer is opened to close around the conductor carrying current to be measured. The conductor can be treated as the primary winding of the transformer having a single turn. The secondary winding of the transformer can then be used to determine the current level. The 1 mA AC scale of multimeter can be converted to 1 A scale on using this probe.

High-voltage probe

The high-voltage probe is used to extend the measurable voltage range of the multimeter. This probe is also known as **voltage multiplier**. It is basically a well insulated voltage divider as shown in Figure 4.21. The voltage under measurement gets divided by a factor of 1000. Hence, the scales of the multimeter get multiplied by the same factor. In this way, a 50 V scale can be converted to a 50 kV scale.

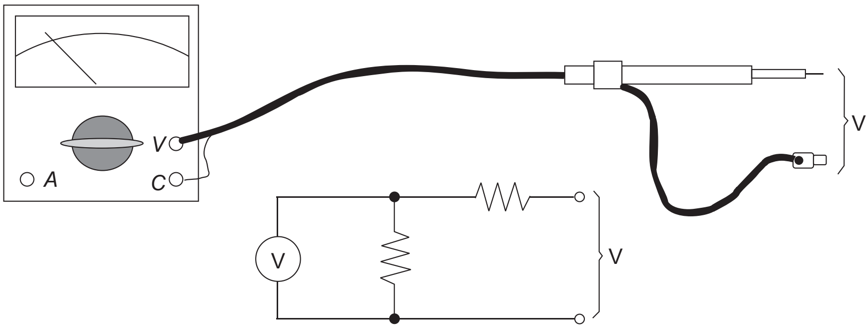


Fig. 4.21 High-voltage Probe

Radio-frequency (RF) probe

The upper frequency limit of an electronic multimeter is not high enough to directly measure the RF voltages. Thus, an RF probe is used for this purpose. It is also known as **peak detector probe**. This probe is meant to clamp and convert the RF waveform into a DC voltage which is equal to its peak level. A peak detector circuit is shown in Figure 4.22 in which capacitor C_1 is connected in series with the input terminal to block any DC component in the input voltage. Thus, only AC voltages are allowed to pass through C_1.

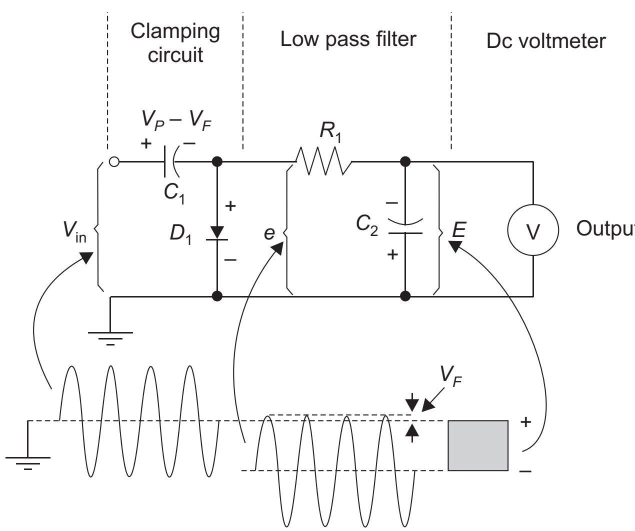


Fig. 4.22 Peak Detector Circuit

It can be seen that a peak detector circuit essentially consists of three sections, namely, a *clamping circuit*, *low pass filter*, and a *DC voltmeter*. When a positive-going input voltage attains its peak value, it makes diode D_1 forward-biased and capacitor C_1 to charge up to a voltage level of $V_{peak} - V_F$. Note that D_1 can never develop a voltage above $+V_F$ level with respect to the ground. When a negative half-cycle occurs at the input, D_1 becomes reverse-biased. At the peak of the negative input, the voltage across the negative terminal of the diode becomes the sum of the input voltage and the voltage across C_1. It can be mathematically given as:

$$e = -V_{peak} - V_{C1}$$

As we know that voltage across the capacitor is $V_{C1} = V_{peak} - V_F$

Thus, we get:

$$e = -V_{peak} - (V_{peak} - V_F)$$

$$= -2\,V_{peak} + V_F$$

The waveform and the amplitude of the diode voltage are identical to the input voltage, just the positive peak of the waveform gets clamped to $+V_F$ above the ground level. Thus, we may say that a clamping circuit is constituted by capacitor C_1 and diode D_1.

The alternating voltage passed through resistance R_1 and capacitor C_2 is identical to the input voltage with a single variation that the AC waveform is virtually below the ground level. This wave has a negative average value of half of its peak-to-peak value provided the voltage drop across the diode is neglected. The waveform is shown in Figure 4.22. The combination of R_1 and C_2 constitutes a low-pass filter in order to block the alternating component of the input voltage. The DC component equal to the peak input value is allowed to pass through this filter which then goes to the DC voltmeter for measurement. The meter reads the rms value of the quantities, thus, this average voltage must be divided to 0.707 of its peak value. Accuracy can be achieved only when the input is a pure sinusoidal waveform.

When the RF probe is used with an electronic multimeter, it extends its frequency range to about 500 MHz. The detector circuitry of the instruments that incorporate peak detection is on a separate probe on one end of coaxial cable. This cable is connected to the terminals of the voltmeter. This ensures that the detector is placed at the point of measurement rather being placed at the other end of the cable where sufficient amount of capacitance and inductance might highly effect the measurement. In addition, conversion of high-frequency voltage to DC voltage eradicates the effect of the cable impedance.

Hall-effect probe

This probe has a Hall-effect transducer which works on the principle that when a flat current-carrying conductor is placed in a magnetic field, a small amount of voltage is produced at its edges. The Hall-effect probe appears similar to the transformer-type AC current probe and is used to produce an output for both DC and AC currents in a conductor (see Figure 4.23).

This output is connected to the voltage terminals of the meter and is indicated on a milli-volt scale which is considered as a current scale. Thus, 1 mV on this scale represents 1 A.

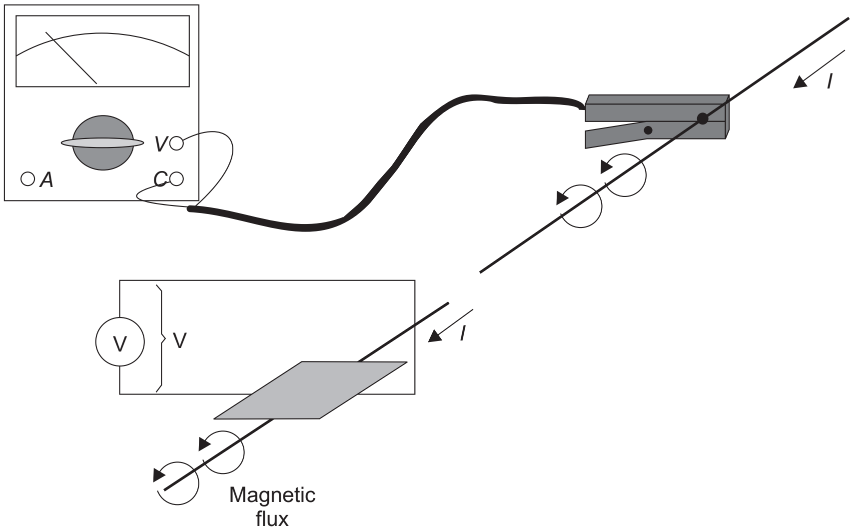


Fig. 4.23 Hall-effect Probe

Let us Summarize

1. The instruments like moving-iron type instruments, dynamometers, thermocouples, and electronic multimeter are used to measure both AC and DC quantities.
2. The most common ammeters and voltmeters used for power frequency applications are moving-iron type instruments. They are classified as two types based on the number of iron vanes used and the principle of operation, namely, repulsion type instruments, and attraction type instruments.
3. A thermocouple consists of two dissimilar metals or wires forming a junction. It is a temperature measuring device that produces voltage when its junction is heated by operating current.
4. The various types of thermo-elements include contact type, non-contact type, vacuum type and bridge type.
5. The dynamometer instrument, also referred to as electrodynamic or electrodynamometer instrument is a transfer instrument that can measure AC quantities with no modification after being calibrated with a DC source.
6. All the factors that cause errors in dynamometer instruments are low torque/weight ratio, strong external magnetic field, eddy currents, changes in temperature, and operating frequency.
7. The different types of transistor voltmeter circuits, include emitter-follower voltmeter, FET-input voltmeter, and difference amplifier voltmeter.
8. The instruments that incorporate rectification technique are called rectifier type instruments. A rectifier element, preferably silicon diode is employed in such instruments which converts conventional AC signal to DC which is then indicated by a DC meter (PMMC).
9. The AC voltmeters can be categorized on the basis of the rectification technique employed as full-wave rectifier voltmeter, half-wave rectifier voltmeter, and half-bridge full-wave rectifier voltmeter.

10. All the instruments can be configured into a single unit which is capable of measuring different quantities, that is, current, voltage, and resistance of an electronic circuit. The resultant single unit circuit is known as an electronic multimeter.
11. Multimeters use a number of probes and adapters for measuring some other quantities like temperature. The range of measurements can be extended using different probes, namely, high-current probe, high-voltage probe, radio-frequency probe, and hall-effect probe.

EXERCISES

Fill in the Blanks

1. Moving-iron type instruments can be used to measure _________ quantities.
2. The peak detector probe is used to extend the _________ range of an electronic multimeter.
3. An FET is connected at the input of an emitter-follower circuit to provide _________.
4. The scale of a moving iron instrument is _________.
5. An astatic system is used for a dynamometer instrument to protect it against _________.

Multiple Choice Questions

1. Moving-iron type instruments can be used as
   (a) Standard instrument for calibration of other instruments
   (b) Transfer type instruments
   (c) Indicator type instruments as on panels
   (d) All of these
2. To perform AC measurement which of the following should be connected in series with a PMMC meter?
   (a) Resistor (b) Diode
   (c) Inductor (d) Capacitor
3. Electrodynamic type instruments can be used as
   (a) Standard instruments only (b) Transfer instruments only
   (c) Indicator type instruments (d) Both (a) and (b)
4. The torque/weight ratio of a dynamometer instrument is
   (a) Small (b) High
   (c) Medium (d) None of these
5. Thermocouple instruments can be used for the frequency range
   (a) Up to 100 Hz (b) Up to 5000 Hz
   (c) Up to 1 MHz (d) 50 MHz and above

State True or False

1. The movement of the disc in moving-iron type instruments is always in the same direction regardless to the direction of the coil current.
2. The moving-iron voltmeters indicate the same value for AC and DC quantities.
3. Eddy current damping cannot be used in moving-iron type instruments.
4. The heater wire of thermocouples is made very thin in order to reduce skin effects at high frequencies.

5. Electrodynamometers are incorporated with an astatic movement so that the operating torque of the instrument gets increased.

Descriptive/Numerical Questions

1. Classify and explain the moving-iron instruments. List the incurred errors and methods to compensate them.
2. A moving-iron instrument has a coil current of 300 mA for which it deflects 50° on scale. Determine the self inductance increment from zero current level if the controlling spring constant is 5 μNm/degree.
3. What are the advantages and disadvantages of electrodynamometer instruments?
4. Derive the general torque equation for a moving-iron instruments and explain how they are different from moving-coil instruments.
5. The mutual inductance between the fixed and moving coils of an electrodynamometer instrument changes by 15 mH for the pointer deflection from 0° to 60°. Determine the coil current if the spring-controlling constant is 2.5 μNm/degree.
6. Describe the construction and working of thermocouples. Also, discuss their advantages and disadvantages.
7. Explain the working of a true rms reading voltmeter.
8. Explain the working of an electronic AC ammeter and a voltmeter with neat diagrams.
9. A PMMC instrument has FSD = 60 μA and coil resistance of 1500 Ω. This instrument is to be used as a half-wave rectifier to indicate 60 V rms at full scale. The silicon diodes are used in the circuit; D_1 must have a minimum (peak) forward current of 120 μA when the measured voltage is 15% of FSD.
10. Explain how emitter-follower structure reduces loading effect in voltmeters?
11. Describe the factors which are taken into consideration while selecting an analog electronic voltmeter.
12. Explain the working principle of an electronic multimeter. Write short notes on high-current probes.

5

Digital Instruments

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Differentiate between analog and digital instruments
- Discuss different types of digital voltmeters including ramp type, staircase type, dual slope integrating type, and successive approximation type
- Discuss various advantages and disadvantages of each type of digital voltmeter
- Describe the functioning and specifications of digital multimeters
- Explain the fundamentals of digital frequency meter system along with its applications

5.1 INTRODUCTION

Electrical instruments used as indicating instruments measure current, voltage, power, and resistance. These instruments are categorized as *analog instruments*, and *digital instruments*. The analog indicating instruments have been discussed in previous chapters which display the measured quantity in terms of deflection of the pointer on a scale. However, digital instruments are analog-to-converters which convert the analog values into digital or numeric values and present a digital display of a quantity being measured in the form of decimal number.

With the use of modern technology, the size and cost of digital instruments can be reduced. Also, their reliability and versatility have been significantly improved along with a reduction in power consumption. Thus, digital instruments have many advantages over analog instruments. This chapter focuses on digital instruments such as digital voltmeter and its types, digital multimeters, and digital frequency meters.

5.2 ADVANTAGES OF DIGITAL INSTRUMENTS

Digital instruments have many advantages over analog instruments as discussed here.

- The errors due to human factors like parallax and approximation errors are eliminated by digital instruments as the readings are directly indicated in decimal form.

- These instruments consume very little power.
- The readings in digital instruments can be carried to any number of significant figures simply by placing a decimal point.
- The output from the digital instruments being in digital form can be directly fed into memory devices like printers, floppy discs, digital computers, and tape recorders for storage and future computations.

5.3 COMPARISON OF ANALOG AND DIGITAL INSTRUMENTS

Depending upon requirements, one can choose between analog and digital instruments. Analog instruments are cheaper and simpler in comparison to digital instruments but when the cost and complexity of the circuit are not important parameter, a digital instrument can be chosen. In addition to these, there are many other factors which determine whether a digital or an analog instrument should be selected for a particular measurement. These factors are discussed as follows.

- **Resolution:** The limit of an instrument below which the changes in the readings cannot be differentiated is known as resolution. For digital instruments, it is one part in several thousands while for analog instruments it is one part in only several hundreds.
- **Portability:** Digital instruments require an external source of power and are not easily portable while analog instruments require no such source and thus can be moved easily from one place to other. However, with the advancements in technology, digital instruments can be made portable.
- **Power requirements:** When power requirements of an instrument are considered, digital instruments are preferable over analog instruments as they require negligible power for their operation. The range of input impedance of a digital instrument is of the order of 10 MΩ or even higher while the power requirement of analog instruments is quite high; it may load the circuit and cause it to indicate erroneously.
- **Observational errors:** In analog instruments, various observational errors exist as the readings are measured on the basis of deflection of pointers on a scale. These include parallax and approximation errors. However, in digital instruments the measured quantity is directly indicated in decimal numbers through digital display and thus, these observational errors do not exist in such instruments.
- **Environmental factors:** The environmental factors such as humidity and temperature may affect the performance of digital instruments as they consist of very complex parts. On the other hand, analog instruments are simpler and can operate in a wide range of environmental situations. However, digital instruments are made without moving parts so it is advantageous that errors owing to movements are removed.
- **Accuracy:** Digital instruments have very high accuracy in comparison to analog instruments which are usually specified within $\pm 0.1\%$ of full scale.
- **Range and polarity:** Most digital instruments come with automatic range indication and polarity selection feature resulting in less damage due to overload, reduced measurement error and operator training. Many digital instruments are used for measuring DC values up to the range of 100 V which can be extended up to 1000 V using range attenuators.

5.4 DIGITAL VOLTMETER

A **digital voltmeter**, abbreviated as **DVM**, is a measuring instrument that displays the value of an AC or DC voltage directly in decimal numbers instead of a pointer deflection on a continuous scale. The data output from a digital voltmeter may be fed directly into memory devices for further computations and storage. DVM is an accurate and versatile instrument used in many laboratory applications. The size, power requirements and cost of this instrument is reduced with the development of integrated circuit (IC) modules. A digital voltmeter essentially has all the advantages associated with digital instruments. Digital voltmeters can measure current, resistance, and voltage ratios by including additional circuitry to it.

Some characteristics of the digital voltmeter are as follows.

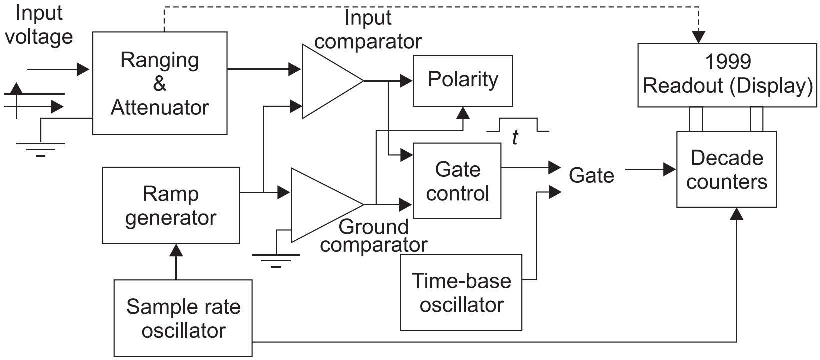
- Its input capacitance and resistance are 40 pF and 10 MΩ, respectively.
- Its resolution is 1 part in a million.
- Its accuracy is given by $\pm 0.005\%$ of the measured value.
- Automatic range selection and overload indication features are also incorporated.
- The range of the input value is defined from $+1$ V to $+1000$ V.
- Calibration is done internally from stabilized reference sources which are independent of the circuit.

Depending upon various parameters, such as accuracy, speed, number of digits, number of measurements, and type of digital output, the DVMs are classified as *ramp type DVM*, *staircase ramp type DVM*, *dual slope integrating type DVM*, and *successive approximation type DVM*. It is to be noted here that a digital voltmeter essentially consists of an analog-to-digital converter which performs the basic conversion of analog values to their digital equivalent.

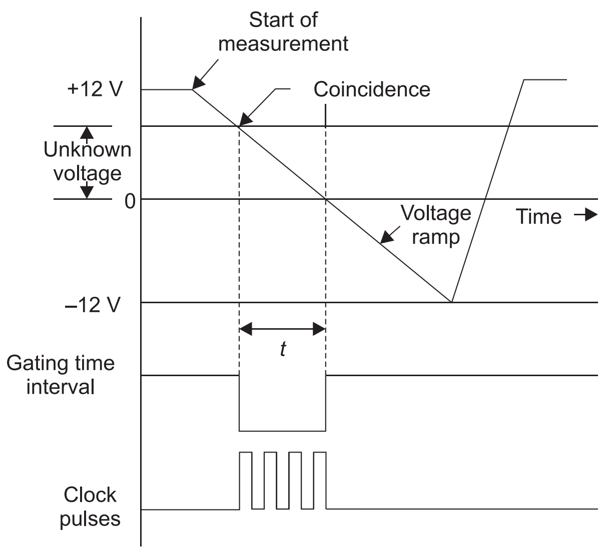
5.4.1 Ramp Type Digital Voltmeter

The operating principle of a ramp type digital voltmeter is to measure the time interval using an electronic time-interval counter. The time interval measured is the time taken by a ramp voltage to change from input voltage level to the zero voltage level or vice versa. This time interval is in proportion to the measured voltage and the count is displayed as a number of digits on the readout of the voltmeter. The block diagram of ramp type digital voltmeter along with its timing diagram is shown in Figure 5.1.

At the start of the measurement, the counter is reset to zero and a pulse from sample rate oscillator initiates the ramp generator which further generates a ramp voltage. A ramp voltage may be negative-going or positive-going. A negative-going ramp is considered in the timing diagram shown in Figure 5.1(b). This ramp voltage is compared continuously with the input voltage being measured (that is, an unknown voltage) with the help of input comparator. When both the voltages become equal, an input comparator acting as a coincidence circuit generates a pulse which derives the gate control circuit and opens the gate. The ramp voltage continues to decrease to 0 V (or ground potential). The ramp voltage retains its zero level until it is sensed by a second comparator, that is, ground comparator which generates an output pulse and closes the gate. The time elapsed between the opening and closing of the gate is represented as t. During this time interval, the pulses from the time-base oscillator passes



(a) Block diagram



(b) Timing diagram

Fig. 5.1 Ramp Type Digital Voltmeter

through the gate which drives the counter and the count is then displayed on the readout. The magnitude of this readout represents the magnitude of input voltage. Thus, the voltage is converted into time which is then measured and displayed.

The ranging and attenuation section of the digital voltmeter is used to scale the input voltage so that it remains in the dynamic range. The ranging circuit positions the decimal point in the output display of the voltmeter thus enabling automatic ranging. The polarity circuit gets input from the comparator pulses to detect the polarity of the signal. If no sign is indicated, then the signal is of positive polarity. For negative polarity, a negative sign is indicated prior to the numeric value. The output remains displayed for some time, that is, around three seconds and then refreshed by a sample rate oscillator and a new ramp voltage is generated.

Advantages and disadvantages

The advantages of a ramp type DVM are as follows.

- It is easy to design.
- It has low cost.
- Output can be transferred over long feeder cables.

The disadvantages of a ramp type DVM are as follows.

- Large errors may occur due to noise.
- It requires additional circuitry.
- The linearity of the ramp signal is must.
- The time measurement requires excellent characteristics.

5.4.2 Staircase Ramp Type Digital Voltmeter

The staircase ramp type DVM is a modification of the ramp type DVM. In this DVM, the voltage measurements are done on the basis of comparison between input voltage and an internally generated staircase ramp voltage. The block diagram of a staircase ramp type DVM consists of an input attenuator, generally of 10 MΩ and provides full scale five input ranges from 100 mV to 1000 V as shown in Figure 5.2. The DC amplifier having a gain of 100, provides 10 V voltage to the comparator, at any of the full-scale voltage settings of the input divider. When the staircase ramp voltage and the amplified input voltage become equal, the input comparator senses this coincidence and the input voltage gets displayed on the digital display.

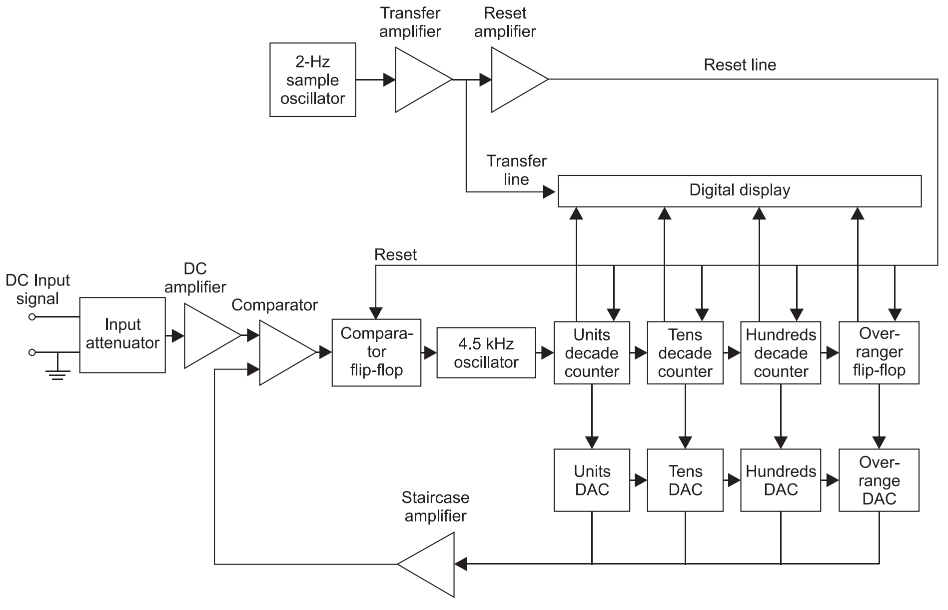


Fig. 5.2 Block Diagram of Staircase Ramp Type Digital Voltmeter

On initiation of a measurement cycle, a 4.5 kHz relaxation oscillator used as a clock provides pulses to the three decade counting units (DCUs), connected in cascade. After every tenth input pulse, a carry pulse is generated by the units counter and sent to the tens decade counter. The tens decade counter in turn counts these pulses. When the count reaches ten, this decade counter provides its own carry pulse to the hundreds decade counter. Now, a carry pulse from the hundreds decade counter is passed onto an over-range circuit which warns the operator that the applied input is more than the specified range by lighting up a front panel indicator. The operator should then change the setting on input attenuator to the next higher level.

A digital-to-analog converter (abbreviated as DAC) is connected in parallel to each of the DCUs. The shunt connected outputs of these DACs provide an output current which is proportional to the current count of DCUs. This output current is then applied to a staircase amplifier which converts it into a staircase voltage. This staircase voltage is then applied to the comparator which detects the coincidence of this voltage to the input voltage. When the coincidence is detected, the comparator provides a trigger pulse to stop the oscillator so that the content of the counter becomes proportional to the magnitude of the input voltage.

A simple relaxation oscillator is used to control the sample rate. It provides a sampling rate of two samples per second at which it resets and triggers the transfer amplifier. A pulse provided by the transfer amplifier performs the function of displaying the stored information of decade counting units to the display unit. This reading is stored in the display unit until a new reading is completed without any blinking or counting during the computation. When this pulse ends, it triggers the reset amplifier which further resets the value of decade counters to zero and starts the master oscillator to start a new measurement cycle.

Staircase ramp type DVM finds application in laboratories, repair shops, at inspection stations, and on production test-stands.

Advantages and disadvantages

The advantages of the staircase ramp type DVM are as follows.

- It is simpler in overall design as compared to ramp type DVM.
- It has low cost.
- It can be used as a general purpose instrument for many applications.

The disadvantages of the staircase ramp type DVM are as follows.

- The precision circuits are required to achieve accuracy.
- It requires additional circuitry, that is a combination of a capacitor and resistor in integrator to maintain the quality of ramp.
- It is critical to set the offset voltages and currents of the op-amp used in the integrator.

5.4.3 Dual Slope Integrating Type Digital Voltmeter

In the previous section, we have observed the dependency of the staircase voltmeter on the capacitor, resistor, and operational amplifier to maintain the accuracy of conversion. Here, we will describe a technique known as dual slope integrator which reduces this dependency. It is shown in Figure 5.3.

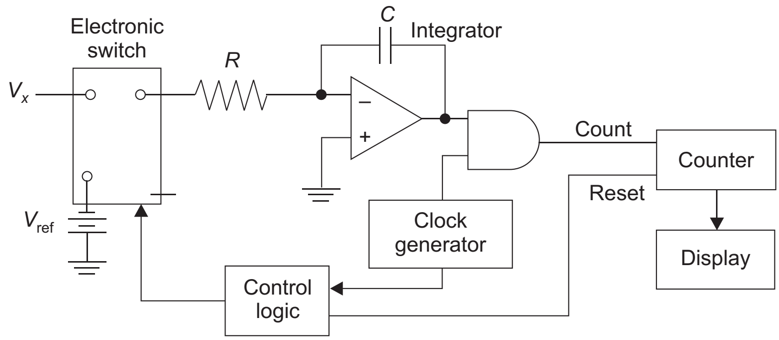


Fig. 5.3 Block Diagram of Dual Slope Integrating Type Digital Voltmeter

The block diagram of a dual slope integration type DVM consists of an integrator which is used to integrate an accurate reference voltage V_{ref} and input voltage V_x with the reverse slope (see figure). The two integrations can be carried out in any order and are performed for a fixed time interval. The time period required to return to the starting voltage is measured.

The input voltage V_x to be measured is applied to the integrator through an electronic switch. The counter is also initiated by control logic at the same time as integration starts. The integration stops when the counter reaches a predetermined value. The output of the integrator V_o is given by the relation:

$$V_o = -\frac{V_x t}{RC}$$

where V_x represents the input voltage, t is the time elapsed from when the integration began, and R and C are the time constant components of the integrator. Here, it is to be noted that the integrator output V_o is in opposite polarity to that of input voltage V_x. Also, the integrator capacitor had no charge in the beginning and integrator output started at zero volt. Let the integration be done for a time period t_1, then the output voltage V_{o1} is given as:

$$V_{o1} = -\frac{V_x t_1}{RC} \qquad ...(1)$$

From Equation (1), we can say that a negative integrator output is produced by a positive input voltage.

The counter is reset. Then, the integrator input is connected to a reference voltage V_{ref} through an electronic switch (usually JFET device) which is of opposite polarity to that of unknown input voltage, the integrator will start to discharge to zero value with a discharging rate of $\frac{V_{ref}}{RC}$. The output voltage V_{o2} can be represented as:

$$V_{o2} = V_{o1} + \frac{V_{ref} t}{RC} \qquad ...(2)$$

Since the integrator does not start at zero, the term V_{o1} is present in the output V_{o2}. Equating the output voltage of the integrator to zero and solving Equations (1) and (2), the input voltage V_x comes out to be:

$$V_x = \frac{t_2}{t_1} V_{\text{ref}} \qquad ...(3)$$

where t_2 is the time taken by the integrator to ramp down from output level V_{o1} to zero level. The respective waveforms are shown in Figure 5.4.

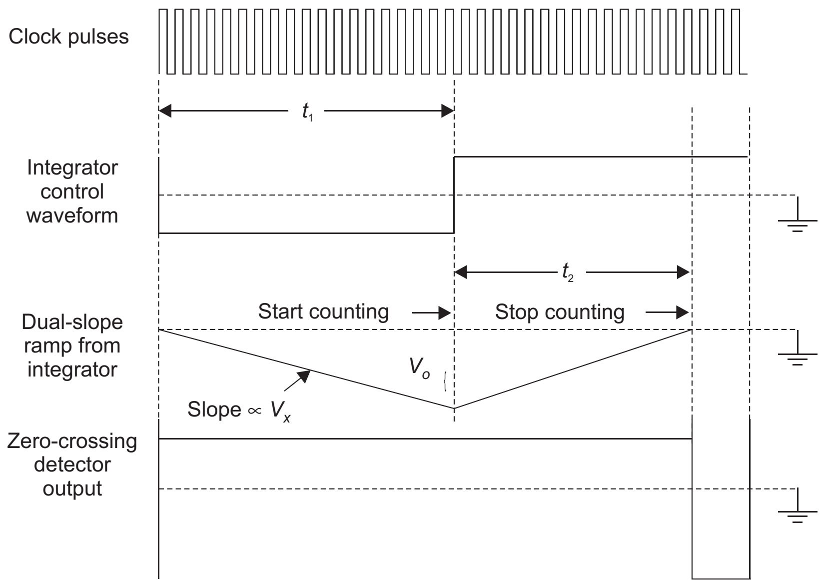


Fig. 5.4 Waveforms of Dual Slope Integrating Type DVM

The dual slope integrating type DVM responds to the average value of the unknown input voltage due to the integration process. Thus, this circuit does not require a sample and hold circuit. The integration here represents the end value obtained by a voltage that equals the average of the unknown input voltage even if the integrator output is not a linear ramp. From Equation (3), we can say that the input voltage V_x depends only upon the two times taken by the integrator to go up ramp and down ramp and not on its time constants R and C. Also, there is no need to incorporate a very accurate clock to measure this time period as the relation between two times is in proportion. However, the clock must be stable enough such that the frequency remains same for the two ramps.

Zero correction

One of the most significant features of dual slope integrating type DVM is automatic zero correction, that is, no output will be shown in the absence of input voltage which otherwise may not be the case with analog systems. In analog systems, offset currents, bias currents, leakage currents, and amplifier offset voltages may cause errors. The zero correction in this DVM can be implemented by connecting an auto zero capacitor to the output of the integrator through an electronic switch, and grounding the input to the converter. The feedback circuit provides zero voltage at the output of the integrator which further results in an equivalent offset voltage on the automatic zero capacitor ensuring no integration. This offset voltage also counteracts the effects of offset voltages of the input circuit during conversion. The changes in offset currents and voltages can be compensated by performing this automatic zero function before each conversion.

Advantages and disadvantages

The advantages of dual slope integrating type DVM are as follows.

- Zero correction can be made automatically using this DVM.
- It gives better accuracy and is adequate for laboratory instruments.
- It has better noise rejection capabilities.

The disadvantages of dual slope integrating type DVM are as follows.

- In dual slope ADCs, the leakage current of capacitor may cause error in integration which thereafter reflects in the meter readings.
- It is slow and quite expensive.

Example 1 A dual slope integrating type analog-to-digital converter has an integrating capacitor of 0.1 μF and a resistance of 100 kΩ. If the reference voltage is 2 V, and the output of the integrator is not to exceed 10 V, what is the maximum time the reference voltage can be integrated?

Solution: Given that: $R = 100$ kΩ and $C = 0.1$ μF

Thus, time constant $RC = 0.01\ s = 10$ ms

Reference voltage $V_{\text{ref}} = 2$ V

$$V_{o2} = 10 \text{ V}$$

The maximum time t for which the reference voltage V_{ref} can be integrated, is given by Equation (2) as:

$$V_{o2} = V_{o1} + \frac{V_{\text{ref}}t}{RC}$$

where V_{o1} is equated to zero, and substituting values in the above equation we get:

$$10 = \frac{2t}{10 \times 10^{-3}}$$

$$\Rightarrow \qquad t = \frac{10}{200} = 50 \text{ ms}$$

5.4.4 Successive-Approximation Conversion Type Digital Voltmeter

The successive-approximation conversion type DVM increases the speed of operation and decreases the number of comparisons. In this convertor, the analog input is compared with a digital-to-analog converter (DAC) reference voltage which is divided over and over into half. For this purpose, a comparator is used which compares input voltage with a sequence of binarily related voltages.

Let E_r be the full-scale input voltage. Firstly, the input is compared with $E_r/2$ and if the input voltage is greater, then the reference voltage is changed to $(E_r/2 + E_r/4)$. On the contrary, for lower input voltage, the reference voltage is changed to $(E_r/2 - E_r/4)$. Until the desired accuracy is obtained, this procedure of comparison is repeated.

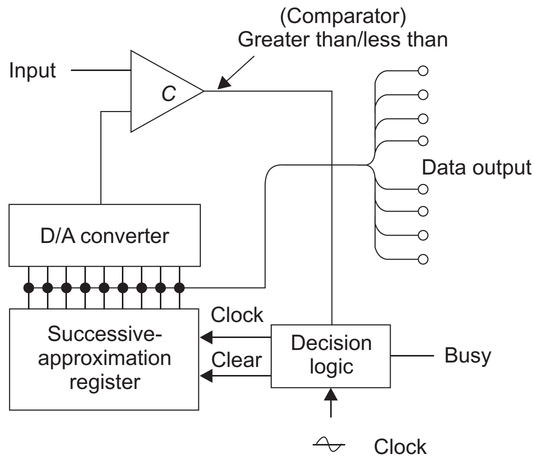


Fig. 5.5 Block Diagram of Successive Approximation Type Digital Voltmeter

Figure 5.5 shows a schematic block diagram for this conversion DVM technique. It consists of a comparator which compares the estimates provided by the digital-to-analog converter and compares it with the input signal. Based on the comparison, it makes the *equal to*, *less than* or *greater than* decision. Also, a special register is provided which controls the DAC. This register is a shift register known as **successive approximation register** (abbreviated as SAR). All outputs from SAR are at logic zero at the beginning of the conversion. The comparator output is high when the input signal is lower than the estimate. Then, the first SAR output reverses state and the second output attains a logic one. However, in the other case, the comparator output is low when the input signal is greater than the estimate value. Then, the first output retains its state of logic one and the second output changes to logic one state. This is repeated for each state till the conversion gets completed.

This type of analysis can be done for any number of bits and the number of bits is equal to the number of estimates required. At the edge of each clock pulse, an estimate is generated. The actual value of input will be known after N clocks for an N-bit conversion. The LSB (least significant bit) of SAR register represents the state of comparator. In some systems, the clock stores the last bit in SAR for which an additional clock pulse, $(N + 1)$ is required for an N-bit conversion.

Advantages and disadvantages

The advantages of successive approximation type DVM are as follows.

- It is highly effective.
- SAC technique is comparatively less expensive.

- The conversion time does not depend upon the amplitude of analog signal and is constant.
- The conversion time is short. For an n-bit ADC, it is equal to n-clock cycle period. Thus, it increases the speed of operation.

The disadvantages of successive approximation type DVM are as follows.

- When the resolution is higher, the successive approximation ADC will be slow.
- It is complex in design.
- SAR may increase the cost.

5.5 CHARACTERISTICS OF DIGITAL VOLTMETER

In the previous section, we had studied the different types of DVM. Now we will discuss the various characteristics of DVM, namely, *auto ranging*, *automatic polarity indication*, and *auto zeroing*. The circuits that employ these characteristics are explained with their working.

5.5.1 Auto Ranging

The range of a DVM can be varied from its fixed voltage range of 1 V to 1000 V by adding an input attenuator. The input attenuator consists of four resistors R_1, R_2, R_3, and R_4 and can be controlled in order to alter the range of the instrument thereby improving its overall performance (see Figure 5.6).

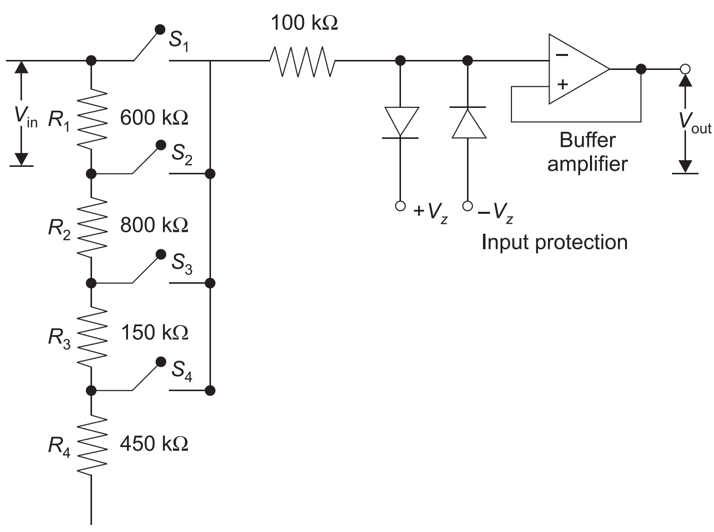


Fig. 5.6 Auto Ranging Circuit

The input attenuator provides a constant input resistance to the DVM which is equal to the sum of all resistances present in the attenuator. The input resistance provided by this attenuator is 2 MΩ. It can be increased by adding more resistances in series with the input terminals of the buffer. For example, by adding a resistor of 8 MΩ, the input resistance can be increased to 10 MΩ. The resistors used must be very precise with low temperature coefficient and high stability like metal-film type to ensure proper division of the input voltage. The input voltage fed to the buffer is controlled by setting the switches $S_1 - S_4$ which can be ON one at a time. When the switch S_1 is ON, a 1 V range will be provided, which is the fixed range of the DVM. Different ranges may be provided by making their corresponding switches ON manually or automatically, one at a time.

As the voltage range is changed, the decimal point on the readout also gets switched. For instance, if an input voltage of 1.999 V is applied and the 2 V range is selected, the decimal point after '1' is energized while if the 20 V range is selected, the decimal point shifts after '19' (see Figure 5.7).

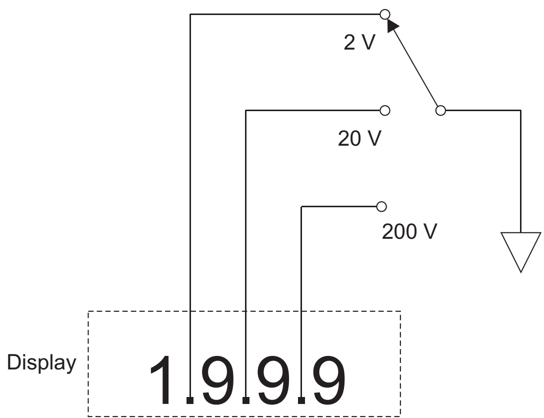


Fig. 5.7 Diagram Showing Different Decimal Points for Different Voltage Ranges

Automatic ranging is advantageous since it eradicates the need of an operator to switch between different ranges. However, by changing the range manually we can achieve a high degree of accuracy by choosing the range displaying the largest possible number. Also, the DVM display flashes ON and OFF continuously if the selected range is too low for the applied input voltage.

5.5.2 Automatic Polarity Indication

The automatic polarity indication circuit consists of two voltage comparators as shown in Figure 5.8. They handle opposite polarity signals and the reference voltage of both the comparators is set to zero volt. The input to these comparators is provided by the buffer amplifier. The Op-amps (µA741) or IC µA710 can be used as voltage comparators. The high sensitivity of comparators results in their excellent performance.

When the applied input signal is positive, comparator 1's output will be in logic 1 state, whereas when the negative input signal is applied, comparator 2's output will be in logic 1 state. The comparator outputs are applied to the lamp driver circuits to indicate the polarity automatically. Any other single-element display device or light emitting diodes (LEDs) can be used as an alternative to lamp circuits.

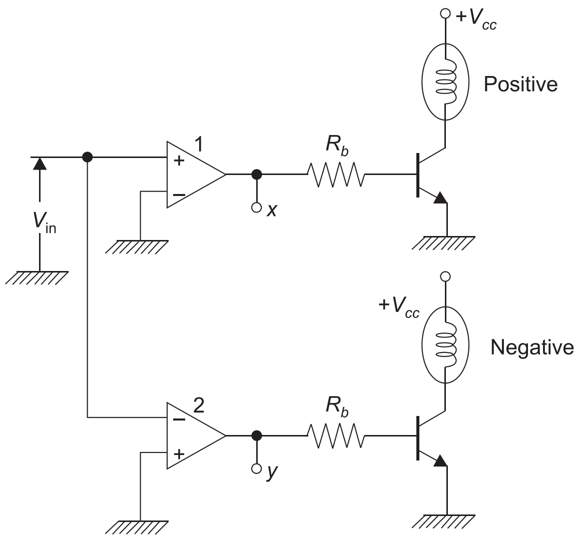


Fig. 5.8 Automatic Polarity Indication Circuit

5.5.3 Auto Zeroing

When a conversion is started using a DVM, a zero is indicated on its display. This is known as **auto zeroing**. The circuit used for this purpose is shown in Figure 5.9.

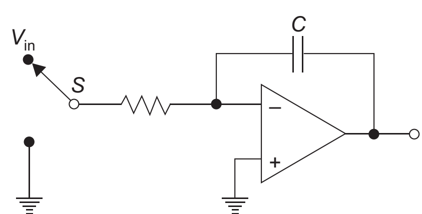


Fig. 5.9 Auto Zeroing Circuit

It consists of an integrator. To achieve auto zeroing, the switch S is connected from V_{in} to the ground potential and the capacitor C of the dual slope integrator discharges to the ground.

5.6 DIGITAL MULTIMETERS

Multimeter is a device used to measure multiple quantities in a circuit. Earlier, we had studied analog multimeters in Section 4.7. They give visual indication of changes and face less of isolation and electric noise problems. They require no power supply whereas the **digital multimeters** (abbreviated as **DMM**) have many advantages in comparison to analog multimeters. DMMs give highly precise results, have high input impedance, much greater resolution and are smaller in size. They indicate overload and show negative sign for reverse connection, and can switch to various appropriate ranges also. Since this multimeter does not have a pointer to indicate the measured values, there is no observation error. The main components used in a digital multimeter are shown in Figure 5.10.

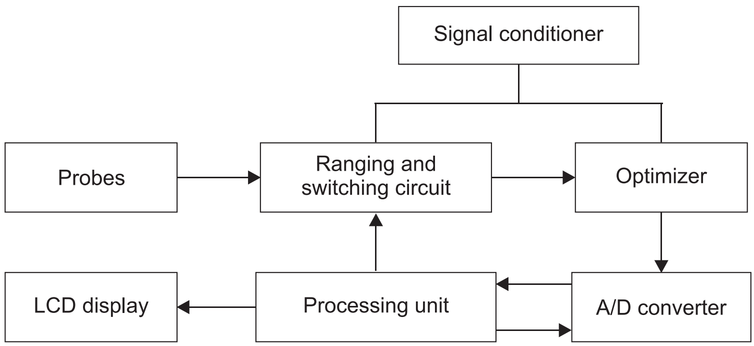


Fig. 5.10 Block Diagram of a Digital Multimeter

This block diagram represents the various steps involved in the working of a digital multimeter. The analog input enters the internal circuit through the probes in a waveform. After selecting a suitable range via ranging and switching circuit, the signal is then passed through a signal conditioner which performs the conditioning of the signal. The various linear processes involved in signal conditioning are attenuation, integration, addition, subtraction, differentiation, and amplification as well as some non-linear processes like modulation, demodulation, clipping, clamping, squaring, sampling so that a faithful reproduction of the input signal can be obtained. The signal is then measured by the measurement circuitry after which it is optimized by the optimizer and then forwarded to an analog-to-digital converter (that is, ADC) which converts the input voltage into its discrete equivalent. Usually, dual slope integration method is used in ADC for the conversion. The processing unit takes the input from the output of ADC, decodes the magnitude of the values and then passes it onto the LCD display.

Digital multimeter is a voltage sensing meter. Most of the measurements done are voltage based. The *current* and *resistance measurements* are performed where each quantity is converted to voltage and the DVM circuitry measures the voltage.

5.6.1 Current Measurement

Two different circuits can be utilized to measure current in a digital multimeter. One of them is based on voltage measurement. In this circuit, a digital voltmeter is used to measure the voltage drop across a constant resistance when the current is passed through it. In the other circuit, the measurement of current is done by frequency method. A current-to-frequency converter is shown in Figure 5.11.

The circuit consists of an integrator, pulse generator, comparator, and a current source. The capacitor C of the integrating circuit is charged by the constant current source I and the integrator output is compared with the reference voltage $-V_{ref}$ by applying it to a comparator as shown in the figure. When the applied voltage V_i is less than the reference voltage $-V_{ref}$,

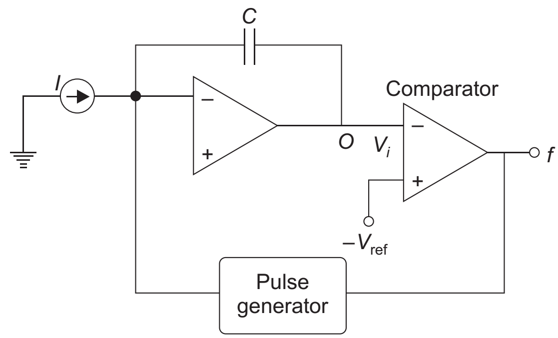


Fig. 5.11 Current-to-Frequency Converter

the pulse generator gets triggered and the capacitor gets discharged. The comparator results in a frequency output which is proportional to the current. The current can be measured in different ranges given as: 0–10 mA, 10 mA–100 mA, 100 mA–1 A, and 1 A–10 A.

5.6.2 Resistance Measurement

The circuit used for measuring resistance consists of a constant current source *I* which is generated by a voltage source as shown in Figure 5.12. For this, the internal resistance of the voltage source is increased to a very high value. The current is then passed through the resistance to be determined and the voltage drop across it is measured by DVM to calculate the value of resistance.

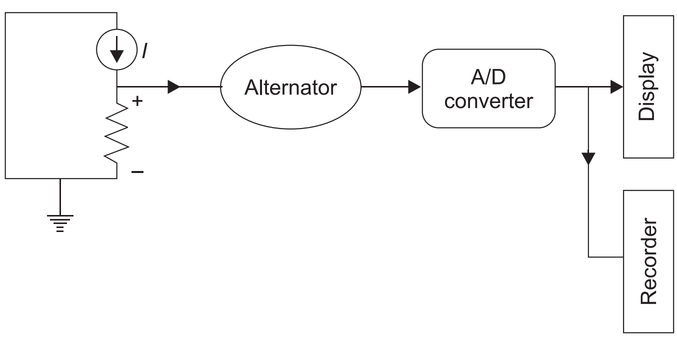


Fig. 5.12 Circuit to Measure Resistance

5.6.3 Specifications of Digital Multimeter

Specifications of a DMM define the functions and ranges of a quantity, such as voltage, current, and resistance. Figure 5.13 shows a basic digital multimeter consisting of a single function switch (that is, rotary selector) which can be rotated to select any one of the available functions. The switch can make any one of these selections: OFF, V (dc), V (ac), A (dc), A (ac), and Ω (resistance). Three terminals for connecting the cords are also provided which are: COM (common), V/Ω (volts/ohm), and A (current). The A terminal is used for measuring current whereas V/Ω terminal is used for measuring both voltage and resistance. The COM terminal is always connected to the ground for all measurements. Auto ranging and auto polarity indication features are also provided in the DMM.

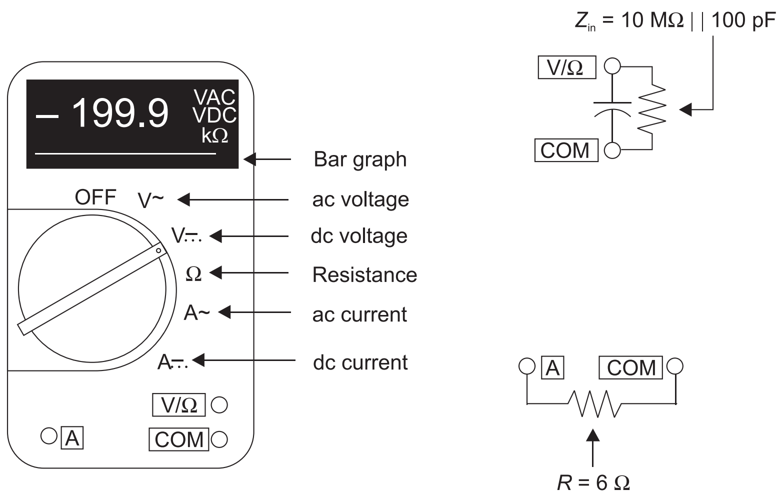


Fig. 5.13 Basic Digital Multimeter

The ranges of various measurements are given as follows.

V (dc): 0 to 1.999/19.99/199.9/1000 V

V (ac): 0 to 1.999/19.99/199.9/750 V

A (dc): 0 to 1.999/19.99/199.9/1999 mA

A (ac): 0 to 1.999/19.99/199.9/1999 mA

Ω: 0 to 199.9 Ω, 0 to 1.999/19.99/199.9 kΩ, 0 to 1.999/19.99 MΩ

Other than these, some more specifications are also provided for DMM as:

Burden voltage for current measurements: 6 mV/mA

Frequency range for AC measurements: 40 to 500 Hz

Input impedance: 10 MΩ || 100 pF

When the multimeter is used as an ammeter, the terminal voltage drop is indicated by the burden voltage. For example, if the burden voltage is 6 mV/mA, the ammeter resistance is 6 Ω, then the terminal voltage drop V_T for a current of 500 mA will be:

$$V_T = 500 \text{ mA} \times 6 \text{ mV/mA}$$

$$V_T = 3 \text{ V}$$

The effect of this voltage drop must be considered on the circuit under test.

Here, it should be noted that the instrument must be operated in the specified frequency range of 40 Hz to 500 Hz for AC measurements on low cost DMMs and the measurements should not be done outside this frequency range by this instrument.

The basic DC accuracy of the multimeter is given to be ±0.7% of the reading or even better. The type of measurement being made affects the accuracy of the multimeter. Accuracy of digital instruments is given as ±(0.5% rdg + 1 *d*) or it can be expressed as (0.5 + 1). Here, rdg means reading, thus 0.5% rdg means 0.5% of the reading while, 1 *d* refers to the least significant (extreme right numeral) digit of the display.

We have considered the most basic type of DMM and discussed its features. Other than this, many more DMMs are also available, namely bench-type DMM, and high performance hand-held DMM. The range of such DMMs can be extended using high-voltage, current, and frequency probes. In addition, these instruments have many more features that include measuring frequency, duty cycle, decibel, conductance, capacitance, and true rms value.

Example 2 An input voltage of 1.255 V is applied to a digital multimeter having an accuracy of ±(0.1% rdg + 1 *d*). Calculate the possible:
(a) minimum indicated voltage
(b) maximum indicated voltage.

Solution: Given that: Accuracy = ±(0.1% rdg + 1 *d*) and input voltage rdg = 1.255 V.

Thus, the error is obtained as:

Error = ±(0.1% of 1.255 V + 1 mV) = ±2.255 mV

(a) Minimum indicated voltage V_{min} = input voltage – error

$$V_{min} = 1.255 \text{ V} - 2.255 \text{ mV} = 1.253 \text{ V}$$

(b) Maximum indicated voltage V_{max} = input voltage + error

$$V_{max} = 1.255 \text{ V} + 2.255 \text{ mV} = 1.257 \text{ V}$$

Example 3 A digital multimeter having an accuracy of ±(0.1% rdg + 1 *d*) is indicating a value of 1.490 V. Determine the maximum measurement error.

Solution: Given that: Accuracy = ±(0.1% rdg + 1 *d*) and reading rdg = 1.490 V.

Thus, we get error as:

$$\text{Error} = \pm(0.1\% \text{ of } 1.490 \text{ V} + 1 \text{ mV}) = \pm 2.49 \text{ mV}$$

$$\text{Percentage error} = \pm 2.49 \text{ mV} \times \frac{100\%}{1.490} \approx \pm 0.17\%$$

5.7 DIGITAL FREQUENCY METER SYSTEM

An instrument which displays the frequency of a periodic electrical signal is known as **digital frequency meter**. The block diagram of a digital frequency meter system, shown in Figure 5.14 consists of an amplifier or an attenuator to which the input signal is applied. This performs the function of amplification or attenuation of the signal as per the requirement before passing it to the wave-shaping circuit. The wave-shaping circuit converts the input signal into a pulse or square waveform without changing its frequency. The presence of this circuit implies that input can be any repetitive-type waveform, such as sinusoidal, triangular or square wave. The output of the wave-shaping circuit is then applied to a two-input AND gate. The other input of the AND gate is the *Q* output from the flip-flop. When the flip-flop

Q terminal is high, pulses pass through the AND gate and are counted. Here, the flip-flop is controlled by a time-base circuit. When the output waveform of time-base circuit goes in the negative direction, the flip-flop changes its state at each instant. For example, if the time-base output frequency is 1 Hz, the flip-flop output Q will be an alternating high and low wave for a time period of 1 s each.

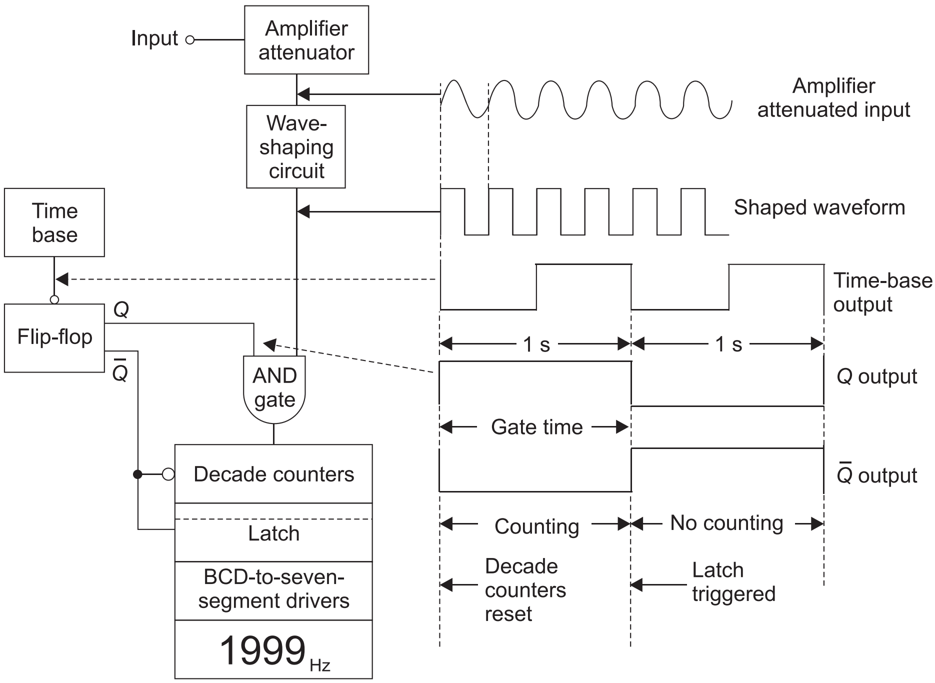


Fig. 5.14 Block Diagram of a Digital Frequency Meter System

Now, the output from the wave-shaping circuit will pass through AND gate when the flip-flop Q output is high for a period of 1 s. Thus, the total count represented by the counters indicates the frequency of the input in hertz. The negative-going edge of $\bar{Q}$ output of the flip-flop is used to reset the counters so that a count always begins from zero. The digital display is made readable by employing latch or display/enable circuits. The latch is triggered by the positive-going edge of $\bar{Q}$ output of the flip-flop at the end of counting time. The output of the display remains constant until the latch is again triggered and if required any changes can be made in the output at this instant.

When the flip-flops are connected in cascade, they can be used for frequency division purpose. Figure 5.15 shows three flip-flops connected in cascade and the respective waveforms for each of them are shown.

From the figure, we can say that everytime two input pulses are applied, there occurs a change in the state of flip-flop 1 and hence, the output wave from Q_1 has a time period twice to that of the triggering or clock input time period. If T_{in} is the input time period, then the time period of different waves are related as:

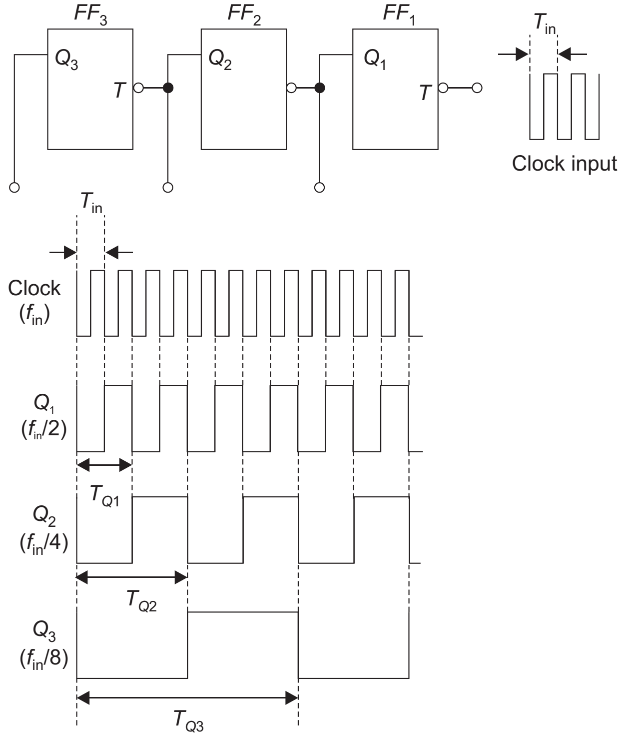


Fig. 5.15 Digital Frequency Division using Cascaded Flip-Flops

$$T_{Q1} = 2T_{\text{in}} \Rightarrow f_{Q1} = \frac{f_{\text{in}}}{2}$$

$$T_{Q2} = 4T_{\text{in}} \Rightarrow f_{Q2} = \frac{f_{\text{in}}}{4}$$

$$T_{Q3} = 8T_{\text{in}} \Rightarrow f_{Q3} = \frac{f_{\text{in}}}{8}$$

Therefore, the output frequency triggered by a clock input at any point in a digital register is given as $f_{(n)} = f_{\text{in}}/2n$, where n is the number of flip-flops from the clock input to the output or bit number at the output.

Similarly, by using decade counters, the frequencies can be divided by tens. This concept is utilized to create multi-range frequency meter. By using crystal oscillator with a number of decade counters, accurate time periods of 10 μs, 100 μs, 1 ms and many more can be generated and thus several ranges can be obtained. In Figure 5.14, the unit Hz is displayed along with the numerical value. Now, if the decimal point after first number is energized, the unit displayed will be kHz. Further, if a different range, say 100 ms is selected, the decimal point is shifted to the right displaying a value of 19.99 kHz. Thus, as the range is changed, the decimal point also gets shifted accordingly by a switching arrangement as shown in Figure 5.16. The maximum pulse count that can be obtained is 1999 pulses per ms, or 1.999 MHz.

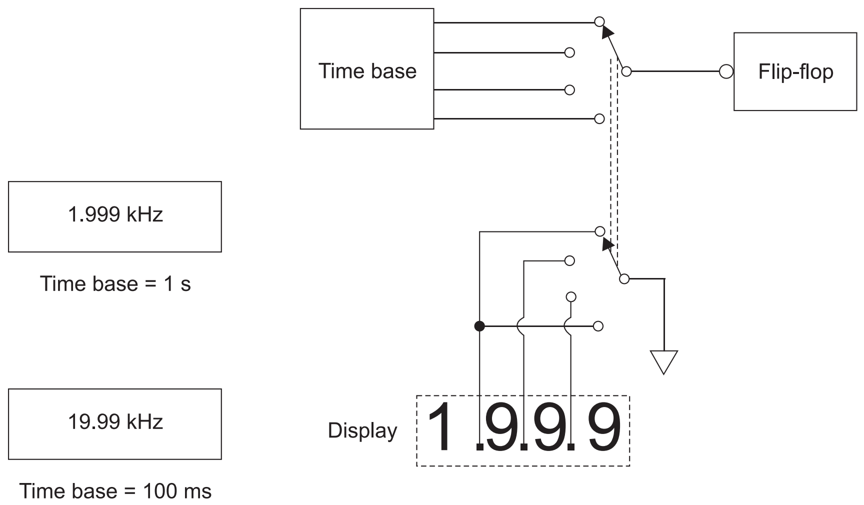


Fig. 5.16 Range Selection for Frequency Meter

A frequency meter incorporating decade counters is shown in Figure 5.17 in which a Schmitt trigger is used as an amplifier or an attenuator and 1 MHz frequency is converted to 1 Hz, 10 Hz, 100 Hz, gating pulses by decade counter in cascade arrangement.

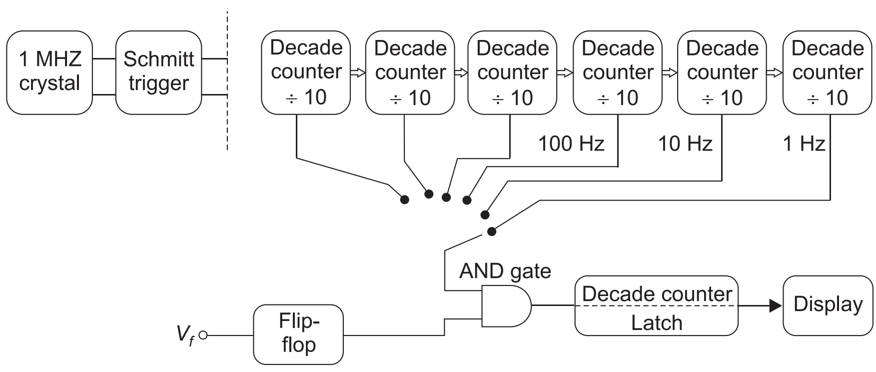


Fig. 5.17 Frequency Digital Meter With Cascaded Arrangement of Decade Counters

5.7.1 Accuracy Specifications

The accuracy of a frequency meter is specified as ±(1 LSD + time-base error). Here, a possible gating error of ±1 cycle occurs in the LSD of the count during the timing period. The AND gate can be switched on or off by the time-base circuit when a pulse from wave-shaping circuit is applied to it. This results in partial pulses at the output of AND gate which may or may not trigger the counter. Thus, an error is always present in the count.

The time-base error corresponds to the errors generated by the crystal-controlled oscillator due to supply voltage changes, variations in temperature, and aging of crystals. The total amount of error in the measurement depends upon the frequency measured as shown in Figure 5.18.

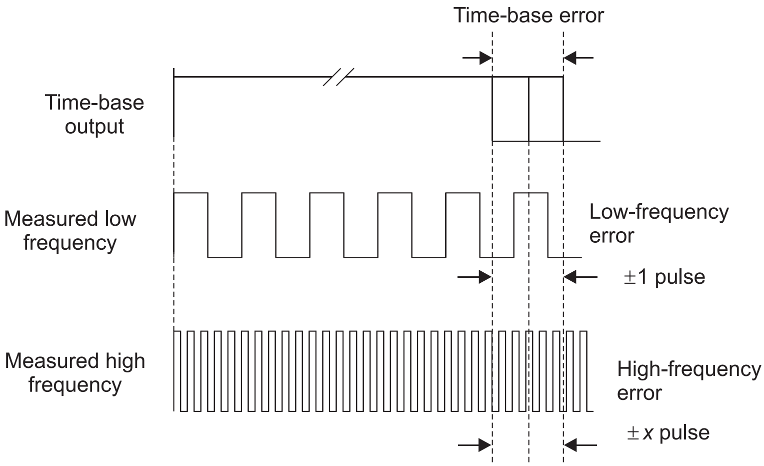


Fig. 5.18 Depiction of Time-base Error

However, adopting some precaution methods and using high quality time bases, this error can be reduced significantly to less than 1×10^{-6}. While measuring a frequency using frequency meter, the lowest possible range should be selected to ensure highest accuracy.

5.7.2 Reciprocal Counting

At low frequencies, ±1 count error is greater than the time-base error, whereas at high frequencies, ±1 count error is smaller than time-base error. At frequencies less than 100 Hz, the percentage error due to ±1 count is greater than 1%. Therefore, at low frequencies, the greatest measurement error occurs which can be reduced by a technique known as **reciprocal counting technique** as discussed here.

Figure 5.14 is modified to rearrange the frequency meter system as shown in Figure 5.19 in which the reshaped input wave whose frequency is to be measured is applied from wave-shaping circuit to the flip-flop to toggle it and oscillator frequency from time base is applied directly to the input of AND gate. The pulses from the time base, that is, 1 MHz oscillator frequency are passed to the counters during a time period of input wave. Now, when a 100 Hz input wave to be measured is applied, the AND gate will pass the pulses for a time period of 1/100 Hz = 10 ms and the time period of each cycle from 1 MHz oscillator is 1 μs. Hence, during time period T, the number of counted pulses n is given as:

$$n = 10 \text{ ms}/1 \text{ μs} = 10000$$

which is displayed as 10000 μs and its reciprocal determines the frequency. Here, we can say that the accuracy of measurement is 0.01% (that is, ±1 count in 10000) for a measurement of 100 Hz which is lower than the straight counting technique having error greater than 1%.

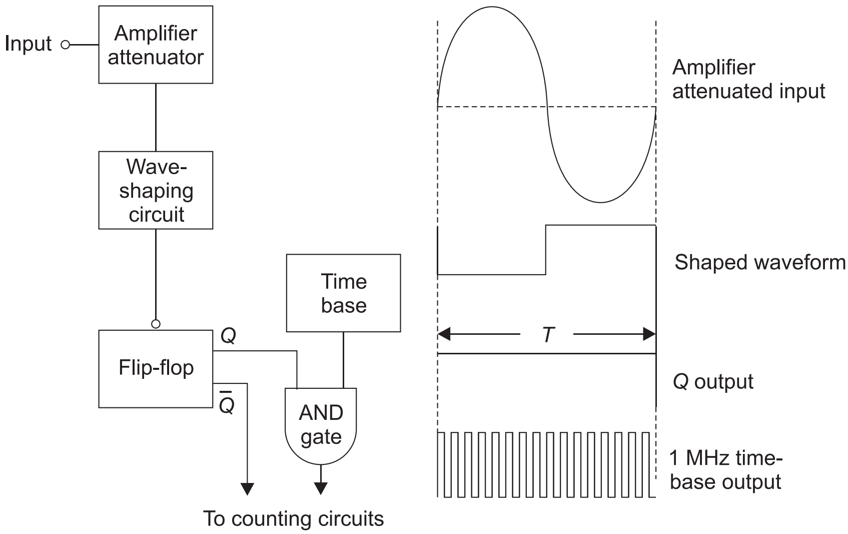


Fig. 5.19 Frequency Meter for Reciprocal Counting with System Waveforms

For frequencies lower than 100 Hz, this technique gives even better results. However, the straight counting method is well suited for high frequency measurements.

5.7.3 Applications of Digital Frequency Meter

The digital frequency meter system can be used to measure many more quantities in addition to frequency as discussed here.

Measuring pulse time period and width

The reciprocal counting technique can be used to measure the time period and width of a pulse. To measure the time period, no modification is required in the existing circuit (see Figure 5.19). The circuit can be used to measure the time period. The waveform is shown in Figure 5.20(a).

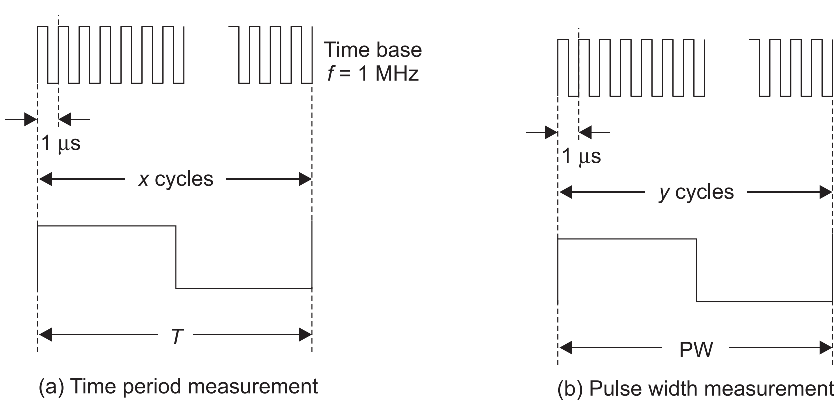


Fig. 5.20 Measurement using a Frequency Meter

However, to measure the width of the input pulse, the flip-flop is made to toggle on both positive- and negative-going pulses as shown in Figure 5.20(b). The time between the two events can be measured by some digital counters that have a start and stop input.

Measuring frequency ratio

The digital frequency meter is used for measuring the ratio of two frequencies. The AND gate is applied with the higher frequency input wave via an appropriate wave-shaping circuit in place of time base, whereas the lower frequency is applied as an input to the circuit in Figure 5.19. Now, the number of high-frequency cycles is counted during the time period of low frequency. Ten cycles are counted during the time period of a low frequency cycle if the ratio between both the waves is 10 and is displayed as the result. The waveforms are shown in Figure 5.21. These methods can be utilized to measure the phase difference between two waveforms.

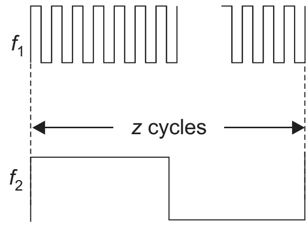


Fig. 5.21 Frequency Ratio Measurement

Example 4 Calculate the percentage error in each measurement, if a frequency meter having an accuracy of ± 1 LSD $\pm(1 \times 10^{-5})$ is used to measure the following frequencies:

(a) 30 Hz

(b) 30 MHz

(c) 300 MHz.

Solution: Given that: Accuracy = ± 1 LSD $\pm(1 \times 10^{-5})$

(a) For f = 30 Hz

The error can be obtained as:

Error = $\pm$[1 count + (30 Hz $\times$ 10^{-5} counts)] $\approx \pm 1$ count [Refer to Section 5.7.1]

Thus,

$$\text{Percentage error} = \pm\left(1 \times \frac{100\%}{30}\right) = \pm 3.3\%$$

(b) For f = 30 MHz

The error can be obtained as:

Error = $\pm$[1 count + (30 MHz $\times$ 10^{-5} counts)] $\approx \pm 301$ counts [Refer to Section 5.7.1]

Thus,

$$\text{Percentage error} = \pm\left(301 \times \frac{100\%}{30 \times 10^6}\right) = \pm 0.001\%$$

(c) For f = 300 MHz

The error can be obtained as:

Error = $\pm$[1 count + (300 MHz × 10^{-5} counts)] ≈ $\pm$3001 counts

[Refer to Section 5.7.1]

Thus,

$$\text{Percentage error} = \pm\left(3001 \times \frac{100\%}{300 \times 10^6}\right) = \pm 0.001\%$$

Let us Summarize

1. The electrical instruments used as indicating instruments measure current, voltage, power, and resistance. These instruments are categorized as analog instruments, and digital instruments.
2. The analog indicating instruments display the measured quantity in terms of deflection of pointer on the scale. However, digital instruments are analog-to-converters which convert the analog values into digital or numeric values and present a digital display of a quantity being measured in the form of decimal number.
3. Analog instruments are cheaper and simpler in comparison to digital instruments but when the cost and complexity of the circuit are not important parameters, a digital instrument can be chosen.
4. There are many factors which determine whether a digital or an analog instrument should be selected for a particular measurement. These factors are resolution, portability, power requirements, observational errors, environmental factors, accuracy, and range and polarity.
5. A digital voltmeter, abbreviated as DVM, is a measuring instrument that displays the value of AC or DC voltage being measured directly in decimal numbers instead of a pointer deflection on a continuous scale.
6. Depending upon various parameters, such as accuracy, speed, number of digits, number of measurements, and type of digital output, the DVMs are classified as ramp type DVM, dual slope integrating type DVM, and successive approximation type DVM.
7. Various characteristics of DVM include auto ranging, automatic polarity indication, and auto zeroing.
8. Multimeter is a device used to measure multiple quantities in a circuit. They can be analog or digital.
9. Analog multimeters give visual indication of changes and face less of isolation and electric noise problems. They require no power supply whereas the digital multimeters (abbreviated as DMM) give highly precise results, have high input impedance, much greater resolution and are smaller in size.
10. The digital multimeter is a voltage sensing meter. Most of the measurements done are voltage based. The current and resistance measurements are performed where each quantity is converted to voltage and the DVM circuitry measures the voltage.
11. The specifications of DMM define functions and ranges of a quantity, such as voltage, current, and resistance.

12. An instrument which displays the frequency of a periodic electrical signal is known as digital frequency meter.
13. The digital frequency meter system can be used to measure many more quantities, such as pulse time period and width and frequency ratio.

EXERCISES

Fill in the Blanks

1. The resolution of digital voltmeter is _________.
2. The voltage to time conversion principle is used in _________ DVM.
3. Accuracy of digital multimeter is specified as _________.
4. The value of burden voltage of a digital multimeter is _________.
5. _________ is selected by a time-base selector.

Multiple Choice Questions

1. DVM is the abbreviation of:
   (a) digital voltmeter (b) divider voltage meter
   (c) digital volume meter (d) digital vacuum meter
2. A digital voltmeter having a relatively fixed conversion-time independent of the input applied voltage uses an analog-to-digital converter which is:
   (a) digital ramp converter (b) dual slope integrating type converter
   (c) successive approximation converter (d) all the above
3. The slope used to perform the measurement in ramp type DVM is:
   (a) positive (b) negative
   (c) both of the above (d) none of the above
4. Schmitt trigger is used in digital frequency meter for:
   (a) scaling of input waveforms
   (b) converting input waveforms into rectangular wave
   (c) providing time base
   (d) all of the above
5. The slope which is used to perform the measurement in dual slope integrating type DVM is:
   (a) rising (b) falling
   (c) both of the above (d) none of the above

State True or False

1. Reciprocal counting is used to remove errors at high frequencies.
2. A frequency meter is used to measure phase.
3. Frequency dividers are used in digital frequency meters to divide the frequency by a factor of 20.
4. Frequency ratio can be measured using a frequency meter.
5. In accuracy specification of digital multimeter, *d* refers to MSD.

Descriptive/Numerical Questions

1. Explain the advantages of digital meters over analog meters.
2. Explain the principle of operation of a digital frequency meter with the help of a block diagram.
3. Explain the working of successive approximation type digital voltmeter with suitable diagram.
4. Define and compare the ramp type voltmeter and integrating type voltmeter with respect to working principle, advantages, and disadvantages.
5. Explain digital multimeter.
6. Define reciprocal counting. Draw the basic block diagram of the digital frequency meter rearranged for reciprocal counting. Explain its operation, and show why reciprocal counting is sometimes used in preference to the straight counting method.
7. With a neat circuit diagram, explain the working of ramp type DVM.
8. Explain various specifications of digital multimeter (DMM) which are important while selecting for any application.
9. With the help of a block diagram, explain the working of dual slope DVM.
10. Distinguish between an analog and digital multimeter.
11. Consider a digital frequency meter using a time base consisting of a 1 MHz clock generator frequency divided by six decade counters. If the applied input frequency is 6 kHz, determine the meter indication for the time-base output at the fifth decade counter.

6

Power and Energy Measurement

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Discuss the various types of wattmeter—electrodynamic, low power factor and induction type
- Explain the sources of errors in a wattmeter and their compensation techniques
- Determine power in a three phase circuit using one-, two- or three-wattmeter method, and three-phase wattmeter
- Extend the range of wattmeter by using different techniques including use of transformers
- Discuss the measurement of energy in AC and DC circuits
- Measure energy in a three phase system using three-element and two-element energy meter
- Employ phantom loading method to test an energy meter
- Describe maximum demand indicators and trivector meters

6.1 INTRODUCTION

In *Chapter 4*, we studied the electrodynamic instruments which have been used as ammeter, voltmeter, and wattmeter for many years. Wattmeters are used to measure electric power in a circuit. It is a deflecting type instrument which consists of current and voltage coils whose pointer produces a deflection proportional to the product of the current and voltage passing through it. Wattmeters are widely adopted for AC power measurements and work on the combined principle of ammeter and voltmeter. A single-phase wattmeter can measure power in a single-phase load and to measure power in a three-phase load, a combination of two single-phase wattmeters may be used. In addition to this, portable wattmeters are also available in multi-range. With some precautions to avoid overloading, these instruments can work as multi-range wattmeters.

An electromechanical energy meter also has voltage and current coils as it measures electric energy of a system which is equal to average power over a time period. An energy meter is not a deflecting type of instrument, rather it consists of a disc whose speed of rotation is proportional to the power of the load. The number of disc rotations are registered by a counting mechanism and then indicated on a register. Similar to wattmeter, energy can be measured by single-phase energy meters in a single-phase load and a combination of single-phase energy meters may measure energy in three-phase loads.

6.2 ELECTRODYNAMIC WATTMETER

In addition to measuring current and voltage, electrodynamic instruments also measure power in a circuit. They may be used for both DC and AC power measurement for any voltage and current waveform, not only sinusoidal waveform.

The construction and design of these instruments is somewhat similar to those discussed in Section 4.4. The only difference lies in the coil arrangement. The fixed and movable coils in dynamometer voltmeters and ammeters are connected in series so as to measure the effect of current squared. Figure 6.1 depicts the configuration of an electrodynamic wattmeter. The fixed coil, also known as **field coil**, is divided into two parts which are connected in series with the load to carry current in the circuit. Thus, these are known as **current coils** or simply **CC** of the wattmeter. The fixed coils are used as current coils because they can be made more massive. They are wound with heavy wires. The wires could be laminated or stranded to avoid eddy current losses in conductors while carrying heavy currents. The movable coil is used to carry a current proportional to the applied voltage by connecting it across the voltage, that is, in parallel with the load. It is located in the magnetic field of fixed coils. To limit the value of current in the movable coil, a high non-inductive resistance is connected in series with it. Therefore, the movable coil is known as **pressure or voltage or potential coil** or simply **PC** of the wattmeter, since it carries current proportional to the voltage. The movable coil is controlled by spring control and is mounted on a pivoted spindle. A small current is carried by this coil without any healing. The current is limited to 100 mA as series resistance is used with voltage circuit. The fixed and movable coils both are air cored. The damping used is air friction damping and the moving system carries a light aluminium vane that moves in a sector-shaped box. The reading errors due to parallax can be removed by using mirror type scales and knife edge pointers.

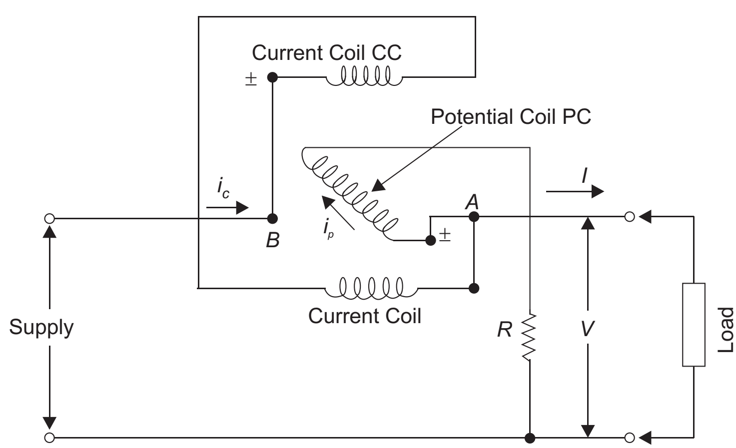


Fig. 6.1 Arrangement of Coils in Electrodynamic Wattmeter

Now, we know that the instantaneous torque of an electrodynamic instrument is given by the relation:

$$T_i = i_{FC} i_{MC} \frac{dM}{d\theta} \qquad \text{(as studied in } \textit{Section 4.4}\text{)}$$

where i_{FC} and i_{MC} are the instantaneous values of currents flowing through fixed coil and movable coil, respectively. Here,

$$i_{FC} = i_c \text{ and } i_{MC} = i_p \qquad \text{[Refer to } \textit{Figure 6.1}\text{]}$$

Thus, the expression for instantaneous torque can be written as:

$$T_i = i_c i_p \frac{dM}{d\theta} \qquad ...(1)$$

Now, let the rms value of current and voltage to be measured be denoted by I and V, respectively. We may write as:

$$V = \frac{v_p}{\sqrt{2} \sin \omega t} \qquad ...(2)$$

where v_p is the voltage across potential coil.

Since a very high resistance is connected in series with the pressure coil, it can be considered as purely resistive yielding inphase current and voltage. Thus, we get:

$$i_p = \frac{v_p}{R_p} = \frac{\sqrt{2} V \sin \omega t}{R_p} \qquad \text{[Refer to Eqn. (1)]} \quad ...(3)$$

Here, R_p is the resistance of the pressure coil circuit. Let the rms value of the current in pressure coil circuit be denoted by I_p, such that:

$$I_p = \frac{V}{R_p} \qquad ...(4)$$

Substituting Equation (4) into Equation (3), we get:

$$i_p = \sqrt{2}\, I_p \sin \omega t \qquad ...(5)$$

Now, if voltage in the current coil lags the current by a phase angle of ϕ, then the instantaneous current through the current coil is given as:

$$i_c = \sqrt{2} I \sin (\omega t - \phi) \qquad ...(6)$$

Substituting values from Equations (5) and (6) in Equation (1), we get:

$$T_i = \sqrt{2}\, I \sin (\omega t - \phi) \times \sqrt{2}\, I_p \sin \omega t \frac{dM}{d\theta} = 2 I_p I \sin \omega t \sin (\omega t - \phi) \frac{dM}{d\theta}$$

Or, it can be written as:

$$T_i = I_p I [\cos \phi - \cos (2\omega t - \phi)] \frac{dM}{d\theta} \qquad ...(7)$$

Here, Equation (7) contains a term $2\omega t$ which shows that a power component varies in accordance with twice the frequency of voltage and current. Now, the average deflecting torque is given as:

$$T_D = \frac{1}{T}\int_0^T T_i d\,(\omega t)$$

Substituting the value from Equation (7), we get:

$$T_D = \frac{1}{T}\int_0^T I_p I\,[\cos\phi - \cos(2\omega t - \phi)]\,\frac{dM}{d\theta_i}\,d\,(\omega t) \qquad \text{...(8)}$$

On solving, we obtain:

$$T_D = I_p I \cos\phi\,\frac{dM}{d\theta} = \left(\frac{VI}{R_p}\right)\cos\phi\,\frac{dM}{d\theta} \qquad \text{[Refer to Eqn. (4)]}$$

Also, we know that the controlling torque is given as:

$$T_C = K\theta$$

where K and θ are spring constant and final steady deflection, respectively. Due to the presence of double frequency component, the moving system of the instrument cannot match up the rapid variations in the torque. Thus, it acquires a position where average deflection torque and restoring torque of the springs are equal. Therefore, at balance position we have:

$$K\theta = I_p I \cos\phi\,\frac{dM}{d\theta}$$

Thus, the deflection θ is obtained as:

$$\theta = \frac{I_p I \cos\phi\,\dfrac{dM}{d\theta}}{K} = \frac{VI\cos\phi\,\dfrac{dM}{d\theta}}{R_p K}$$

Or, it can be written as:

$$\theta = K_1 VI \cos\phi\,\frac{dM}{d\theta}$$

where $K_1 = \dfrac{1}{R_p K}$

Let P be the power being measured, then we have:

$$\theta = K_1 P\,\frac{dM}{d\theta} \qquad \text{...(9)}$$

where $P = VI\cos\phi$ and $\cos\phi$ is the power factor. The significance of power factor in this expression implies that for power measurements, a wattmeter must be used rather than a voltmeter and an ammeter, since there is no power factor term in the expression of ammeter and voltmeter.

The circuit of Figure 6.1 can be modified to a simpler form by using a single coil to represent two field coils, connected in series as shown in Figure 6.2.

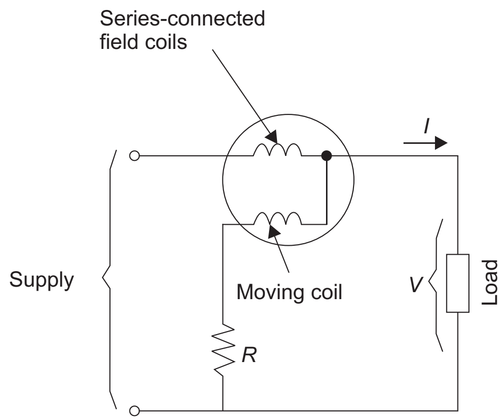


Fig. 6.2 Simplified Arrangement of Electrodynamic Wattmeter

Here, it is to be noted that the electrodynamic wattmeter measures the true power of a circuit. The wattmeter discussed above is used for single-phase power measurements.

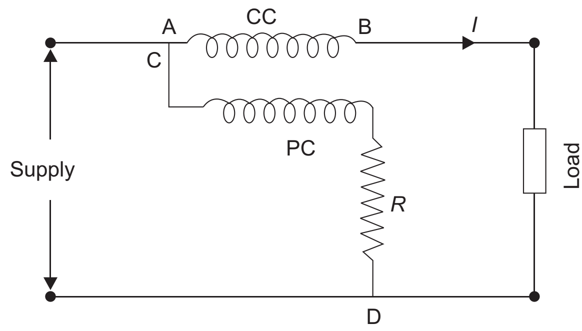
6.2.1 Wattmeter Connection and Scale

A wattmeter can be connected in a circuit in two ways as shown in Figure 6.3. The current coil is connected across the terminals *AB* while the pressure coil is connected across terminals *CD*. The supply voltage is applied across terminal *A*, while a load is connected across terminal *B*.

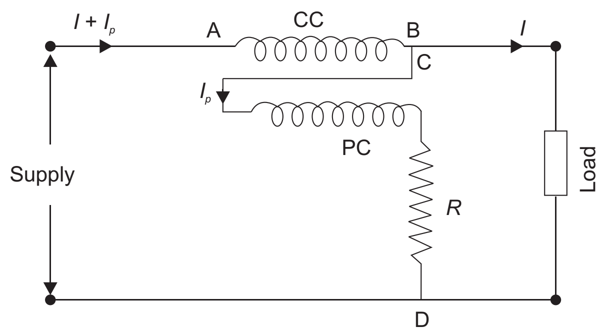
Figure 6.3(a) shows that the pressure coil is connected on the supply side and thus, the voltage across pressure coil is greater than that applied across load with an amount equal to the voltage drop across current coil. The instrument with this configuration measures the power loss across current coil, denoted by I^2R_{CC}. This connection is suitable for small value of load current as low value of load current results in low voltage drop across the load, resulting in a small error.

Figure 6.3(b) shows that the current coil is connected at the supply side and thus, the current coil carries a larger current as compared to the load current with an amount equal to the current drawn by the pressure coil current. In this configuration, the wattmeter measures the power loss in pressure coil as well as the power across the load. This method of connection is preferred when the value of load current is large which results in a smaller pressure coil current as compared to load current. Due to this, the power loss in pressure coil will be very small resulting in a small error.

Equation (9) shows that the deflection of the wattmeter is in direct proportion to the power. The range of the instrument for which $dM/d\theta$ is constant, the scale will be uniform.



(a) First configuration



(b) Second configuration

Fig. 6.3 Connections of a Wattmeter

The scale of wattmeter is so designed that M varies linearly with θ over a range of 40° to 50° on either side of zero mutual inductance position as shown in Figure 6.4. Now, to cover the entire scale range, the position of zero mutual inductance is kept at its centre and thereby, making it uniform for a range of 80° to 100°.

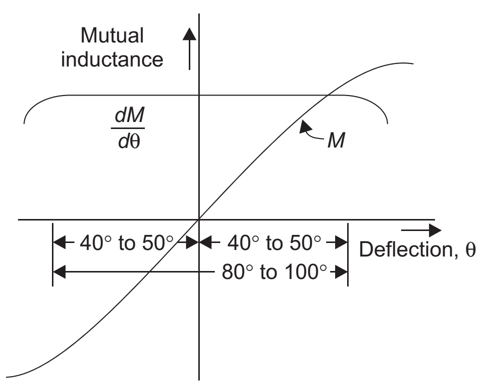


Fig. 6.4 Variation of M with θ

Example 1 Calculate the phase angle between the voltage and current of a wattmeter if AC power delivered to the load is 200 W for an applied voltage of 150 V. Assume the load current to be 2.1 A.

Solution: Given that: $P = 200$ W, $V = 150$ V, and $I = 2.1$ A

We know that the power for electrodynamometer wattmeter is given by the relation:

$$P = VI \cos \phi \qquad \text{[Refer to Eqn. (9)]}$$

Substituting the given values, we get:

$$200 = (150)\,(2.1) \cos \phi$$

$$\Rightarrow \qquad \cos \phi = \frac{200}{(150)\,(2.1)} = 0.635$$

$$\Rightarrow \qquad \phi = \cos^{-1}(0.635) = 50.58°$$

Example 2 A wattmeter is designed to give FSD with a current coil and voltage coil current of 1.5 A and 1 mA, respectively. Calculate the:

(a) Maximum power which can be measured by the meter.

(b) Voltage applied to the moving coil circuit when power dissipated in a load of 2 A is measured and the meter indicates FSD. The phase angle is 55°. The fixed coil and moving coil resistances of the meter are given as 0.7 Ω and 150 kΩ, respectively.

Solution: Given that: $i_c = 1.5$ A, $i_p = 1$ mA, $R_c = 0.7\ \Omega$, and $R_p = 150$ kΩ

(a) Voltage across load can be calculated as:

$$V = i_p \times R_p$$

Substituting the given values, we get:

$$V = 1 \text{ mA} \times 150 \text{ k}\Omega = 150 \text{ V}$$

Now, the maximum power can be obtained when $\cos \phi = 1$. In that case,

$$P = VI$$

where $I = i_c$

Substituting the given values, we get:

$$P = 150 \text{ V} \times 1.5 \text{ A} = 225 \text{ W}$$

(b) Given that: $\phi = 55°$ and $I = 2$ A

We know that the power for electrodynamometer wattmeter is given by the relation:

$$P = VI \cos \phi \qquad \text{[Refer to Eqn. (9)]}$$

Substituting the given values, we get:

$$225 = V \times 2 \text{ A} \times \cos 55°$$

$$\Rightarrow \qquad V = \frac{225}{2\cos 55°} = \frac{225}{2(0.574)}$$

$$\Rightarrow \qquad V = 195.99 \text{ V}$$

6.2.2 Errors and their Compensation in Wattmeter

Practically, some errors always exist in the instruments. These errors may be due to various factors such as *pressure coil inductance* or *capacitance* and *connections of different elements*, *mutual inductance effects*, *temperature variations*, *eddy currents*, *stray magnetic fields*, and *vibrations of moving system*. The factors causing errors in electrodynamometer wattmeters are discussed in this section along with the methods to compensate them.

Errors due to pressure coil inductance

We know that for ideal wattmeters, the applied voltage and the current through the pressure coil are in phase with each other. However, if an inductance is present in the pressure coil, the current and voltage no longer remain in the same phase. The pressure coil current will lag the voltage by some angle. Due to this, the angle difference between the current through pressure coil and the current through current coil no longer remains equal to ϕ, resulting in lagging or leading loads.

The wattmeter will read high for lagging power factor of the load as the pressure coil inductance in this case decreases the phase angle between pressure coil current and load current. For leading loads, the pressure coil inductance increases the phase angle between pressure coil current and load current resulting in the wattmeter to read low.

To compensate the errors caused due to pressure coil inductance, a capacitor is connected across a part of the series resistance r as shown in Figure 6.5. This compensation technique can be applied to frequencies at which $\omega^2C^2r^2 << 1$ and is very useful for the frequency range above 10 kHz.

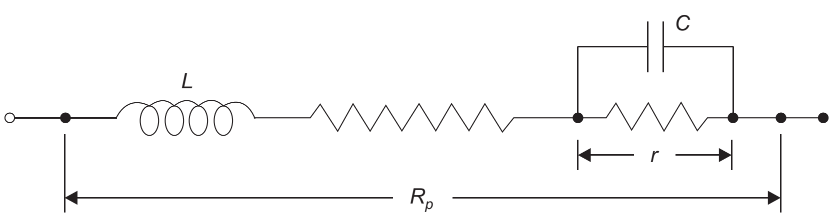


Fig. 6.5 Pressure Coil Inductance Compensation

Errors due to pressure coil capacitance

The pressure coil may have some capacitance due to inter-turn capacitance of series resistance. This capacitance results in some error in the wattmeter measurements. The effects generated by the pressure coil capacitance are opposite to those generated by pressure coil inductance. Hence, for leading power factor of the load, the wattmeter will read high.

The reactance of the pressure coil circuit determines the phase angle between applied voltage and pressure coil current. This phase angle varies in direct proportion with the frequency as the inductive reactance is higher than the capacitive reactance.

Note: *If the inductive and capacitive reactances are equal, they both neutralize the effect of each other resulting in zero error.*

Connection errors

Two configurations of a wattmeter are shown in Figure 6.3. As stated earlier, the wattmeter reading includes the power loss in the coil which is connected on the load side. If the power loss is neglected then the two types of connections can be used according to different applications. Connection in Figure 6.3(a) is appropriate for small current connections while that shown in Figure 6.3(b) is suitable for applications where large current connections are made. However, if power loss is taken into account, for accuracy constraints, the connection shown in Figure 6.3(b) proves to be more appropriate in which the loss occurring in pressure coil is given as:

$$\frac{V^2}{R_p}$$

For a constant voltage V, this loss is also constant and can be subtracted from the wattmeter reading to obtain the true power. This is the reason why connection (b) is preferable when power loss is to be taken into account.

Errors due to mutual inductance effects

The mutual inductance between pressure and current coils of the wattmeter results in some errors. These errors are directly proportional to frequency. At power frequencies, these errors are very low but become prominent with an increase in frequency. These errors change the phase angles of wattmeters. When the pressure coil is connected on load side, the phase angle increases, while it decreases when the current coil is connected on load side. The change in phase angle can be expressed mathematically as: $\tan^{-1}(\omega M/R_p)$ where M is mutual inductance between the coils, ω is the angular frequency and R_p is the resistance of pressure coil. However, these errors can be compensated by using specialized instruments designed with coils so arranged such that the mutual inductance between them is always zero. Here, zero mutual inductance (that is, $M = 0$) implies that $dM/d\theta$ is at its maximum, thus, for any value of current, such instruments provide the maximum torque. An example of such an instrument is Drysdale Torsion-head wattmeter.

Errors due to temperature variations

Variations in room temperature have a great impact on the stiffness of springs and the resistance of the pressure coil. This causes the meter to read erroneously. However, since these effects are opposite in nature so by using a resistance alloy (with negligible resistance temperature coefficient) as well as copper to construct the pressure coil circuit, these effects can be neutralized.

Errors due to eddy currents

Eddy currents are induced by the alternating magnetic field of the current coil. These are induced in the thickness of conductors and solid metal parts. Due to these currents, a field is produced which causes the error by altering the magnitude and phase of current coil field.

Due to eddy current errors, the wattmeter reads high for leading power factors and low for lagging power factors. These errors cannot be determined easily and can be very serious, thus, it is necessary to eliminate them.

Eddy current errors can be avoided by removing solid metal parts from the current coil. When a large current is to be carried, stranded conductors can be used to reduce the eddy currents produced in the coil.

Errors due to stray magnetic fields

Owing to weak operating fields of electrodynamic wattmeter, they are very much affected by strong stray magnetic fields resulting in great errors. To avoid such errors, shielding must be provided. These shields are made up of laminated iron in case of portable laboratory instruments, whereas for switch board instruments steel cases are used as shields. However, the permanent magnetism of the shield causes some DC and eddy current errors. Thus, precision type wattmeters do not incorporate shielding but use an astatic system for the purpose.

Errors caused due to vibration of moving system

We can say from Equation (8) that there is double frequency component in the torque equation, that is, the torque on the moving system varies cyclically with twice that of the frequency of the voltage. If some part of the moving system, say pointer or spring, possesses a natural frequency in approximation with the torque pulsation frequency, it vibrates with a significant magnitude. Due to these vibrations, the readings become difficult to observe as well as the mean position of the pointer gets displaced from respective power reading and points to the average power reading.

To eliminate such errors, the instruments are so designed that the natural frequency of the moving system is very much away from twice the frequency of the system on which the instrument is supposed to work.

6.3 LOW POWER FACTOR WATTMETER

The ordinary electrodynamic wattmeters cannot be used for measuring power with precision in the circuits which have low power factor (power factor less than 0.5) due to reasons given as follows.

- Due to low power factor, the deflecting torque on the moving system is very small, regardless of the fully excited pressure and current coils.
- At low power factors, the inductance of the pressure coil is large which induces errors.

Low power factor meters are none but electrodynamic wattmeters with some additional features incorporated. These features are:

- **Pressure coil current:** The current through pressure coil is increased so as to increase the operating torque. This is achieved by designing the pressure coil circuit possessing a low value of resistance. The value of pressure coil current for a low power factor wattmeter may be 10 times that of a high power factor wattmeter.

- **Compensation for pressure coil current:** In the previous section, we have studied two types of wattmeter connections in Figure 6.3. Among these, when connection (b) is used, the meter reading comprises of power loss in pressure coil circuit. However, for low power factor circuits, connection (a) may result in very large errors. Due to low power factor, the value of the current in the circuit is high while the power to be measured is small making the power loss considerably large. Thus, for low power factor wattmeters, the compensation of pressure coil current is mandatory. A compensating coil is used for this purpose (see Figure 6.6).

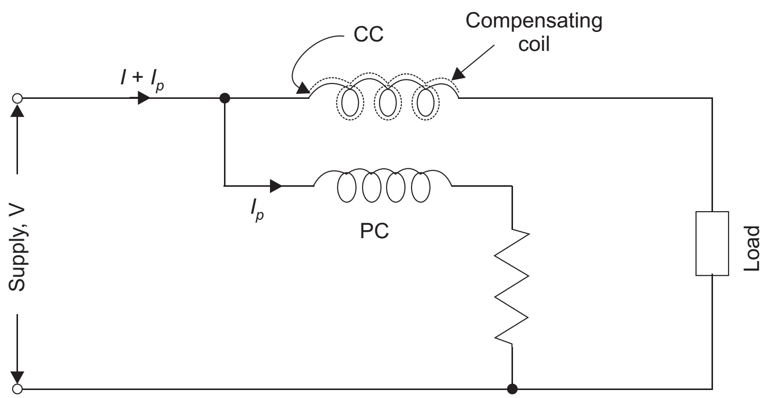


Fig. 6.6 Pressure Coil Current Compensation

Let the current flowing through the current coil in the connection in Figure 6.3(a) be $I + I_p$. This current will produce a corresponding field. This field must be eliminated by means of a compensating coil, connected in series with the pressure coil circuit (see Figure 6.6). This coil is made identical to the current coil and carry a current I_p which produces a field opposing the current coil field. Thus, these two fields cancel each other out and only the field corresponding to current I is left. In this way, the error caused due to pressure coil current is removed using a compensating coil.

- **Compensation for pressure coil inductance:** Large errors are caused by pressure coil inductance in low power factor circuits. To compensate for these errors, a capacitance is connected in parallel to a part of series resistance of pressure coil circuit as shown in Figure 6.7.

- **Smaller control torque:** The control torque of a low power factor wattmeter must be very small such that for very low power (equal to 0.1), the instrument can give full scale deflection.

A diagram of low power factor wattmeter, incorporating all the changes mentioned above, is shown in Figure 6.7.

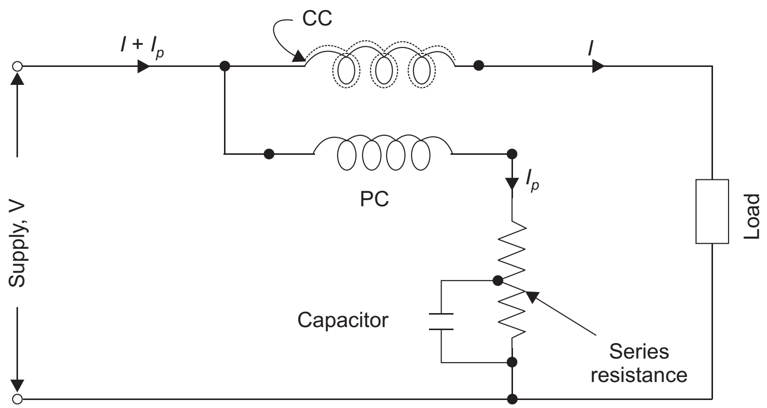


Fig. 6.7 A Low Power Factor Wattmeter

6.4 INDUCTION TYPE WATTMETER

We have already studied electrodynamic wattmeters. Those instruments can measure both AC and DC powers. However, induction type wattmeter can measure only AC power. The main advantage of induction type instruments is that they can give a full scale deflection of up to 300°. Also, they have a greater working torque in comparison to electrodynamic wattmeters.

Induction type wattmeters consist of two laminated electromagnets, one of which is energized by the current proportional to the load voltage, known as **shunt magnet**. The other known as **series magnet**, is energized by the load current or a part of load current. A thin disc made up of aluminium is mounted such that it cuts the fluxes from both the electromagnets. These fluxes induce the eddy currents in the aluminium disc which then interacts with the fluxes to generate deflecting torque. The two types of induction type wattmeter are shown in Figure 6.8.

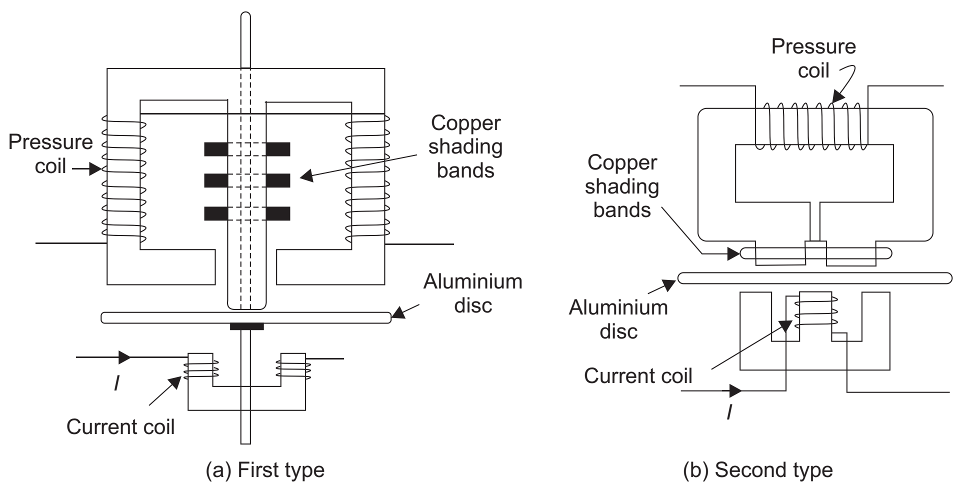


Fig. 6.8 Induction Type Wattmeter

From the figure, we can say that the upper electromagnet, that is, shunt magnet, consists of a pressure coil and one or more copper rings on its one limb. These rings help in keeping a 90° phase difference between the net flux in the magnet and the applied voltage, where the applied voltage is leading. The series magnet consisting of the current coil is placed below the aluminium disc. Here, the control torque is provided by the use of springs.

Figure 6.8 shows two different types of wattmeters whose basic construction is the same as mentioned above. However, the wattmeter shown in Figure 6.8(a) consists of two pressure coils which are connected in series. The coils are so wound that the flux due to both of them passes through the middle limb. The series magnet also consists of two current coils, connected in series. The coils are wound in such a way that they both magnetize the core in the same direction. The copper bands are movable and their positions can be adjusted to obtain the correct phase displacement between the series and shunt magnet fluxes.

The wattmeter shown in Figure 6.8(b) consists of one pressure coil and one current coil. In this case, the copper shading band is surrounding the two projecting pole pieces of the shunt magnet. Here also, its position is adjustable to correct the phase of the flux of shunt magnet.

Let us now derive the torque equation for an induction wattmeter. Let the applied voltage be V and the current through the coil be denoted by I. The flux generated in the shunt magnet is represented by Φ_{sh} and the flux generated in the series magnet is represented by Φ_s. Let the voltage induced due to Φ_{sh} and Φ_s be denoted by e_{sh} and e_s, respectively. Here, the phase difference between the applied voltage V and flux through the shunt magnet Φ_{sh} is 90° and the phase difference between the flux Φ_{sh} and the induced voltage e_{sh} is also 90°. The current i_{sh} is in phase with e_{sh} and the current i_s with e_s, assuming the aluminium disc to be resistive. The corresponding phasor diagram is shown in Figure 6.9.

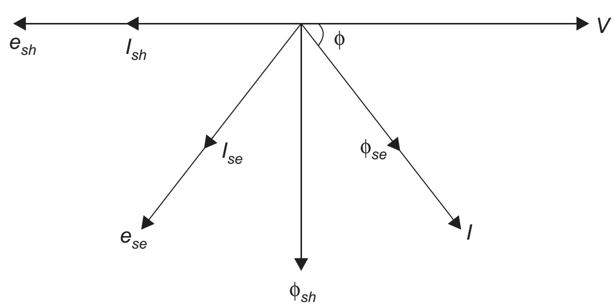


Fig. 6.9 Phasor Diagram of Induction Wattmeter

Here, it is to be noted that to keep a phase difference of 90° between the flux of the shunt magnet Φ_{sh} and the applied voltage V, the pressure coil circuit is made as inductive as possible, whereas in dynamometer wattmeter the current is kept in phase with voltage by connecting a high resistance in series with pressure load. Let the voltage and current be expressed as:

$$V = V_m \sin \omega t$$

$$I = I_m \sin (\omega t - \phi)$$

Now, the flux induced in the series coil Φ_s is directly proportional to the coil current I. Thus, we have:

$$\Phi_s \,\alpha\, I_m \sin(\omega t - \phi)$$

The relation between the flux through the shunt magnet Φ_{sh} and the applied voltage V is given by the relation:

$$V \,\alpha - \frac{d\Phi_{sh}}{dt}$$

Or,
$$\Phi_{sh} = -\int V_m \sin \omega t dt$$

$\Rightarrow$
$$\Phi_{sh} = -\frac{V_m}{\omega} \cos \omega t$$

The relation between the flux Φ_{sh} and the voltage induced e_{sh} in shunt magnet is given as:

$$e_{sh} \,\alpha\, \frac{d\Phi_{sh}}{dt} \,\alpha\, \frac{V_m}{\omega} \omega \sin \omega t$$

$\Rightarrow$
$$e_{sh} \,\alpha\, V_m \sin \omega t$$

In the similar way, the induced voltage due to series coil e_s is given as:

$$e_s \,\alpha\, \frac{d\Phi_s}{dt} \,\alpha\, I_m \,\omega \cos(\omega t - \phi)$$

And,

$$i_s \,\alpha\, I_m \,\omega \cos(\omega t - \phi)$$

Now, the net torque is proportional to $\Phi_s i_{sh} - \Phi_{sh} i_s$.

Substituting the values from the above equations, we get the deflecting torque T_D as:

$$T_D \,\alpha\, I_m \sin(\omega t - \phi) V_m \sin \omega t + \frac{V_m}{\omega} \cos \omega t I_m \,\omega \cos(\omega t - \phi)$$

$$\alpha\, V_m I_m [\sin(\omega t - \phi) \sin \omega t + \cos \omega t \cos(\omega t - \phi)]$$

$$\alpha\, V_m I_m \cos(\omega t - \phi - \omega t)$$

$\Rightarrow$
$$T_D \,\alpha\, V_m I_m \cos \phi$$

Thus, we get:

$$T_D \,\alpha\, VI \cos \phi$$

The above result states that the deflecting torque T_D is proportional to the power in the circuit. Here, the deflecting torque is proportional to the power in the circuit. Also, springs are used to generate torque leading to a uniform scale as in dynamometer wattmeters.

In addition, they can measure only AC voltages. Some other disadvantages of induction type wattmeters are:

- Less accuracy.
- High power consumption.
- Greater weight of moving parts.

6.5 THREE-PHASE POWER MEASUREMENT

Before going in the detail of various methods employed to measure power in a three-phase system, let us first discuss **Blondel's theorem** which is used to specify the number of wattmeters required to measure power in a system of electrical conductors. This theorem states that the power of a network, consisting of n conductors is given by the algebraic sum of the readings of n wattmeters. These wattmeters are arranged in such a way that the current coil of each wattmeter is connected in each line to measure the current level in each individual conductor and the corresponding pressure coil is connected between that line and a common point. This measurement can be done using $n - 1$ wattmeters if the common point is situated on one of the lines. The different methods used to measure power are *three-wattmeter*, *two-wattmeter*, and *single wattmeter methods*.

6.5.1 Three-wattmeter Method

Consider the circuit shown in Figure 6.10 consisting of three wattmeters to measure power in a three-phase system. The system used is a four-wire system.

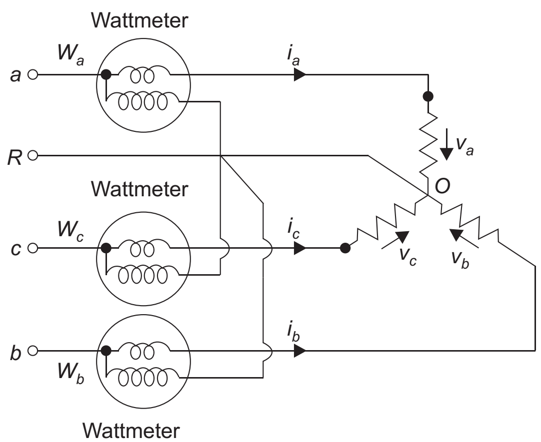


Fig. 6.10 Three-wattmeter Method

For the circuit shown, the instantaneous power in the load is given by the relation:

$$v_a i_a + v_b i_b + v_c i_c$$

Reading of the wattmeter, W_a is given as:

$$P_a = v_a i_a$$

Similarly, readings of the wattmeters W_b and W_c are respectively given to be:

$$P_b = v_b i_b \quad \text{and} \quad P_c = v_c i_c$$

Thus, the sum of the readings of the wattmeters is given as:

$$P = P_a + P_b + P_c$$

$$\Rightarrow \qquad P = v_a i_a + v_b i_b + v_c i_c$$

This total reading is equal to the instantaneous power of the load. Thus, these wattmeters measure the power consumed by the load.

6.5.2 Two-wattmeter Method

According to Blondel's theorem, if the common point of the pressure coils is made to coincide with one of the lines, then only $n - 1$ wattmeters are required for a n-phase system. This is employed in two-wattmeter method. Here, for a 3-phase system, $n - 1 = 2$ wattmeters are used. This is shown in Figure 6.11.

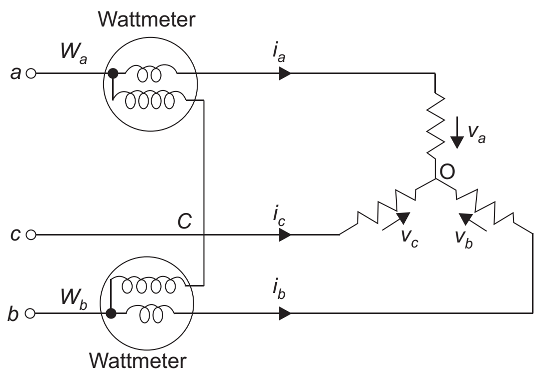


Fig. 6.11 Two-Wattmeter Method for Star Connection

The instantaneous power in the load is given by the relation:

$$v_a i_a + v_b i_b + v_c i_c$$

Reading of the wattmeter, W_a for a star connection is given as:

$$P_a = i_a(v_a - v_c)$$

Similarly, the reading of the wattmeter W_b is given as:

$$P_b = i_b(v_b - v_c)$$

The sum of the readings of the wattmeters is given as:

$$P = P_a + P_b$$

$$= i_a(v_a - v_c) + i_b(v_b - v_c)$$

$$\Rightarrow \qquad P = v_a i_a + v_b i_b - v_c(i_a + i_b) \qquad \text{...(10)}$$

Applying Kirchoff's current law at point O, we get:

$$i_a + i_b + i_c = 0$$

$$\Rightarrow \quad i_a + i_b = -i_c$$

Substituting this value in Equation (10), we get:

$$P = v_a i_a + v_b i_b + v_c i_c$$

This total reading is equal to the instantaneous power of the load. Thus, these two wattmeters measure the power consumed by the load. This discussion is for the three-phase circuit in which the conductors are connected in star connection. Let us now discuss the power measurement for a delta connected circuit as shown in Figure 6.12.

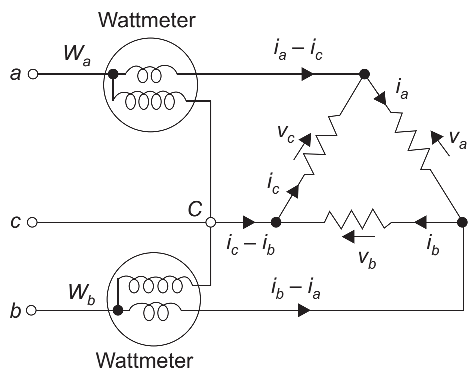


Fig. 6.12 Two-wattmeter Method for Delta Connection

The instantaneous power in the load is same as for star connection; it is given as:

$$v_a i_a + v_b i_b + v_c i_c$$

Reading of the wattmeter W_a is given as:

$$P_a = -v_c(i_a - i_c)$$

Reading of the wattmeter W_b is given to be:

$$P_b = v_b(i_b - i_a)$$

The sum of the readings of the wattmeters is given as:

$$P = P_a + P_b$$

$$= -v_c(i_a - i_c) + v_b(i_b - i_a)$$

$$\Rightarrow \quad P = v_b i_b + v_c i_c - i_a(v_b + v_c) \quad ...(11)$$

Applying Kirchoff's voltage law, we get:

$$v_a + v_b + v_c = 0$$

$$\Rightarrow \quad v_b + v_c = -v_a$$

Substituting this value in Equation (11), we get:

$$P = v_a i_a + v_b i_b + v_c i_c$$

This total reading is equal to the instantaneous power of the load. Thus, these two wattmeters measure the power consumed by the load.

Note: *The above results are valid for both balanced and unbalanced load.*

Now consider a **balanced star-connected load** with its phasor diagram as shown in Figure 6.13. Let I_a, I_b, and I_c be the rms values of the phase currents and V_a, V_b, and V_c be the rms values of phase voltages.

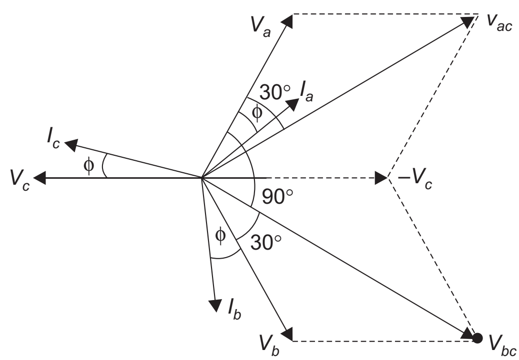


Fig. 6.13 Phase Diagram of a Balanced Load (Star Connection)

For balanced load, we may write as:

$$V_a = V_b = V_c = V$$

$$I_a = I_b = I_c = I$$

And, the line voltages can be expressed as:

$$V_{ac} = V_{bc} = V_{ab} = \sqrt{3}\ \mathrm{V}$$

As already known, the power factor is equal to cos ϕ, where ϕ is the angle by which phase voltages lead the corresponding phase currents. For wattmeter W_a, the current is I_a and the voltage across pressure coil is V_{ac}. From phasor diagram, it can be seen that the current I_a leads the voltage V_{ac} by an angle of $(30° - \phi)$. Thus, the reading of wattmeter W_a is given as:

$$P_a = V_{ac} I_a \cos(30° - \phi) = \sqrt{3}\ VI \cos(30° - \phi)$$

Now, voltage V_{bc} through the wattmeter W_b, leads the current I_b by an angle of $(30° + \phi)$. Thus, the reading can be written as:

$$P_b = V_{bc} I_b \cos(30° + \phi) = \sqrt{3}\ VI \cos(30° + \phi)$$

The sum of the readings of the wattmeters is given as:

$$P = P_a + P_b$$

$\Rightarrow\ P = \sqrt{3}\ VI \cos(30° - \phi) + \sqrt{3}\ V I \cos(30° + \phi) = \sqrt{3}\ V I\, [\cos(30° - \phi) + \cos(30° + \phi)]$

Or, $$P = 3\ V I \cos \qquad ...(12)$$

Equation (12) represents the total power consumed by the load. Now, let us derive the required relation for power factor. The difference between two readings of the wattmeter can be written as:

$P_a - P_b$

$$= \sqrt{3}\ V I \cos(30° - \phi) - \sqrt{3}\ V I \cos(30° + \phi) = \sqrt{3}\ V I [\cos(30° - \phi) - \cos(30° + \phi)]$$

Or, $$= \sqrt{3} V\, I \sin \phi \qquad ...(13)$$

Dividing Equation (13) by (12), we get:

$$\frac{P_a - P_b}{P_a + P_b} = \frac{\sqrt{3}\text{ V} \sin\phi}{3\text{ V}\cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

Or, it can be written as:

$$\phi = \tan^{-1}\left(\sqrt{3}\,\frac{P_a - P_b}{P_a + P_b}\right)$$

Now, since the power factor is equal to cos ϕ we may write:

$$\cos\phi = \cos\left[\tan^{-1}\left(\sqrt{3}\,\frac{P_a - P_b}{P_a + P_b}\right)\right] \qquad ...(14)$$

6.5.3 Single Wattmeter Method

The circuit for the single wattmeter method is shown in Figure 6.14. Here, the current coil is connected to one of the lines on which one end of the pressure coil is also connected. The other end of the pressure coil is made to alternate between the remaining two lines via a switch.

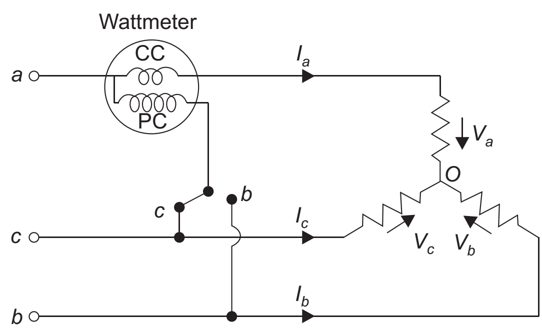


Fig. 6.14 Single Wattmeter Method

Here, it is to be noted that this method is applicable only for balanced load. The phasor diagram for this method is shown in Figure 6.15.

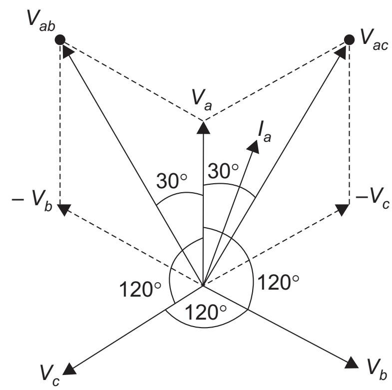


Fig. 6.15 Phasor Diagram for the Single Wattmeter Method

Now, when the switch is at point *c*, the reading of the wattmeter is given as:

$$P_a = V_{ac}\, I_a \cos(30° - \phi) \qquad \text{(Refer to Section 6.5.2)}$$

It can be written as:

$$P_a = \sqrt{3}\ V\, I \cos(30° - \phi)$$

Similarly, the reading of the wattmeter, when the switch is at point *b* is given as:

$$P_b = V_{ab} I_a \cos(30° + \phi) = \sqrt{3}\ V\, I \cos(30° + \phi)$$

The sum of these two readings is:

$$P = P_a + P_b$$

$$\Rightarrow \qquad P = \sqrt{3}\ V\, I \cos(30° - \phi) + \sqrt{3}\ V\, I \cos(30° + \phi)$$

Or, $$P = 3\ V\, I \cos \phi$$

This is the total power consumed by the load. The power factor for the single wattmeter method is also the same as for the two-wattmeter method. Thus, we have:

$$\cos\phi = \cos\left[\tan^{-1}\left(\sqrt{3}\,\frac{P_a - P_b}{P_a + P_b}\right)\right] \qquad \text{[Refer to Eqn. (14)]}$$

Example 3 The readings of two wattmeters connected to measure the power of a balanced three-phase circuit are 2400 W and 310 W. Determine the power factor of the circuit.

Solution: Given that: P_a = 2400 W and P_b = 310 W

For a balanced circuit employing two-wattmeter method, the power factor is given as:

$$\cos\phi = \cos\left[\tan^{-1}\left(\sqrt{3}\,\frac{P_a - P_b}{P_a + P_b}\right)\right] \qquad \text{[Refer to Eqn. (14)]}$$

Substituting the given values, we get:

$$\cos\phi = \cos\left[\tan^{-1}\left(\sqrt{3}\,\frac{2400-310}{2400+310}\right)\right]$$

$$\Rightarrow \qquad \cos\phi = \cos[\tan^{-1} 1.335]$$

$$\Rightarrow \qquad \cos\phi = 0.599$$

Example 4 The power factor of a three-phase 440 V load is given to be 0.51 and the power measured by the two-wattmeter method is 43 kW. Calculate the reading of each wattmeter.

Solution: Given that: power factor $\cos\phi = 0.51$ and $P_a + P_b = 43$ kW

Now, we have:

$$\phi = \cos^{-1}(0.51) = 59.34°$$

$$\Rightarrow \qquad \tan\phi = \tan 59.34° = 1.68$$

We have the relation:

$$\phi = \tan^{-1}\left(\sqrt{3}\,\frac{P_a - P_b}{P_a + P_b}\right) \qquad \text{[Refer to Eqn. (14)]}$$

$$\Rightarrow \qquad \tan\phi = \sqrt{3}\,\frac{P_a - P_b}{P_a + P_b}$$

Substituting the given values, we get:

$$1.68 = \sqrt{3}\,\frac{P_a - P_b}{43}$$

$$\Rightarrow \qquad P_a - P_b = 41.70 \text{ kW} \qquad ...(4a)$$

Also given, $$P_a + P_b = 43 \text{ kW} \qquad ...(4b)$$

From Equations (4a) and (4b), we get:

$$P_a = 42.35 \text{ kW} \quad \text{and} \quad P_b = 0.645 \text{ kW}$$

6.6 THREE-PHASE WATTMETER

In the previous section, we have studied the different methods to measure power in a three-phase circuit. Now, we will study an electrodynamic three-phase wattmeter which is a single instrument constructed for the same purpose. It consists of two wattmeters having two moving coils mounted together on a single pivoting spindle and encased in a single chest.

The combination of a current coil and pressure coil is termed as an **element**. The three-phase wattmeter consists of two such combinations, hence, it is also called a **three-phase two-element wattmeter** as shown in Figure 6.16.

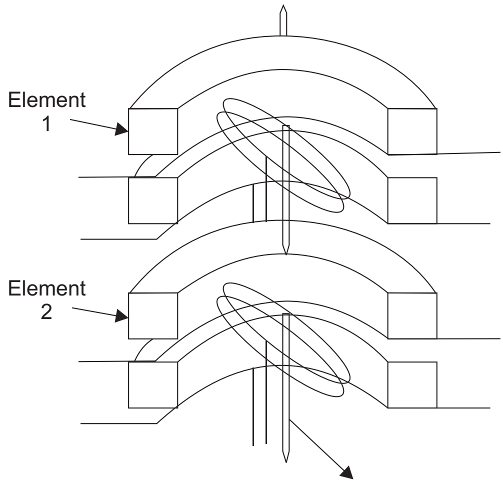


Fig. 6.16 A Three-phase Wattmeter

Here, both the elements are connected in the same way as two single-phase wattmeters are connected in a two-wattmeter method. Torque is exerted by both the moving coils on the spindle to produce deflection. The torque on each element is in proportion to the power being measured by it. Thus, we may write as:

For element 1, deflecting torque α P_a
For element 2, deflecting torque α P_b

The total deflecting torque is equal to the algebraic sum of the torques exerted by the two coils and is proportional to the total power measured by them. Thus, we have:

Total deflecting torque α $(P_a + P_b)$ α P

Due to mutual interference between the two elements, there may occur some errors in the reading of the wattmeter. To eliminate these effects, the elements are shielded from each other by using a laminated iron shield. Another method is known as **Wetson's method** and is shown in Figure 6.17. The value of the resistance R can be adjusted as per the requirement.

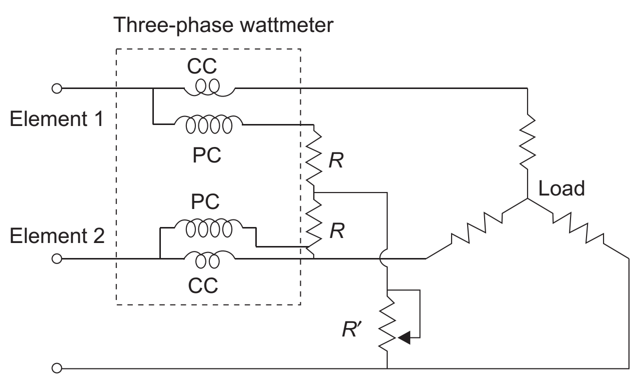


Fig. 6.17 Wetson's Method to Remove Mutual Interference

6.7 EXTENSION OF WATTMETER RANGE

Different valued multipliers can be used to change the range of applied voltages in the same way, as we studied in the case of voltmeter (refer to Section 4.4). Switches are used to select between different values. To change the current ranges, the field coils are switched from series to parallel connection. The scale, controls, and circuit of an electrodynamic multi-range wattmeter are shown in Figure 6.18.

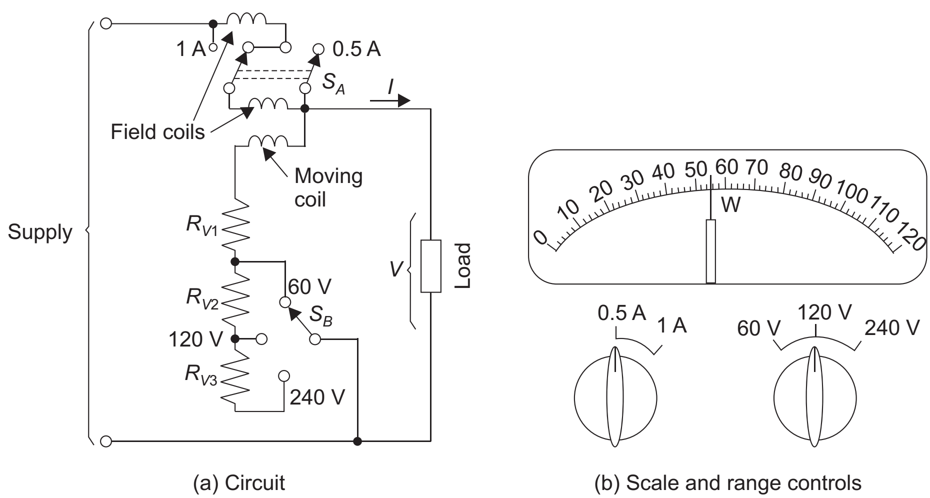


Fig. 6.18 Multi-range Wattmeter

As can be seen from Figure 6.18(a), any value of voltage among 60 V, 120 V, and 240 V can be selected using switch S_B and multiplier resistors connected in series. Switch S_A is provided to select the current range. When the switch is on the right side, the field coils get connected in series and when the switch is set on the left side, the field coils get connected in parallel.

When range switches are set to 120 V and 1 A, the scale indicates the value directly in watts and a full scale deflection of 120 W occurs. Similarly, when 240 V and 0.5 A ranges are selected, the scale again indicates the reading in watts. To determine the FSD power, the selected current and voltage ranges are multiplied together.

Now, consider an example where the current and voltage ranges for a wattmeter are set at 1 A and 60 V, respectively. The instrument will give FSD equal to 60 W (that is, 1 A × 60 V). However, let the actual voltage and current applied to the instrument be 120 V and 0.5 A, respectively. Then the power for actual readings comes out to be 60 W as 0.5 A × 120 V = 60 W.

The above example shows that although the meter indicates a reasonable deflection, the insulation on the moving coil will be destroyed due to overheating as twice the maximum current flows through the moving coil. However, generally the load for which power measurement is being done has a constant supply voltage which does not change with a change in load current. Thus, the moving coil carries a constant current which is in proportion to the supply voltage. As a result, the instrument deflection becomes proportional to the load current and thus, the calibration of the scale can be done linearly.

When load current and voltage are much higher than the normal wattmeter limits, the instrument transformers are used to extend the range of wattmeter. It needs a current transformer (CT) to reduce the load current and a voltage transformer (VT) to reduce the load voltage to a suitable level. The wattmeter circuit with these transformers is shown in Figure 6.19.

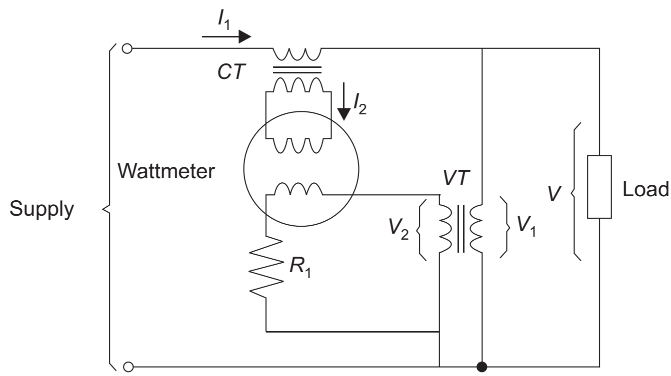


Fig. 6.19 Wattmeter's Range Extension using CT and VT

The ratios of current and voltage transformers are given as:

$$\text{CT ratio} = \frac{I_1}{I_2} \quad ...(15)$$

$$\text{VT ratio} = \frac{V_1}{V_2} \quad ...(16)$$

To determine the actual value of load power, a factor is multiplied to the wattmeter reading since the wattmeter indicates the product of CT and VT outputs. This multiplication factor is given by the relation:

$$\text{Multiplication factor} = \frac{I_1}{I_2} \times \frac{V_1}{V_2} \quad ...(17)$$

The transformers are subjected to phase angle errors induced due to transformers. Hence, these errors must be considered while measuring power to prevent inaccurate results. In addition, the values of transformer ratio must be known accurately for precise measurements.

Example 5 The range of an electrodynamic wattmeter is extended by using instrument transformers having CT ratio = 5 and VT ratio = 3. If load voltage and current are given to be 230 V and 6.2 A and the phase angle between them is equal to 33°, calculate the following:

(a) Multiplication factor

(b) Power measured by wattmeter

Solution: Given that: $I_1 = 6.2$ A, $V_1 = 230$ V, CT ratio = 5, VT ratio = 3, and $\phi = 33°$

The current transformer ratio CT is given by the relation:

$$\text{CT ratio} = \frac{I_1}{I_2} \qquad \text{[Refer to Eqn. (15)]}$$

$$\Rightarrow \qquad I_2 = \frac{I_1}{\text{CT ratio}}$$

Substituting the values, we get:

$$I_2 = \frac{6.2}{5} = 1.24 \text{ A}$$

Similarly, we have voltage transformer ratio *VT* as:

$$V_2 = \frac{V_1}{\text{VT ratio}} \qquad \text{[Refer to Eqn. (16)]}$$

$$V_2 = \frac{230}{3} = 76.67 \text{ V}$$

(a) The multiplication factor is given by Equation (17) as:

$$\text{Multiplication factor} = \frac{I_1}{I_2} \times \frac{V_1}{V_2}$$

Substituting the values, we get:

$$\text{Multiplication factor} = 5 \times 3 = 15$$

(b) The power in an electrodynamic wattmeter is given by:

$$P = VI \cos \phi$$

Substituting the given values, we get:

$$P = 76.67 \times 1.24 \cos 33°$$

$$\Rightarrow \qquad P = 79.73 \text{ W}$$

6.8 ENERGY METERS

Earlier, we have studied the wattmeters which indicate the value of power at a particular instant. However, energy meter also known as **integration meter** is incorporated with some type of registration mechanism to sum up all the instantaneous readings of power for a time interval. That is, it measures the amount of electric energy. **Electric energy** is defined as the amount of total power consumed or delivered to a circuit over a specific time period. It is given as:

$$\textit{Energy} = \text{Power} \times \text{Time}$$

Thus, the energy consumed or delivered will be 1 kWh if a power of 1 kW is supplied for 1 hour. Some other higher (Megajoules) or smaller (watt second or watt hour) units can be used to measure the energy whose consumption rate varies for different time periods with corresponding variation in power consumption.

Some characteristics of energy meter are given as follows.

- The design must be simple and the use of rapidly degenerating parts must be avoided.
- The multiplication factors must be avoided and direct dial readings must be taken.
- Dust, water, and insect proof casing must be used.
- Lightweight moving parts and best quality jewels and pivots should be used to ensure minimum and constant friction losses over long periods of time.
- The value of the torque must be high so as to eliminate inaccuracies due to unavoidable errors caused by friction irregularities.
- Internal dissipation of energy must be very small.
- For reasonably varying load and voltage, the accuracy of the instrument must be maintained.
- The friction at the pivots and the retarding torque of magnetic brakes must not vary in order to keep calibration of the instrument permanent. Also, the magnets must be placed in such a way that their strength remains unaffected by the magnetic field of current coil.

Generally energy meters are of three types, namely, *electrolytic, clock,* and *motor meters*. Out of these, the most widely used are motor meters that are discussed here.

6.8.1 Motor Meters

Motor meters can be employed to measure energy in both AC as well as DC circuits. The meter may be a watt hour meter or ampere hour meter for DC circuits. The motor meter consists of a small motor which revolves continuously. In case of ampere hour meter, the speed of rotation is proportional to the circuit current while it is proportional to the power of the circuit in case of watt hour meters. Thus, the total number of revolutions of watt hour meter indicates the total amount of energy supplied whereas for ampere hour meter it indicates the total amount of electricity supplied. The meter is marked with the number of revolutions completed per kilowatt hour (kWh), known as **meter constant**. This meter constant along with full load current and supply voltage is marked on the dial of the energy meter to specify its ratings.

The motor meter consists of a braking device which is responsible for controlling the speed of the moving system. The device comprises a permanent magnet known as **braking magnet** due to which eddy currents get induced in aluminium disc, mounted on the moving system. In turn a braking torque is produced by these eddy currents which is retarding in nature. This torque is proportional to the speed of the moving system, as torque is proportional to the induced currents which is being produced by them and thus, the induced currents are proportional to moving system speed. Now, as the moving system revolves, the field of the permanent magnet is cut by the disc resulting in an emf generated in it. This emf e is given as:

$$e = K_1 \Phi_n \qquad \text{...(18)}$$

where K_1 is a constant, n is the speed of rotation, and Φ is the flux of the permanent magnet. Let the resistance of eddy current paths be denoted by R, then the expression for eddy current can be given as:

$$i = \frac{e}{R} = \frac{K_1 \Phi n}{R} \qquad \text{[Refer to Eqn. (18)]} \qquad ...(19)$$

Now, the braking torque is given by the expression:

$$T_B = K_2 \Phi i r \qquad ...(20)$$

where r is the effective radius from the axis of aluminium disc. Equation (20) represents that the braking torque is directly proportional to the product of magnitude of eddy currents, the flux of permanent magnet, and effective radius r. Substituting the value of i from Equation (19) into Equation (20), we get:

$$T_B = \frac{K_1 K_2 \Phi^2 \, nr}{R}$$

$$= \frac{K_3 \Phi^2 \, nr}{R}$$

where $K_3 = K_1 K_2$, another constant. Now, if the radius of the disc is considered to be constant, the above expression becomes:

$$T_B = \frac{K_4 \Phi^2 \, n}{R} \qquad ...(21)$$

where $K_4 = K_3 r$ is again a constant. Now, at steady speed N, the breaking torque is given as:

$$T_B = \frac{K_3 \Phi^2 \, Nr}{R} = K_5 \, N \qquad ...(22)$$

where $K_5 = \frac{K_3 \Phi^2 r}{R}$ is another constant.

At steady speed, the braking torque becomes equal to the driving torque. Thus, we have:

$$T_B = T_D$$

Substituting the value from Equation (22), we get:

$$\frac{K_3 \Phi^2 Nr}{R} = T_D$$

On rearranging, the steady speed can be written as:

$$N = \left(\frac{R}{K_3 \Phi^2 r} \right) T_D \qquad ...(23)$$

$$= K_6 T_D$$

where $K_6 = \frac{R}{K_3 \Phi^2 r}$, is a constant.

From Equation (23), it can be seen that at steady speed, N is directly proportional to R and inversely proportional to r and square of Φ. It is desired that at steady speed, N must be low. By keeping disc resistance R small, flux Φ and effective radius r large, this can be achieved. The radius of the disc, however, cannot be increased excessively as its size and weight are limited by frictional forces. Thus, strong magnets with large pole areas are used to obtain low steady speeds. The air gap is also made as small as possible. It must be noted here that since the resistance per unit weight of aluminium is smaller as compared to that of copper, the disc is made up of aluminium.

It is necessary that R and Φ should not change to keep the calibration of the meter same. Thus, it is essential to maintain a constant strength of the braking magnet throughout the meter's life. This requires proper treatment and careful designing of the magnet during manufacture. The temperature changes must also be avoided as R increases with increase in temperature and, thereby, reduces the braking torque. By changing the distance between braking magnet and magnetic shunt, the braking torque can be adjusted. The braking torque increases when the distance between them is increased because on moving the magnetic shunt away from the magnet, it will bypass a less amount of flux and vice versa. Equation (20) also reveals that the braking torque is directly proportional to r. Thus, increasing r by placing the magnet at a larger radius results in a larger braking torque.

There are three types of motor meters, namely, *mercury motor meters, commutator motor meters,* and *induction motor meters*. Out of these, mercury motor and commutator motor meters work on DC circuits. Induction motor meter works on AC circuits and is discussed in the next section.

Example 6 Calculate the kWh registered by a 230 V ampere hour type meter if it completes 227 revolutions in 13 minutes. The meter is carrying a current of 12.5 A and is connected to a DC supply of 230 V and the timing constant of the meter is given to be 33 A-s per revolution.

Solution: Given that: timing constant = 33 A-s/revolution

In 227 revolutions, the meter would register = 33 × 227 = 7491 A-s

Now, the time taken is given as 13 minutes. Or, it can be written as 13 × 60 = 780 s.

Thus, the total current registered is given to be:

$$\frac{7491}{780} = 9.60 \text{ A}$$

Now, we know that the energy recorded by the meter in kWh is given as:

$$\text{Energy} = \text{Power} \times \text{Time} = \frac{VIt}{1000} \text{ kWh} \qquad \text{(Since power = voltage × current)}$$

Substituting the values, we get:

$$E = \frac{230 \times 9.60 \times 13}{1000 \times 60} \qquad \text{(as time } t \text{ is in hours)}$$

$$\Rightarrow \qquad E = 0.478 \text{ kWh}$$

6.9 SINGLE PHASE INDUCTION TYPE ENERGY METER

The single phase induction type energy meter has almost similar construction as that of induction type wattmeter (refer to Section 6.4). The only difference is that here only one pressure coil and one copper shading band are provided on the central limb of the shunt magnet. The friction error is compensated by placing two copper bands obliquely on the remaining two limbs of this magnet.

Similar to the motor meter, this meter also consists of a braking system to provide braking torque. The brake magnet replaces the spring and pointer present in wattmeter circuits. The brake magnet (or **permanent magnet**) is placed near the aluminum disc, opposite to the electromagnet which is used for providing braking torque. Due to this brake magnet, currents get induced in the disc and in turn produce a retarding torque which is in proportion to their magnitude. This retarding torque is in proportion to the speed of the moving system and forces the system to attain a steady state when it becomes equal to the driving torque. The speed of the disc depends upon the position of the brake magnet with respect to the position of the disc spindle. The braking torque increases if the brake magnet is moved away from the spindle while it decreases when the magnet is moved towards the spindle. The construction of a single phase induction type energy meter is shown in Figure 6.20.

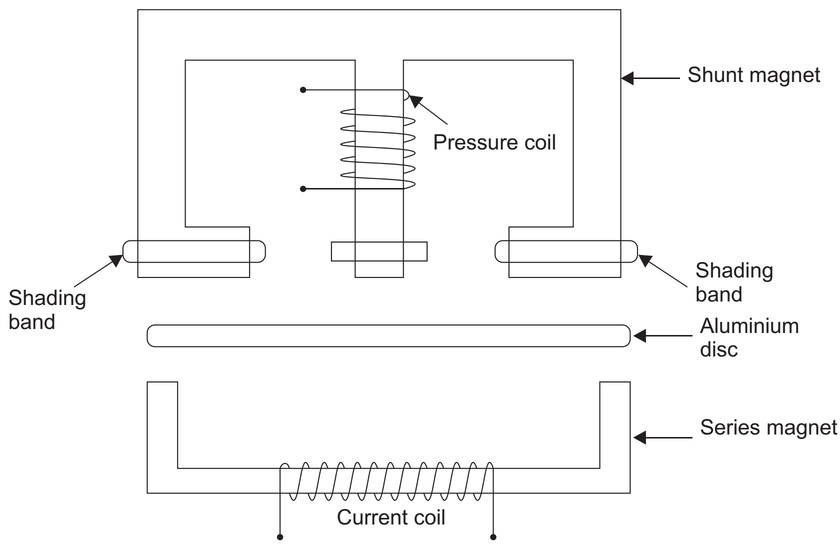


Fig. 6.20 Single Phase Induction Type Energy Meter

A counting or registering mechanism consisting of a train of wheels is used to record the number of revolutions made by the meter. The spindle of the moving system is geared to this counting mechanism. A pointer type of registering mechanism is shown in Figure 6.21 which consists of round dials marked with ten equal divisions. Other than this, cyclo-meter register can be used.

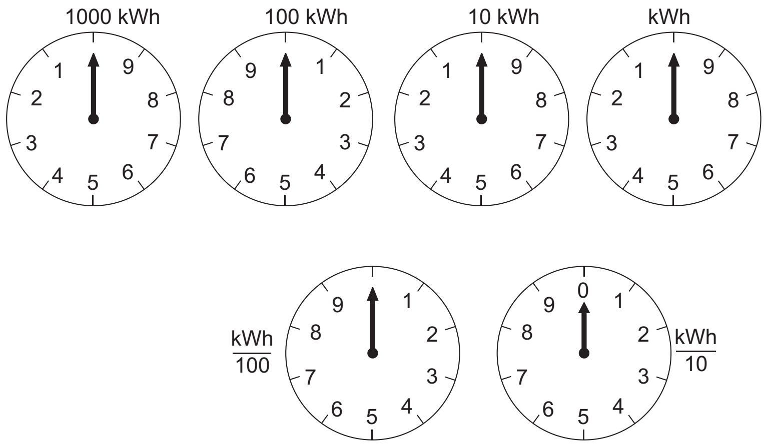


Fig. 6.21 A Pointer Type of Register

Let Φ be the flux of the brake magnet, then the braking torque T_B is given by the relation:

$$T_B \,\alpha\, \Phi i \qquad ...(24)$$

where i is the current induced when the moving system rotates in the field of brake magnet. Let e be the voltage induced in the aluminium disc, we have:

$$e \,\alpha\, \Phi n$$

where n is the speed of rotation of the disc.

Let the resistance of eddy current paths be denoted by R, then the expression of eddy current is given as:

$$i = \frac{e}{R} \qquad ...(25)$$

Substituting Equation (25) in Equation (24), we get:

$$T_B \,\alpha\, \Phi \frac{e}{R}$$

Or, it can be written as:

$$T_B \,\alpha\, \Phi^2 \frac{n}{R}$$

Now, at steady speed N, the braking torque T_B' is given as:

$$T_B' \,\alpha\, \frac{\Phi^2 N}{R} \qquad ...(26)$$

As we know that, at steady speed, the braking torque becomes equal to the driving torque. Thus, we have:

$$T'_B = T_D \propto \frac{\Phi^2 N}{R} \quad ...(27)$$

Or, it can be written as:

$$N \propto \frac{R}{\Phi^2} T_D$$

Now, as we know that:

$$T_D \propto VI \cos \phi \quad \text{[Refer to } \textit{Section 6.4}\text{]} \quad ...(28)$$

where V is the supplied voltage, I is the load current, and ϕ is the phase angle between them. Combining Equations (26), (27), and (28), we get:

$$T_B \propto N$$

$$\Rightarrow \quad N \propto VI \cos \phi \quad ...(29)$$

Equation (29) implies that the power of the system is proportional to N and the total number of revolutions is proportional to the energy supplied to the system which can be written as:

$$\int N dt \propto \int VI \cos \phi$$

6.9.1 Error Sources and Compensation

Temperature compensation

Due to an increase in temperature, the resistance of all aluminium and copper parts gets increased which may cause the following errors.

- Torques produced by all shading bands gets decreased.
- The potential coil flux gets reduced.
- A decrease in the phase angle between the potential coil flux and supply voltage occurs.
- Phase angle of eddy currents also gets reduced.

The errors which occur due to rise in temperature tend to neutralize one another's effects and hence, are very small. These errors are negligible with non-inductive loads. However, for low power factor loads, these errors become serious and cause the meter to register high and run fast.

The temperature rise effects can be compensated by using a temperature shunt on the brake magnet. Some special magnetic materials such as Mutemp can also be used whose permeability decreases with an increase in temperature.

Voltage compensation

In an energy meter, certain amount of variation from its specified value of supply voltage is permitted and thus, the meters should be compensated. The errors occur due to voltage variations because of two reasons:

- On account of saturation in iron parts, the relationship between the supply voltage and shunt magnet flux is non-linear.
- A dynamically induced emf is produced in the disc by the shunt magnet flux which further generates a self-braking torque in proportion to the square of the supply voltage.

This variation in voltage is compensated by diverting a large portion of the flux into the active path, whenever a rise in voltage occurs. This diversion is performed by a saturable magnetic shunt. Another method is to cut the holes in the side limbs of the shunt magnet so that its reluctance can be increased.

Friction compensation

The errors occurring due to friction at the top and bottom of the spindle due to its bearing equipments are very serious and cannot be removed merely by careful designing. These errors hold greater importance at low loads and, thus, some arrangements must be made to ensure accurate readings. Friction compensation is also known as **light load compensation**.

The flux contained in the two outer limbs of the shunt magnet is held by the two shading bands. Due to this, eddy currents are generated in the shading bands resulting in a phase displacement between the main gap flux and the enclosed flux. As a consequence, a small driving torque is exerted on the disc which can be made equal in magnitude to friction torque by varying the position of the two shading bands. This torque acts in the opposite direction of rotation and does not depend on the load of the meter.

The correctness of the friction compensation method can be checked by running the meter at low loads at which it should run properly and the registrations must be free from errors.

Creep

A meter is said to be creeped when the supply voltage causes a slow, continuous rotation in the meter even when the load current is reduced to zero. The causes of creeping are over friction compensation, vibrations, excessive voltage across the voltage coil, and stray magnetic fields. Among all these, the compensation for friction is the major one.

The creep must be prevented to ensure meter accuracy by cutting two holes or slots in the disc. These holes are diametrically opposite being on opposite sides of the spindle. When one of the holes comes under the edge of a pole of the shunt magnet, the disc becomes stationary. Thus, the rotation of the disc is limited to half of a revolution. Sometimes, creeping is prevented by attaching a small piece of iron to the edge of the disc. The brake magnet then exerts a force to attract towards it and thus, no creeping is observed under no load condition.

Overload compensation

A dynamically induced emf is produced in the disc due to its continuous revolution in the field of series magnet when a load is applied. Due to this emf, eddy currents are produced and interact with the field of series magnet to generate a braking torque. This self-braking torque is in proportion to the square of the load current and increases with an increase in the load. This results in a lower registration when the load current is high. This self-braking action can be minimized by keeping the speed of the disc as low as possible, approximately equal to 40 rpm at full load. The dynamically induced emf (responsible for production of braking torque) can be made small in comparison to statistically induced emf (responsible for producing the driving torque) by making the current coil flux small in comparsion to pressure coil flux.

Usually an overload compensating device is added to compensate for overload. It is a magnetic shunt for the series magnets core and approaches saturation at overloads to reduce its permeability. Thus, only a small portion of the series magnetic flux gets diverted at large values of currents and the remaining larger portion appears in the air gap of the disc contributing to driving torque.

Phase angle compensation

From Equations (28) and (29), it can be seen that both the driving torque and the disc speed are proportional to $VI \cos \phi$. This quantity also represents the power applied to the load and when multiplied by time, it gives the energy consumed. However, this equation is valid for energy meter if the supply voltage leads the voltage coil current by a phase angle of 90°.

To achieve this phase lead, the voltage coil circuit is made highly inductive. Thus, the swamping or multiplier resistors must not be connected in series with the voltage coil of the energy meter. By this method, an approximation of the desired 90° phase difference is obtained. Additional compensation method, like **pole shading** method should also be employed. In this method, the pole of the voltage coil core is wound closely with a few turns of heavy conducting wire with the coil ends connected with an adjustable low resistance. This method brings the coil current within exact 90° phase lag with the applied voltage and thereby provides **lag adjustment**.

In addition to the errors discussed above, the driving system and braking system can also cause errors in an energy meter. These errors are:

- Due to temperature changes, changes in the strength of brake magnet and disc resistance may arise.
- Phase errors may occur due to reduction in power factor.
- Moving parts may have abnormal friction.
- The series magnetic flux causes the self-braking effect.
- The values of retarding torque may get altered due to the movement of disc in the field of current coils.
- Abnormal load currents and voltages may lead to wrong magnitude of fluxes.
- Due to lack of symmetry in the magnetic circuit, the disc may continue rotating even when no current is drawn and only the pressure coils are excited. This results in creeping of the meter.
- Changes in resistances of the disc occur due to change in temperature.
- Defective lagging, abnormal frequencies, changes in iron losses may result in improper phase relations of fluxes.

Example 7 Calculate the power in the circuit if a meter completes 120 revolutions in 32 seconds. The dial on the meter indicates "1 kWh = 20000 revolutions".

Solution: Given that: 1 kWh = 20000 revolutions

For 120 revolutions, the power is metered as:

$$1 \times \frac{120}{20000} = 0.006 \text{ kWh} \qquad \text{...(7a)}$$

Let the power in the circuit be P. Then the energy consumed in 32 seconds is given as:

$$\frac{P \times 32}{3600} = 0.0088\, P \text{ kWh} \qquad \text{...(7b)}$$

The two amounts of energy must be equal from Equations (7a) and (7b), and thus, we get power P as:

$$P = \frac{0.006}{0.009} = 0.675 \text{ kW} = 675 \text{ W}$$

Example 8 The meter constant of a 25 A, 230 V meter is given as 440 revolutions/kWh. Calculate the error if the disc made 36 revolutions in 62 seconds for a load of 5 kW.

Solution: The energy consumed by the load in 62 seconds is given as:

$$5 \times \frac{62}{3600} = 0.086 \text{ kWh}$$

The energy registered by the meter is given as:

$$\frac{36}{440} = 0.081 \text{ kWh}$$

From these two readings, we can see that the meter is slow by:

$$\frac{0.086 - 0.081}{0.086} \times 100 = 5.81\%$$

6.10 ENERGY METERS IN POLYPHASE CIRCUITS

Similar to wattmeters, Blondel's theorem is applicable on energy meters as well. Thus, a three-element energy meter is required for a three-phase four-wire system and for a three-phase three wire system, a two-element energy meter is required.

All polyphase wattmeters consist of a single spindle on which various elements are mounted. This spindle is used to drive the registering mechanism thereby recording the effect of all elements. These meters can be single disc type or multi-disc type. In single disc type, a single disc is driven by all the elements while in multi-disc type, a separate disc is there for each element.

Let us now discuss the power measurements in a three-phase circuit. In this case, any one of the three-phase four-wire system and three-phase three-wire system can be employed. Therefore, these meters are categorized as three-element energy meter and two-element energy meter.

6.10.1 Three-element Energy Meter

A three-element energy meter as shown in Figure 6.22 is used for three-phase four-wire system. The fourth wire is a neutral wire. Each element has the same constructional details as that of a single phase energy meter. Three discs are provided for each element, thus, it is a multi-disc type wattmeter. The pressure and current coils are labelled as P_1, P_2, P_3, and C_1, C_2, C_3, respectively. The pressure coils are connected across a neutral and a line whereas the current coils are connected in series with the lines. Therefore, the net torque produced is the algebraic sum of the torques produced by each element.

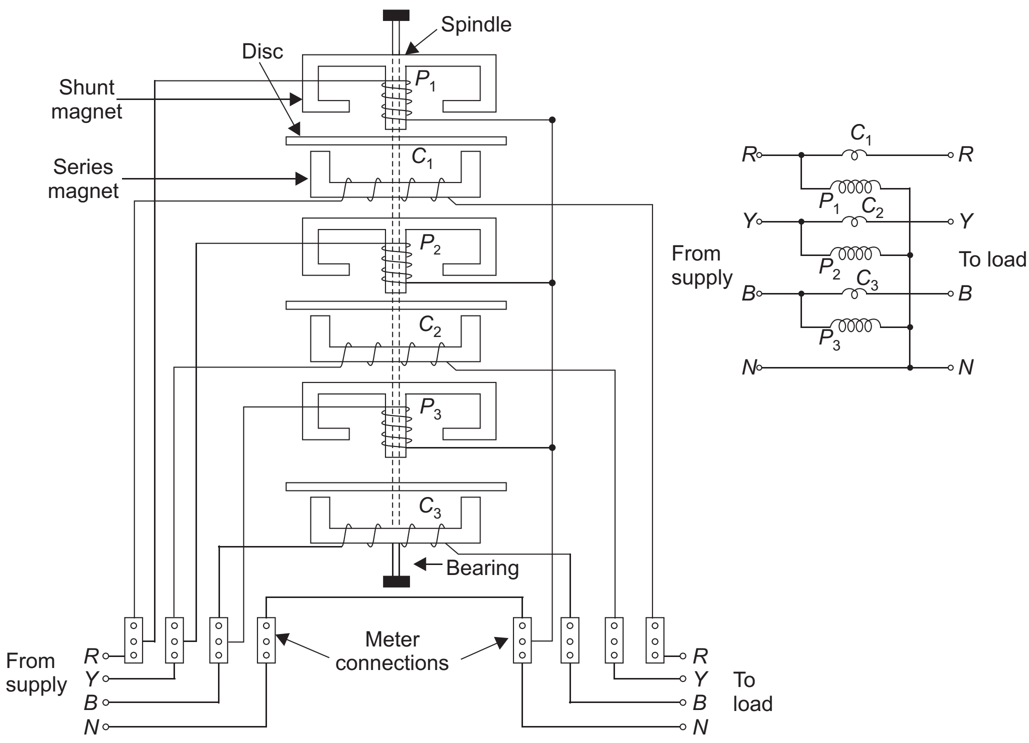


Fig. 6.22 Three-element Energy Meter

6.10.2 Two-element Energy Meter

This meter is also a multi-disc type wattmeter as two separate discs are provided for each element. It is used to measure energy for a three-phase three-wire system as shown in Figure 6.23. The connections of the meter are similar to that of two wattmeter method of power measurement. The pressure coils, carried by shunt magnet are connected in parallel and the current coils, carried by series magnet are connected in series. The total torque generated is equal to the sum of the torques generated by each element and the method of torque generation is similar to the single-phase energy meter.

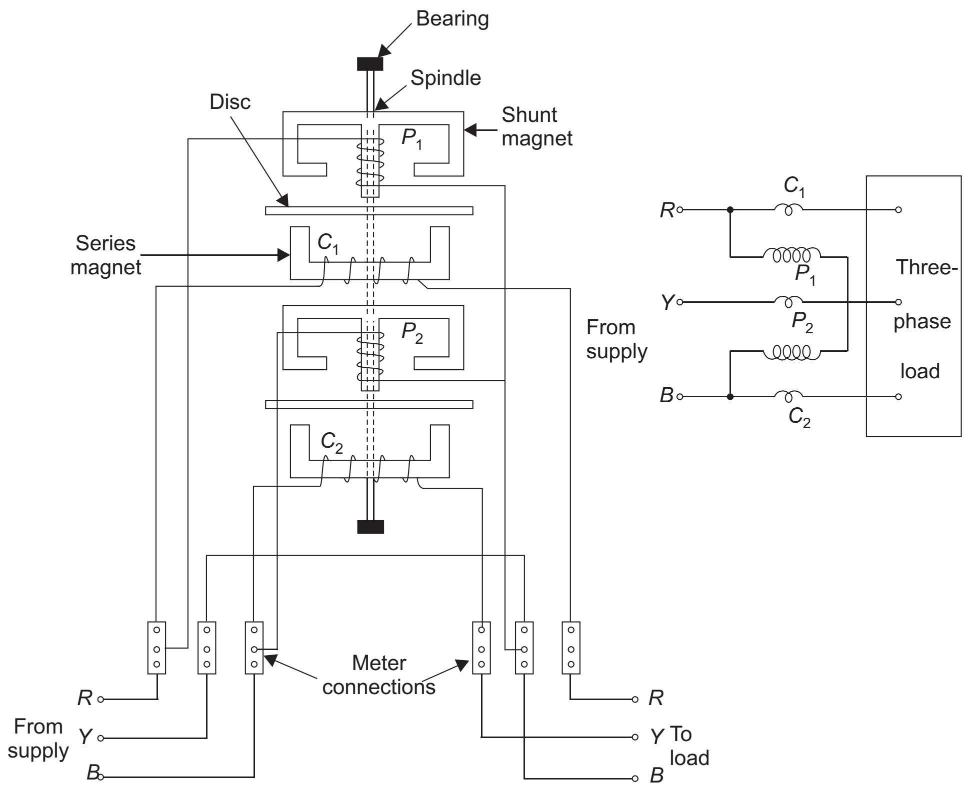


Fig. 6.23 A Two-element Energy Meter

6.11 TESTING OF ENERGY METERS

Before energy meters are released for measuring purposes, their actual registration is checked. Also, the errors are kept within reasonable limits by doing some adjustments. This is known as **testing** of the energy meter. Some of the conditions for which energy meters are tested are given below.

- Power factor is unity and the current is 5% of the marked value.
- Power factor is unity and the current is 100% or 125% of marked value.
- Current is equal to the marked value and 0.5 lagging power factor.
- Power factor is unity with one intermediate load.

Other than these tests, two more tests are performed on the meters which are as follows.

- **Starting test:** This test does not check the accuracy of the meter, rather it is performed to check if the meter starts and runs properly. According to this test, the meter should start and run with normal voltage and a current of 0.5% of the marked value.
- **Creep test:** This test states that the meter should not creep, that is, it must not undergo more than one revolution for a voltage of 110% of its marked value and open current circuit.

6.11.1 Phantom Loading

Phantom loading is also known as **fictitious loading**. The testing of a high current rating meter involves a significant power loss, if the test is performed using actual loading arrangements. Thus, phantom loading is used to avoid this. In this technique, the voltage applied across the pressure coil is from a circuit of required normal voltage while the voltage across the current coil is supplied by a separate low voltage source. This is represented by Figure 6.24. Here, the total power supplied by this arrangement is due to the circuit current at low voltage in addition to the small current in pressure coil at normal voltage. Thus, the amount of total power required for the testing is very small.

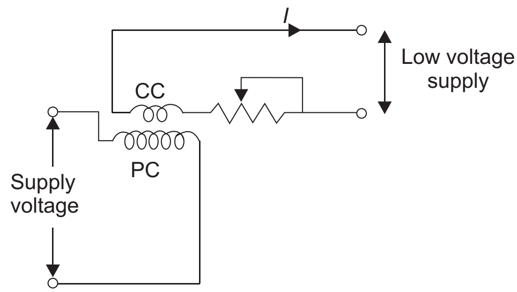


Fig. 6.24 Circuit for Phantom Loading

Example 9 The resistances of pressures coil circuit and current coil of a 230 V, 20 A energy meter are given to be 9000 Ω and 0.2 Ω, respectively. This meter is tested on its marked ratings. Determine the amount of power consumed if the meter is tested with:

(a) direct loading arrangements

(b) phantom loading method, where a 5 V battery is used to excite current circuit.

Solution: Given that: Resistance of pressure coil circuit is 9000 Ω

Resistance of current coil circuit is 0.2 Ω

Voltage supply = 230 V

(a) In direct loading arrangement as shown in Figure 6.24(a), we can calculate the power as:

Power consumed by current coil circuit is given as:

$$VI = 230 \times 20 = 4600 \text{ W} \qquad \text{...(9a)}$$

Power consumed by pressure coil circuit is given as:

$$\frac{V^2}{R} = \frac{(230)^2}{9000} = 5.88 \text{ W} \qquad \text{...(9b)}$$

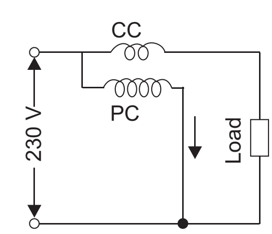


Fig. 6.24 (a)

Thus, we get total power consumed by the meter by adding Equations (9a) and (9b) as:

$$4600 + 5.88 = 4605.88 \text{ W}$$

(b) In phantom loading arrangement as shown in Figure 6.24(b), we calculate the power as:

Power consumed by current coil circuit is given as:

$$VI = 5 \times 20 = 100 \text{ W} \qquad \text{...(9c)}$$

Power consumed by pressure coil circuit is given as:

$$\frac{V^2}{R} = \frac{(230)^2}{9000} = 5.88 \text{ W} \qquad \text{...(9d)}$$

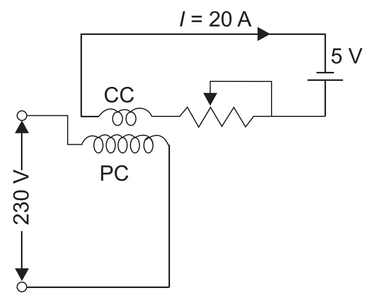


Fig. 6.24 (b)

Thus, we get total power consumed by the meter by adding Equations (9c) and (9d) as:

$$100 + 5.88 = 105.88 \text{ W}$$

6.12 MAXIMUM DEMAND INDICATOR

Maximum demand indicators are the instruments that are used to determine the optimum value of electricity required by a specific consumer during a given time period. These instruments are designed in such a way that they record the maximum power consumption during a particular period for a particular consumer. However, they do not record any sudden momentary increase in loads which may occur due to high starting currents of motors. Also, short circuits are not considered in their designing. Thus, it is so designed that the average power is recorded at regular intervals of 15 or 30 minutes. The essential parts of a maximum demand indicator are indicating pin, reset device, a pointer on dial, a moving system to which a dial is connected, and fraction device. The mathematical expression to calculate maximum demand is given as:

Maximum demand (kW) = maximum energy recorded (kWh)/time (hours)

There are four types of maximum demand indicators which include *recording demand indicators*, *thermal type maximum demand indicators*, *average demand indicators*, and *digital maximum demand indicators*.

6.13 TRIVECTOR METER

A trivector meter is a compact unit which is used to measure reactive, active, and apparent power along with maximum demand indicators. It consists of three recording elements namely, rkVAh, kWh, and kVAh. The records rkVAh and kWh have the same principle as that of 3-phase meters while kVAh element uses different gearing systems and records mechanically.

The five different gear systems used are described below.

- At normal speed of watt hour meter, it is driven alone, providing power factor to be one.
- Speed of watt hour meter is reduced slightly and speed of reactive meter is reduced considerably. This corresponds to a power factor of 0.925 and phase angle of 22.5°.
- Speed of both meters is decreased by the same amount corresponding to a power factor of 0.707 and phase angle of 45°.
- Speed of watt hour meter is reduced considerably and speed of reactive meter is reduced slightly. This corresponds to a power factor of 0.38 and phase angle of 67.5°.
- Speed of the reactive meter is normal and it is driven alone, providing power factor to be zero.

The advantages of this meter are as follows.

- It produces full and very accurate data.
- It is very reliable, compact, light, and robust.
- It can be used for both bi-directional and uni-directional metering.
- There is no need for any adjustment for a long period of time.
- Since trivector meter is a compact unit which performs various recordings, it replaces many instruments.
- It is available for both HT (high tension) and LT (low tension) applications.

Let us summarize

1. Electrodynamic instruments, in addition to measuring current and voltage, also measure power in a circuit. They may be used for both DC and AC power measurement for any voltage and current waveform, not only sinusoidal waveform.
2. The errors present in the instruments may be due to various factors such as pressure coil inductance or capacitance and connections of different elements, mutual inductance effects, temperature variations, eddy currents, stray magnetic fields, and vibrations of moving systems.
3. Ordinary electrodynamic wattmeters cannot be used for precise measurement of power in the circuits which have low power factor (power factor less than 0.5).
4. Low power factor meters are none but electrodynamic wattmeters with some additional features incorporated, such as pressure coil current, compensation for pressure coil current, compensation for pressure coil inductance, and smaller control torque.
5. Induction type wattmeter consists of two laminated electromagnets, one of which is energized by the current proportional to load voltage, known as shunt magnet. The other one, known as series magnet is energized by the load current or a part of load current.
6. Blondel's theorem is used to specify the number of wattmeters required to measure power in a system of electrical conductors. This theorem states that the power of a network, consisting of *n* conductors, is given by the algebraic sum of the readings of *n* wattmeters.
7. The different methods used to measure power are three-wattmeter, two-wattmeter, and single wattmeter methods.
8. An electrodynamic three-phase wattmeter consists of two wattmeters having two moving coils mounted together on a single pivoting spindle and encased in a single chest.

9. Energy meters, also known as integration meters, are incorporated with some type of registration mechanism to sum up all the instantaneous readings of power for a time interval, that is, they measure the amount of electric energy.
10. Electric energy is defined as the amount of total power consumed or delivered to a circuit over a specific time period.
11. Motor meters can be employed to measure energy in both AC as well as DC circuits. The meter may be a watt hour meter or ampere hour meter for DC circuits. The motor meter consists of a small motor which revolves continuously.
12. The motor meter consists of a braking device which is responsible for controlling the speed of the moving system. The device comprises a permanent magnet known as braking magnet due to which eddy currents get induced in the aluminium disc, mounted on the moving system.
13. The single phase induction type energy meter has almost similar construction as that of induction type wattmeter. The only difference is that here only one pressure coil and one copper shading band are provided on the central limb of the shunt magnet.
14. Similar to the motor meter, the single phase induction type energy meter also consists of a braking system to provide braking torque. The brake magnet replaces the spring and pointer, present in wattmeter circuits.
15. All polyphase wattmeters consist of a single spindle on which various elements are mounted. This spindle is used to drive the registering mechanism thereby recording the effect of all elements.
16. A three-element energy meter is used for a three-phase four-wire system. The fourth wire is a neutral wire. Each element has the same constructional details as that of a single phase energy meter.
17. A two-element energy meter is also a multi-disc type wattmeter as two separate discs are provided for each element. It is used to measure energy for a three-phase three-wire system.
18. Prior to the implementation of energy meters for measuring purposes, their actual registration is checked. Also, the errors are kept within reasonable limits by doing some adjustments. This is known as testing of the energy meter.
19. Maximum demand indicators are the instruments used to determine the optimum value of electricity required by a specific consumer during a given time period.
20. A trivector meter is a compact unit which is used to measure reactive, active, and apparent power along with maximum demand indicators. It consists of three recording elements namely, rkVAh, kWh, and kVAh.

EXERCISES

Fill in the Blanks

1. The full scale deflection of induction type wattmeter is _________.
2. Number of revolutions per kilowatt hour is known as _________.
3. Trivector meter is used to measure _________, _________, and _________ power.
4. When only pressure coils are excited and no current is drawn, the disc may go on rotating. This phenomenon is known as _________.
5. Ampere hour meters are used to measure the total amount of _________ supplied.

Multiple Choice Questions

1. In an electrodynamic type wattmeter:

(a) the current coil is fix
(b) both current coil and pressure coil are moving
(c) the pressure coil is fix
(d) any of the current or pressure coil may be fix

2. Power in a three-phase four-wire circuit can be measured by using:
   (a) two wattmeters (b) one wattmeter
   (c) three wattmeters (d) four wattmeters
3. On bringing the brake magnet towards the spindle of the disc, the braking torque:
   (a) increases (b) remains the same
   (c) decreases (d) none of the above
4. To compensate the error due to pressure coil in electrodynamometer wattmeter, a capacitor is connected across:
   (a) pressure coil (b) pressure coil and series resistance
   (c) series resistance (d) a portion of series resistance
5. Creeping is observed in:
   (a) watt hour meter (b) voltmeter
   (c) ammeter (d) wattmeter

State True or False

1. The household energy meter is an indicating instrument.
2. The torque produced in a wattmeter is proportional to the average value of supply voltage.
3. In testing of energy meters, phantom loading is used for meters having large current ratings.
4. Trivector meters can be used for rkVAh metering.
5. Creep test is done with open voltage circuit and 0.5% of the marked current.

Descriptive/Numerical Questions

1. What are the special features of a wattmeter suitable for working on low power factor circuits?
2. Discuss the various types of error and their methods of compensation in the dynamometer type wattmeter.
3. With a neat diagram, explain the construction and working of a single phase wattmeter. What is the importance of deflection torque in these instruments? Discuss about its shape also.
4. How is the reading of a wattmeter affected if pressure coil inductance is taken into consideration?
5. Show how you can measure power in a three-phase balanced load using just one wattmeter.
6. Explain why compensating coil is used in electrodynamometer wattmeter.
7. Prove that in a 3-phase 3-wire system, the two-wattmeter method of total power measurement is valid for both balanced and unbalanced loads.
8. Explain the construction of induction type energy meter. Explain the adjustments done in energy meter to read accurately.
9. Explain testing of single phase energy meter by phantom method.
10. State Blondel's theorem.
11. Write short notes on the following:
    (a) Maximum demand indicator

(b) Trivector meter
(c) Three-phase wattmeter
(d) Extension of wattmeter range using transformers

12. A single phase kWh meter makes 500 revolutions per kWh. It is found on testing to be making 40 revolutions in 58.1 seconds at 5 kW full load. Find the percentage error.
13. Consider a 3-phase, 500 V motor load which has a power factor of 0.4. Two wattmeters are connected to measure the power input to the motor. The input to the motor is found to be 30 kW. What are the readings of the two meters?
14. What is creep and how creep adjustment is made in a single phase induction type energy meter?

7

DC and AC Measurement Bridges

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:
- Explain the concept of DC and AC bridges
- Describe different types of DC bridges including Wheatstone bridge, and Kelvin bridge
- Measure high resistances by an instrument, called megger
- Describe fall-of-potential method for measuring earth resistance
- Describe different types of AC bridges for measuring unknown capacitance, resistance, frequency, inductance, and mutual inductance
- Describe different miscellaneous bridges, such as parallel *T* network, bridge *T* network, *Q* meter, Wagner earthing device

7.1 INTRODUCTION

In the previous chapters, we have studied the measurement of current, voltage, power, and energy. For this, a number of direct current (DC) and alternating current (AC) measuring circuits have been employed. In this chapter, we will proceed to measure several other parameters of an electrical circuit which include resistance, inductance, capacitance, and frequency. In addition, some other quantities like quality factor of a coil in the circuit and dissipation factor of a capacitor are also sometimes needed to be calculated. These measurements are accomplished using bridge circuits.

Bridge circuits are employed to eliminate errors occurring due to temperature effects, improper shielding and grounding of the circuit, and parasitic values. Thus, more accurate measurements can be made using bridge circuits rather than using the meter alone. These bridge circuits are operative from DC level to several GHz level. Hence, these can be used to measure both DC and AC quantities and are thus, named as DC and AC bridges, respectively. **DC bridges** are capable of measuring a wide range of resistances in the DC circuits with a significant precision level. On the other hand, **AC bridges** can measure capacitance, inductance, and frequency in addition to impedance in the AC circuits. Moreover, the measurement of quality factor of the coil and dissipation factor of a capacitor can also be done using AC bridges.

7.2 DC BRIDGES

A very useful way to measure the unknown values of resistances in DC circuits is to use **DC bridge circuits** or **DC bridges**. A bridge circuit is basically a four-armed circuit, each arm containing a component. One of these components is to be measured and the remaining are known to a precise degree. Two arms together form a voltage divider arrangement making it a circuit having two branches with a meter connected between them as a bridge that indicate a balance at zero volt and the branches are connected across the same voltage source. The meter may be a galvanometer, milli-voltmeter, or a micro-ammeter. The most basic configuration of a bridge is shown in Figure 7.1.

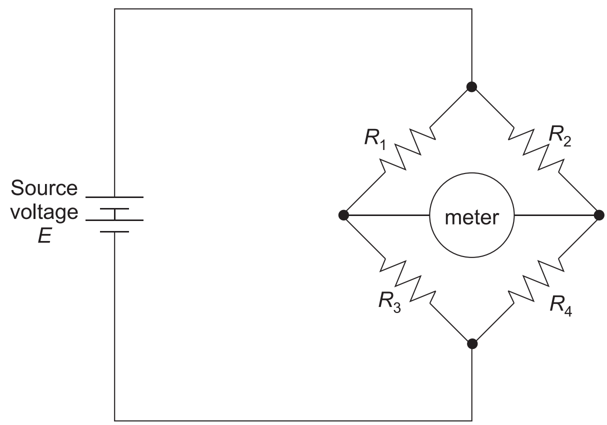


Fig. 7.1 A Basic Bridge Circuit

When the bridge is balanced, the meter indicates zero volt. The balanced condition of a bridge is given in terms of resistances R_1, R_2, R_3, and R_4 as:

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \qquad ...(1)$$

Equation (1) shows that the value of the unknown resistance can be easily obtained from the ratio of the remaining three calibrated resistances which include two fixed resistances and an adjustable resistance. It should be noted here that more the applied voltage, more precise will be the measurement. However, it does not imply that a lower or higher voltage supply would cause fundamental errors. It may be counted as a definite advantage of bridge circuits over other resistance measurement techniques.

There are broadly two types of DC bridge circuits which include *Wheatstone bridge* and *Kelvin bridge*.

7.2.1 Wheatstone Bridge

The most basic DC bridge used to measure the resistance of a DC circuit is known as **Wheatstone bridge**. This bridge provides very accurate measurements. The configuration of a Wheatstone bridge is shown in Figure 7.2 in which two fixed resistances R_1 and R_2, an adjustable resistance R_3, and an unknown resistance R_4 are connected to a constant voltage source or battery. This battery causes the current flow through the resistances. Here, a current sensitive meter, that is, a galvanometer is used to indicate the null deflection.

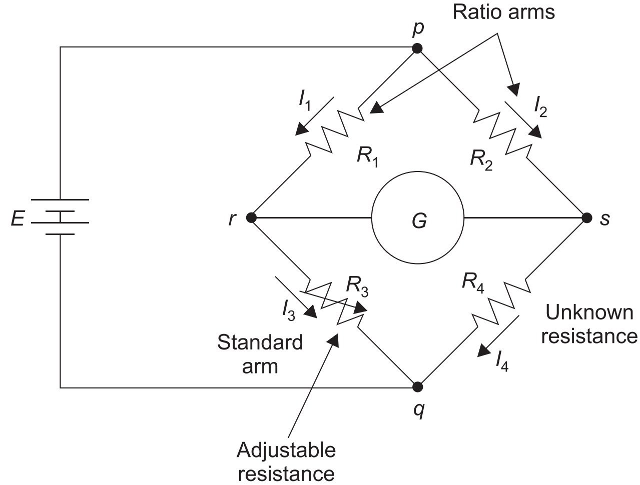


Fig. 7.2 Wheatstone Bridge

The adjustable resistance R_3 is called **standard arm** while the combination of fixed resistances is called **ratio arms** of the bridge. When an unknown resistance R_4 is to be measured, the variable resistance R_3 is adjusted or rotated until a null voltage is indicated on the galvanometer. Another possible configuration of this bridge is shown in Figure 7.3 which shows the respective currents and voltages through each of these resistances.

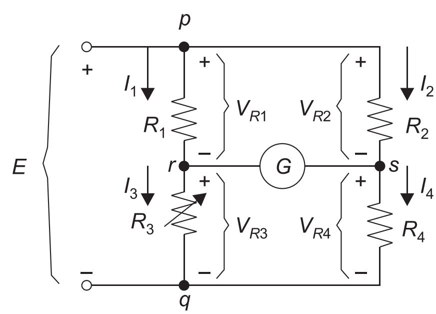


Fig. 7.3 Wheatstone Bridge Showing Currents and Voltages through Resistances

The galvanometer must be initially shunted lest it may be damaged due to extreme levels of current. The shunting resistance of the galvanometer must be steadily increased till the instant zero indication is obtained by the meter with open-circuited resistor, as null is approached. The potential difference between points r and s determines the amount of current flowing through the galvanometer. Thus, a 0 V potential difference yields 0 A current. To attain the zero or null deflection on the meter, the voltage between points r and p is the same as that between points s and p. The same can also be stated between points r and q and points q and s. Therefore, this leads to the conclusion that the essential condition for the bridge to be balanced is given as:

$$I_1R_1 = I_2R_2 \quad ...(2)$$

When a null deflection occurs, the following conditions must also exist:

$$I_1 = I_3 = \frac{E}{R_1 + R_3} \quad ...(3)$$

and

$$I_2 = I_4 = \frac{E}{R_2 + R_4} \quad ...(4)$$

Substituting the values of I_1 and I_2 from Equations (3) and (4) in Equation (2), we get:

$$\frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4}$$

Or

$$R_1R_4 = R_2R_3 \quad ...(5)$$

Equation (5) represents the balance expression of the bridge. The value of the unknown resistance R_4 can be readily obtained if values of all other resistances are known. Thus, it may be written as:

$$R_4 = R_3\frac{R_2}{R_1} \quad \text{or} \quad R_x = R_3\frac{R_2}{R_1} \quad (\text{Since } R_4 \text{ is sometimes represented as } R_x) \quad ...(6)$$

In order to indicate the precise measurements, the galvanometer must be sensitive enough to accurately show the balance position of the bridge. No other characteristics of the meter influence its reading. The sensitivity of the bridge circuit can be defined as the minimum change in the variable resistance that causes the pointer to clearly move from zero position on the scale. The pointer gets back to zero when the variable resistor is readjusted. Mathematically, it can be obtained as:

$$\Delta R_x = \Delta R_3\frac{R_2}{R_1} \quad ...(7)$$

Equation (7) reveals that a change in adjustable resistance R_3 would cause a proportional change in the unknown resistance.

The Wheatstone bridge can precisely measure resistances from 1 Ω to low range of megohms. However, sometimes the limiting error of the known resistances becomes the major source of measurement error. In addition, some other errors which may occur during a measurement are as follows.

- The sensitivity of the meter is unsatisfactory.
- The resistances of the contacts and connecting leads may cause errors in low resistance measurements.
- Generation of thermal emfs in the meter or bridge may introduce errors in low resistance measurement. Sensitive meters are therefore used with copper suspension systems and copper coils to avoid such emfs.

- The current flowing through the resistances of the bridge circuit may generate heat in them. An excessive heat may cause a permanent change in their resistive values. This effect is more serious when small resistances are to be measured. To eliminate this, the current must be limited to a certain level and power dissipation in the bridge arms must be checked prior to the measurements.

Thevenin equivalent circuit

A number of galvanometers are available with different current sensitivities and internal resistances. It is difficult to opt for a particular galvanometer for bridge circuit without performing prior computations regarding its sensitivity since it must possess the required level of sensitivity to detect a condition of imbalance. Thus, the sensitivity must be checked by calculating the current through the meter for a small imbalance. This requires the Thevenin equivalent circuit of the bridge.

The current can now be obtained by looking into terminals *r* and *s* of the circuit shown in Figure 7.1. For this, first remove the galvanometer and calculate the equivalent voltage appearing at these terminals; which comes out as:

$$E_{rs} = E_{pr} - E_{ps}$$

Or

$$E_{rs} = I_1 R_1 - I_2 R_2 \quad ...(8)$$

Here,

$$I_1 = \frac{E}{R_1 + R_3}$$

and

$$I_2 = \frac{E}{R_2 + R_4}$$

Substituting the values of I_1 and I_2 in Equation (8), we get:

$$E_{rs} = E_{Th} = E\left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4}\right) \quad ...(9)$$

Equation (9) represents the required Thevenin equivalent voltage.

Once the voltage is obtained, replace the battery with its internal resistance and find the equivalent resistance of the circuit looking into the same terminals. The circuit can be redrawn as shown in Figure 7.4.

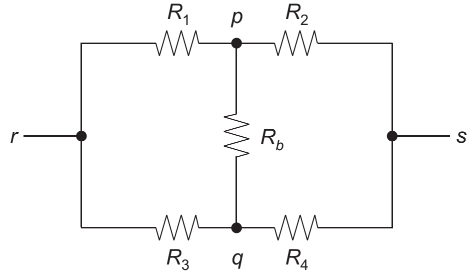


Fig. 7.4 Circuit for Determining Thevenin's Equivalent Resistance

In the figure, the internal resistance of the battery is assumed to be 0 Ω since it is much lower than the bridge resistances. Therefore, if there is a short circuit between terminals *p* and *q*, Thevenin resistance R_{Th} yields:

$$R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4} \qquad ...(10)$$

Now, the circuit of Wheatstone bridge can be converted into its Thevenin equivalent which consists of an emf E_{Th} with its internal resistance R_{Th}. The resultant circuit is shown in Figure 7.5.

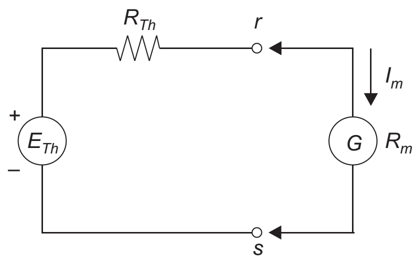


Fig. 7.5 Thevenin's Equivalent Circuit

Notice that the galvanometer is also connected between *r* and *s* terminals of the circuit, through which the current can be calculated as:

$$I_m = \frac{E_{Th}}{R_{Th} + R_m} \qquad ...(11)$$

where I_m and R_m represent the galvanometer current and resistance, respectively. It can be concluded from Equation (11) that the sensitivity of the bridge circuit can be improved either by increasing the Thevenin equivalent emf E_{Th} which can be increased by increasing the supply voltage or by using a more sensitive galvanometer.

From the above discussion it is quite clear that the Thevenin equivalent circuit is a useful method for inspecting the response of a galvanometer. The accuracy of measurements depends upon the sensitivity of the galvanometer as well as upon the accuracies of individual components used in the circuit. The Wheatstone bridge is capable of finding resistance values ranging from a few ohms to several megohms. Here, the upper limit for the measurement is decided by the fact that for high resistance measurement using this method, Thevenin resistance becomes higher, causing a reduced meter current and thereby reduced sensitivity of the meter. On the contrary, if much smaller resistances are measured, the same order contact resistances of the connecting leads and wires may cause some errors. These contact resistances are difficult to determine and therefore, Wheatstone bridge is not suitable for very low resistance measurements. Another bridge circuit, known as Kelvin bridge is used for this purpose.

Example 1 An unbalanced Wheatstone bridge is given with following specifications:

R_1 = 2 kΩ, R_2 = 2.5 kΩ, R_3 = 6 kΩ, R_4 = 8 kΩ, E_b = 5 V, and R_m = 200 Ω

Determine the current flowing through the detector.

Solution: The Thevenin equivalent potential between terminals *r* and *s* can be expressed as:

$$E_{Th} = E\left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4}\right) \qquad \text{[Refer to Eqn. (9)]}$$

Substituting the given values, we get:

$$E_{Th} = 5\left(\frac{2 \times 10^3}{2 \times 10^3 + 6 \times 10^3} - \frac{2.5 \times 10^3}{2.5 \times 10^3 + 8 \times 10^3}\right) = 0.059 \text{ V}$$

Thevenin equivalent resistance can be found as:

$$R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4} \qquad \text{[Refer to Eqn. (10)]}$$

Substituting the values, we obtain:

$$R_{Th} = \frac{2 \times 10^3 \times 6 \times 10^3}{2 \times 10^3 + 6 \times 10^3} + \frac{2.5 \times 10^3 \times 8 \times 10^3}{2.5 \times 10^3 + 8 \times 10^3} = 3.4 \text{ k}\Omega$$

Now, current through the detector can be obtained as:

$$I_m = \frac{E_{Th}}{R_{Th} + R_m} \qquad \text{[Refer to Eqn. (11)]}$$

Substituting the required values, we get:

$$I_m = \frac{0.059}{3.4 \times 10^3 + 0.2 \times 10^3} = 0.016 \ \mu\text{A}$$

7.2.2 Kelvin Bridge Circuit

As explained earlier, Wheatstone bridge circuits are not capable of measuring low resistances. Thus, an improvement was overlaid on these circuits so that they could be used to measure very small values of resistances, generally below 1 Ω. This modified version of Wheatstone bridge is known as **Kelvin bridge** circuit and provides a significant accuracy in low resistance measurements. The basic configuration of a bridge circuit is shown in Figure 7.6 which justifies the concept of a Kelvin bridge circuit.

The circuit shown in the figure is similar to a Wheatstone bridge circuit except that it takes into account the resistances of connecting leads. A connecting lead with its internal resistance R_c is shown between variable resistance R_3 and unknown resistance R_x. Points *a* and *b* indicate two possible connections of the galvanometer. The galvanometer can be connected to either of these points. If it is connected to point *a*, the connecting lead resistance R_c will be added to the unknown resistance R_x indicating a considerably large value of R_x. On the contrary, resistance R_c is added to resistance R_3 which increases the actual value of R_3 for the galvanometer located at point *b*. This in turn causes the meter to indicate a comparatively lower value of resistance R_x.

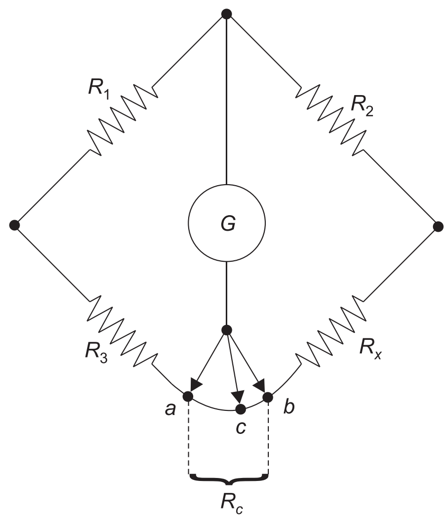


Fig. 7.6 Bridge Circuit Showing Connecting Leads Resistance

Now consider the case when the galvanometer is connected exactly between points a and b, that is, at point c. Let the connection be such that the ratio of resistances from point b to c and from point a to c and of resistances R_2 to R_1 are equal, then we get:

$$\frac{R_{bc}}{R_{ac}} = \frac{R_2}{R_1} \qquad ...(12)$$

If Equation (12) is satisfied, the balance equation can be written for the circuit as:

$$R_x + R_{bc} = \frac{R_2}{R_1}(R_3 + R_{ac}) \qquad ...(13)$$

Thus, we may write as:

$$\frac{R_{ac}}{R_{ac} + R_{bc}} = \frac{R_1}{R_1 + R_2}$$

Or,

$$R_{ac} = \left(\frac{R_1}{R_1 + R_2}\right) R_{ab} \qquad (\text{since } R_{ac} + R_{bc} = R_{ab}) \quad ...(14)$$

and

$$\frac{R_{bc}}{R_{bc} + R_{ac}} = \frac{R_2}{R_1 + R_2}$$

Or,

$$R_{bc} = \left(\frac{R_2}{R_1 + R_2}\right) R_{ab} \qquad ...(15)$$

Substituting the values of R_{ac} and R_{bc} from Equations (14) and (15) into Equation (13), it becomes:

$$R_x + \left(\frac{R_2}{R_1 + R_2}\right) R_{ab} = \frac{R_2}{R_1}\left[R_3 + \left(\frac{R_1}{R_1 + R_2}\right) R_{ab}\right]$$

On solving, it comes out to be:

$$R_x = \frac{R_2}{R_1} R_3 \qquad ...(16)$$

Note that Equation (16) is exactly similar to Equation (6) which represents the balance equation of a Wheatstone bridge circuit. This shows that the impact of connecting leads resistances can be removed when the galvanometer is connected at point *c*.

Kelvin double bridge circuit

The Kelvin bridge circuit shown in Figure 7.6 is practically a difficult approach to obtain the result as it becomes difficult to locate an appropriate point for galvanometer connection. Thus, it is modified by introducing an additional set of ratio arms as illustrated in Figure 7.7. This is known as **Kelvin double bridge circuit** that is signified by the second set of ratio arms in the bridge circuit.

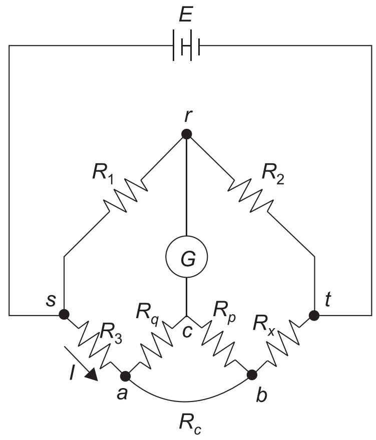


Fig. 7.7 Kelvin Double Bridge Circuit

From the figure, we can say that the new arms *p* and *q* are connected to each other at point *c*. The galvanometer is also connected at the same point. This reduces the error in determining a suitable point for connecting the galvanometer. This configuration ensures that the connecting leads or yoke resistance R_c does not affect the measurements. The prerequisite condition for the circuit is that the ratio of the resistance arms *p* and *q* must be equal to the ratio of R_2 and R_1. Thus, we may write as:

$$\frac{R_2}{R_1} = \frac{R_p}{R_q} \qquad ...(17)$$

The balance condition for the circuit is attained when the potential at point *r* is equal to the potential at point *c*. In other words, the potential difference between points *r* and *s* must be equal to that between points *s* and *c* to achieve balance or null of the meter. Mathematically, it can be written as:

$$E_{rs} = E_{sac}$$

where $$E_{rs} = \left(\frac{R_1}{R_1 + R_2}\right)E$$

and $$E = I\left\{R_3 + R_x + \frac{(R_p + R_q)R_c}{R_p + R_q + R_c}\right\}$$

$$\Rightarrow \qquad E_{rs} = I\left\{R_3 + R_x + \frac{(R_p + R_q)R_c}{R_p + R_q + R_c}\right\}\left(\frac{R_1}{R_1 + R_2}\right) \qquad \text{...(18)}$$

Similarly, $$E_{sac} = I\left[R_3 + \frac{R_q}{R_p + R_q}\left\{\frac{(R_p + R_q)R_c}{R_p + R_q + R_c}\right\}\right]$$

Or, $$E_{sac} = I\left[R_3 + \left(\frac{R_q R_c}{R_p + R_q + R_c}\right)\right] \qquad \text{...(19)}$$

Equating Equations (18) and (19), we can get the value of resistor R_x as:

$$I\left\{R_3 + R_x + \frac{(R_p + R_q)R_c}{R_p + R_q + R_c}\right\}\left(\frac{R_1}{R_1 + R_2}\right) = I\left[R_3 + \left(\frac{R_q R_c}{R_p + R_q + R_c}\right)\right]$$

Or, $$\left\{R_3 + R_x + \frac{(R_p + R_q)R_c}{R_p + R_q + R_c}\right\} = \left(\frac{R_1 + R_2}{R_1}\right)\left[R_3 + \left(\frac{R_q R_c}{R_p + R_q + R_c}\right)\right]$$

$$\left\{R_3 + R_x + \frac{(R_p + R_q)R_c}{R_p + R_q + R_c}\right\} = R_3 + \left(\frac{R_2 R_3}{R_1}\right) + \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_q R_c}{R_p + R_q + R_c}\right)$$

$$\left\{R_3 + R_x + \frac{(R_p + R_q)R_c}{R_p + R_q + R_c}\right\} = R_3 + \left(\frac{R_2 R_3}{R_1}\right) + \left(\frac{R_2}{R_1}\right)\left(\frac{R_q R_c}{R_p + R_q + R_c}\right) + \left(\frac{R_q R_c}{R_p + R_q + R_c}\right)$$

Now, solving for R_x gives:

$$R_x = R_3 + \left(\frac{R_2 R_3}{R_1}\right) + \left(\frac{R_2}{R_1}\right)\left(\frac{R_q R_c}{R_p + R_q + R_c}\right) + \left(\frac{R_q R_c}{R_p + R_q + R_c}\right) - R_3 - \frac{(R_p + R_q)R_c}{R_p + R_q + R_c}$$

Or, $$R_x = \left(\frac{R_2 R_3}{R_1}\right) + \left(\frac{R_2}{R_1}\right)\left(\frac{R_q R_c}{R_p + R_q + R_c}\right) - \frac{(R_p + R_q)R_c}{R_p + R_q + R_c} + \left(\frac{R_q R_c}{R_p + R_q + R_c}\right)$$

Or, $$R_x = \left(\frac{R_2 R_3}{R_1}\right) + \left(\frac{R_2}{R_1}\right)\left(\frac{R_q R_c}{R_p + R_q + R_c}\right) - \left(\frac{R_p R_c}{R_p + R_q + R_c}\right)$$

Taking term $\frac{R_q R_c}{R_p + R_q + R_c}$ common, we get:

$$R_x = \left(\frac{R_2 R_3}{R_1}\right) + \left(\frac{R_q R_c}{R_p + R_q + R_c}\right)\left(\frac{R_2}{R_1} - \frac{R_p}{R_q}\right)$$

Here, using relation from Equation (17) as $\frac{R_2}{R_1} = \frac{R_p}{R_q}$, we obtain:

$$R_x = \frac{R_2 R_3}{R_1} \qquad \text{...(20)}$$

Equation (20) again represents the balance equation of a Wheatstone bridge circuit and ensures that the resistance of connecting leads does not affect the measurements. Thus, this equation is also referred to as working equation for Kelvin bridge.

Kelvin bridge is a useful tool for measuring resistances of extremely low values ranging from as low as 10 μΩ to 1 Ω with an accuracy of about ±0.2% (depending on the accuracies of individual components). The required circuit for such measurements is shown in Figure 7.8. In addition, even smaller resistances, say 0.1 μΩ can also be measured with this bridge at the cost of reduced accuracy.

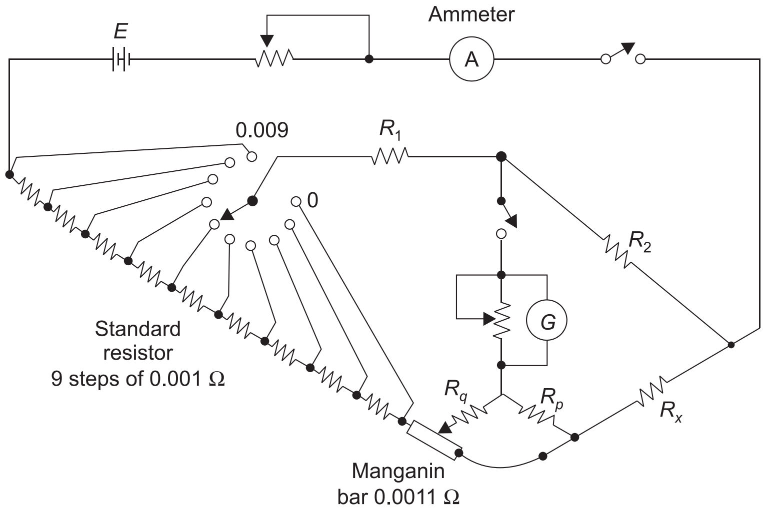


Fig. 7.8 Kelvin Double Bridge Circuit for Extremely Low Resistance Measurements

In the figure, resistance R_3 is represented by a standard variable resistance having a number of decade steps. This reduces the possibility of large errors that might be generated due to contact potential drops. It can be seen that there are nine steps starting from 0.001 Ω to 0.009 Ω, each step being of 0.001 Ω. Resistances R_1 and R_2 can be switched to any of these steps. However, the ratio of these two resistances must be so selected that the appreciable amount of standard resistance is used. This ensures more precise and accurate measurements. A manganin bar with a sliding contact, having a resistance of 0.0011 Ω, is also incorporated in the circuit. This makes a total resistance of 0.0101 Ω in the R_3 arm of the circuit which can be varied in steps of 0.001 Ω along with sliding contact fractions of 0.0011 Ω. If both contacts are switched in order to select an appropriate value of standard resistance, the voltage difference between the points of connection of ratio arm gets varied. However, this does not alter the total resistance in the battery circuit. In this way, the resistance in the ratio arms gets connected in series with contact resistance. Note that the ratio arms resistance is relatively higher than the contact resistance. Thus, the effect of contact resistance is made negligibly small.

Example 2 A balanced Kelvin bridge (see *Figure 7.7*) is given with following specifications: $R_1 = R_2 = R_p = R_q = 1.2$ kΩ, $R_x = 0.001$ Ω, $R_m = 400$ Ω, $E = 100$ V with source resistance $R_b = 10$ Ω.

Determine:

(a) resistance R_3

(b) current flowing through R_3

Solution:

(a) The unknown resistance R_x can be found using the following relation:

$$R_x = \frac{R_2 R_3}{R_1} \qquad \text{[Refer to Eqn. (20)]}$$

Substituting the given values, we obtain:

$$0.001 = \frac{1.2 \times 10^3 R_3}{1.2 \times 10^3}$$

On solving, this gives:

$$R_3 = 0.001\ \Omega$$

(b) For calculating the current through R_3, we must know the value of battery current. Therefore, let us first determine I_b as:

$$I_b = \frac{E}{R_b + R_3 + R_x} \qquad \text{[Refer to Figure 7.7]}$$

Substituting the required values, we obtain:

$$I_b = \frac{100}{10 + 0.001 + 0.001} = 9.99\,\text{A}$$

Now, current I can be found as:

$$I = \frac{I_b(R_1 + R_2)}{(R_1 + R_2) + (R_3 + R_x)} \qquad \text{[Refer to Figure 7.7]}$$

Substituting the required values, we get:

$$I = \frac{9.9(1.2\times10^3 + 1.2\times10^3)}{(1.2\times10^3 + 1.2\times10^3) + (0.001 + 0.001)} = 9.9 \text{ A}$$

7.2.3 High Resistance Measurement

The Wheatstone and Kelvin bridges studied earlier cannot be used for measuring high resistances in the range of hundreds or thousands of megaohms. However, it is often customary to find high resistances such as leakage resistance of a capacitor, insulation resistance of a cable, or surface or volume resistivity of a material. It is difficult when a leakage current gets induced in the instrument, or at the binding posts connecting the component to the instrument, or in the component under measurement. This leakage current depends upon the atmosphere and hence varies with the humidity conditions. This undesired current may enter the measuring circuit and greatly hamper its accuracy. This is more profound in high resistance measurements where satisfactory deflection sensitivity requires high voltages. Thus, it needs to be dealt with effectively. For this, special circuits called **guard circuits** are employed in the unknown resistance arm of the Wheatstone bridge as shown in Figure 7.7.

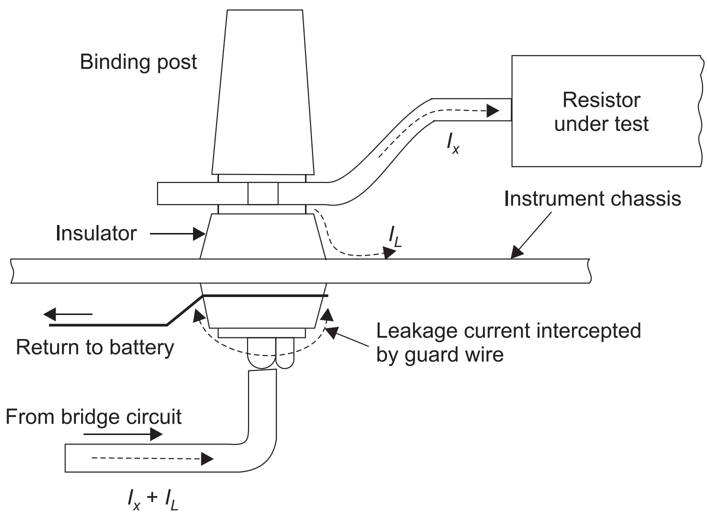


Fig. 7.9 Basic Concept of a Guard Circuit

It can be seen from the figure that a leakage current I_L flows along the insulated surface of the binding post and another current I_x flows through the unknown resistor. In the absence of the guard circuit, I_L gets added to I_x resulting in a significantly larger total circuit current than the actual device current. However, when a guard wire is placed in such a way that it surrounds the insulated surface of the binding post, it catches the leakage current I_L and returns it to the battery. In this way, the leakage current cannot enter the bridge circuit and affect its measurements. Note that the guard wire must be placed such that the leakage current always meets some portion of it.

Guarded Wheatstone bridge

The Wheatstone bridge circuit which consists of a guard wire around the binding post of its unknown resistance R_x is known as a **guarded Wheatstone bridge**. However, it is equally applicable on any internal part of the bridge which suffers leakage. The other end of the guard wire is directly connected to the source battery terminal. Note that the wire should not be in contact with any other part of the circuit. Figure 7.10 shows the basic configuration of a guarded Wheatstone bridge circuit. As can be seen, a guard wire is connected to the terminal of the unknown resistance R_x at one of its ends and to the battery at its other end which intercepts leakage current and sends it back to the battery.

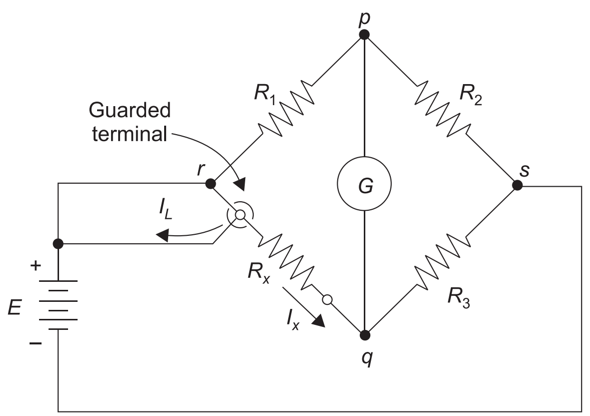


Fig. 7.10 Basic Guarded Wheatstone Bridge Circuit

Now consider Figure 7.11 in which a three-terminal resistance having a high resistance is used. Terminal p, that is, the junction of ratio arms R_1 and R_2, is generally extended to the front panel of the instrument and considered as a guard terminal in order to eliminate the

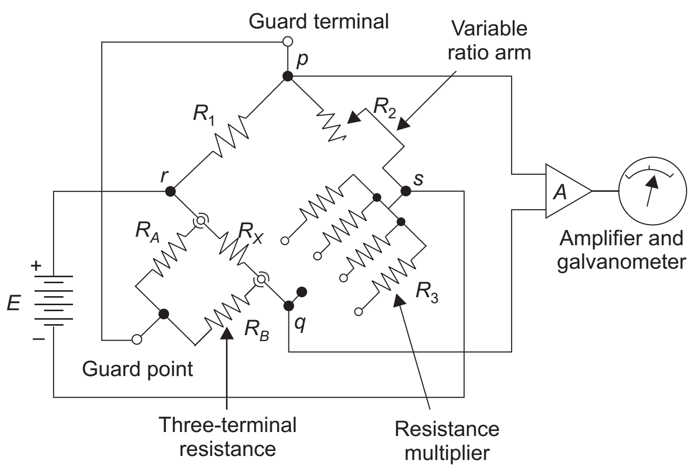


Fig. 7.11 Guarded Wheatstone Bridge Circuit Incorporating a Three-terminal Resistance

effects of external leakage current. This guard terminal is connected to the three-terminal resistance which is mounted over two insulated posts which are tied up with a metal plate. The two main terminals of this resistance are connected to the terminals of the unknown resistance R_x. Two additional resistances R_A and R_B are connected to these terminals as shown in the figure. These additional resistances represent the paths of leakage current via insulating posts to the metal plate or guard and their common point is the third terminal of the three-terminal resistance. This terminal is referred to as **guard point** and is connected to the guard terminal on the front panel of the bridge.

The resistance R_A is connected to bridge resistance R_1 in parallel but since R_A is comparatively much larger than R_1, the shunting effect becomes insignificant. In the same way, resistance R_B is connected with the galvanometer in parallel. Due to high value of the resistance R_B, the shunt effect has no significance, however, it reduces the galvanometer sensitivity to some extent. Therefore, the external leakage paths are eliminated using this circuit. Thus, it may be concluded that in the absence of guard circuit, leakage resistances R_A and R_B would be connected directly across the unknown resistance R_x resulting in measurement errors.

7.3 RESISTANCE MEASUREMENT USING MEGGER

Megger, or megohmmeter, is an instrument that is used to measure extremely high resistances, such as insulation resistances of cables of the order of mega ohms. It is capable of measuring resistances with high potential or breakdown voltage. It is also used for testing purpose in electrical power systems to test ground, short circuit and continuity conditions. It works on the basic principle that when two coils perpendicular to each other are suspended in a magnetic field, the resulting deflection depends upon the ratio of the coil currents. Figure 7.12 depicts the circuit of a megger.

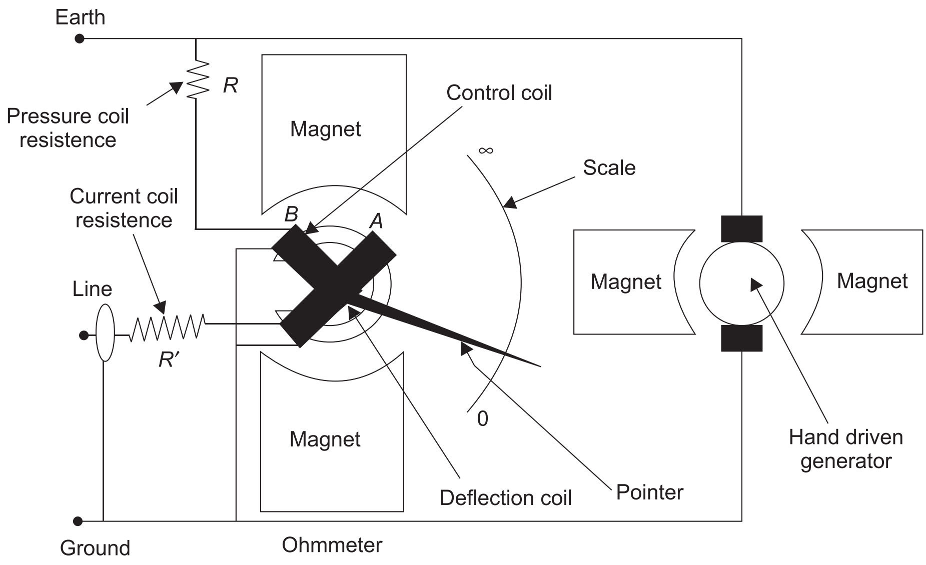


Fig. 7.12 Megger Circuit

It can be seen from the figure that the circuit incorporates a guard wire at its *line* terminal so that the leakage current may not produce measurement errors. The major elements of the circuit are an ohmmeter and a hand driven DC generator. The generator is meant to supply the current necessary for measurements while the ohmmeter reads the value of resistance under measurement. The moving part of the instrument consists of two coils (marked as A and B in the figure) which are mounted on the same spindle such that they are perpendicular to each other. This arrangement is suspended in the magnetic field produced by a permanent magnet where it is free to move as a single unit over the C-shaped core.

As long as the generator is not supplying any current, the pointer is free to move and can point at any position on the scale. When an operating current is provided by the generator, it flows through coil B which endeavours to settle down at an angle of 90° with respect to the field of the permanent magnet. If the test terminals are open, coil A does not carry any current. Hence, the movement of the entire moving element is controlled by coil B which attains a position such that the pointer moves in a counter-clockwise direction and indicates infinity on the scale. On the other hand, when the test terminals are short-circuited, a current starts flowing through coil A which in turn generates a clockwise torque to overcome the torque generated by coil B. Now, the pointer moves in clockwise direction and indicates a zero. Under such condition, resistance R is used to protect coil A against the excess current while resistance R' is used to protect the instrument.

Now, consider the case when an unknown resistance is connected between test terminals, *earth* and *line* of the instrument. Both the coils tend to move the pointer in opposite directions, as a consequence of which the pointer rests at a position where the forces of coils A and B are balanced. This position on the scale gives the value of the unknown resistance. It should be noted here that any changes in the voltage affect coil A and B equally and thus, the position of the moving element of the instrument remains unaffected by such changes.

7.4 MEASUREMENT OF EARTH RESISTANCE: FALL-OF-POTENTIAL METHOD

All electrical systems must incorporate parts like switch boxes, metal conduits, machine casings, transformer tanks, and circuit breakers which must be connected to ground or earth potential. This is to ensure that in case a fault occurs, the fault current flows directly to the earth or ground and protects the system as well as the operator. The earth electrodes may be used for this purpose as they provide a low resistance path for leakage current so that it can flow to the earth. This leakage current trips the circuit breaker and hence isolates the faulty circuit from the supply. Thus, whenever a situation of overvoltage occurs, the earthed parts of the system do not possess extremely high potentials and are protected from electric shocks, machinery damage, or fire.

The grounding method uses a large electrode of metal, a metal plate, or several electrodes connected together forming a pattern buried under the ground. The low resistance path of this grounding system must be checked at the time of installation as well as at regular intervals to ensure that it can carry large amounts of current without degradation. This is achieved by measuring resistance of the electrode since it is difficult to measure the current carried by it. The typical value of ground resistance is set up to 5 Ω. There are several factors upon which the resistance of a grounding system depends, which are:

- The depth of the electrodes buried under the ground.
- The specific resistance of the soil around the electrode. It varies for different type of soils and moisture level in the soil which in turn varies the electrode resistance.
- The material used in manufacturing of electrodes, and their shape.

The resistance of the earth is measured by *fall-of-potential* test method. This method demonstrates the ability of a ground system or an electrode to carry leakage current. Figure 7.13 illustrates the method of fall-of-potential.

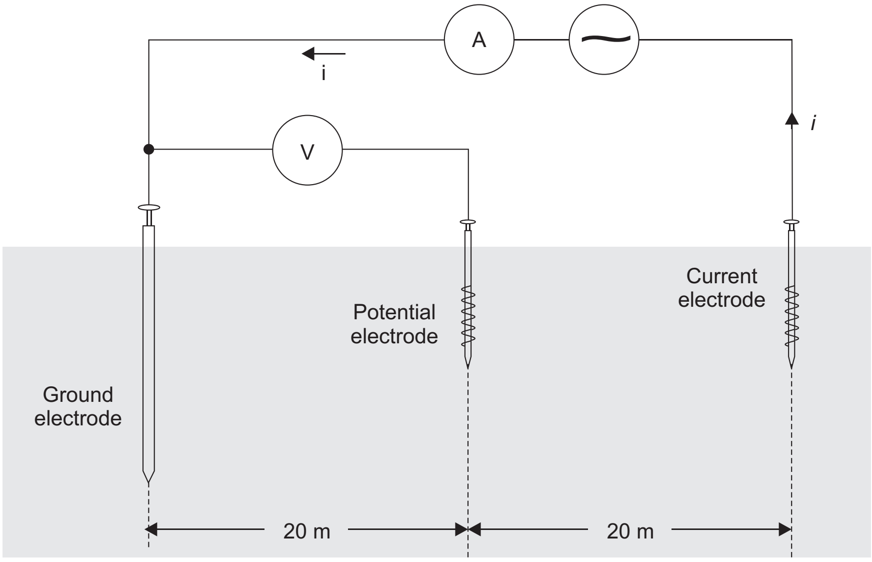


Fig. 7.13 Illustration of Fall-of-Potential Method

It can be seen from the figure that three electrodes, namely, *ground electrode*, *potential electrode*, and *current electrode* are placed equally apart under the ground. The distance between the electrodes depends upon the system used as well as upon the depth of the ground electrode under the ground. An AC voltage is applied such that a measureable current gets produced between the ground electrode and current electrode and is measured by the connected ammeter. A voltmeter is connected between the ground electrode and potential electrode which is meant to measure the potential drop between the two electrodes. This potential drop is then divided by the ammeter current to obtain the ground resistance.

The current density increases in ground and current electrodes as current electrode is moved towards the ground electrode which in turn causes distortion in resistance measurement. The potential electrode, apparently carries no current but a small current to operate the voltmeter. Thus, potential electrode can be moved towards either electrode to some extent without causing any distortion to the measured resistance.

It can be seen from the graph (shown in Figure 7.14) that at the initial point, the ground resistance increases abruptly and attains a constant value when the distance between the ground and potential electrodes is increased. At the instant, potential electrode approaches current

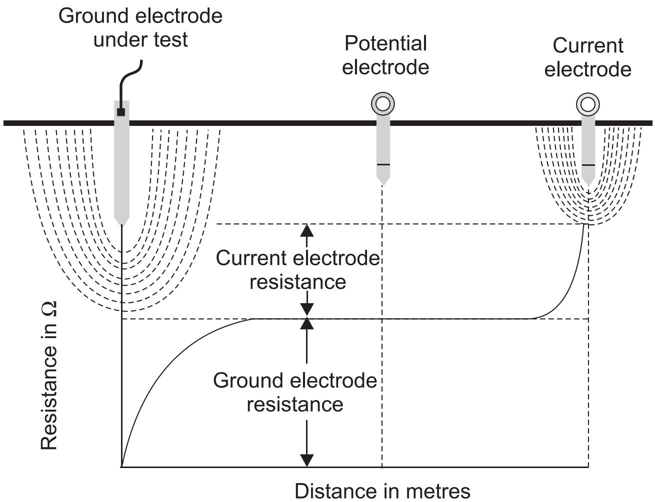


Fig. 7.14 Resistance Versus Distance between Ground and Potential Electrodes

electrode, the resistance again rises rapidly. This reveals that the ground resistance varies with the distance of potential electrode with respect to ground electrode. For different positions of potential electrode, measurements of the earth resistance are made.

The electrodes must be placed at appropriate positions, otherwise serious errors may be produced. The potential electrode should be placed at a distance for which a constant resistance can be obtained. In addition, the current electrode must be placed at a sufficient distance from potential electrode to achieve error-free measurements. The distance may be as large as a few hundred metres for a low resistance.

7.5 AC BRIDGES

AC bridges are used to determine various AC quantities, such as self and mutual inductance, capacitance, dissipation factor (D) of a capacitance, and quality factor (Q) of an inductance. The AC bridges use an AC supply voltage as well as an AC null detector instrument. A basic AC bridge circuit is shown in Figure 7.15 which consists of impedances instead of resistances.

It can be seen from the figure that an AC supply feeds the bridge containing an impedance in each of its arms. An electronic galvanometer, a tunable amplifier detector, an oscilloscope, or a pair of headphones can be used as an AC null detector or meter. The procedure to balance an AC bridge is somewhat complex as compared to that of a DC bridge. The voltage across terminals 1 and 2 must be zero so that no current flows through the meter and it indicates zero. For this, the voltage across impedances Z_1 and Z_2 must have the same amplitude and phase. This is equally true for impedances Z_3 and Z_4. Thus, we may write the balance condition of the bridge as:

$$V_{Z1} = V_{Z2} \quad \text{or} \quad i_A Z_1 = i_B Z_2 \qquad ...(21)$$

and

$$V_{Z3} = V_{Z4} \quad \text{or} \quad i_A Z_3 = i_B Z_4 \qquad ...(22)$$

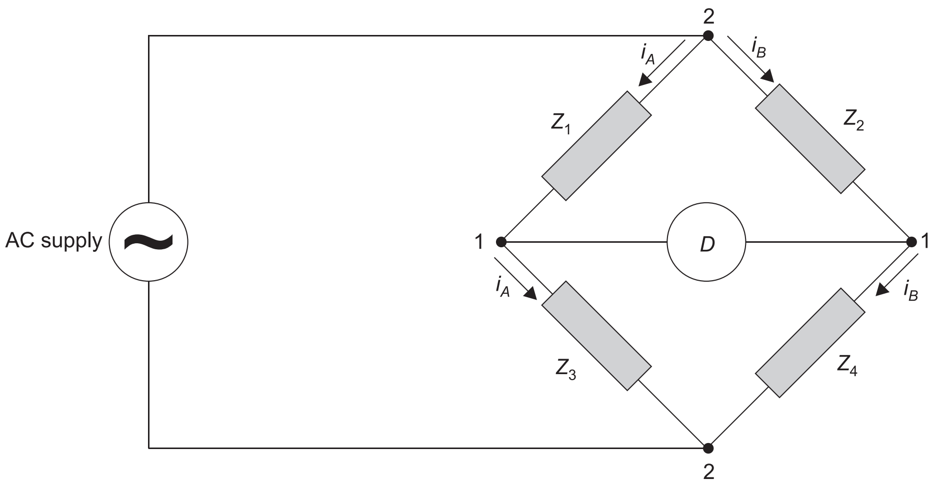


Fig. 7.15 A Basic AC Bridge Circuit

Dividing Equation (21) by (22), we get:

$$\frac{i_A Z_1}{i_A Z_3} = \frac{i_B Z_2}{i_B Z_4} \Rightarrow Z_1 Z_4 = Z_2 Z_3 \qquad ...(23)$$

Similar results can be obtained if admittances Y were used instead of impedances Z. Thus, we may write:

$$Y_1 Y_4 = Y_2 Y_3 \qquad ...(24)$$

Equations (23) and (24) are known as **balance equations of an AC bridge**. Here, it should be noted that the real and imaginary parts of impedances and admittances must be taken into account while using these equations. This leads to the formation of two balance equations since equating real parts yields one equation and equating imaginary parts results in another equation.

Note: *The impedances and admittances are easily expressed in rectangular form instead of polar form.*

Null deflection

In AC bridges, the balance condition through a null detector can be obtained by adjusting two components. These two adjustments are dependent on each other. To understand this concept, consider the circuit shown in Figure 7.16.

As shown in the figure, impedance Z_4 consists of a series combination of a variable resistor R_4 and a variable capacitor C_4. Both of these components need to be adjusted to make voltages across impedances Z_3 and Z_4 equal in amplitude as well as in phase. Adjusting C_4 may result in equal amplitude of both voltages but the phase may still differ. In the same way, adjusting R_4 may bring the voltages in phase but it may alter previously adjusted magnitude.

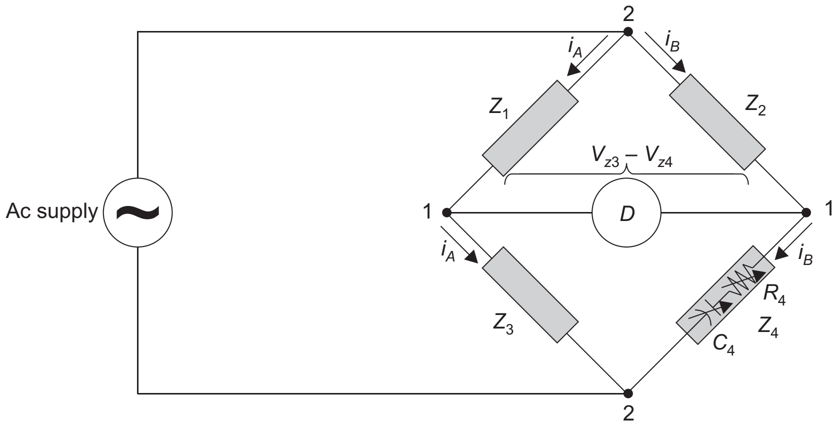


Fig. 7.16 Modified AC Bridge to Obtain Balance Condition

Let the capacitor be varied first so that the amplitude of V_{Z4} becomes equal to that of V_{Z3}. At this moment, the meter would deflect the potential difference between both the voltages, that is, $V_{Z3} - V_{Z4}$ since the phase is still different. Now, when the resistance R_4 is adjusted, it results in the reduced phase difference which thereby reduces the overall potential difference. These adjustments must be repeated over and over again to obtain the null deflection (or smallest possible indication) through the meter. The variation in V_{Z3}, V_{Z4}, and $V_{Z3} - V_{Z4}$ are shown in steps as C_4 and R_4 are adjusted.

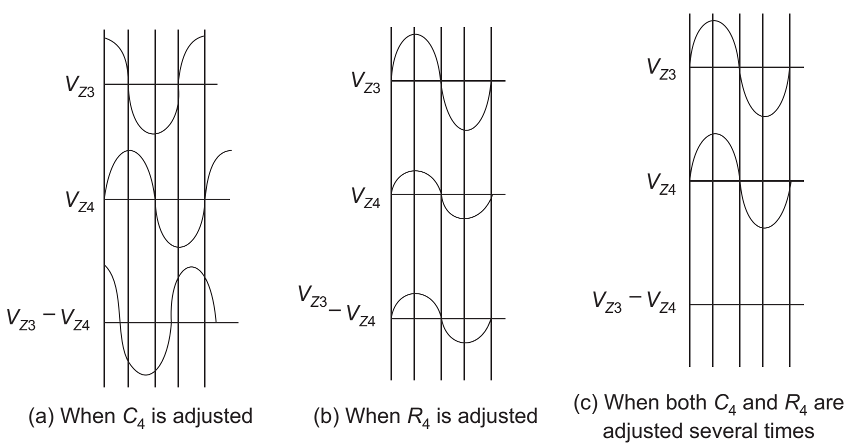


Fig. 7.17 Effect of Adjusting C_4 and R_4 to Obtain Null Deflection

The accuracy and sensitivity considerations of an AC bridge are exactly similar to DC bridges. Similar analytical approach is used to examine these factors.

7.5.1 Resistance Capacitance Bridges—Capacitance Measurement

The bridges consisting of combinations of resistances and capacitances in one or more of their arms are termed as **resistance-capacitance bridges** or **RC bridges**. These bridges are meant to measure the unknown capacitance in one of their arms. *De Sauty bridge*, *modified*

De Sauty bridge, *parallel-resistance capacitance bridge*, *Schering bridge*, and *Wien bridge* are all various categories of RC bridges.

Before going into the detail of RC bridges, let us discuss the terms *power factor* and *dissipation factor*. **Power factor** of a series RC circuit is defined as the cosine of the phase angle between resistance and impedance of the circuit. Mathematically, it is given as:

$$\text{PF} = \frac{R_s}{Z_x}$$

When the phase angle approaches 90°, the reactance becomes almost equal to the impedance. The power factor PF can be given as:

$$\text{PF} = \frac{R_s}{X_x} = \omega C_x R_s \qquad ...(25)$$

Now, a capacitor can be judged on its quality in terms of its power dissipation. An extremely pure capacitor has practically zero power dissipation while a lossy capacitor dissipates some power. Thus, the quality of a capacitor can be defined by the **dissipation power factor**. At a given frequency, the dissipation power factor of a capacitor can be expressed as the ratio of reactance to resistive component of the impedance. It can be specifically given for both series and parallel RC circuits as follows.

For series configuration:

$$D = \frac{R_s}{X_x} = \omega C_x R_s \qquad ...(26)$$

Equation (26) indicates that dissipation power factor of a series RC circuit is exactly the same as that of its power factor.

For shunt configuration:

$$D = \frac{X_x}{R_{sh}} = \frac{1}{\omega C_x R_{sh}} \qquad ...(27)$$

It should be noted that R_{sh} in Equation (27) must be significantly larger than $\frac{1}{\omega C_x}$ in order to obtain considerably low dissipation power factor.

The value of dissipation power factor depends upon frequency and lies in the range of 0.1 (in electrolytic capacitors) to even less than 10^{-4} (in capacitors with plastic film dielectrics).

Now, let us discuss each one of RC bridges in detail and obtain their balance equations.

De Sauty bridge

De Sauty bridge is also known as **simple capacitance bridge** and is used to measure an unknown capacitance value whose configuration is shown in Figure 7.18.

From the figure, we can say that the circuit of a De Sauty bridge consists of two purely capacitive impedances, Z_1 and Z_x (with zero leakage current through dielectrics) and two

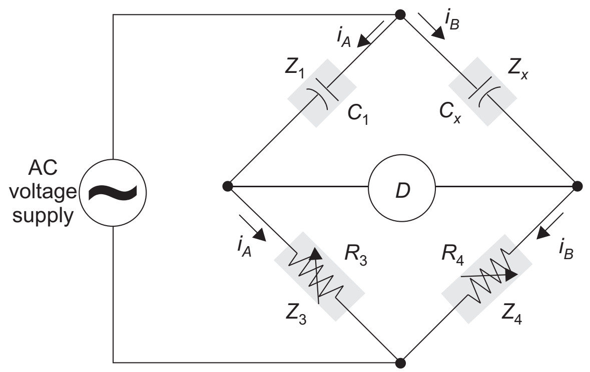


Fig. 7.18 De Sauty Bridge

purely resistive impedances, Z_3 and Z_4. Notice that Z_1 is a standard capacitance while Z_x is an unknown capacitance to be measured. Also, Z_3 and Z_4 are known as variable resistances which can be replaced by a voltage divider resistance box if their ratio is given. Thus, in complex notation, we may write as:

$$Z_1 = -\frac{j}{\omega C_1}$$

$$Z_x = -\frac{j}{\omega C_x}$$

$$Z_3 = R_3$$

and $$Z_4 = R_4$$

For the bridge to balance, it must hold the relation:

$$Z_1 Z_4 = Z_x Z_3 \qquad \text{[Refer to Eqn. (23)]}$$

Substituting the values of impedances in the above relation, we get:

$$-\frac{j}{\omega C_1} R_4 = -\frac{j}{\omega C_x} R_3$$

Or, $$C_x R_4 = C_1 R_3$$

$\Rightarrow$ $$C_x = \frac{C_1 R_3}{R_4} \qquad \text{...(28)}$$

Now consider that a resistance is connected in parallel or series with the unknown capacitance while the rest of the circuit remains the same. This combination is equivalent to the circuit of a leaky capacitor. The leakage currents so produced will cause currents i_A and i_B to go out of phase, which in turn causes the potential drops $i_A R_3$ and $i_B R_4$ to become out of phase with each other. Thus, it is impossible to achieve the balance condition in such a case. Hence, it may be concluded that a De Sauty bridge should be employed only to determine high resistance dielectric capacitors.

Example 3 A De Sauty bridge has a standard capacitance of 0.1 μF and the adjustable range of R_3 and R_4 from 1 kΩ to 200 kΩ. What are the minimum and maximum values of capacitance that can be measured using this bridge?

Solution: Given that: $C_1 = 0.1\ \mu F$, $R_{3\,min} = R_{4\,min} = 1\ k\Omega$, $R_{3\,max} = R_{4\,max} = 200\ k\Omega$

The unknown capacitance C_x can be measured as:

$$C_x = \frac{C_1 R_3}{R_4} \qquad \text{[Refer to Eqn. (28)]}$$

Thus, the maximum measurable value of C_x can be determined as:

$$C_{x\max} = \frac{C_1 R_{3\max}}{R_{4\min}}$$

On substituting the required values, it gives:

$$C_{x\max} = \frac{0.1 \times 10^{-6} \times 200 \times 10^3}{1 \times 10^3} = 20\ \mu F$$

The minimum measurable value of C_x can be determined as:

$$C_{x\min} = \frac{C_1 R_{3\min}}{R_{4\max}}$$

Substituting the required values, we get:

$$C_{x\min} = \frac{0.1 \times 10^{-6} \times 1 \times 10^3}{200 \times 10^3} = 500\ pF$$

Modified De Sauty bridge

The De Sauty bridge studied earlier is used to measure the capacitance with zero leakage current. However, if there is some leakage current flowing through the capacitor with a high-resistance dielectric, the bridge is modified as shown in Figure 7.19. This modified circuit is also known as **series-resistance capacitance bridge**.

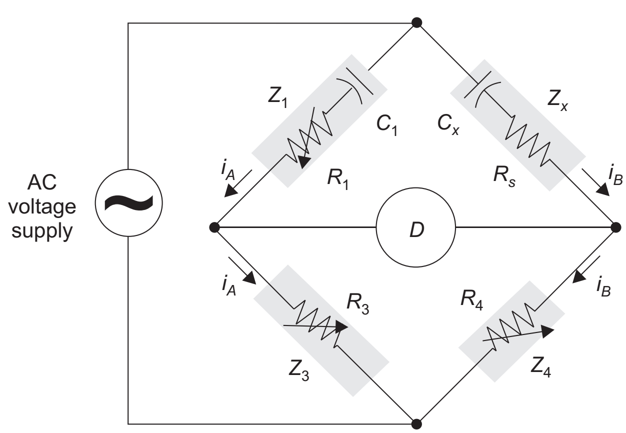


Fig. 7.19 Modified De Sauty Bridge

The $R_s C_x$ combination in the circuit represents the high-resistance dielectric capacitance possessing low dissipation factor and an extremely low leakage current. Note that C_x is a pure capacitance. The standard variable resistance R_1 is connected serially with standard capacitance C_1. Under balance condition, the potential drop across this resistance R_1 balances the potential drops across Z_x branch. The inconvenient small values of R_1 can be eliminated by connecting another resistance R_s in Z_x branch to increase its total resistive component. It should be noted here that when each capacitance branch possesses a considerable resistive component, the minimum voltage deflection by the null detector can be achieved easily. Variable resistances R_3 and R_4 are alternatively adjusted along with R_1 in order to attain balance of the bridge. Here, we know that the balance occurs when the following condition is satisfied:

$$Z_1 Z_4 = Z_x Z_3 \qquad \text{[Refer to Eqn. (23)]}$$

In complex notation, it can be written as:

$$\left(R_1 - \frac{j}{\omega C_1}\right) R_4 = \left(R_s - \frac{j}{\omega C_x}\right) R_3$$

Now, separating and equating real and imaginary parts of the above equation, we get:

$$R_s = \frac{R_1 R_4}{R_3} \quad \text{and} \quad C_x = \frac{C_1 R_3}{R_4} \qquad \text{...(29)}$$

Phasor diagram

Under the balanced condition, we know that the voltage drop across opposite arms of the bridge must be the same in amplitude as well as in phase. This implies:

$$V_{Z1} = V_{Zx} \quad \text{and} \quad V_{Z3} = V_{Z4}$$

Or,

$$i_A R_1 = i_B R_s \quad \text{and} \quad i_A R_3 = i_B R_4$$

The relationship between voltage drops in a modified De Sauty bridge can be shown in a phasor diagram as shown in Figure 7.20.

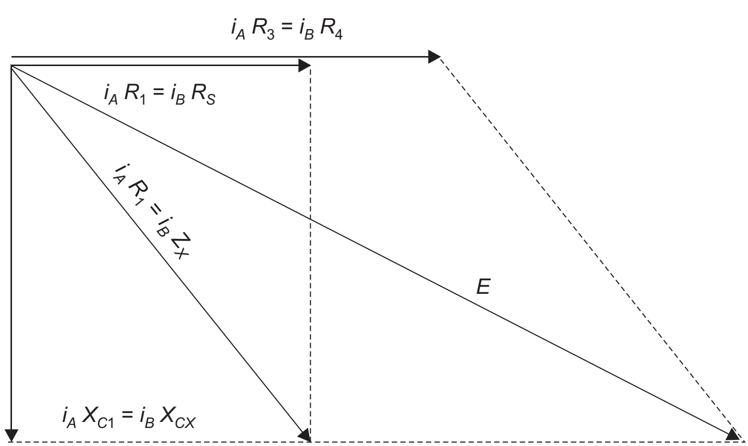


Fig. 7.20 Phasor Diagram for Modified De Sauty Bridge

The voltage drops across pure resistances R_3 and R_4 are equal in phase and amplitude. The current flowing through them is in phase with their respective voltage drops, that is i_A is in phase with $i_A R_3$ while i_B is in phase with $i_B R_4$. However, the case would not be the same with impedances Z_1 and Z_x since they comprise both resistance and capacitance. The current i_A and i_B through resistances R_1 and R_S are equal to voltage drops $i_A R_1$ and $i_B R_s$, respectively, as shown in the figure. Now, as C_1 and C_x are purely capacitive, currents i_A and i_B through them lead their respective capacitive voltage drops $i_A X_{C1}$ and $i_B X_{Cx}$ by 90°. The phasor sum of resistive and capacitive voltages $i_A R_1$ and $i_A X_{C1}$ is the voltage drop across impedance Z_1, that is, $i_A Z_1$, while that of $i_B R_2$ and $i_B X_{Cx}$ is the voltage drop across Z_x, that is, $i_B Z_x$. The phasor sum of $i_A Z_1$ and $i_A R_3$, or $i_B R_4$ and $i_B Z_x$ gives the supply voltage E.

Example 4 A modified De Sauty bridge is operating at a frequency of 120 Hz and has C_1 = 0.5 μF and R_3 = 12 kΩ. The bridge attains balance when R_1 = 130 Ω and R_4 = 15 kΩ.

Determine:

(a) capacitive impedance

(b) dissipation factor

Solution: Given that: R_1 = 130 Ω, C_1 = 0.5 μF, R_3 = 12 kΩ, R_4 = 15 kΩ, and f = 120 Hz

(a) The capacitance under measurement C_x can be obtained as:

$$C_x = \frac{C_1 R_3}{R_4} \qquad \text{[Refer to Eqn. (29)]}$$

Substituting the required values, we get:

$$C_x = \frac{0.5 \times 10^{-6} \times 12 \times 10^3}{15 \times 10^3}$$

This gives:

$$C_x = 0.4 \ \mu\text{F}$$

The unknown resistance R_s can be found as:

$$R_s = \frac{R_1 R_4}{R_3} \qquad \text{[Refer to Eqn. (29)]}$$

Substituting the required values, it becomes:

$$R_s = \frac{130 \times 15 \times 10^3}{12 \times 10^3} = 162.5 \ \Omega$$

(b) The dissipation factor D can be calculated as:

$$D = \omega C_x R_S = 2\pi f C_x R_S \qquad \text{[Refer to Eqn. (26)]}$$

On substituting the required values, we get:

$$D = 2\pi \times 120 \times 0.4 \times 10^{-6} \times 162.5$$

Or,

$$D = 0.049$$

Parallel-resistance capacitance bridge

Parallel-resistance capacitance bridge circuit is the most appropriate bridge when capacitances with low dielectric resistance are to be measured. Such capacitances have high dissipation factor and a reasonably high leakage current. In this bridge circuit, the unknown capacitor is represented by a parallel combination of pure capacitance C_x and resistance R_{sh}. The circuit of such a bridge is shown in Figure 7.21.

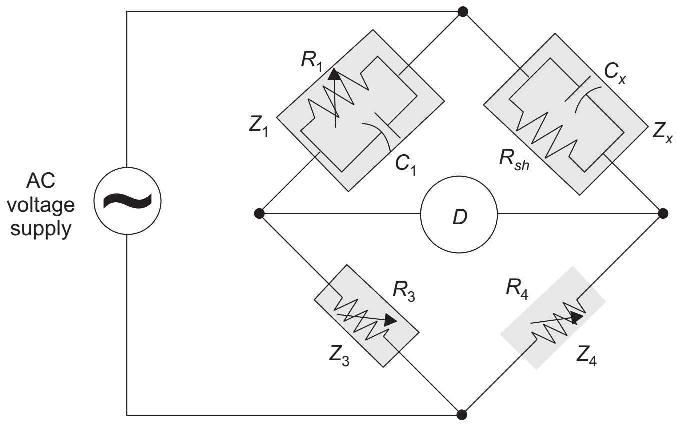


Fig. 7.21 Parallel-Resistance Capacitance Bridge Circuit

The standard capacitance C_1 is also connected in parallel with variable resistance R_1. The resistances R_3 and R_4 are configured in the same way, they were configured in the case of a series-resistance capacitance bridge. These resistances are adjusted alternatively along with R_1 to balance the bridge. We know that at balance the following condition is satisfied:

$$Z_1 Z_4 = Z_x Z_3 \qquad \text{[Refer to Eqn. (23)]}$$

In complex notations, these impedances may be written as:

$$\frac{1}{Z_1} = \frac{1}{R_1} - \frac{1}{j/\omega C_1}$$

Or,

$$\frac{1}{Z_1} = \frac{1}{R_1} + j\omega C_1$$

Thus,

$$Z_1 = \frac{1}{1/R_1 + j\omega C_1}$$

Similarly, Z_x can be given as:

$$Z_x = \frac{1}{1/R_{sh} + j\omega C_x}$$

While Z_3 and Z_4 are purely resistive, thus, we may write as:

$$Z_3 = R_3$$

$$Z_4 = R_4$$

Substituting these relations in the balance equation, we get:

$$\frac{R_4}{1/R_1 + j\omega C_1} = \frac{R_3}{1/R_{sh} + j\omega C_x}$$

Or, $$R_3(1/R_1 + j\omega C_1) = R_4(1/R_{sh} + j\omega C_x)$$

Equating and separating real and imaginary terms, we get:

$$R_{sh} = \frac{R_1 R_4}{R_3} \quad \text{and} \quad C_x = \frac{C_1 R_3}{R_4} \qquad ...(30)$$

Equation (30) reveals that the result is exactly the same using series-resistance or parallel-resistance capacitance bridges.

Note: *The capacitances with moderate leakage resistance should be measured by a parallel-resistance capacitance bridge circuit since it provides a direct deflection of leakage resistance capacitance.*

Example 5 A parallel-resistance capacitance bridge possess C_1 = 0.2 μF and R_3 = 12 kΩ. The bridge achieves balance when f = 150 Hz, R_1 = 345 kΩ, and R_4 = 15 kΩ. Determine:

(a) resistive and capacitive components

(b) dissipation factor

Solution:

(a) The unknown capacitance C_x can be measured as:

$$C_x = \frac{C_1 R_3}{R_4} \qquad \text{[Refer to Eqn. (30)]}$$

Substituting the required values, we get:

$$C_x = \frac{0.2 \times 10^{-6} \times 12 \times 10^3}{15 \times 10^3}$$

On solving, it gives:

$$C_x = 0.16\ \mu\text{F}$$

The unknown resistance R_{sh} can be found as:

$$R_{sh} = \frac{R_1 R_4}{R_3} \qquad \text{[Refer to Eqn. (30)]}$$

Substituting the required values, we get:

$$R_{sh} = \frac{345 \times 10^3 \times 15 \times 10^3}{12 \times 10^3} = 431.25\ \text{k}\Omega$$

(b) The dissipation factor D of the capacitor can be determined as:

$$D = \frac{1}{\omega C_x R_{sh}} = \frac{1}{2\pi f C_x R_{sh}} \qquad \text{[Refer to Eqn. (27)]}$$

Substituting the values, we get:

$$D = \frac{1}{2\pi \times 150 \times 0.16 \times 10^{-6} \times 431.25 \times 10^{3}} = 0.0154$$

Schering bridge circuit

Schering bridge circuit is basically a modification of series-resistance capacitance bridge or series RC bridge. The circuit of a Schering bridge is depicted in Figure 7.22.

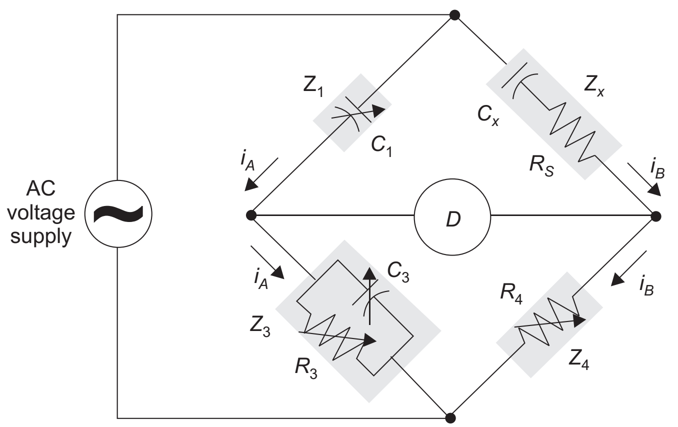


Fig. 7.22 Schering Bridge

The unknown leaky capacitance, that is, a series combination of C_x and R_s is compared with the standard capacitance C_1. The standard capacitance is usually a pure capacitance with mica or air as dielectric which provides an extremely high dielectric resistance. The parallel combination of C_3 and R_3 is meant to adjust the phase angle of current i_A flowing through impedances Z_1 and Z_3. This ensures balance in the voltage drop between Z_1 and Z_x thereby, balancing the bridge. Thus, the combination of C_3 and R_3 eradicates the need of a resistance in the C_1 arm. Small capacitance values at low AC voltages can be measured precisely using a Schering bridge. In addition, the dielectric and insulation properties of capacitances at high voltages can be accurately analyzed with this bridge.

The balance equations of the bridge can be derived in the usual manner. The complex notation of the impedances can be written as:

$$Z_1 = \frac{-j}{\omega C_1}$$

$$Z_x = R_s - \frac{j}{\omega C_x}$$

$$Z_3 = \frac{1}{1/R_3 + j\omega C_3}$$

$$Z_4 = R_4$$

We know that the balance condition is given as:

$$Z_1 Z_4 = Z_x Z_3 \qquad \text{[Refer to Eqn. (23)]}$$

Substituting the complex values in the above equation, it becomes:

$$\left(\frac{-j}{\omega C_1}\right) R_4 = \left(R_s - \frac{j}{\omega C_x}\right)\left(\frac{1}{1/R_3 + j\omega C_3}\right)$$

Or,

$$R_s - \frac{j}{\omega C_x} = R_4 \left(\frac{-j}{\omega C_1}\right)\left(\frac{1}{R_3} + j\omega C_3\right)$$

$$R_s - \frac{j}{\omega C_x} = \frac{-jR_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1}$$

Now, equating real and imaginary parts, we get:

$$R_s = \frac{R_4 C_3}{C_1} \quad \text{and} \quad C_x = \frac{C_1 R_3}{R_4} \qquad \text{...(31)}$$

Equation (31) reveals that the unknown capacitance in Schering bridge is obtained by the same relation as that of previous RC bridges while the resistance is calculated in terms of capacitors.

Now, we can easily express the dissipation power factor of a Schering bridge shown in Figure 7.22 by substituting Equation (31) into (26), we get:

$$D = \omega C_3 R_3 \qquad \text{...(32)}$$

If resistance R_3 of the circuit is a fixed value resistance, then the dissipation power factor becomes proportional to capacitance C_3. This enables the calibration of capacitance dial such that the dissipation factor of the unknown capacitance can be directly read on it. The term ω in the above relation indicates that the dial calibration is applicable only for the frequency at which it has been calibrated. Thus, some corrections have to be made at all other frequencies. For this, the ratio of the two frequencies must be multiplied by the dial reading.

When measurements are to be done at high voltages, a step up transformer (operating at power frequency) is used to provide the supply voltage to the bridge. Also, variable capacitance C_1 is replaced by a fixed one with a guard ring. This is to isolate the leakage currents at such high voltages; grounded screens are also used to eradicate the stray capacitance effects. Now, only components C_3, R_3, and R_4 are adjusted to attain the balance condition. The voltage drop across impedances Z_1 and Z_x will be relatively more than those of other impedances to ensure the safety of the operator.

Determination of dielectric material

Schering bridge can be used to determine the properties of dielectric material of a capacitor. A guard ring is incorporated in the capacitor to avoid surface leakage. The combination of C_x and R_s in Z_x arm is replaced by a standard variable capacitance C_s. In addition, another capacitor C_4 is connected in parallel with resistance R_4 as shown in Figure 7.23.

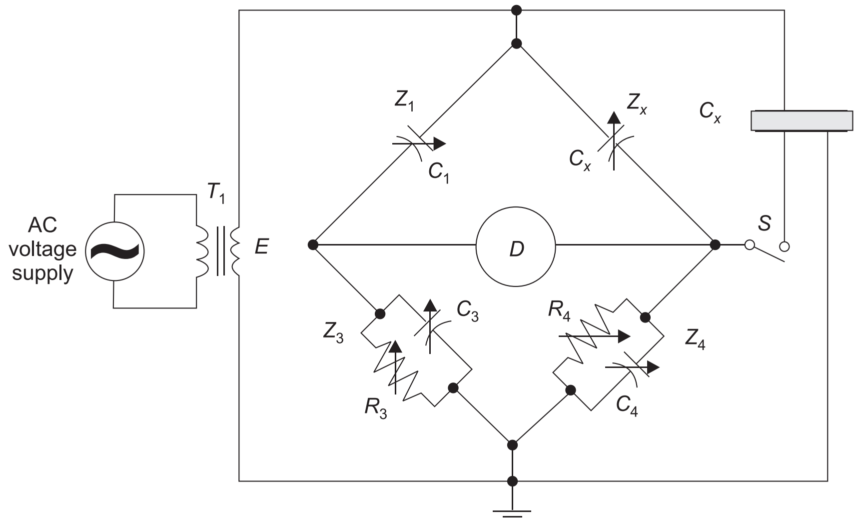


Fig. 7.23 Schering Bridge for Determining Dielectric Material

It can be seen from the figure that a switch S has also been included in the circuit. This switch is opened when the balance is attained first. At this instant, the value of C_s is measured and noted as $C_{s,\text{open}}$. Now, the switch is closed and the balance is attained once again by adjusting capacitances C_s and C_4. The new value of C_s is measured and noted as $C_{s,\text{close}}$. The unknown capacitance C_x can be obtained as:

$$C_x = C_{s,\text{close}} - C_{s,\text{open}} \qquad ...(33)$$

Once C_x has been measured, its relative permittivity ε can be found easily provided the dimensions of metal plates or electrodes are known. The general relation for relative permittivity of a parallel plate capacitance is given as:

$$\varepsilon_r = \frac{C_x d}{\varepsilon_o A} \qquad ...(34)$$

where d, A, and ε_o represent the distance between metal plates, the area of plates, and permittivity of free space or vacuum, respectively.

Another method to determine the relative permittivity is the one in which the dielectric material is removed after measuring the value of C_x, and the distance between the metal plates is adjusted so that the value of capacitance remains the same. When the thickness of the dielectric material and adjustment of the distance between metal plates are considered, the effective capacitance C_{eff} is given by the relation:

$$C_{eff} = \frac{\varepsilon_o A}{t + d - x} \qquad ...(35)$$

where t represents the thickness of dielectric, d represents dielectric–metal plate gap, and x represents the reduction in distance between the plates in two measurements.

However, the effective capacitance can also be found as the series combination of C_x and capacitance due to space between dielectric and metal plate C_o. Thus, we may write it as:

$$C_{eff} = \frac{C_x C_o}{C_x + C_o}$$

Or,
$$C_{eff} = \frac{(\varepsilon_o \varepsilon_r A/d)(\varepsilon_o A/t)}{(\varepsilon_o \varepsilon_r A/d) + (\varepsilon_o A/t)} = \frac{\varepsilon_o \varepsilon_r A}{\varepsilon_r t + d} \qquad \text{...(36)}$$

Now, equating Equations (35) and (36), we get:

$$\frac{\varepsilon_o A}{t + d - x} = \frac{\varepsilon_o \varepsilon_r A}{\varepsilon_r t + d}$$

Therefore, the relative permittivity ε_r is given as:

$$\varepsilon_r = \frac{d}{d - x} \qquad \text{...(37)}$$

Equation (37) gives the required relation for relative permittivity of capacitance under measurement.

Example 6 A Schering bridge is operated on 800 V with a frequency of 500 Hz in order to measure a high voltage capacitor. The following data is given about the balanced bridge $C_1 = 0.2\ \mu F$, $C_3 = 2200$ pF, $R_3 = 1.5\ k\Omega$, and $R_4 = 12\ k\Omega$.

Determine:

(a) value of unknown capacitor
(b) dissipation factor

Solution:

(a) The value of unknown capacitance C_x can be found as:

$$C_x = \frac{C_1 R_3}{R_4} \qquad \text{[Refer to Eqn. (31)]}$$

Substituting the given values, we get:

$$C_x = \frac{0.2 \times 10^{-6} \times 1.5 \times 10^3}{12 \times 10^3}$$

This gives:

$$C_x = 0.025\ \mu F$$

(b) The dissipation factor D can be computed as:

$$D = \omega C_3 R_3 = 2\pi f C_3 R_3 \qquad \text{[Refer to Eqn. (32)]}$$

Substituting the required values, we obtain:

$$D = 2\pi \times 500 \times 2200 \times 10^{-12} \times 1.5 \times 10^3 = 0.01$$

Wien bridge–frequency measurement

Wien bridge is also RC bridge consisting of a series as well as parallel RC combination. Unlike other RC bridges, this bridge is used to determine the supply frequency. The circuit of a Wien bridge is depicted in Figure 7.24.

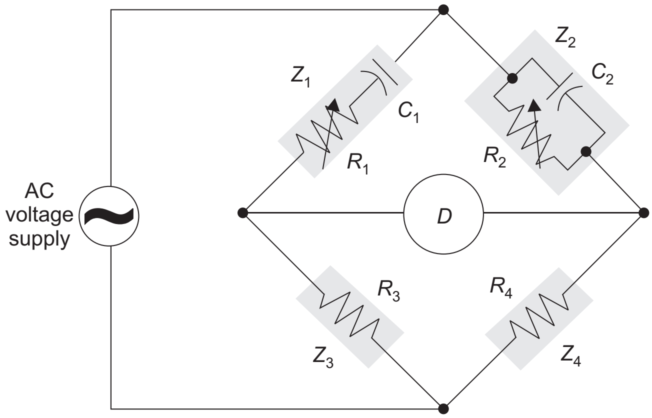


Fig. 7.24 Wien Bridge

From the figure, we can say that the series and parallel RC combinations are connected in adjoining arms of the bridge. Also, the capacitances used in the bridge are of fixed values. However, R_1 and R_2 are variable, and thus, adjusted in order to attain the balance of the bridge. The controls of these resistances are usually connected mechanically such that they can be adjusted simultaneously. The impedances can be written in their complex notations as:

$$Z_1 = R_1 - \frac{j}{\omega C_1}$$

$$Z_2 = \frac{1}{1/R_2 + j\omega C_2}$$

$$Z_3 = R_3$$

and

$$Z_4 = R_4$$

The balance condition of the bridge is given as:

$$Z_1 Z_4 = Z_2 Z_3 \qquad \text{[Refer to Eqn. (23)]}$$

Substituting the complex values of impedances, we get:

$$\left(R_1 - \frac{j}{\omega C_1}\right) R_4 = \left(\frac{1}{1/R_2 + j\omega C_2}\right) R_3$$

Equating the real and imaginary terms, we get:

$$\frac{R_3}{R_4} = \frac{R_1}{R_2} + \frac{C_2}{C_1} \quad \text{and} \quad \omega C_2 R_1 R_4 = \frac{R_4}{\omega C_1 R_2} \qquad \text{...(38)}$$

The angular frequency ω is expressed as $2\pi f$ in imaginary part, it yields:

$$(2\pi f)^2 C_2 R_1 C_1 R_2 = 1$$

Or,

$$f = \frac{1}{2\pi\sqrt{C_2 R_1 C_1 R_2}} \qquad \text{...(39)}$$

Wien bridge attains balance only at a specific supply frequency which can be obtained using Equation (39). We have discussed this bridge in its basic form which measures the frequency. In addition, it can be used as a frequency determining element in audio range and high frequency oscillators and as a notch filter in harmonic distortion analyzers.

Example 7 A Wein bridge at balance has the following components given as:

$R_1 = R_2 = 820\ \Omega$, $C_1 = 0.2\ \mu F$, $C_2 = 0.4\ \mu F$, and $R_3 = 1.5\ k\Omega$. Find the following:

(a) frequency

(b) value of resistance R_4

Solution:

(a) The frequency of a Wein bridge can be found as:

$$f = \frac{1}{2\pi\sqrt{C_2 R_1 C_1 R_2}} \qquad \text{[Refer to Eqn. (39)]}$$

Substituting the given values we get:

$$f = \frac{1}{2\pi\sqrt{0.4 \times 10^{-6} \times 0.2 \times 10^{-6} \times 820 \times 820}}$$

On solving, it gives:

$$f = 686 \text{ Hz}$$

(b) The value of resistance R_4 can be determined using relation:

$$R_4 = \frac{R_3}{\dfrac{R_1}{R_2} + \dfrac{C_2}{C_1}} \qquad \text{[Refer to Eqn. (38)]}$$

Substituting the values, we get:

$$R_4 = \frac{1.5 \times 10^3}{\dfrac{820}{820} + \dfrac{0.4 \times 10^{-6}}{0.2 \times 10^{-6}}}$$

On solving, it comes out as:

$$R_4 = 500\ \Omega$$

7.5.2 Inductance Measuring Bridges

As the name suggests, the inductance bridges are AC bridges used to measure the unknown inductances connected in one of their arms. *Inductance comparison bridge*, *Maxwell bridge*, *Hay bridge*, *Anderson bridge*, and *Owen bridge* are all explained under this category. Each one of these bridges is discussed in detail.

Inductance comparison bridge

The inductance comparison bridge is similar to a modified De Sauty bridge in its construction, except that capacitances are replaced by inductances as shown in Figure 7.25.

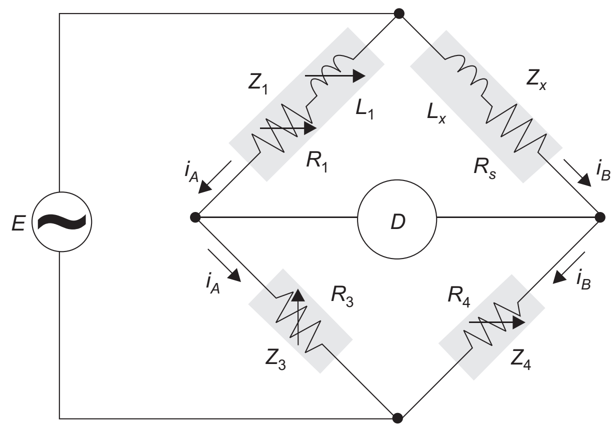


Fig. 7.25 Inductance Comparison Bridge

The series combination of inductance L_x and resistance R_s represents the unknown leaky inductance. The standard variable resistance R_1 is adjusted along with variable resistance R_3 and R_4 alternately to attain the balance of the bridge. The unknown inductance L_x is measured in terms of standard variable inductance L_1. The impedances can be written in their complex form as:

$$Z_1 = R_1 + j\omega L_1$$

$$Z_x = R_s + j\omega L_x$$

$$Z_3 = R_3$$

And, $$Z_4 = R_4$$

We know that the balance condition is given as:

$$Z_1 Z_4 = Z_x Z_3 \qquad \text{[Refer to Eqn. (23)]}$$

Substituting the above relations, we get:

$$(R_1 + j\omega L_1)\, R_4 = (R_s + j\omega L_x)\, R_3$$

Or,

$$\frac{R_1 + j\omega L_1}{R_3} = \frac{R_s + j\omega L_x}{R_4}$$

$$\frac{R_1}{R_3} + \frac{j\omega L_1}{R_3} = \frac{R_s}{R_4} + \frac{j\omega L_x}{R_4}$$

Equating real and imaginary terms, it gives:

$$\frac{R_1}{R_3} = \frac{R_s}{R_4} \quad \text{and} \quad \frac{\omega L_1}{R_3} = \frac{\omega L_x}{R_4}$$

Thus, we get:

$$R_s = \frac{R_1 R_4}{R_3} \quad \text{and} \quad L_x = \frac{L_1 R_4}{R_3} \qquad \text{...(40)}$$

Equation (40) represents the two balance equations of an inductance comparison bridge.

Example 8 An inductance comparison bridge, operating at a frequency of 6 kHz, attains balance when:

R_1 = 50 kΩ, R_3 = 12 kΩ, R_4 = 100 kΩ, and L_1 = 8 mH. Determine the value of inductive impedance.

Solution: The series resistance R_s of the bridge can be found as:

$$R_s = \frac{R_1 R_4}{R_3} \qquad \text{[Refer to Eqn. (40)]}$$

Substituting the given values, we get:

$$R_s = \frac{50 \times 10^3 \times 100 \times 10^3}{12 \times 10^3} = 416.67 \text{ k}\Omega$$

The unknown inductance L_x can be determined as:

$$L_x = \frac{L_1 R_4}{R_3} \qquad \text{[Refer to Eqn. (40)]}$$

Substituting the required values, we get:

$$L_x = \frac{8 \times 10^{-3} \times 100 \times 10^3}{12 \times 10^3}$$

On solving, it comes out as:

$$L_x = 66.67 \text{ mH}$$

Maxwell bridge

Maxwell bridge measures the unknown inductance in terms of a known standard capacitance. This is because an accurate standard capacitance is much easier to manufacture than an accurate standard inductance. Maxwell bridge is sometimes referred to as **Maxwell-Wien bridge**. The circuit of the bridge is illustrated in Figure 7.26.

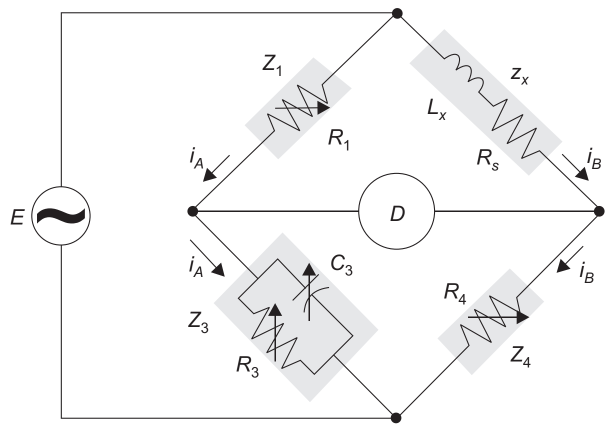


Fig. 7.26 Maxwell Bridge Circuit

The leaky unknown inductance is represented by series combination of L_x and R_s. The parallel combination of resistance R_3 and capacitance C_3 is connected in the Z_3 arm of the bridge. Resistances R_1, R_3, and R_4 are all variable and are adjusted to obtain the balance of the bridge. We know that balance is achieved when the following condition is satisfied:

$$Z_1 Z_4 = Z_x Z_3 \qquad \text{[Refer to Eqn. (23)]}$$

Let us write the complex form of impedances as:

$$Z_1 = R_1$$

$$Z_x = R_s + j\omega L_x$$

$$Z_3 = \frac{1}{1/R_3 + j\omega C_3}$$

$$Z_4 = R_4$$

Substituting these values in the balance condition, we get:

$$R_1 R_4 = \frac{(R_s + j\omega L_x)}{1/R_3 + j\omega C_3}$$

On solving, it gives:

$$\frac{R_1}{R_3} + j\omega C_3 R_1 = \frac{R_s}{R_4} + \frac{j\omega L_x}{R_4}$$

Equating real and imaginary terms, we get:

$$R_s = \frac{R_1 R_4}{R_3} \quad \text{and} \quad L_x = C_3 R_1 R_4 \qquad ...(41)$$

Let us introduce a new term which expresses the quality of an inductor, known as **quality factor** or ***Q* factor** of an inductor defined as the ratio of its inductive reactance ωL and resistance R at the operating frequency. Thus, the quality factor Q of the unknown inductance L_x in Maxwell bridge can be given as:

$$Q = \frac{\omega L_x}{R_s} \qquad ...(42)$$

Maxwell bridge is able to determine the values of inductances with medium quality factor in the range of $1 < Q < 10$. This can be clearly understood by considering balance condition in context of phase angles, given as:

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3 \qquad ...(43)$$

where θ_1, θ_2, θ_3, and θ_4, are the phase angles of respective arms.

Equation (43) reveals that the bridge balance can also be attained if the sum of phase angles of opposite arms is equal to the sum of the phase angles of other two opposite arms. Since impedances Z_1 and Z_4 are purely resistive, they possess a sum of 0° phase angle. Thus, the sum of phase angles of Z_1 and Z_4 should also be 0°. It should be noted here that the phase angle of high Q inductance L_x is about +90° which indicates that the phase angle of capacitance C_3 must be –90° to make a total of 0° phase angle in the two arms, thereby balancing the bridge. However, –90° phase angle of C_3 implies that resistance R_3 must be significantly large which is practically impossible. Hence, Maxwell bridge is not suitable for measuring high Q inductances.

In addition, inductances with $Q < 1$ cannot be measured using Maxwell bridge. In Equation (41), resistance R_1 is present in both the expressions. Thus, it has to be adjusted for L_x as well as R_s which proves to be difficult. The inductive balance achieved by adjusting R_1 disturbs the resistive balance achieved by adjusting R_3. Resistance R_1 is then again adjusted to compensate the adjustment of resistance R_3. For low Q inductances, these resistances are to be adjusted alternately to attain the bridge balance. The two adjustments are done repeatedly and influence each other by altering their selected values. This results in slow convergence to attain balance and the effect is known as **sliding balance**. The sliding balance is not prominent in medium Q inductances. Therefore, Maxwell bridge is well suited for measuring such inductances.

Example 10 In a Maxwell bridge, one arm has a resistance of 1 kΩ, another arm has the resistance of 5 kΩ. The third arm has a resistance of 4.7 kΩ in shunt with a capacitor of 1 μF. The bridge is excited at frequency of 1 kHz. Determine the value of an unknown inductance L_x in the fourth arm.

Solution: Given that: R_1 = 1 kΩ, R_3 = 4.7 kΩ, C_3 = 1 μF, R_4 = 5 kΩ, and f = 1 kHz

The resistance of the inductance R_s to be measured can be determined as:

$$R_s = \frac{R_1 R_4}{R_3} \qquad \text{[Refer to Eqn. (41)]}$$

Substituting the given values, we get:

$$R_s = \frac{1\times 10^3 \times 5\times 10^3}{4.7\times 10^3} = 1.063\ \text{k}\Omega$$

The unknown inductance L_x can be found as:

$$L_x = C_3 R_1 R_4 \qquad \text{[Refer to Eqn. (41)]}$$

$$L_x = 1 \times 10^{-6} \times 1 \times 10^3 \times 5 \times 10^3 = 5\ \text{H}$$

Hay bridge

The **Hay bridge** is required for the measurement of high Q factor which cannot be measured by Maxwell bridge. This bridge is similar to Maxwell bridge. The only difference in Hay bridge is that the parallel combination of standard resistance R_3 and standard capacitance C_3 is replaced with their series combination. However, the unknown leaky inductance can be represented either by a series combination or a parallel combination of R_s and L_x. The circuit of Hay bridge is illustrated in Figure 7.27.

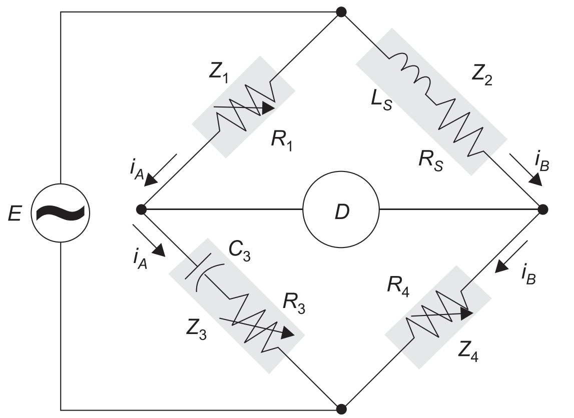


Fig. 7.27 Hay Bridge Circuit

The figure shows the usual configuration of Hay bridge in which the inductance to be measured is represented by the series combination of R_s and L_x. Let us derive the balance equations for the bridge. The impedances in complex form are:

$$Z_1 = R_1$$

$$Z_x = R_s + j\omega L_x$$

$$Z_3 = R_3 - \frac{j}{\omega C_3}$$

$$Z_4 = R_4$$

Substituting these values in the balance condition, we get:

$$R_1 R_4 = (R_s + j\omega L_x)\left(R_3 - \frac{j}{\omega C_3}\right)$$

Or,
$$R_1 R_4 = R_s R_3 + j\omega L_x R_3 - \frac{jR_s}{\omega C_3} + \frac{L_x}{C_3}$$

Separating real and imaginary terms, we get:

$$R_1 R_4 = R_s R_3 + \frac{L_x}{C_3} \quad \text{and} \quad \frac{R_s}{\omega C_3} = \omega R_3 L_x \qquad ...(44)$$

Notice that both of the relations consist of terms R_s and L_x. Thus, both equations must be solved concurrently.

From the second relation of Equation (44), we get:

$$L_x = \frac{R_s}{\omega^2 R_3 C_3} \qquad ...(45)$$

Putting this value in the first relation gives:

$$R_1 R_4 = R_s \left[R_3 + \frac{1}{\omega^2 R_3 C_3^2} \right]$$

Or

$$R_1 R_4 = R_s \left[\frac{\omega^2 R_3^2 C_3^2 + 1}{\omega^2 R_3 C_3^2} \right]$$

$\Rightarrow$

$$R_s = \frac{\omega^2 R_1 R_4 R_3 C_3^2}{1 + \omega^2 R_3^2 C_3^2} \qquad ...(46)$$

Equation (46) represents the required relation for the series resistance R_s of the unknown inductance L_x. Once R_s is calculated, L_x can be easily obtained by substituting its value in Equation (45) as:

$$L_x = \frac{\omega^2 R_1 R_4 R_3 C_3^2}{1 + \omega^2 R_3^2 C_3^2} \times \frac{1}{\omega^2 R_3 C_3}$$

Or,

$$L_x = \frac{R_1 R_4 C_3}{1 + \omega^2 R_3^2 C_3^2} \qquad ...(47)$$

Equations (46) and (47) together represent the balance equations of a **series–Hay bridge**. However, the balance equations for a **parallel–Hay bridge** remain the same as those in Maxwell bridge. Thus, we may write from Equation (41) as:

$$R_{sh} = \frac{R_1 R_4}{R_3} \quad \text{and} \quad L_x = C_3 R_1 R_4 \qquad ...(48)$$

Notice that Equations (46) and (47) possess angular velocity ω. Therefore, the operating frequency must be known. Now, recollecting the balance condition given by Equation (43), we may conclude that the inductive and capacitive phase angles, θ_L and θ_C, respectively, of a Hay bridge should be equal to each other; since the resistive phase angles are always zero. Thus, their tangents should also be equal and we may write as:

$$\tan \theta_L = \tan \theta_C$$

where $\tan\theta_L = \dfrac{\omega L_x}{R_s}$ and $\tan\theta_C = \dfrac{1}{\omega C_3 R_3}$

From Equation (42), we have:

$$Q = \frac{\omega L_x}{R_s}$$

Substituting this in the above equation we obtain:

$$Q = \tan\theta_L = \tan\theta_C = \frac{1}{\omega C_3 R_3} \qquad ...(49)$$

Using Equation (49) in (47), we get:

$$L_x = \frac{R_1 R_4 C_3}{1 + (1/Q)^2} \qquad ...(50)$$

For inductances with $Q > 10$, the term $\left(\frac{1}{Q}\right)^2$ comes out to be less than $\frac{1}{100}$ and hence can be neglected. Equation (50) is thus reduced to:

$$L_x = R_1 R_4 C_3 \qquad ...(51)$$

Notice that Equation (51) is exactly similar to the inductive balance equation given by Equation (48).

The phase angles of high Q inductors are considerably large which requires a significantly small value of resistance R_3. Hay bridge satisfies this requirement. Therefore, it is appropriate for measurement of such inductances possessing $Q > 10$.

Example 11 A Hay bridge consists of the following constants: $R_1 = 400\,\Omega$, $R_3 = 150\ \Omega$, $C_3 = 0.2\ \mu F$, $R_s = 100\ \Omega$, $L_x = 10$ mH, and $f = 1$ kHz. Determine the remaining constants.

Solution: It is clear from the given data that resistance R_4 is to be calculated. It can be determined using Equation (46) as:

$$R_s = \frac{\omega^2 R_1 R_4 R_3 C_3^2}{1 + \omega^2 R_3^2 C_3^2}$$

Substituting the given values in the above relation, we obtain:

$$100 = \frac{(2\pi \times 1 \times 10^3)^2 \times 400 \times R_4 \times 150 \times (0.2 \times 10^{-6})^2}{1 + (2\pi \times 1 \times 10^3)^2 (150)^2 (0.2 \times 10^{-6})^2}$$

Solving this, we get:

$$R_4 = 1093\ \Omega$$

Owen bridge

Owen bridge is another modification of Maxwell bridge used for a wide range of inductance measurements. The bridge circuit consists of two additional capacitors—a variable capacitor C_1 in Z_1 arm and a fixed standard capacitor C_3 in the Z_3 arm. Capacitor C_1 is used to provide the required phase angle adjustments. Capacitor C_3 replaces the parallel combination of C_3 and R_3 as in Maxwell bridge (refer to Figure 7.26). The configuration of an Owen bridge is depicted in Figure 7.28.

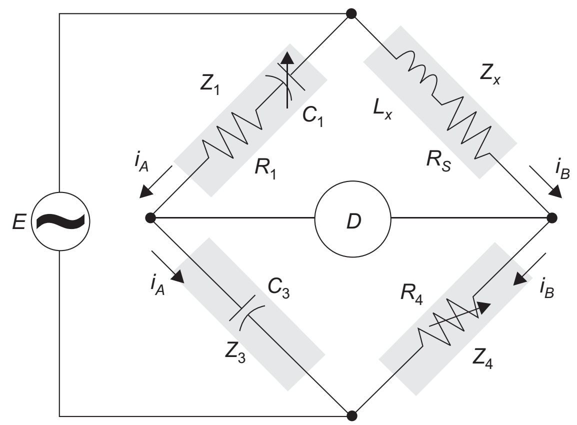


Fig. 7.28 Owen Bridge Circuit

If an additional resistance R_s is serially connected with the unknown inductance L_x in the Z_x arm, then a fixed capacitor can be used as capacitor C_1. In that case, resistances R_s, R_1, and R_4 are to be adjusted to attain balance of the bridge.

Let us derive the balance equations for the bridge. The impedances in complex form are:

$$Z_1 = R_1 - \frac{j}{\omega C_1}$$

$$Z_s = R_s + j\omega L_s$$

$$Z_3 = \frac{-j}{\omega C_3}$$

$$Z_4 = R_4$$

Thus, at balance, we have:

$$\left(R_1 - \frac{j}{\omega C_1}\right)R_4 = (R_s + j\omega L_x)\left(\frac{-j}{\omega C_3}\right)$$

Separating the real and imaginary components, we get:

$$R_s = \frac{R_4 C_3}{C_1} \text{ and } L_x = R_1 R_4 C_3 \quad \text{...(52)}$$

Equation (52) represents the required balance equations of Owen bridge.

Example 12 An Owen bridge is used to measure the properties of a sample of sheet steel at 2 kHz. At balance, arm *ab* is test specimen, arm *bc* is 100 Ω, arm *cd* is 0.1 μF, and arm *da* is a resistance of 839 Ω in series with a capacitor of 0.124 μF. Calculate from the fundamentals, the effective impedance of the specimen under test conditions.

Solution: Given that: $R_1 = 839\,\Omega$, $C_1 = 0.124\ \mu F$, $C_3 = 0.1\ \mu F$, $R_4 = 100\ \Omega$, and $f = 2$ kHz.

The unknown resistance R_S can be calculated as:

$$R_s = \frac{R_4 C_3}{C_1} \qquad \text{[Refer to Eqn.(52)]}$$

Substituting the given values, we get:

$$R_s = \frac{100 \times 0.1 \times 10^{-6}}{0.124 \times 10^{-6}} = 80.64\ \Omega$$

The value of unknown inductance L_x can be obtained as:

$$L_x = R_1 R_4 C_3 \qquad \text{[Refer to Eqn.(52)]}$$

Substituting the values, it becomes:

$$L_x = 839 \times 100 \times 0.1 \times 10^{-6} = 8.39\ \text{mH}$$

Anderson bridge

The bridge similar to Maxwell bridge which also can measure low Q inductances like it, is an **Anderson bridge**. The only difference in Anderson bridge is the addition of resistance R_5 connected serially to capacitance C_3 as shown in Figure 7.29. The null detector is connected at the junction of R_5 and C_3, marked as t.

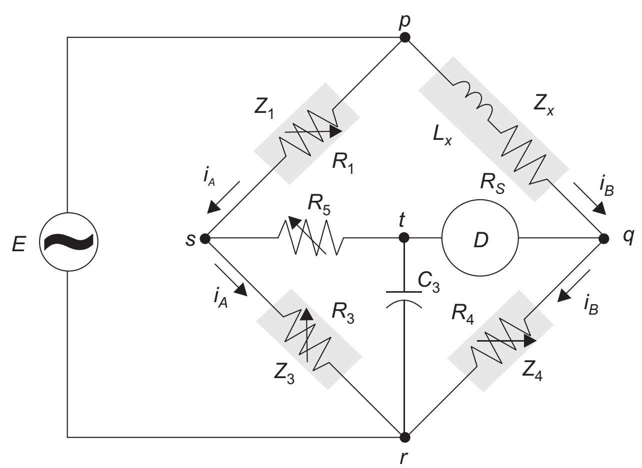


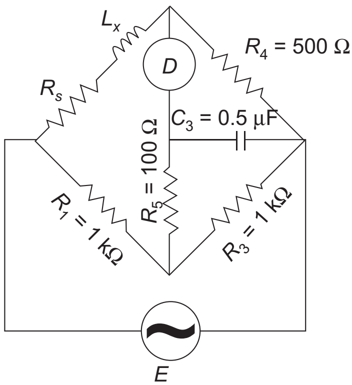
Fig. 7.29 Anderson Bridge

Unlike Maxwell bridge, C_3 can be a standard fixed capacitance in Anderson bridge configuration. These modifications lead to easier balancing of the bridge. The balance or null deflection is achieved first using DC supply and detector by adjusting resistances R_1, R_3, and R_4. Another adjustment is then made to resistance R_5 using AC supply and detector. Thus, two alternate adjustments on DC and AC supply are required to attain balance in an Anderson bridge. The balance equations of this bridge are difficult to derive and thus, final result can be given as:

$$L_x = \frac{C_3 R_4}{R_3}[R_5(R_3 + R_1) + R_1 R_3] \quad \text{and} \quad R_s = \frac{R_1 R_4}{R_3} \qquad ...(53)$$

Equations (53) represent the balance equations of Anderson bridge.

Example 13 For the bridge circuit shown in the following figure, calculate the unknown inductance and resistance at balance. The values of all other components are marked in the figure.



Solution: Given that: R_1 = 1 kΩ, R_3 = 1 kΩ, C_3 = 0.5 μF, R_4 = 500 Ω, and R_5 = 100 Ω.

The unknown resistance R_s can be calculated as:

$$R_s = \frac{R_1 R_4}{R_3} \qquad \text{[Refer to Eqn. (53)]}$$

On substituting the given values, it comes out to be:

$$R_s = \frac{1 \times 10^3 \times 500}{1 \times 10^3} = 500\ \Omega$$

The inductance under measurement L_x can be obtained as:

$$L_x = \frac{C_3 R_4}{R_3}\left[R_5(R_3 + R_1) + R_1 R_3\right] \qquad \text{[Refer to Eqn. (53)]}$$

Substituting the given values, we get:

$$L_x = \frac{0.5 \times 10^{-6} \times 500}{1 \times 10^3}\left[100(1 \times 10^3 + 1 \times 10^3) + 1 \times 10^3 \times 1 \times 10^3\right]$$

On solving, it gives:

$$L_x = 0.3 \text{ H}$$

7.5.3 Bridges Measuring Mutual Inductance

Mutual inductance measuring bridges are designed to measure mutual inductance of the circuits. These bridges may be resistance-inductance (*RL*) bridges or resistance-inductance-capacitance (*RLC*) bridges. These bridges work on the principle that if two inductances are connected in series, then their effective inductance L_{eff} can be calculated by using the following relations.

In case, the flux through them is in the *same* direction:

$$L_{eff1} = L_1 + L_2 + 2\,M \qquad \text{(positive coupling)}$$

In case, the flux through them is in *opposite* direction:

$$L_{eff2} = L_1 + L_2 - 2\,M \qquad \text{(negative coupling)}$$

The mutual inductance can be obtained from the above two relations as:

$$M = \frac{1}{4}(L_{eff1} - L_{eff2}) \qquad ...(54)$$

Equation (54) holds good for circuits possessing a fairly high coupling between the inductances. In case of low coupling circuits, this equation is not suitable and results in poor accuracy since the two terms L_{eff1} and L_{eff2} become nearly equal to each other.

Heaviside bridge, *Campbell bridge*, and *Carey-Foster bridge* are all mutual inductance measuring bridges discussed here.

Heaviside bridge

Heaviside bridge is used to measure the mutual inductance in terms of a known value of self-inductance. The configuration of this bridge is shown in Figure 7.30 where mutually coupled coils represent the standard mutual inductance M, while L_1 represents the self-inductance of this mutual inductance standard.

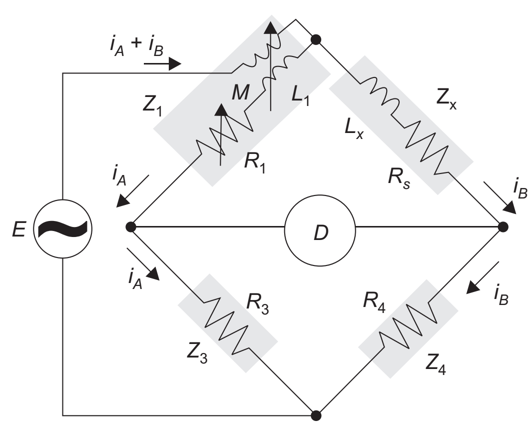


Fig. 7.30 Heaviside Bridge Circuit

The mutual inductance M along with variable resistance R_1 is adjusted to balance the bridge. The balance condition for Heaviside bridge requires that:

- Resistances R_3 and R_4 must be equal to each other;
- Resistances R_1 and R_S must be equal to each other;
- Inductance L_x must be equal to the total inductance of mutually coupled inductances, that is, $L_1 + M$.

Thus, at balance, we may write as:

$$i_A R_3 = i_B R_4$$

and
$$i_A(R_1 + j\omega L_1) + j\omega M(i_A + i_B) = i_B(R_S + j\omega L_x)$$

When $i_A = i_B$, the above equation becomes:

$$(R_1 + j\omega L_1) + 2j\omega M = (R_S + j\omega L_x)$$

Separating the real and imaginary terms, we get:

$$R_S = R_1 \quad \text{and} \quad L_x = L_1 + 2M \qquad ...(55)$$

Equations (55) represents the required balance equations for Heaviside bridge. It must be noted here that if the value of inductance L_x is known, then the mutual inductance M can be found in terms of inductances L_1 and L_x using the relation:

$$M = \frac{1}{2}(L_x - L_1) \qquad ...(56)$$

Equation (56) reveals that the bridge measures mutual inductance in terms of self inductance.

Campbell's modified heaviside bridge

The Heaviside bridge which measures the mutual inductance in terms of self-inductance was modified by Campbell so as to measure self-inductance in terms of mutual inductance and, thus, known as **Campbell's Modified Heaviside Bridge**. The modified bridge is illustrated in Figure 7.31.

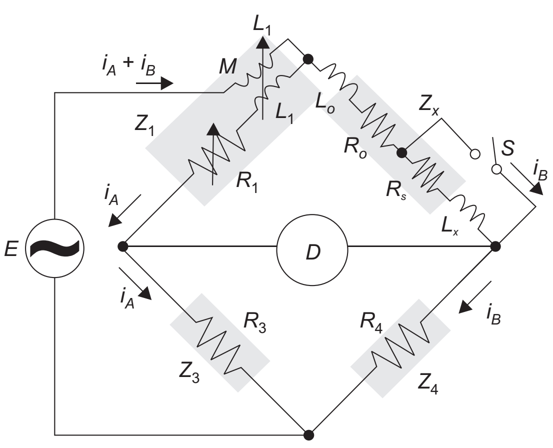


Fig. 7.31 Campbell's Modified Heaviside Bridge

An additional inductance represented by L_o and R_o is connected serially with the inductance under test and is known as **balancing coil**. It can be seen from the figure that a short-circuit switch S is also incorporated across the unknown inductance L_x. This modified circuit eliminates the possible errors due to resistances and inductances of connecting leads.

The bridge is balanced by adjusting resistance R_1 and mutual inductance M. The balance of the bridge is achieved twice, first when the switch S is opened, and then when it is closed. The difference between two resultant sets of measurements gives the required values of L_x and R_S. Hence, we may write:

$$R_S = R_{1OC} - R_{1SC} \quad \text{and} \quad L_x = 2(M_{OC} - M_{SC}) \qquad \text{...(57)}$$

where R_{OC} and R_{SC} are open circuit and short circuit resistances, and M_{OC} and M_{SC} are open circuit and short circuit mutual inductances, respectively.

Equation (57) is useful for calculating self-inductance of the inductor under measurement in terms of mutual inductance.

Heaviside Campbell equal ratio bridge

The modified Heaviside bridge studied in the preceding section possesses a balancing coil which affects its sensitivity. Thus, an improved version of this bridge was designed which is known as **Heaviside Campbell equal ratio bridge**. The configuration of this bridge is depicted in Figure 7.32 which provides an improved sensitivity.

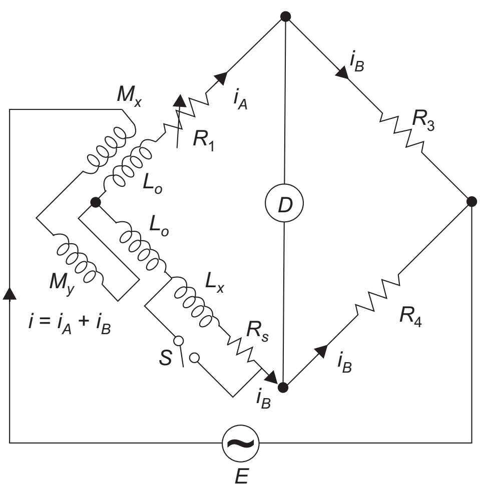


Fig. 7.32 Heaviside Campbell Equal Ratio Bridge

It can be seen that the two equal coils, each with a self-inductance, are used for the secondary winding of the mutual inductance. These inductances, marked as L_o are connected in Z_1 and Z_x arms of the bridge. The primary winding of the mutual inductance reacts with both of these secondary windings. The resistances R_3 and R_4 are made equal to each other. By adjusting the variable resistance R_1 as well as the mutual inductance M, the bridge can be balanced. Thus, at balance, we may write from Equations (55) and (57) as:

$$R_S = R_1 \quad \text{and} \quad L_x = 2(M_x + M_y) \qquad ...(58)$$

where $M_x = M_y = M$. Thus, the above equation becomes:

$$R_S = R_1 \quad \text{and} \quad L_x = 4\,M \qquad ...(59)$$

Equation (59) indicates that using the improved circuit shown in Figure 7.32, the unknown inductance L_x can be measured as high as twice the mutual inductance M. However, the result consists of errors due to resistances and inductances of connecting leads. Thus, two measurements are done—one with open switch and another with close switch. The difference between the two readings gives the value of inductance under measurement L_x. Balance equations of the bridge can then be given as:

$$R_S = R_{1OC} - R_{1SC} \quad \text{and} \quad L_x = 2(M_{OC} - M_{SC}) \qquad ...(60)$$

where R_{OC} is open circuit resistance and R_{SC} is short circuit resistance, and M_{OC} and M_{SC} are open circuit and short circuit mutual inductances, respectively.

It should be noted here that Equation (60) is exactly the same as Equation (57).

Campbell bridge

Campbell bridge is used to measure the unknown mutual inductance in terms of a standard mutual inductance. The configuration of this bridge is shown in Figure 7.33.

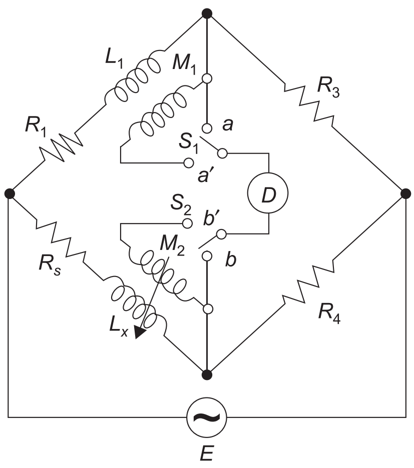


Fig. 7.33 Campbell Bridge Circuit

In the circuit, M_1 and M_2 represent the unknown and standard variable mutual inductances, respectively, while L_1 and L_x represent their respective self-inductances. The bridge can be balanced in two steps as described below.

- When switches S_1 and S_2 are connected at points a and b, respectively, it results in a simple self-inductance comparison bridge for which the balance can be easily obtained by adjusting resistances R_1 or R_2 and R_3 or R_4. The condition is given as:

$$\frac{L_1}{L_x} = \frac{R_1}{R_s} = \frac{R_3}{R_4} \qquad ...(61)$$

- When switches S_1 and S_2 are connected at points a' and b', respectively. The adjustments done on resistances in the above step are maintained. In addition, standard mutual inductance M_2 is adjusted to attain the balance of the bridge. Thus, we get:

$$\frac{M_1}{M_2} = \frac{R_3}{R_4} \quad \text{or} \quad M_1 = \frac{M_2 R_3}{R_4} \qquad ...(62)$$

Equation (62) is used to calculate the unknown mutual inductance M_1 in terms of known standard mutual inductance M_2.

Carey-Foster bridge

The Carey-Foster bridge, also known as **Heydweiller bridge** is used to measure a mutual inductance in terms of a standard capacitance. That is, it can measure the value of a capacitance in terms of a standard mutual inductance. The basic configuration of this bridge is shown in Figure 7.34.

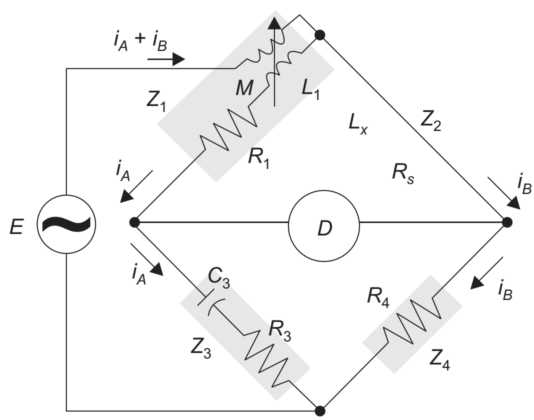


Fig. 7.34 Carey-Foster Bridge Circuit

It can be seen from the figure that Z_2 arm of the bridge is short-circuited. This implies that under balance condition, the voltage drop across this arm is substantially zero volts which further implies that the same potential difference should exist across arm Z_1 to maintain balance. To accomplish this, mutually coupled coils are connected to the arm Z_1 such that their mutual inductance M is in opposition to their self-inductance L_1. Thus, by subtracting M from L_1, we get a zero volt drop across Z_1. Now, let us derive the balance equation for the bridge.

When $V_{Z1} = V_Z = 0$, we have:

$$i_A(R_1 + j\omega L_1) - (i_A + i_B)\, j\omega M = 0$$

$$\Rightarrow \quad i_A(R_1 + j\omega L_1) = (i_A + i_B)\, j\omega M \qquad ...(63)$$

When $V_{Z3} = V_{Z4} = 0$, we have:

$$i_A\left[R_3 - \frac{j}{\omega C_3}\right] = i_B R_4$$

which gives,

$$i_B = i_A\left[\frac{R_3 - \dfrac{j}{\omega C_3}}{R_4}\right] \qquad \text{...(64)}$$

Substituting the value of i_B from Equation (64) into (63), we get:

$$i_A(R_1 + jwL_1) = \left(i_A + i_A\left[\frac{R_3 - \dfrac{j}{\omega C_3}}{R_4}\right]\right) jwM$$

Equating real and imaginary parts, we get:

$$M = C_3 R_1 R_4 \quad \text{and} \quad L_1 = M\left(1 + \frac{R_3}{R_4}\right) \qquad \text{...(65)}$$

Example 14 An inductor is measured on a Heaviside bridge with following specifications at balance: $R_1 = 270\ \Omega$, $L_1 = 100$ mH, $R_3 = R_4 = 5$ kΩ, and $M = 55$ mH. If the operating current is given to be 1 mA with a frequency of 3 kHz, determine the unknown inductive impedance.

Solution: Given that: $f = 3$ kHz, $i = 1$ mA the unknown inductance L_x can be computed as:

$$L_x = L_1 + 2\text{ M} \qquad \text{[Refer to Eqn. (55)]}$$

Substituting the required values, we obtain:

$$L_x = 100 \times 10^{-3} + 2 \times 55 \times 10^{-3}$$

$$\Rightarrow \qquad L_x = 210 \text{ mH}$$

The impedance R_s is obtained as:

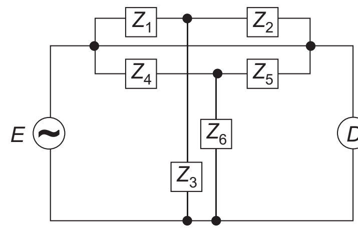
$$R_S = R_1 = 270\ \Omega \qquad \text{[Refer to Eqn. (55)]}$$

7.5.4 Miscellaneous Bridges

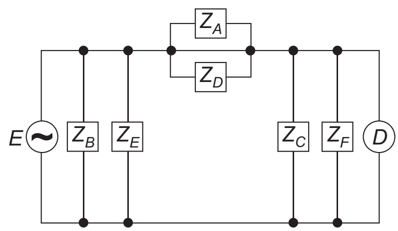
So far the bridges we have discussed are designed for the measurement of a specific component. However, there are some other bridges, like *T networks* and *Q meters* that are capable of determining any component, say resistance, inductance, capacitance, and frequency at high frequency ranges. In addition, *Wagner earthing device* which removes the stray capacitance errors from measurements and thereby provides highly accurate results is also included in this section.

Parallel T network

Parallel T network, also known as **twin T network** is depicted in Figure 7.35. This network is suitable for the applications which require a common ground terminal for their source and output.



(a) Simple configuration



(b) Star-delta transformation

Fig. 7.35 Different Configurations of a Parallel T network

It can be seen from Figure 7.35 (a) that two Ts are formed in a parallel T network. Impedances Z_1, Z_2, and Z_3 together make one T while impedances Z_4, Z_5, and Z_6 make another T. These two Ts are parallel to each other. Figure 7.35 (b) shows another form of parallel T network derived using star-delta transformation for analysis. The null detector deflects zero when the total admittance of parallel impedances Z_A and Z_B becomes equal to zero. Thus, at balance, we may write as:

$$\frac{1}{Z_A} + \frac{1}{Z_D} = 0 \quad \text{or} \quad Z_A + Z_D = 0$$

This gives,

$$Z_A = -Z_D \qquad \cdots(66)$$

However, from transformation, we have:

$$Z_A = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

and

$$Z_D = Z_4 + Z_5 + \frac{Z_4 Z_5}{Z_6}$$

Substituting these values in Equation (66), we get the null point at following condition:

$$Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3} = -Z_4 - Z_5 - \frac{Z_4 Z_5}{Z_6} \qquad \text{...(67)}$$

When Equation (67) is satisfied, the null detector in Figure 7.35(a) deflects to zero, indicating the balancing of the bridge.

The parallel *T* network in Figure 7.35(a) can be redrawn for frequency measurement as shown in Figure 7.36.

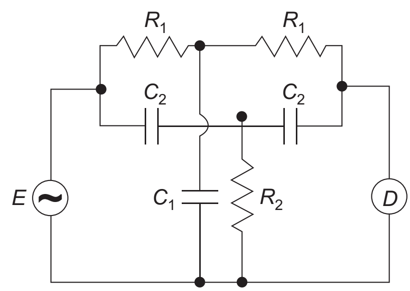


Fig. 7.36 Parallel T Network for Frequency Measurement

Comparing Figure 7.36 and Figure 7.35(a), we get:

$$Z_1 = R_1$$

$$Z_2 = R_1$$

$$Z_3 = \frac{-j}{\omega C_1}$$

$$Z_4 = \frac{-j}{\omega C_2}$$

$$Z_5 = \frac{-j}{\omega C_2}$$

and
$$Z_6 = R_2$$

To attain balance of the bridge, apply Equation (67) and substitute the respective resistive and capacitive values from the above relations. Thus, we get:

$$R_1 + R_1 + \frac{R_1^2}{\frac{-j}{\omega C_1}} = \frac{j}{\omega C_2} + \frac{j}{\omega C_2} + \frac{1}{\omega^2 C_2^2 R_2}$$

On solving, it gives:

$$2R_1 + jR_1^2 \omega C_1 = \frac{2j}{\omega C_2} + \frac{1}{\omega^2 C_2^2 R_2}$$

Separating real and imaginary terms, we get:

$$2R_1 = \frac{1}{\omega^2 C_2^2 R_2} \quad \text{and} \quad R_1^2 \omega C_1 = \frac{2}{\omega C_2}$$

$$\Rightarrow \qquad \omega^2 = \frac{1}{2R_1 R_2 C_2^2} \quad \text{and} \quad \omega^2 = \frac{2}{R_1^2 C_1 C_2} \qquad \text{...(68)}$$

From Equation (65), we obtain:

$$\frac{1}{2R_1 R_2 C_2^2} = \frac{2}{R_1^2 C_1 C_2}$$

This requires that the following relations must be satisfied.

$$R_1 = 4R_2$$

and

$$C_1 = C_2$$

If these relations are fulfilled, then the network shown in Figure 7.36 becomes balanced and high frequencies can be easily measured.

Bridge T network

If impedance Z_3 is infinite in parallel T network and the combination of impedances Z_1 and Z_2 is replaced by only impedance Z_1', the simplified network is called a **bridge *T* network**. Thus, we may write that:

$$Z_1 + Z_2 = Z_1'$$

and

$$Z_3 = \infty$$

Figure 7.37 illustrates the configuration of a bridge T network.

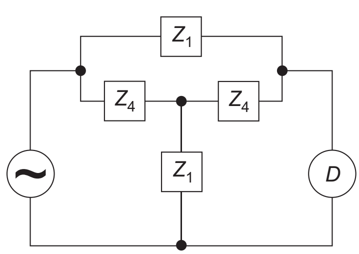


Fig. 7.37 A Bridge T Network

The balance equation for this network may be derived from Equation (67) as:

$$Z_1' = -Z_4 - Z_5 - \frac{Z_4 Z_5}{Z_6} \qquad \text{...(69)}$$

The bridge T network shown above can be redrawn for measuring effective resistance R, inductance L, and Q factor of the coil as shown in Figure 7.38.

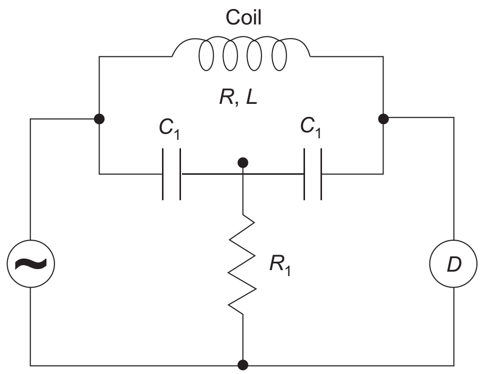


Fig. 7.38 Bridge T Network for Measuring Effective Resistance, Inductance, and Q Factor

Comparing the above network with that of Figure 7.35 (a), we get:

$$Z_1' = R + j\omega L$$

$$Z_4 = \frac{-j}{\omega C_1}$$

$$Z_5 = \frac{j}{\omega C_1}$$

$$Z_6 = R_1$$

Substituting these values in Equation (69), we get:

$$R + j\omega L = \frac{j}{\omega C_1} + \frac{j}{\omega C_1} + \frac{1}{\omega^2 C_1^2 R_1}$$

Separating real and imaginary components, we obtain:

$$R = \frac{1}{\omega^2 C_1^2 R_1} \quad \text{and} \quad L = \frac{2}{\omega^2 C_1} \qquad ...(70)$$

The quality factor Q of the circuit can be easily found using Equation (42), given as:

$$Q = \frac{\omega L}{R}$$

Substituting the values of L and R from Equation (70), we get:

$$Q = 2\omega C_1 R_1 \qquad ...(71)$$

Equation (71) gives the required relation for the quality factor of the circuit. The common ground terminal in T networks ensures easy screening of the bridges to eliminate the stray capacitance errors and to minimize the inductive and capacitive coupling effects.

Example 15 A bridge T network with $R_1 = 4.5\ \Omega$, $C_1 = 50$ pF is operated at a frequency of 6 MHz. Determine the value of inductance and impedance.

Solution:

(a) The unknown resistance R can be found as:

$$R = \frac{1}{\omega^2 C_1^2 R_1} = \frac{1}{(2\pi f)^2 C_1^2 R_1} \qquad \text{[Refer to Eqn. (70)]}$$

Substituting the required values, we get:

$$R = \frac{1}{(2\pi \times 6 \times 10^6)^2 (50 \times 10^{-12})^2 \times 4.5}$$

On solving, it comes out as:

$$R = 0.062\ \text{M}\Omega$$

(b) The unknown inductance L can be measured as:

$$L = \frac{2}{\omega^2 C_1} = \frac{2}{(2\pi f)^2 C_1} \qquad \text{[Refer to Eqn. (70)]}$$

Substituting the required values, we get:

$$L = \frac{2}{(2\pi \times 6 \times 10^6)^2 \times 50 \times 10^{-12}}$$

It gives:

$$L = 28.14\ \mu\text{H}$$

Q meter

The measurement of resistances, inductances, and capacitances at radio frequencies cannot be done adequately using bridges that operate on lower frequencies. At high frequencies, Q meter is used to perform these measurements. In addition, it can also measure the Q factor of a coil. The basic circuit of the Q meter is shown in Figure 7.39.

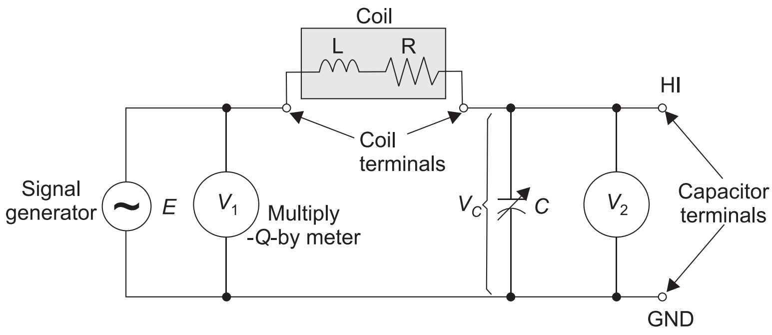


Fig. 7.39 Basic Q Meter Circuit

The AC supply in the circuit possesses variable frequency components. Voltmeters V_1 and V_2 keep track of the source voltage and voltage across variable capacitor C, respectively. The source voltage signal is kept at a convenient magnitude and set to the frequency at which measurement is to be performed. The capacitance C is varied so that the voltage across it becomes equal to that across inductance L. The circuit is then said to be at **resonance**. Thus, at resonance, we may write:

$$V_L = V_C$$

From Equation (42), we have:

$$Q = \frac{\omega L}{R}$$

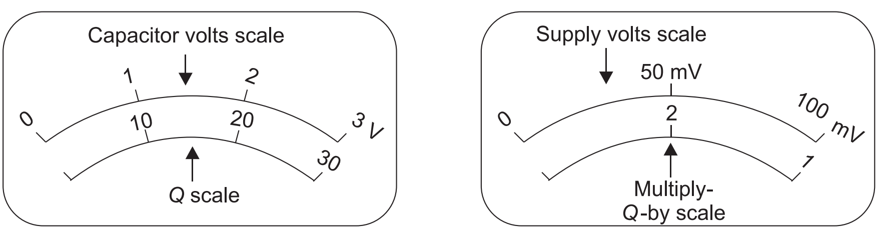
At resonance, it can be written as:

$$Q = \frac{\omega L}{R} = \frac{1}{\omega CR} \qquad ...(72)$$

Or, differently, we may write it as:

$$Q = \frac{V_L}{E} = \frac{V_C}{E} \qquad ...(73)$$

It should be noted that voltmeter V_2 can be calibrated to directly read the value of Q factor while voltmeter V_1 is calibrated as a *multiply-Q-by meter*. The scales of the two voltmeters are shown in Figure 7.40.



(a) Voltmeter V_2 scale calibrated to directly indicate Q factor

(b) Voltmeter V_1 scale calibrated as a multiply-Q-by meter

Fig. 7.40 Voltmeter Scales of a Q Meter

All the values of Q measured on voltmeter V_2 scale get multiplied by the value indicated on the voltmeter V_1 scale. For instance, if voltmeter V_1 scale indicates "2", then each value of Q measured by voltmeter V_2 gets doubled. The input voltage of the circuit must be highly stable; otherwise the changes in the supplied voltage directly appear in the circuit current since at resonance it is given as:

$$I = \frac{E}{R}$$

This results in erroneous indication of Q on the scale. Thus, the voltage level must always be set at the correct level. It must be noted here that since these effects are not present, if the supply voltage is stabilized, there is no need of monitoring the input voltage. Thus, the voltmeter V_1 is not incorporated in such a case. If the variable capacitor C is calibrated and its values are marked on the scale dial, then the inductance of the coil can be measured as:

$$L = \frac{1}{\omega^2 C} \qquad \text{[Refer to Eqn. (72)]}$$

Or, it may written as:

$$L = \frac{1}{(2\pi f)^2 C} \qquad ...(74)$$

It should be noted here that the value of inductance L is indicated for either a change in capacitance C or in frequency f. However, it is more convenient to vary the frequency for a fixed value of capacitance.

Let us now proceed to study different measurement methods using a Q meter. There are three ways to connect the component to be measured with the meter, namely *direct connection*, *series connection*, and *parallel connection*. Based upon the type and size of the component, the method of connection is determined.

Direct connection

This connection is appropriate for the measurement of medium range inductances, that is, up to 100 mH. These inductances can be directly connected to the inductance terminals of the meter. Keeping frequency level at the desired value, the output of the signal generator is so adjusted that it gives a suitable range of Q factor. The value of coil inductance is indicated on the C/L dial which indicates the capacitance and inductance of the circuit. The dial indications can be varied by adjusting capacitance control C/L. However, for all frequencies, circuit inductance and Q factor can be obtained using Equations (70) and (74).

There is a ΔC control on the meter panel which provides fine adjustments of variable capacitor C. When it is set to zero, an adjustment of Q capacitor control gives maximum deflection on the meter which in turn gives the direct indication of the Q factor of the coil. The circuit resistance can also be determined using these equations for known values of Q and L.

Series connection

This connection is suitable for measuring low impedance components, such as large capacitances, very small resistances, and small inductances. These components are connected serially with the reference inductance or coil for measurement. Initially, the component is shorted by a low resistance shorting strap. The inductance of the internal coil of the meter is varied to obtain resonance and the corresponding values of quality factor and capacitance are noted as Q_1 and C_1, respectively. Now, again a measurement is done after removing the shorting strap and re-adjusting the meter inductance to attain resonance. Let the new values be noted as Q_2 and C_2.

If a serially connected resistance R_s (to be measured) is pure, only the value of Q gets reduced while the resonance of the circuit remains unaffected by the removal of shorting strap. The value of this resistance can then be found by using the relation:

$$R_s = \frac{\Delta Q}{\omega C_1 Q_1 Q_2} \qquad ...(75)$$

where ΔQ represents the variation in Q_2 with respect to Q_1.

Similarly, in case a series capacitance C_S is to be measured, it is first shorted by a shorting strap and the values of Q and C of the circuit are noted as Q_1 and C_1, respectively. Then, the strap is removed and resonance is obtained by adjusting capacitance C of the circuit. Again the values of Q and C are noted as Q_2 and C_2, respectively. It is to be noted here that the quality factor Q of the circuit remains unaffected in this case. Thus, the value of C_S is obtained in terms of circuit capacitance as:

$$C_s = \frac{C_1 C_2}{C_2 - C_1} \qquad ...(76)$$

A serially connected inductance L_S can be measured in the same way. It is also initially shorted by a shorting strap and the circuit is resonated. The value of circuit capacitance is noted as C_1 and the strap is removed. The value of capacitance C is again noted as C_2 after again resonating the circuit. The inductance can then be calculated as:

$$L_s = \frac{C_1 - C_2}{\omega^2 C_1 C_2} \qquad ...(77)$$

Parallel connection

This connection is appropriate for measuring high impedance components. The component to be measured is the capacitance less than 400 pF, inductance greater than 100 mH, and resistance of high values, are connected parallel to the capacitor terminals.

If an inductance L_P is to be measured, the circuit is resonated by adjusting the reference inductance prior to connecting the parallel inductance. The values of Q and C are measured and noted as Q_1 and C_1, respectively. The inductance to be measured is then connected and the circuit is again resonated. The new values of Q and C are noted as Q_2 and C_2, respectively. The inductance L_p can now be determined as:

$$L_P = \frac{1}{\omega^2 (C_2 - C_1)} \qquad ...(78)$$

The effective Q factor is given as:

$$Q = \frac{Q_1 Q_2 (C_2 - C_1)}{C_1 (Q_2 - Q_1)} \qquad ...(79)$$

The similar procedure is followed for measuring a parallel capacitance C_P. The circuit is resonated by a reference inductance and the values of Q and C are measured and noted as Q_1 and C_1, respectively. Now, the unknown capacitance is connected and the circuit is again resonated by adjusting the circuit capacitance. Let this value be noted as C_2. The parallel capacitance C_P can then be found as:

$$C_P = C_1 - C_2 \qquad ...(80)$$

Here, Q remains the same even after connecting the capacitor.

When a resistance R_P, connected in parallel is to be measured, the circuit is again resonated using a reference inductance and the values of Q and C are noted as Q_1 and C_1, respectively. Now, the resistance to be measured is connected and the circuit is again resonated. In this case, only the value of Q is affected and the new value of Q is noted as Q_2 and the difference between Q_1 and Q_2 is noted as ΔQ. The resistance is then obtained by using the relation:

$$R_P = \frac{Q_1 Q_2}{\omega C_1 \Delta Q} \qquad ...(81)$$

Sources of errors

Among the factors that introduce errors in the measurements, the residual inductance and resistance can considered as the most pronounced source of error when the supplied voltage is not metered. If there is some source resistance R_E in the signal generator, it gets added to the circuit resistance R. This affects the circuit current as well as the Q factor of the circuit. Thus, at resonance, the current flowing through the circuit can be given as:

$$I = \frac{E}{R + R_E}$$

and the Q factor can be given as:

$$Q = \frac{\omega L}{R + R_E}$$

The above two equations show that the meter indicates erroneous values of current and quality factor due to the presence of R_E. For precise measurements, the residual resistance and inductance should be considerably smaller than the resistance and inductance of components to be measured. The practical Q meters have R_E of about 0.02 Ω and residual inductance of about 0.015 μH.

Example 16 For a resonant Q meter, determine the Q factor of inductor and meter indication, if:

(a) $E = 150$ mV, $\omega L = 1/\omega C = 150\ \Omega$, and $R = 6\ \Omega$

(b) $R = 12\ \Omega$ keeping rest of the components same

Solution: The current flowing through the circuit can be found as:

$$I = \frac{E}{R}$$

(a) Substituting the required values to calculate I:

$$I = \frac{150 \times 10^{-3}}{6} = 25 \text{ mA}$$

Now, we must know the voltage across inductor in order to obtain its Q factor. Thus, calculating V_L as:

$$V_L = V_C = I\omega L$$

Substituting the given values, we get:

$$V_L = 25 \times 10^{-3} \times 150 = 3.75 \text{ V}$$

Therefore, the Q factor of the coil can now be obtained as:

$$Q = \frac{V_L}{E} = \frac{3.75}{150 \times 10^{-3}} = 25 \qquad \text{[Refer to Eqn. (73)]}$$

The value of I can be calculated as:

$$I = \frac{150 \times 10^{-3}}{12} = 12.5 \text{ mA}$$

The voltage across inductance comes out as:

$$V_L = I\omega L = 12.5 \times 10^{-3} \times 150 = 1.875 \text{ V}$$

Therefore, the Q factor of the coil is computed as:

$$Q = \frac{V_L}{E} = \frac{1.875}{150 \times 10^{-3}} = 12.5 \qquad \text{[Refer to Eqn. (73)]}$$

Wagner earthing device

Wagner earthing device is a bridge circuit that eliminates the effects of stray capacitances between detector terminals and the ground (or earth capacitances). It basically reduces the potential difference between the ground and detector terminals. The bridge circuit of the device is shown in Figure 7.41.

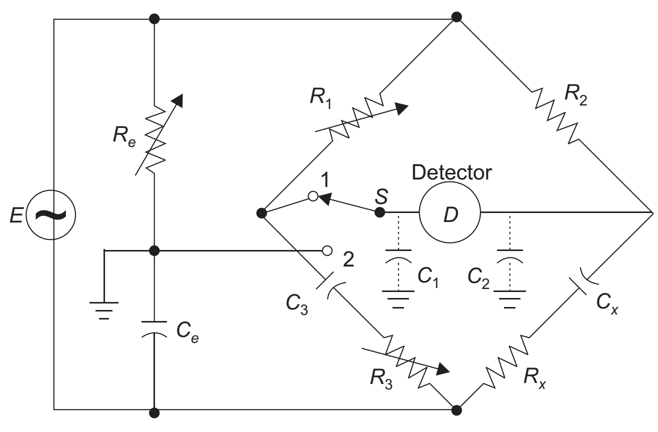


Fig. 7.41 Bridge Circuit of Wagner Earthing Device

At higher frequencies, while measuring large inductors, or small capacitors, stray capacitances C_1 and C_2 cause errors. A method known as **Wagner ground connection** is used to eliminate the effects of these capacitances. The oscillator is removed from its ground connection and a series combination of resistor R_e and capacitor C_e is used to bridge the oscillator whose junction is grounded. This ground point is known as **Wagner ground point**. The bridge is balanced by connecting the detector to point 1 and adjusting resistance R_1 to null. Then the switch is thrown to point 2 and resistance R_e is adjusted to obtain null position. Again, the switch is thrown to point 1 and null is obtained by adjusting R_1 and R_3. At null position, points 1 and 2 are at ground potential. Hence, stray capacitances C_1 and C_2 are shorted and no longer affect the normal bridge balance.

Note: *The capacitances in bridge arms still remain same to obtain the accuracy of the measurement and are not eliminated by Wagner ground connection.*

Let us Summarize

1. Bridge circuits are employed to remove errors occurring due to temperature effects, improper shielding and grounding of the circuit, and parasitic values.
2. A very useful way to measure the unknown values of resistances in DC circuits is to use DC bridge circuits or DC bridges.
3. The most basic DC bridge used to measure the resistance of a DC circuit is known as Wheatstone bridge which provides very accurate measurements.
4. The circuit used for measuring very small values of resistances, generally below 1 Ω is known as Kelvin bridge circuit. This provides a significant accuracy in low resistance measurements.
5. Wheatstone and Kelvin bridges cannot be used for measuring high resistances in the range of hundreds or thousands of megaohms. To find high resistances such as leakage resistance of a capacitor, insulation resistance of a cable, or surface or volume resistivity of a material, special circuits called guard circuits are employed in the unknown resistance arm of Wheatstone bridge.
6. Megger or megohmmeter is an instrument used to measure extremely high resistances, such as insulation resistances of cables of the order of megaohms.
7. All electrical systems must incorporate parts like switch boxes, metal conduits, machine casings, transformer tanks, and circuit breakers which must be connected to the ground or earth potential. This is to ensure that in case a fault occurs, the fault current flows directly to the earth or ground and protects the system as well as the operator. The resistance of the earth is measured by fall-of-potential test method which demonstrates the ability of a ground system or an electrode to carry leakage current.
8. AC bridges are used to determine various AC quantities, such as self and mutual inductance, capacitance, dissipation factor (D) of a capacitance, and quality factor (Q) of an inductance.
9. The bridges consisting of combination of resistances and capacitances in one or more of their arms are termed as resistance-capacitance bridges or RC bridges. De Sauty bridge, modified De Sauty bridge, parallel-resistance capacitance bridge, Schering bridge, and Wien bridge are all various types of RC bridges.
10. Inductance bridges are AC bridges that are used to measure the unknown inductances connected in one of their arms. Inductance comparison bridge, Maxwell bridge, Hay bridge, Anderson bridge, and Owen bridge are all various categories of these bridges.
11. Mutual inductance measuring bridges are designed to measure mutual inductance of the circuits. Heaviside bridge, Campbell bridge, and Carey-Foster bridge are all mutual inductance bridges.
12. There are some other bridges, like T networks and Q meters that are capable of determining any component, say resistance, inductance, capacitance, and frequency at high frequency ranges.
13. Wagner earthing device removes the stray capacitance errors from measurements and provides highly accurate results.

EXERCISES

Fill in the Blanks

1. The DC bridges are used to find the _________ in DC circuits.
2. A guard wire is incorporated in Wheatstone bridge in order to eliminate _________.
3. Frequency of operation can be found using _________ bridge.
4. Fall-of-potential method is used to measure _________.
5. Using Schering bridge, the relative permittivity of an unknown capacitance is obtained by the relation _________.

Multiple Choice Questions

1. For measurement of low impedance by Q meter, the component is connected in
   (a) parallel (b) series
   (c) directly (d) none of these
2. Which of the following bridges is preferred for the measurement of inductance having high Q-factor?
   (a) Maxwell bridge (b) Hay bridge
   (c) Owen bridge (d) De Sauty bridge
3. Campbell bridge is used to measure the unknown mutual inductance of an AC circuit in terms of
   (a) standard mutual inductance (b) standard self inductance
   (c) standard capacitance (d) both (a) and (c)
4. In AC circuits, Wagner earthing device is used to
   (a) eliminate the effects of earth capacitances
   (b) shield the components of the bridge
   (c) eliminate the effect of stray static electric fields
   (d) eliminate the effect of inter-component capacitances
5. The quality factor of an inductor is given as
   (a) $Q = \dfrac{R_s}{\omega L_x}$ (b) $Q = \dfrac{\omega L_x}{R_s}$
   (c) $Q = \dfrac{L_x}{R_s}$ (d) none of the above

State True or False

1. The inductive and capacitive phase angles of a Hay bridge should be equal to each other.
2. A wide range of high Q inductance measurements can be done using Anderson bridge.
3. The slow convergence to attain balance is known as sliding balance.
4. Precise measurements can be done by Q meter if the meter residual resistance and inductance are much higher than those of the components under measurement.
5. The unknown inductance can be found using Heaviside bridge as $L_x = \dfrac{1}{2}(M_{OC} + M_{SC})$.

Descriptive/Numerical Questions

1. State the advantages of using bridge circuits for measurement. Also define the sensitivity of Wheatstone bridge.

2. List the applications of AC bridges.
3. What are the difficulties associated with the measurement of low resistances? How are they overcome?
4. With a neat AC bridge network, derive the general equations for bridge balance.
5. A De Sauty bridge has a standard capacitance of 0.5 μF while the variable resistances can be adjusted to values between 5 kΩ to 25 kΩ. Determine the measurable range of the unknown capacitance.
6. Explain the operation of Schering bridge to determine the unknown capacitance. Also, derive the relevant equations.
7. (a) Derive the bridge balance conditions for Maxwell and Wheatstone bridges.
   (b) Explain the need of Wagner earthing device in bridge circuit.
8. A Schering bridge operating at a frequency of 150 Hz has standard capacitance and resistance of 0.4 μF and 15 kΩ, respectively. The bridge attains balance when variable capacitance and resistance are set to 3500 pF and 5 kΩ, respectively. Find the value of unknown capacitance and its dissipation factor.
9. Find the equivalent of the unknown impedance using a Maxwell bridge, if the bridge constants at balance are $C_1 = 0.01$ μF, $R_1 = 470$ kΩ, $R_2 = 5.1$ kΩ, and $R_3 = 100$ kΩ
10. Find the equivalent parallel resistance and capacitance that causes a Wein bridge to null if the following components are given:
    $R_1 = 8$ kΩ, $C_1 = 6$ μF, $R_2 = 30$ kΩ, $R_3 = 1$ kΩ, and $f = 2.5$ kHz
11. Explain practical Q-meter with suitable diagram. Also mention the probable errors in its measurement.
12. A coil of 40 mH has a Q factor of 18 at a frequency of 2 kHz. An Owen bridge with $C_1 = C_3 = 0.2$ μF and $R_4 = 2$ kΩ is used to investigate this coil. Calculate the values of R_1 and R_S.
13. Describe the operation of a basic RC bridge in general with suitable diagram.
14. Write short notes on:
    (a) Megger
    (b) Fall-of-potential method
    (c) Wagner earthing device

8

Methods of Magnetic Measurement

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Discuss the different methods of measuring flux density
- Appreciate the concept of different types of magnetometers and measurement of flux density using ballistic galvanometer
- Differentiate between the two methods of determining B-H curve
- Determine hysteresis loop using step-by-step method and reversal method
- Discuss the different types of permeameters
- Perform AC testing of magnetic materials
- Determine iron losses in magnetic materials using the wattmeter method and bridge method

8.1 INTRODUCTION

While designing and constructing electrical equipments, knowledge of the properties of magnetic materials used for their construction is a must. These properties affect the operating characteristics of electrical machines, instruments, and apparatus owing to close association of electric and magnetic phenomena. Magnetic measurements include determination of parameters, such as magnetic flux density in air, *B-H* curve and hysteresis loop for ferromagnetic materials, eddy current and hysteresis losses when ferromagnetic materials are subjected to alternating magnetic fields, and the parameters of permanent magnets.

To investigate magnetic fields, a test coil, known as **search coil**, is used. When the applied field is stationary, no voltage will be induced in the coil. Thus, alternating or changing fields are required for magnetic measurements. However, some instruments, namely, fluxgate magnetometer and Hall-effect magnetometer can be used to examine stationary magnetic field. A known magnetizing force is applied to the magnetic cores which on reversing generates an output voltage pulse. This voltage pulse is in direct proportion to the change in flux density. Using ballistic galvanometer (as studied in Chapter 3), this voltage pulse can be measured to determine flux density, *B-H* curve, and hysteresis loop. Wattmeter and different AC bridges can be used to determine core losses, which can be further separated into hysteresis and eddy current losses.

The method selected to determine the magnetic parameter could be different but there are always some inherent inaccuracies in the result due to following reasons that include inhomogeneity of magnetic materials, non-uniformity between different batches of specimens, and difference in the assumed and actual conditions of specimens under test.

8.2 DETERMINATION OF MAGNETIC FLUX DENSITY

In this section, we will determine the flux density of a magnetic field using various methods that include *magnetometers* and *ballistic galvanometer*.

8.2.1 Magnetometers

An instrument which is used to measure magnetic flux density is known as **magnetometer**. This instrument is also known as **Gauss meter** or **Tesla meter** since its output can be expressed in Gauss or Tesla. Gauss is the unit of magnetic flux density in CGS system while Tesla is in SI system. The two units can be related as: 1 Tesla = 10^4 gauss.

Magnetometers can also be used to measure the magnetization of a magnetic material, such as ferromagnetic material. They are widely used to detect magnetic anomalies in geophysical surveys and determine the Earth's magnetic field. These instruments can also be used in military applications, such as to detect submarines. In addition, magnetometers can also work as very long range metal detectors. Their detection range is of tens of metres which is much greater than the two metre range of a conventional metal detector. However, these instruments can only detect the magnetic materials. The different types of magnetometers are *induction coil magnetometer*, *fluxgate magnetometer*, and *Hall-effect magnetometer*.

Induction coil magnetometer

An induction coil magnetometer consists of a coil placed in a changing magnetic field which results in an induced voltage at the terminals of the coil. The coil consists of a ferromagnetic core which concentrates the magnetic field that increases the level of induced voltage. However, it may limit the frequency of the field to be measured. An induction coil magnetometer is shown in Figure 8.1.

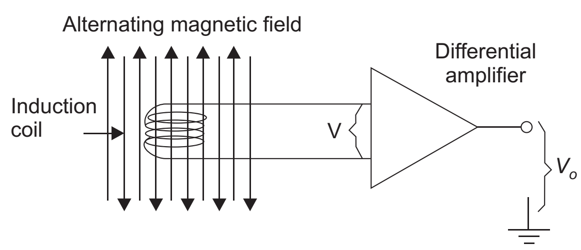


Fig. 8.1 An Induction Coil Magnetometer

The induced voltage, denoted by V, is in direct proportion to the number of turns in the coil, N and the time rate change of the magnetic flux, $\Delta\Phi/\Delta t$. Thus, the expression for induced voltage V can be given as:

$$V = N\frac{\Delta\Phi}{\Delta t}$$

The differential amplifier amplifies the output voltage at the coil terminals to a measurable level. Let the variation in magnetic flux be sinusoidal, then the time taken for its rise from zero level to peak value of the flux is given as:

$$\Delta t = \frac{T}{4} = \frac{1}{4f}$$

where T and f are the time period and frequency of the sinusoidal signal.

Now, the average value of the induced voltage V_{avg} is given by the expression:

$$V_{\text{avg}} = \frac{N\Phi_m}{(1/4f)}$$

where Φ_m is the peak value of the flux. The above equation can also be written as:

$$V_{\text{avg}} = 4fN\Phi_m \quad ...(1)$$

The rms value of the induced voltage is given as:

$$V_{\text{avg}} = 4.44fN\Phi_m \quad ...(2)$$

Substituting the value of V_{avg} from Equation (1) into Equation (2), we get:

$$V_{\text{rms}} = 4.44fN\Phi_m$$

On rearranging the above equation, we get:

$$\Phi_m = \frac{V_{\text{rms}}}{4.44fN} \quad ...(3)$$

The relation between the magnetic flux Φ and magnetic flux density B is given by the relation:

$$B = \frac{\Phi}{A} \quad ...(4)$$

where A is the cross-sectional area of the coil measured in square metre (m^2), flux Φ is in webers (Wb), and the magnetic flux density B is given in Teslas (T).

Fluxgate magnetometer

The induction coil magnetometer discussed in the preceding section works for the alternating magnetic field. However, when both the coil and magnetic field are stationary, there will be no induced voltage in the coil. Thus, it may be concluded that for a steady-state magnetic field, the induction coil magnetometer cannot be used to determine its strength. In fluxgate magnetometer, an excitation field is used which disturbs the effect of steady-state magnetic field on a sense or secondary coil. Due to this, a voltage is induced which is in proportion to the flux density in the static field. The fluxgates are rugged, compact, low in cost, and have low power consumption. There are two types of fluxgate magnetometers to be discussed in detail, namely, *twin-rod fluxgate* and *ring-core fluxgate*.

Twin-rod fluxgate

As the name suggests, this fluxgate consists of two bar-shaped magnets placed adjacent to each other. Each of the magnets is provided with an excitation coil through which an alternating current flows as shown in Figure 8.2.

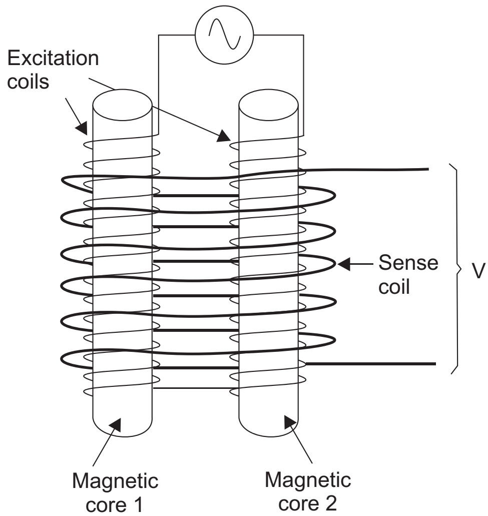


Fig. 8.2 Twin-rod Fluxgate Magnetometer

The alternating current in the coils drives the two rods into positive and negative saturation, that is, if one rod is south at the top, the other will be north at the top and vice versa. The sense coil produces an output voltage on changing the flux of the core. When no external static magnetic field is applied, the flux in one core cancels the flux in the other core due to opposite polarity. As a result, no voltage is induced in the secondary coil in this case. This is illustrated by the waveforms in Figure 8.3. While the cores are saturated, no voltage will be

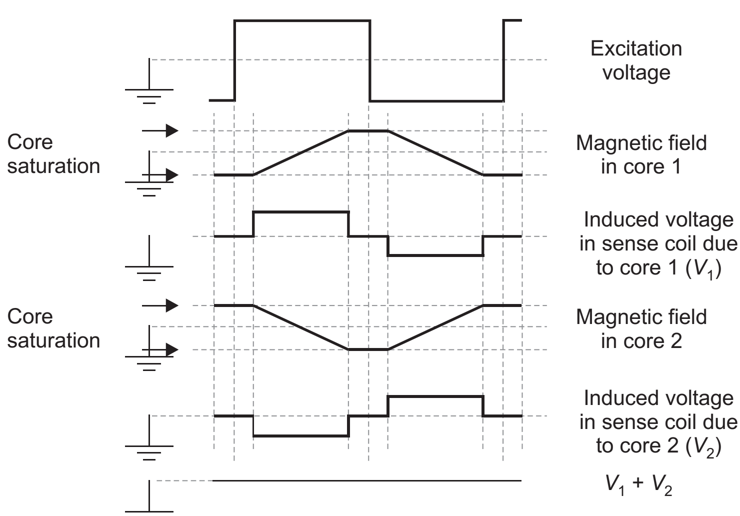


Fig. 8.3 Fluxgate Magnetometer Waveforms in Absence of Static Magnetic Field

induced in the sense coil. However, during the saturation, voltages get induced in the sense coil from the excitation coil. These voltages are represented as V_1 and V_2 in the figure, which are of equal magnitude but opposite polarity. Therefore, these voltages cancel out each other leaving output from the sense coil to be zero.

Now, consider the case when an external magnetic field is applied by placing the fluxgate magnetometer in a static magnetic field. This static field drives the core having same polarity as that of the static field deeper into saturation whereas it opposes the core with opposite polarity. This results in a net flux imbalance in the two rods as well as unsymmetricity in the induced voltages V_1 and V_2. As a consequence, an output voltage proportional to the magnetic flux density of the static magnetic field will be generated in the sense coil. This output is equal to the algebraic sum of V_1 and V_2. The waveforms are shown in Figure 8.4.

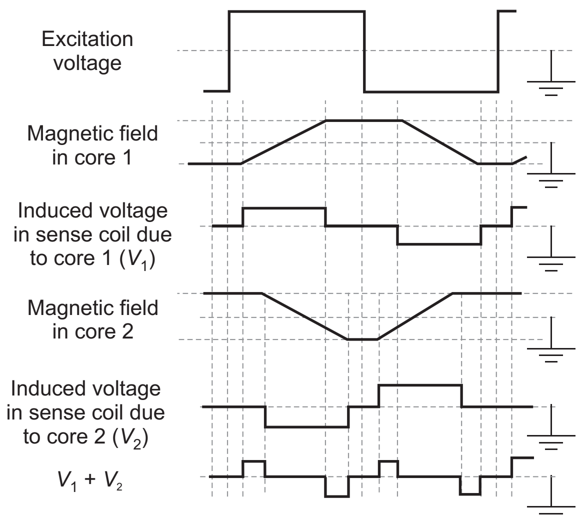
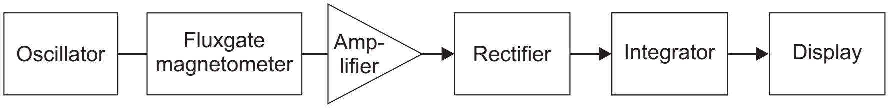


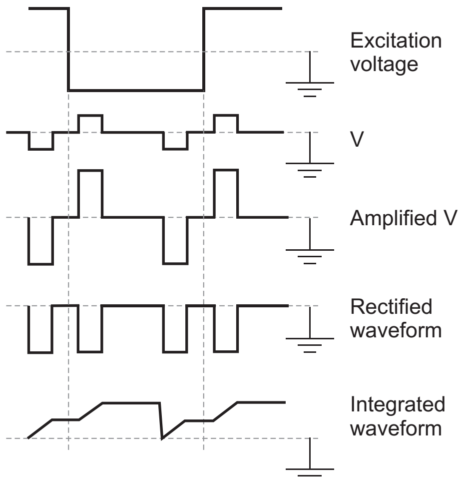
Fig. 8.4 Fluxgate Magnetometer Waveforms in Presence of Static Magnetic Field

The magnitude of the output voltage so produced depends upon the rate of change of the flux which further depends upon the excitation voltage, number of turns in the sense coil, geometry of the sense coil, and magnetic permeability of the core. The difference in the flux densities in the core arises due to flux density of the static field and determines the width of the output pulse. So, we may say that the static magnetic field modulates the output of the fluxgate where the modulation is pulse width modulation. This static field flux density can be obtained using an integrator.

The circuit for obtaining static field flux density along with waveforms at various stages is shown in Figure 8.5. Here the output, before applying to the integrator must be amplified and rectified. Therefore, an amplifier and a rectifier is connected prior to the integrator.



(a) Block diagram



(b) Waveforms of different stages

Fig. 8.5 Flux Density Determination

Ring-core fluxgate

Ring-core fluxgate magnetometer consists of a ring-shaped core on which the excitation coil is wound in a torroidal fashion as shown in Figure 8.6. Its operation is similar to that of twin-rod fluxgate.

The twin-rod fluxgate magnetometer can be converted into a ring-core fluxgate magnetometer by closing the magnetic circuit at top and bottom with two rods.

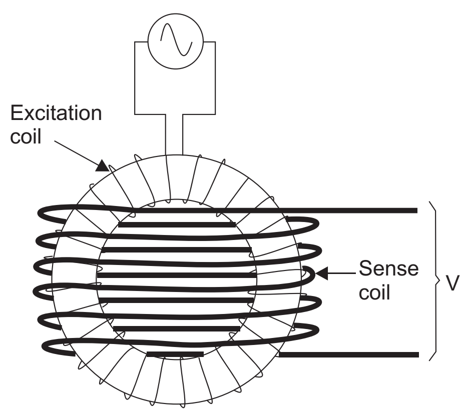


Fig. 8.6 Ring-core Fluxgate

Application of fluxgate magnetometer

The major application of a fluxgate magnetometer is to align it with the Earth's magnetic field since it can measure the flux density of a static field and can also determine the direction and polarity of the field if it is inverted. When a fluxgate is perfectly aligned with the static magnetic field to be measured, it produces the highest output voltage and if a change in the alignment of the fluxgate occurs, the output gets reduced.

Here, it is to be noted that as long as the field frequency is smaller than the excitation frequency, fluxgates can also be used to study alternating magnetic fields.

Hall-effect magnetometer

When a strip of current-carrying conductor is subjected to a transverse magnetic field, a voltage is produced at the edges of the conductor which is perpendicular to the direction of both magnetic field and current as shown in Figure 8.7. This phenomenon is known as **Hall-effect**.

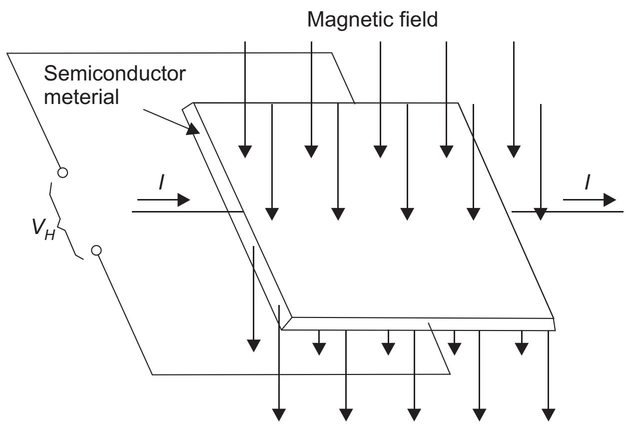


Fig. 8.7 Hall-effect Device

The magnitude of the output voltage V_H is proportional to the magnetic flux density B, current I, the material (generally semiconductor) used for the conductor and its dimensions. Thus, the output voltage V_H is expressed as:

$$V_H = K_H BI \sin \theta \qquad \text{...(4)}$$

where K_H is known as Hall-effect coefficient and represents the combined effect of conductor's dimensions and material, and θ is the angle between the direction of the magnetic field and the Hall-effect device. The relation between output voltage V_H and magnetic flux density B is represented in Figure 8.8. It can be noticed that the output voltage V_H varies linearly with the magnetic flux density B up to a saturation level in either positive or negative direction, provided the current is held constant. The polarity of the magnetic field determines the polarity of the output voltage V_H when the direction of the current does not change.

Generally, a differential amplifier is used to amplify the output voltage from Hall-effect device for measurement as it is of the order of microvolts or millivolts range. A variable resistor is also used to adjust the value of current, however, it must be held constant at a particular value (see Figure 8.9).

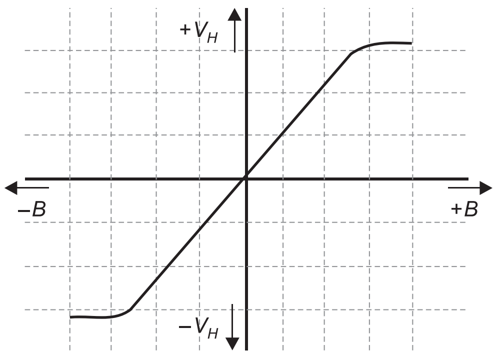


Fig. 8.8 Graph of Magnetic Flux Density versus Output Voltage

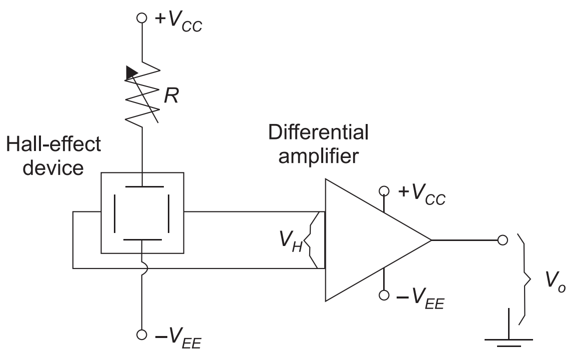


Fig. 8.9 Hall-effect Magnetometer

Hall-effect magnetometers are used in applications with large magnetic field strengths, for example, as anti-lock braking systems in cars.

Example 1 An alternating magnetic field which produces an output of 300 mV rms at 60 Hz is investigated by an inductive coil magnetometer. Calculate the peak field flux density if the diameter and number of turns of search coil are 5 cm and 45, respectively.

Solution: Given that: $V_{\text{rms}} = 300$ mV, $f = 60$ Hz, $N = 45$, and diameter = 5 cm.

From Equation (3), we have the peak value of flux Φ_m as:

$$\Phi_m = \frac{V_{\text{rms}}}{4.44\, fN}$$

Substituting the given values in the above equation, we get:

$$\Phi_m = \frac{300}{4.44(60)\,(45)} = 25\ \mu\text{Wb}$$

Now, we have the magnetic flux density B given from Equation (4) as:

$$B = \frac{\Phi}{A} = \frac{\Phi}{\pi r^2} \qquad \text{[where } r \text{ is the radius of the coil]}$$

Now, $A = \pi(5/2)^2 = \pi(2.5)^2 = 19.634\ \text{cm}^2$

The peak field flux density B_m can be computed as:

$$B_m = \frac{25}{19.634} = 12.5\ \text{mT}$$

Example 2 A Hall-effect magnetometer produces a 10 mV output when used to measure a magnetic field whose flux density is given as 2.7 T. Calculate the required current, if the device coefficient is given to be 0.6×10^{-3} and the magnetic field is placed perpendicular to the device.

Solution: Given that: $K_H = 0.6 \times 10^{-3}$, $B = 2.7$ T, $V_H = 10$ mV, and $\theta = 90°$

Now, we have the output voltage V_H expressed as:

$$V_H = K_H BI \sin\theta \qquad \text{[Refer to Eqn. (5)]}$$

Substituting the given values in the above relation, we get:

$$10 \times 10^{-3} = 0.6 \times 10^{-3} \times 2.7 \times I \times \sin 90°$$

Therefore, the required current I can be determined as:

$$I = \frac{10 \times 10^{-3}}{0.6 \times 10^{-3} \times 2.7} = 6.2\ \text{A}$$

8.2.2 Ballistic Galvanometer

Earlier we have studied the different types of magnetometers which are used to measure flux density. In addition to different magnetometers, the flux density of a magnetic field can also be measured using either a ballistic galvanometer or a flux meter. However, here we will discuss the measurement of flux density using ballistic galvanometer only (as studied in Section 3.4).

Consider a ring-shaped core as shown in Figure 8.10 for which the magnetic flux density is to be determined. The core is wound with a magnetizing (primary) winding carrying a current I_1. This current I_1 is supplied from an adjustable DC supply and can be measured by

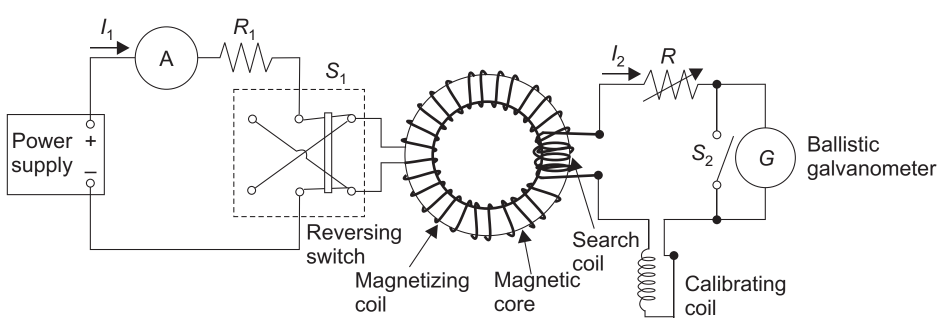


Fig. 8.10 Measurement of Flux Density using Ballistic Galvanometer

the ammeter. A sense coil (also known as **search** or **secondary coil**) with convenient number of turns is also wound on the ring core and connected to a ballistic galvanometer through a calibrating coil and a resistance (see Figure 8.10).

A reversing switch S_1 is connected in the circuit which reverses the direction of the current I_1 passing through the magnetizing coil. Due to this, the flux linkage of the sense coil changes resulting in an induced electromotive force (emf) in it. This induced voltage e is monitored by the ballistic galvanometer and can be given by the relation:

$$e = N\frac{d\Phi}{dt} \qquad ...(6)$$

where N is the number of turns of sense coil and Φ represents the flux in it. Now, the change in the flux for a time t is equal to 2Φ, that is, from $-\Phi$ to $+\Phi$. Thus, we have:

$$\frac{d\Phi}{dt} = \frac{2\Phi}{t} \qquad ...(7)$$

Substituting Equation (7) into Equation (6), we get the induced emf e as:

$$e = \frac{2N\Phi}{t}$$

The current through the galvanometer, that is, I_2 is given as:

$$I_2 = \frac{e}{R} = \frac{2N\Phi}{tR}$$

where R is the resistance of the ballistic galvanometer circuit. The charge passing through the galvanometer can be written as:

$$Q = I_2 t = \frac{2N\Phi}{R} \qquad ...(8)$$

Let the initial peak deflection of the galvanometer be x mm and the deflection constant K_q, measured in coulombs/mm. Then the charge Q applied is given as:

$$Q = K_q x \qquad ...(9)$$

From Equations (8) and (9), we get:

$$\frac{2N\Phi}{R} = K_q x$$

On rearranging the above equation, we have flux Φ as:

$$\Phi = \frac{R}{2N} K_q x \qquad ...(10)$$

When the turns of the coil are perpendicular to the flux density vector for a uniform field, the relation between the magnetic flux Φ and magnetic flux density B is given by:

$$B = \frac{\Phi}{A} \qquad \text{[Refer to Eqn. (4)]}$$

Substituting the value of flux Φ from Equation (10) into the above equation, we get:

$$B = \frac{RK_q x}{2NA} \qquad ...(11)$$

where A is the cross-sectional area of the core.

Air flux correction

While deriving Equation (11), it was assumed that the magnetic flux is uniform and the effective area of the ring-shaped core and cross section of the sense coil are equal. However, the core has smaller area than the sense coil, thus, the observed value of the flux is equal to the sum of the flux in the core and in the air gap between the core and the sense coil. Thus, we have:

Observed value of flux = Flux in core + Flux in air gap

Or, it can be written as:

$$B'A = BA + \mu_o H\,(A_s - A)$$

where A_s is the area of cross section of the sense coil, B' is the observed value and B is the true value of the flux. Therefore, true value of the flux in the core can be written as:

$$B = B' - \mu_o H\left(\frac{A_s}{A} - 1\right)$$

Measurement of H

The strength of core field is given by:

$$H = \frac{I_1 N'}{l} \qquad ...(12)$$

where N' is the number of turns of primary winding and l is the average magnetic path length of core. If l is in m, then the magnetic field strength, H is given in A/m.

Note: *Relative permeability of the core can be determined by the values of B and H using the relation* $\mu_r = \dfrac{B}{\mu_o H}$.

Example 3 Calculate the total core flux of a ring-shaped magnetic core with relative permeability 900, magnetic path length 0.4 m, and cross-sectional area of 270×10^{-6} m^2. The number of turns in primary and secondary windings are given to be 800 and 400, respectively, with a primary winding current of 0.15 A.

Solution: Given that: $N = 400$, $N' = 800$, $l = 0.4$ m, $I_1 = 0.15$ A, $\mu_r = 900$, and $A = 270 \times 10^{-6}$ m^2

We have the relation for magnetic strength H as:

$$H = \frac{I_1 N'}{l} \qquad \text{[Refer to Eqn.(12)]}$$

Substituting the given values, we get:

$$H = \frac{0.15 \times 800}{0.4} = 300 \text{ A/m}$$

Now, the magnetic flux density B is given by:

$$B = H\mu_r\mu_o \qquad \text{[Refer to Section 8.2.2]}$$

Substituting the given values, we get:

$$B = 300 \times 900 \times 4\pi \times 10^{-7} = 339 \text{ mT}$$

Thus, the total core flux Φ is obtained as:

$$\Phi = BA = 339 \times 270 \times 10^{-6} = 91.5\ \mu\text{Wb}$$

8.3 DETERMINATION OF *B-H* CURVE

The curve obtained by plotting the various values of magnetic flux density B of a magnetic material corresponding to different values of its magnetic field intensity H is known as ***B-H* curve**. The *B-H* curve can be determined by two methods, namely, *method of reversals* and *step-by-step method*. These methods are explained here.

8.3.1 Method of Reversals for *B-H* Curve

The ring core specimen whose *B-H* curve is to be determined is connected in the circuit as shown in Figure 8.10. Before determining the values of B and H, the residual magnetism of the core is reduced to zero, that is, it must be demagnetized. Demagnetization of the core is

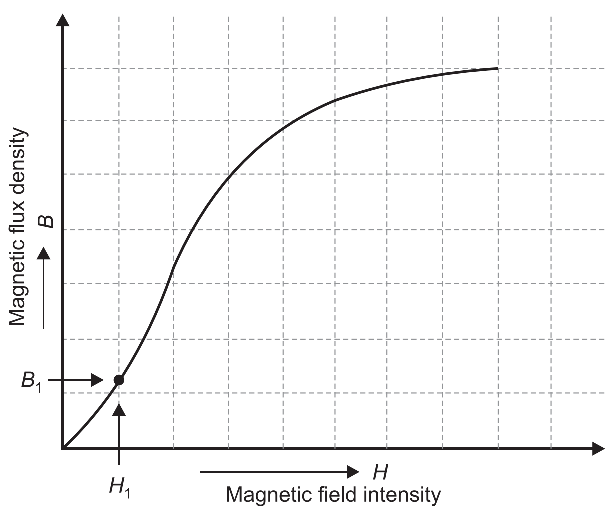


Fig. 8.11 B-H Curve using Method of Reversals

achieved by driving the core into magnetic saturation by increasing the value of magnetizing current I_1 to a sufficient level. This can be done by closing the switch S_2. Now, the switch S_1 is reversed repetitively when current I_1 is gradually reduced to zero.

After demagnetization, the test is performed by increasing the magnetizing current I_1 to a value such that it gives the lowest magnetic field strength H_1. For this value of H_1, the magnetic flux density B_1 is determined from galvanometer throw by opening switch S_2. During this, switch S_1 is reversed repetitively to verify the measured value of flux density. The obtained value of B is then plotted on the B-H graph as shown in Figure 8.11. This procedure is repeated several times to obtain different values of B for various corresponding values of H. The curve is then plotted using those values.

8.3.2 Step-by-Step Method for *B-H* Curve

The circuit for determining the B-H curve by this method is the same as that for reversal method. The only difference is that the current is supplied to the magnetizing winding via a switched potential divider as shown in Figure 8.12. The magnetic strength H can be set to various desired values by means of switch S_2. The desired maximum level of current can be achieved by setting the switch S_2 at position 6 and by adjusting resistance R_1 or power supply. The ring core must be demagnetized prior to the testing.

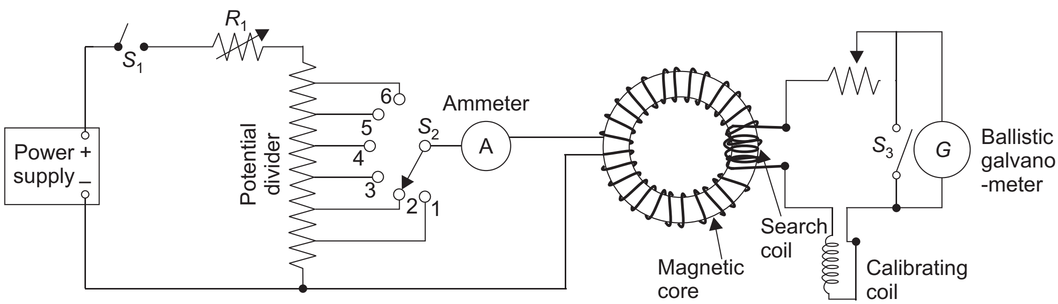


Fig. 8.12 Circuit for Step-by-Step Method

To determine the value of flux density B, the switch S_1 is closed and the switch S_2 is set on position 1 which gives the lowest value of magnetic field strength H. The change in the flux density from zero to some level corresponding to this value of H is determined by similar method used earlier in the method of reversals. The value of the current in magnetizing winding is noted for this position and the magnetic field strength is calculated using this value. The value of magnetic flux density B_1 and magnetic field strength H_1 so obtained are then recorded in a tabular form. Now, the switch S_2 is set to position 2 resulting in an increase in current level to a new value. The corresponding change in magnetic flux density ΔB is noted down from the throw of galvanometer and added to the previous value of the magnetic flux density B_1 to give a new reading B_2. This reading is also recorded along with its corresponding value of magnetic field strength H_2 in the table. This procedure is repeated to obtain a number of values of B and H by selecting different values of H up to its maximum value. These values are plotted to give B-H characteristics as shown in Figure 8.13.

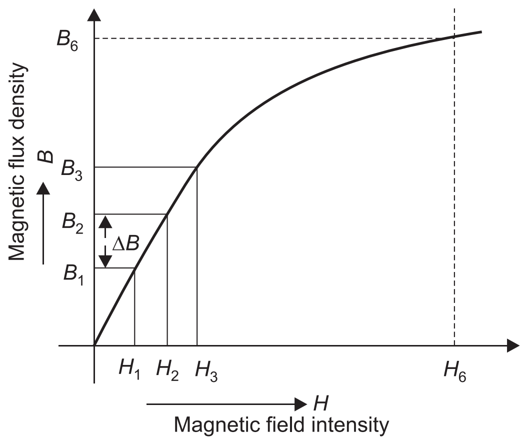


Fig. 8.13 B-H Curve using Step-by-Step Method

8.4 DETERMINATION OF HYSTERESIS LOOP

In magnetic materials, the flux density B always lags behind the magnetic field strength H and forms a loop when we draw the B-H curve. This loop is known as **hysteresis loop** and the area under it gives the amount of heat loss in the material. The hysteresis loop can also be determined by two methods, namely, *method of reversals* and *step-by-step method*.

8.4.1 Method of Reversals for Hysteresis Loop

The circuit for determining the hysteresis loop by this method is shown in Figure 8.14. It consists of three resistances R_1, R_2, and R_4 provided in the galvanometer circuit and magnetizing winding. To control the current through the magnetizing winding, an additional resistor R_3 is provided which can reduce the value of magnetizing winding current from its maximum to any required value. This resistance is a shunting variable resistance whose connection is determined by the position of switch S_2.

Here, it is to be noted that the change in flux density is made from its maximum value B_m down to some lower value. At each step, there is some reduction in the magnetic flux density being measured and then the ring core is magnetized back to its maximum value B_m. The B-H curve determined in the previous section is used to obtain the value of H_m required to generate a flux density B_m.

With switch S_2 in off position, resistances R_1 and R_2 are adjusted such that the current passing through the magnetizing winding may provide magnetic strength equal to H_m. Resistance R_1 is so adjusted that on reversing the maximum value of magnetic strength, the galvanometer shows a convenient deflection. Resistance R_3 is adjusted to reduce the amount of current in magnetizing winding when switch S_2 is opened. Now, with switch S_5 opened and switch S_3 connected to points 1 and 1′, the point is marked for maximum value of current. This point is labeled as a in Figure 8.15.

Now switch S_2 is opened by connecting it to terminal B. This results in a decrease in magnetizing current and thus, magnetic strength. Let the new value of H be denoted by H'.

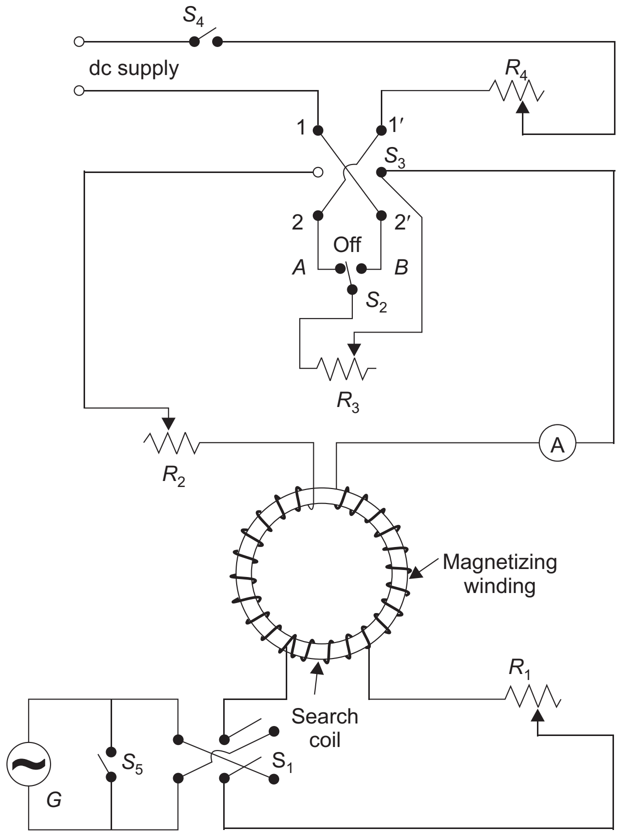


Fig. 8.14 Circuit for Method of Reversals

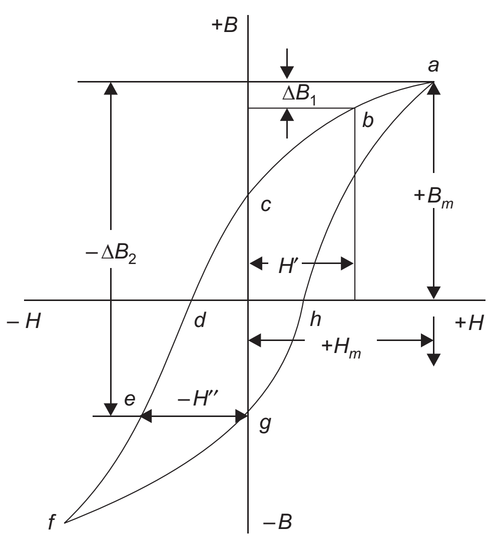


Fig. 8.15 Hysteresis Loop using Method of Reversals

Due to this reduction, the value of magnetic flux density also reduces and the change ΔB_1 is shown by galvanometer deflection. Corresponding to this value, point *b* is plotted on the curve. The ring core is now again magnetized back to its maximum value by closing switch S_5, reversing switch S_3 back to 2 and 2′, and then opening switch S_2. The switch S_3 is then again brought back to position 1 and 1′ to perform the next step of measurement. By continuing this process, we get part *ac* of the loop.

To trace part *cdef* of the loop, switch S_2 and S_5 are closed while switch S_3 remains at terminals 1 and 1′. Now, switch S_5 is opened while reversing switch S_3 rapidly to terminals 2 and 2′. The value of magnetic strength gets changed from its maximum value to some lower value, say $-H''$. The change in magnetic flux density $-\Delta B_2$ corresponding to this reduction can be calculated from the throw of galvanometer and point *e* can be marked on the hysteresis loop using these values. The ring core is then brought back to point *a* of the curve by connecting switch S_3 to terminal 1 and 1′, while switch S_5 is closed. This procedure is followed and part *cdef* is completed. Thus, part *acdef* of the hysteresis loop is obtained and by drawing it in reverse, part *fgha* can be obtained as both parts are symmetrical.

8.4.2 Step-by-Step Method for Hysteresis Loop

In this method, the hysteresis loop is determined simply by continuing the step-by-step method for determining *B-H* curve (refer to Section 8.3.2). When the maximum value of *H* is achieved by setting the switch at position 6, the values of *H* and *B* are determined by decreasing the value of current through magnetizing winding. The switch is now moved consecutively downwards from position 6 to position 1, where position 1 corresponds to the lowest value of *H*. All these readings are recorded.

Now, the polarity of the supply voltage to the potential divider is reversed to obtain negative values of *H*. The switch is now moved upwards from position 1 to position 6 and the corresponding values of *H* and *B* are recorded and then plotted to give the complete hysteresis loop.

8.5 PERMEAMETERS

In the previous sections, we have considered the determination of flux density, *B-H* curve, and hysteresis loop all using ring specimens. These parameters can also be determined using bar specimens which are much easier to construct than ring specimens. However, using bar specimens produces inaccurate results and some difficulties are faced due to following reasons.

- In bar specimens, the air which has very high reluctance acts as the return circuit for flux.
- The poles are produced at the ends of the bar when it is magnetized. Due to these poles, a magnetizing force acts inside the rod from north to south pole. This force acts in opposition to the applied magnetic force. This phenomenon is known as **self-demagnetization** or **end effects**.

Thus, to obtain accurate results, correction for these factors must be applied. In permeameters, the effects of self-demagnetization are reduced or even removed completely in some cases. This is achieved by providing a return path having low reluctance. There are different types of permeameters which may differ in the method of determination of magnetization force, range of magnetizing force, and degree of compensation of leakage flux. However, the return path for flux in all of them is of large cross-sectional area so as to have negligible reluctance. The different types of permeameters discussed here are *Hopkinson permeameter*, *Ewing double bar permeameter*, *Illiovici permeameter*, *Burrow's permeameter*, *Fahy's simplex permeameter*, and *National physical laboratory form of permeameter*.

8.5.1 Hopkinson Permeameter

In Hopkinson type of permeameter, the test coil is wound on the central portion of a bar specimen while the magnetizing coil is wound upon the whole of the specimen. The test coil is wound on its central part as shown in Figure 8.16. The bar specimen is fixed in the middle of two halves of an iron yoke. This yoke has a very large cross-sectional area and a very low reluctance as compared to that of the bar. The return path for the flux is provided by this yoke. Hopkinson permeameter is also known as **bar and yoke method**.

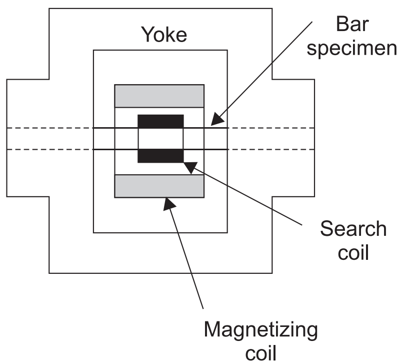


Fig. 8.16 Hopkinson Permeameter

The reluctance of the bar R_s is given as:

$$R_s = \frac{l}{\mu_s A_s}$$

where l is the length of bar specimen, μ_s is the permeability of specimen, and A_s is the area of cross section of the bar. Therefore, flux in the magnetic circuit is given as:

$$\Phi = \frac{\text{mmf}}{\text{Reluctance of magnetic circuit}}$$

Or, it can be written as:

$$\Phi = \frac{NI}{R_y + R_j + (l/\mu_s A_s)} \qquad \text{...(13)}$$

where N is the number of turns of magnetizing winding, I is the current in the magnetizing coil, R_y is the reluctance of yoke, and R_j is the reluctance of joints between bar and yoke.

The flux density B is given by the relation:

$$B = \frac{\Phi}{A_s} \qquad ...(14)$$

Substituting the value from Equation (13) into Equation (14), we get:

$$B = \frac{NI}{A_s[R_y + R_j + (l/\mu_s A_s)]}$$

The magnetic field strength H is given by:

$$H = \frac{B}{\mu_s}$$

Or, we have:

$$H = \frac{NI}{\mu_s A_s[R_y + R_j + (l/\mu_s A_s)]} \qquad ...(15)$$

Let m be a parameter equal to the ratio of reluctances, that is,

$$m = \frac{\text{Reluctance of yoke + joints}}{\text{Reluctance of bar}} = \frac{R_y + R_j}{R_s} = \frac{\mu_s A_s}{l}(R_y + R_j) \qquad ...(16)$$

Using Equation (16), Equation (15) can be written as:

$$H = \frac{NI}{l(1+m)}$$

When m is very small, the above expression becomes:

$$H = \frac{NI}{l}(1-m)$$

Thus, the difference between the actual value and the calculated value of H will be mNI/l. The small value of m can be attained at small values of reluctance of yoke and joints. This is achieved by leaving no air space between the bar and the yoke by fitting the bar properly into it. The large cross-sectional area of the yoke also helps in keeping m small. It is to be noted that the flux density is measured by ballistic galvanometer using the general method.

8.5.2 Ewing Double Bar Permeameter

In Ewing double bar permeameter, two exactly similar bars are used. These bars are made up of the magnetic material to be tested and wound with two pairs of coil each. We have four magnetizing coils, out of which, the length of two coils is half the length of the other two. Each bar is wound with one full length and one half length coil. Here it is to be noted that the number of turns per unit axial length for all four coils are equal. The bars are fitted into the holes of soft iron yokes. This configuration is represented in Figure 8.17. The yokes are mobile and thus their positions on the bars are adjustable.

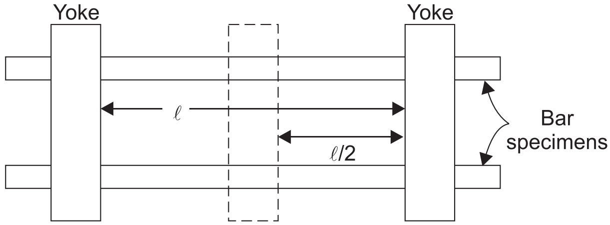


Fig. 8.17 Ewing Double Bar Permeameter

By this arrangement, the reluctance of yoke and joints can be eliminated. The tests are performed for both full length specimen and half length specimen. For a given value of flux density, the reluctance of yokes and joints is assumed to be equal for both positions.

Let H_1 be the apparent magnetic strength for length l and H_2 be the apparent magnetic strength for length $l/2$. Then we have:

$$H_1 = \frac{nlI_1}{l} = nI_1$$

and,

$$H_2 = \frac{n(l/2)I_2}{l/2} = nI_2$$

where n is the number of turns per unit length of magnetizing coil and I_1 and I_2 are the coil currents when length of the specimen is l and $l/2$, respectively. Now, we have:

Total applied mmf = mmf required for iron + mmf required for yoke and joints

Let H be the true magnetic field strength in the iron for a magnetic flux density B (same for both lengths of the specimen), then we have:

$$H_1 l = Hl + F_{mmf} \quad \text{...(17)}$$

where F_{mmf} is the magnetomotive force (mmf) required for the yokes and joints for each case.

Also,

$$H_2(l/2) = H(l/2) + F_{mmf} \quad \text{...(18)}$$

Subtracting Equation (18) from Equation (17), we get:

$$H = 2H_1 - H_2$$

Here, again the flux density measurements are performed by ballistic galvanometer in the usual way. The greatest advantage of this method is the ease it provides for measurements. However, there are some inherent disadvantages of it as listed below.

- Two exactly similar bars are required for the test.
- It is a time-consuming process.
- The reluctance of yokes and joints is not exactly the same for both positions.

8.5.3 Illiovici Permeameter

In Illiovici permeameter, the bar specimen is wound with a magnetizing coil and search coil and is clamped against a yoke with a heavy section. A ballistic galvanometer G_1 is connected across the search coil as shown in Figure 8.18.

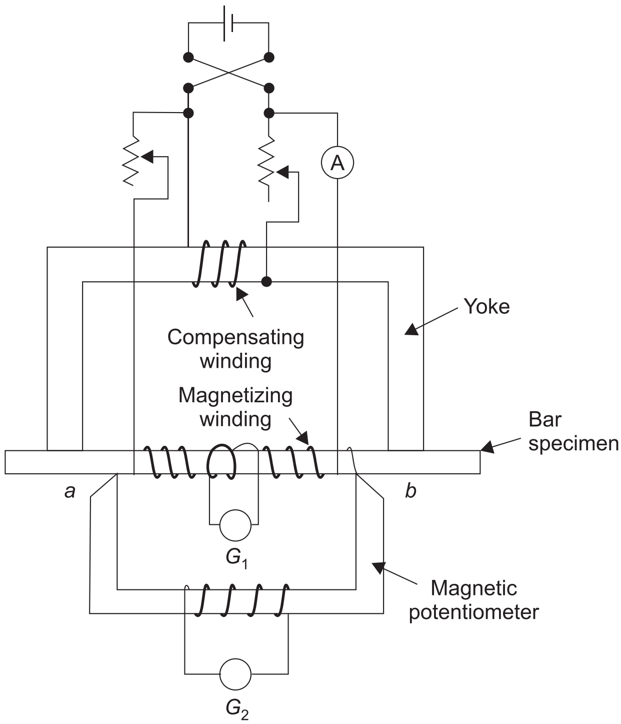


Fig. 8.18 Illiovici Permeameter

As can be seen from the figure, a magnetic potentiometer is connected across terminals *a* and *b* of the bar specimen. A compensating winding is made on the yoke which is connected parallel to the magnetizing winding across a reversing switch. Under zero magnetic potential drop indication by the magnetic potentiometer, the magnetizing winding provides the mmf for part *ab* while the compensating winding provides the mmf for the rest of the bar specimen, yoke, and joints. To attain the zero flux condition, the currents in the magnetizing and compensating windings are adjusted. The current through the magnetizing winding is adjusted to the required test value and then the current through the compensating winding is adjusted so that no throw is indicated by galvanometer G_2 on reversing these currents. Due to this, no magnetic potential drop occurs between points *a* and *b* and thus, the magnetizing force *H* is given as:

$$H = \frac{NI}{l}$$

where l is the length ab, and N and I are the number of turns and current in the magnetizing winding, respectively. When the currents through the magnetizing and compensating windings are reversed simultaneously, the throw of galvanometer G_1 gives the flux density.

The disadvantages of illiovici permeameter are given as below.

- Due to asymmetry in arrangement, satisfactory results are not obtained.
- Uniformity of magnetization along the specimen cannot be tested directly.
- Compensating winding is readjusted for each reading so it makes the entire operation too complicated.
- There may be some leakage of flux between the yoke and the specimen which may affect the magnetic potentiometer and cause some errors.

8.5.4 Burrow's Permeameter

In Burrow's permeameter, a number of compensating coils are used so that compensating mmf can be applied to different parts of the magnetic circuit. These mmfs are applied so that the entire magnetic circuit has a uniform magnetic potential distribution. This results in zero magnetic leakage in the circuit. The magnetizing force at any point can be calculated in terms of the magnetizing mmf per unit length at that particular point. The configuration of Burrow's permeameter is shown in Figure 8.19.

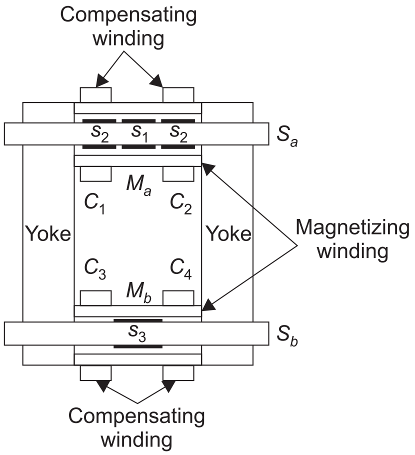


Fig. 8.19 Burrow's Permeameter

From the figure, it can be seen that there are two uniform bar specimens S_a and S_b. Both specimens are uniform, having same dimensions and same magnetic properties. The test is to be performed on S_a whereas S_b is an auxiliary bar. The magnetizing windings M_a and M_b are wound uniformly on bars S_a and S_b, respectively. The effects of leakage at the joints between the yoke and the two bars are eliminated by providing compensating coils C_1, C_2, C_3, and C_4. All search coils have equal number of turns and are denoted by s_1, s_2, and s_3. Coil s_1 is wound on the middle of the bar S_a, coil s_2 is divided into two equal halves and wound on each end of the bar S_a, and coil s_3 is wound on the middle of bar

S_b (see Figure 8.19). The connections of the coils of this permeameter can be represented in the form of an electrical circuit as shown in Figure 8.20. The compensating coils C_1, C_2, C_3, and C_4 are connected serially. The three different batteries are provided in the circuit which feed the serial connection of compensating coils, M_a and M_b.

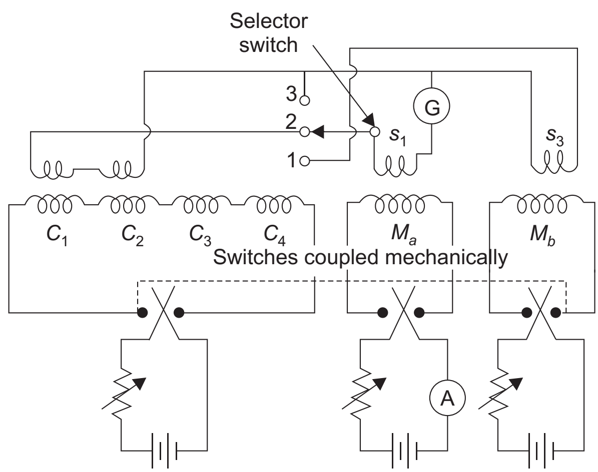


Fig. 8.20 Electrical Circuit Representation of Burrow's Permeameter

Before carrying this test, the flux linkages through all search coils s_1, s_2, and s_3 must be the same for a given value of current I through the coil M_a. This is done to ensure that no magnetic leakage exits near the joints. To attain this condition, the current in the magnetizing winding M_b and compensating windings are adjusted. Since there is no magnetic leakage, compensating windings provide the mmf to the joints and the magnetizing winding M_a provides mmf to the bar S_a. Thus, the magnetizing force H is given as:

$$H = \frac{NI}{l}$$

where l is the length of bar S_a and N is the number of turns in magnetizing winding M_a. To obtain the flux density corresponding to this value of H, the selector switch is put at position 3 (see Figure 8.20). As a result, ballistic galvanometer gets connected across the coil s_1. Now, compensating coils C_1, C_2, C_3, and C_4 and magnetizing coils M_a and M_b are simultaneously reversed and the throw of galvanometer is noted down.

Advantages and disadvantages

The advantages of this permeameter are as follows.

- If the material is uniform, the conditions along the specimen are also uniform.
- Many kinds of magnetic testing can be performed using this method.
- This method can also be used to calibrate specimens which can further be used as reference standards.

The disadvantages of this permeameter are as follows.

- The magnetic inhomogeneities present in the specimen are not considered in this method. Magnetic leakage occurs due to variations in permeability along the specimen. Compensation for this leakage cannot be carried out and thus, errors occur. Hence, this method gives accurate results for only magnetically uniform specimens.
- It is used to determine the properties of uniform bars only as the compensating adjustments are very slow and tedious.

8.5.5 Fahy's Simplex Permeameter

Fahy's Simplex permeameter consists of a single bar specimen, clamped against a laminated steel yoke. The two iron posts are used to clamp the bar against the yoke. The magnetizing coil is wound over the yoke and the search coil is wound over the bar uniformly along its entire length. To measure the flux density, the search coil is connected to a ballistic galvanometer, hence, it is also known as ***B* coil**. Similar to the flux density, magnetic field strength *H* of the specimen can be measured by the ballistic galvanometer. For this, an air cored coil, known as ***H* coil** is placed between the two clamping posts and the galvanometer is connected across it. The construction of this permeameter is shown in Figure 8.21. The values so obtained are made accurate by calibrating *H* coil, that is, by replacing the test specimen by a specimen whose magnetic characteristics are known.

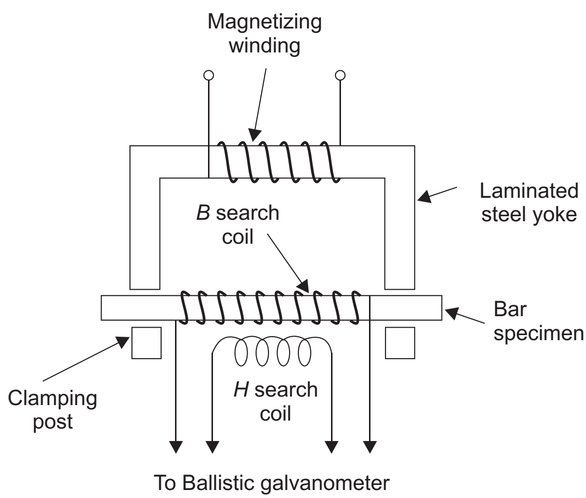


Fig. 8.21 Fahy's Simplex Permeameter

This is the most commonly used permeameter with the following advantages.

- As compared to Burrow's permeameter, this permeameter is simple in construction and operation.
- Magnetic irregularities of the specimen do not affect the results. This is due to the fact that the *B* coil is wound uniformly over the entire length of the specimen and therefore, the average flux density is measured and the effects of local irregularities are nullified.

- The results can be obtained very quickly.
- Only a small test specimen is required.

8.5.6 National Physical Laboratory Form of Permeameter

The construction of National Physical Laboratory form of permeameter is shown in Figure 8.22. It consists of a specimen S made up of a bundle of iron sheets whose dimensions are given as 250 mm × 70 mm × 0.35 mm. The specimen S is clamped between two parallel yokes Y_a and Y_b. These yokes are made of iron and are similar to each other. To provide a low reluctance path to the flux, the cross-sectional area of the yokes is kept large. A bobbin on which the main magnetizing coil M is wound, surrounds the specimen. At both the ends of the magnetizing winding, compensating windings C_1 and C_2 are provided. These compensating windings overcome the reluctance of joints by providing necessary mmfs.

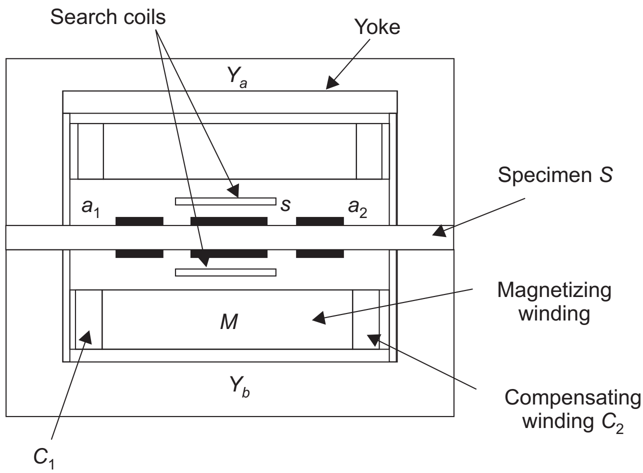


Fig. 8.22 National Physical Laboratory Permeameter

A small search coil s surrounds the middle of the specimen to measure the flux density in the specimen. Two more coils a_1 and a_2 are wound on the specimen to check whether the flux in the specimen is uniform or not. The number of turns in these coils (a_1 and a_2) is equal to half the number of turns in search coil s and the coils are connected in series to carry out the compensation. The current through the magnetizing winding is set to deflect a required value of the magnetic strength H and then the currents in the compensating windings are adjusted so that no indication is shown by the ballistic galvanometer when the magnetizing and compensating winding currents are reversed simultaneously. The flux density over the middle of the specimen is uniform till the compensation is accurate and can be measured by connecting the galvanometer across the coils. When the magnetizing and compensating winding currents are reversed simultaneously, the throw of the galvanometer gives the required value of flux density B. Here it is to be noted that for specimens with low permeability, that is, less than 1000, compensation is not required.

The two H coils, wound on thin rigid flat cards made up of an insulating material, are used to measure the value of H in the specimen. The dimensions of these cards are given as thickness between 1 to 2 mm, length between 50 to 80 mm, and width equal to that of specimen. The H coils are made up of very fine copper and have very large number of turns. The number of turns is so chosen that sufficient sensitivity can be achieved with the ballistic galvanometer. The coils are placed on either side of the specimen. The flux through these coils is proportional to the magnetic field strength H in the specimen, provided the flux density in the specimen is uniform. The total flux linkage in the H coil is given as:

$$\text{Total flux linkage} = \mu_o NAH$$

where H is the strength of the field, A is the mean area of the turns, and N is the total number of turns. To measure the product NA of the H coils, they are calibrated with a long solenoid as it cannot be measured from the coil dimensions.

To carry out the actual measurement of H in the permeameter, the H coils are connected to the ballistic galvanometer after performing the test for flux density B and reversing the current through the magnetizing and compensating windings.

This permeameter is the most accurate and easy to use and may be considered as a modern precision permeameter due to the reasons given below.

- Instead of using the calculated values of magnetizing strength H, it determines them by using H coil. This may be counted as an advantage since the calculated values are usually inaccurate.
- It provides accurate results for a wide range of magnetic force H extending from 10 A/m to 300,000 A/m.
- By adding shaped pole pieces, specimens with circular cross section can be tested.
- The field strength can be doubled easily by using this permeameter. For this, a short specimen is mounted between massive soft-iron pole pieces.

8.6 AC TESTING OF MAGNETIC MATERIALS

In Section 8.2, we determined the flux density of a magnetic field using induction coil magnetometer assuming that the field is present in the air. In this section, we will study AC testing which can be performed using the same search coil technique to calculate the flux density in a magnetic core. The magnetic core to be tested is in the shape of a ring over which two windings are present. The number of turns in primary and secondary windings is N_1 and N_2, respectively. The primary winding is supplied with an AC sinusoidal supply and an ammeter is connected to monitor the input current. A high input impedance voltmeter is connected across the secondary winding to measure the induced output voltage in it. The voltmeter is chosen with high input impedance to nullify the output current. The circuit is shown in Figure 8.23.

From Equation (3) and (4), we have:

$$B_m = \frac{V_2}{4.44 f N_2 A} \qquad \text{...(19)}$$

where B_m is the maximum flux density.

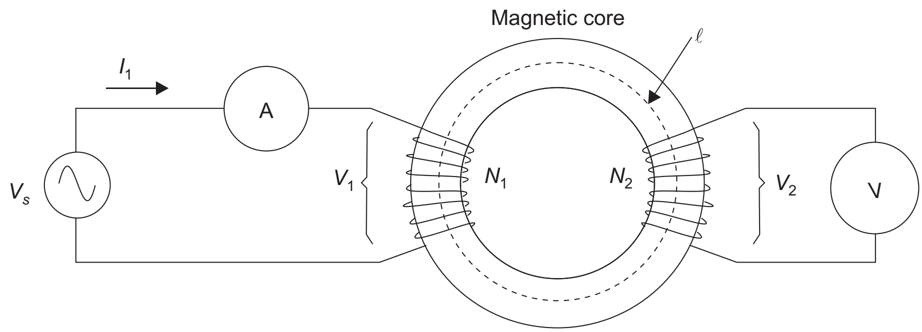


Fig. 8.23 Circuit Arrangement for AC Testing of Magnetic Materials

The magnetic field strength can be given by the relation:

$$H = \frac{I_1 N_1}{l} \qquad ...(20)$$

where I_1 is the input current and l is the length of magnetic path in the core. Here, it is to be noted that since the current I_1 is measured as rms value, the magnetic field strength H is also an rms quantity. Therefore, while calculating the ratio of B and H, either H must be converted to its maximum value or B_m must be converted to its corresponding rms value.

Here, the magnetic flux density is determined in a magnetic core. So, there must be some losses in the core which can be determined using the relation:

$$P = V_1 I_1 \cos \phi$$

Or, it can be written as:

$$P = \frac{N_1}{N_2} V_2 I_1 \cos \phi \qquad ...(21)$$

where cos ϕ can be determined very easily by connecting a resistor serially to the primary winding. The phase angle between the primary voltage and the voltage drop across the resistor are then measured.

Example 4 AC testing is performed on a ring core which has a magnetic path length of 33 cm and a cross-sectional area of 0.5×10^{-3} m^2. The number of turns in primary and secondary windings are given as 1250 and 750, respectively. Determine the relative permeability of the core if the input current is 0.45 A and output voltage is 65 V, 30 Hz.

Solution: Given that: $N_1 = 1250$, $N_2 = 750$, $l = 33$ cm, $A = 0.5 \times 10^{-3}$ m^2, $I_1 = 0.45$ A, $f = 30$ Hz, and $V_2 = 65$ V.

The maximum value of flux density B_m is given as:

$$B_m = \frac{V_2}{4.44 f N_2 A} \qquad \text{[Refer to Eqn. (19)]}$$

Substituting the values in the above relation, we get:

$$B_m = \frac{65}{4.44 \times 30 \times 750 \times 0.5 \times 10^{-3}} = 1.30 \text{ T}$$

The value of flux density obtained is its maximum value. Therefore, it must be converted to its equivalent rms value before further computations. The rms value of flux density is obtained as:

$$B_{\text{rms}} = 0.707 B_m$$

$$\Rightarrow \quad B_{\text{rms}} = 0.707 \times 1.30 = 0.92 \text{ T}$$

Now, the magnetizing force H can be calculated as:

$$H = \frac{I_1 N_1}{l} \qquad \text{[Refer to Eqn.(20)]}$$

Substituting the values in the above relation, we get:

$$H = \frac{0.45 \times 1250}{33 \times 10^{-2}} = 1704.5 \text{ A/M}$$

The relative permeability of core can now be calculated using:

$$\mu_r = \frac{B_{\text{rms}}}{\mu_o H} = \frac{0.92}{4\pi \times 10^{-7} \times 1704.5} = 429.52$$

8.7 MEASUREMENT AND SEPARATION OF IRON LOSSES

Iron loss or core loss in ferromagnetic materials can be measured by two methods, namely, *wattmeter method* and *bridge method*. These methods are described here in detail.

8.7.1 Wattmeter Method

In this method, the iron losses are measured in a square-shaped sheet material or square-shaped magnetic specimen making a closed magnetic circuit. Hence, this arrangement is known as a **magnetic square**. Here, it is to be noted that this method can also be used to measure core losses in ring-shaped specimens. The test set up for wattmeter method is shown in Figure 8.24.

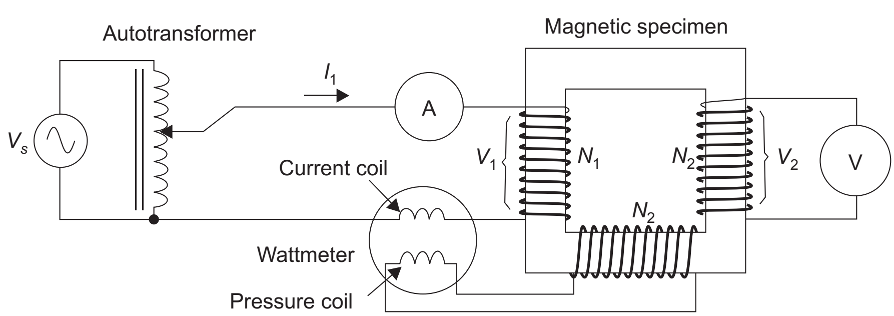


Fig. 8.24 Circuit Arrangement for Wattmeter Method

One primary and two secondary windings (with same number of turns) are wound on the specimen. The number of turns in the primary and secondary windings are N_1 and N_2, respectively. A variable sinusoidal input is applied at the primary winding from an autotransformer and an ammeter measures the current I_1. A wattmeter is also connected in the circuit whose current coil is connected serially to the primary winding and the pressure coil is connected to one of the secondary windings. The wattmeter used is a low power factor wattmeter as the power factor in this case is generally around 0.2. The other secondary winding is connected to a high impedance voltmeter in order to measure the induced voltage in it.

From Equation (19), it can be written as:

$$B_m = \frac{V_2}{4.44 f N_2 A}$$

Thus, the maximum flux density required for this measurement can be obtained by adjusting the voltage in primary winding. The readings of the wattmeter and voltmeter are then recorded. However, the reading indicated by the wattmeter is not accurate. It must be corrected so that the voltage drops due to resistances of the secondary winding and the pressure coil are allowed in the pressure coil of the wattmeter. The corrected reading of the wattmeter P_c is given by:

$$P_c = P\left(1 + \frac{R}{R_p}\right) \qquad ...(22)$$

where P is the reading indicated by the wattmeter, R is the resistance of secondary winding, and R_p is the resistance of pressure coil circuit.

Since the wattmeter pressure coil is connected to the secondary winding, the reading of the wattmeter (above equation) gives the average value of I_1V_2, whereas the actual input power supplied to the core is I_1V_1, and

$$I_1V_1 = I_1V_2\frac{N_1}{N_2} \qquad ...(23)$$

Thus, from Equation (22) and Equation (23), we get the actual input corrected power (P_i) as:

$$P_i = P\left(1 + \frac{R}{R_p}\right)\frac{N_1}{N_2} \qquad ...(24)$$

Here, it is to be noted that, in addition to core losses, the input power also consists of some copper losses in the wattmeter coil and secondary winding which is given as:

$$P_{\text{copper}} = \frac{V_2^2}{R + R_p} \qquad ...(25)$$

Thus, the resultant core losses are obtained by subtracting Equation (25) from Equation (24) as:

$$P_{\text{core}} = P\left(1+\frac{R}{R_p}\right)\left(\frac{N_1}{N_2}\right)-\left(\frac{V_2^2}{R+R_p}\right) \qquad \text{...(26)}$$

Now, the core loss per kg, also known as **specific core loss**, can be determined by dividing the core loss by the weight of the core.

Note: *The copper losses occurring in primary winding are not included in the observed reading and hence, are not present in the final expression.*

8.7.2 Bridge Method

For the sheet materials having very low flux density, the wattmeter method is not suitable. Thus, for such materials AC bridge methods are used. The bridge method has the following advantages.

- Many bridge circuits are available for this purpose.
- It can measure iron losses and AC permeability for the materials with low flux densities.
- It can work at audio or commercial frequencies.
- It can measure quantities even when the quantity of the sheet material is very low.

The various bridge methods described here are *Campbell's bridge method* and *Maxwell's bridge method*.

Campbell's bridge method

The circuit diagram for Campbell's bridge method is shown in Figure 8.25(a). As can be seen, the ring specimen consists of two windings, primary with N_1 turns and secondary with N_2 turns. The cross-sectional area of both windings is equal. A variable mutual inductance M and two variable resistances R_1 and R_2 are also connected in the circuit. Here, M and R_2 are together adjusted to attain the balance.

Let I be the current in the primary winding of the specimen, while V_1 and V_2 are the induced voltages in the primary and secondary of the specimen. The voltage induced in the secondary winding of mutual inductance M is denoted by V_{m}. The phasor diagram of the bridge under balanced conditions is shown in Figure 8.25(b). Here, Φ represents the flux passing through the specimen. As can be seen from the figure, current I leads the flux Φ. This happens due to iron losses. Also, when the sum of phasors V_2 and V_m is equal and opposite to phasor IR_2, the balance is obtained.

Let the mutual inductance between the primary and secondary windings of the ring specimen be denoted by M'. Then, we have:

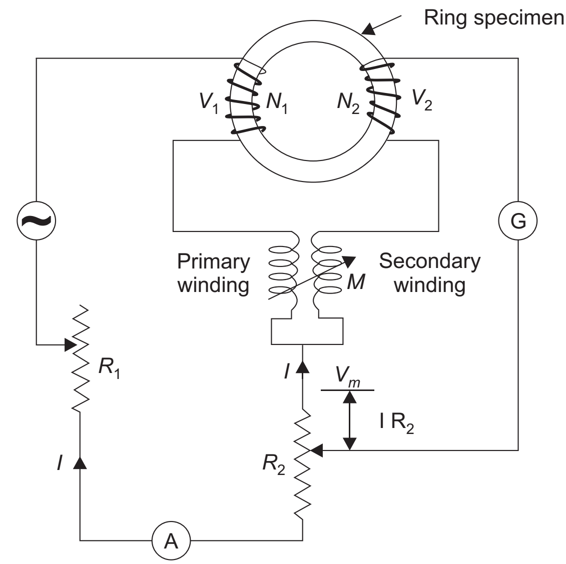
$$V_2 = \omega M'I$$

and,

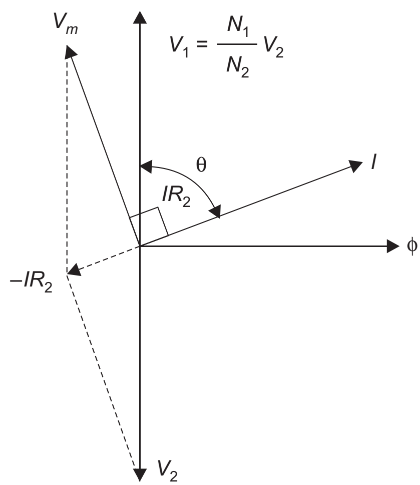
$$V_m = \omega MI$$

As IR_2 is very small, we can also write:

$$V_2 \approx V_m$$



(a) Circuit diagram



(b) Phasor diagram

Fig. 8.25 Campbell's Bridge Method

Or, $$\omega M'I \approx \omega MI$$

$\Rightarrow$ $$M' = M$$

Now, the flux in the specimen is given as:

$$\Phi = \frac{N_1 I}{(l_S / \mu_S A_S)} = \frac{N_1 I \mu_S A_S}{l_S}$$

where l_S is the length of mean flux path in the specimen, A_S is the area of the specimen, and μ_S is its AC permeability.

And, the mutual inductance M is given as:

$$M = \frac{N_2 \Phi}{I}$$

Substituting the value of Φ in the above expression, we get:

$$M = \frac{N_1 N_2 \mu_S A_S}{l_S}$$

On rearranging, AC permeability μ_s can be written as:

$$\mu_S = \frac{M l_S}{N_1 N_2 A_S}$$

Now, core loss is obtained as:

$$P_{\text{core}} = V_1 I \cos\theta$$

Or, it can be written as:

$$P_{\text{core}} = V_1 I \frac{IR_2}{V_2} = I^2 R_2 \frac{V_1}{V_2}$$

Or, $$P_{\text{core}} = I^2 R_2 \frac{N_1}{N_2} \qquad \left[\because \frac{V_1}{V_2} = \frac{N_1}{N_2}\right]$$

Maxwell's bridge method

The circuit diagram for Maxwell's bridge method is shown in Figure 8.26. It can be seen from the figure that the ring-shaped specimen is connected in the arm *ab* of the bridge.

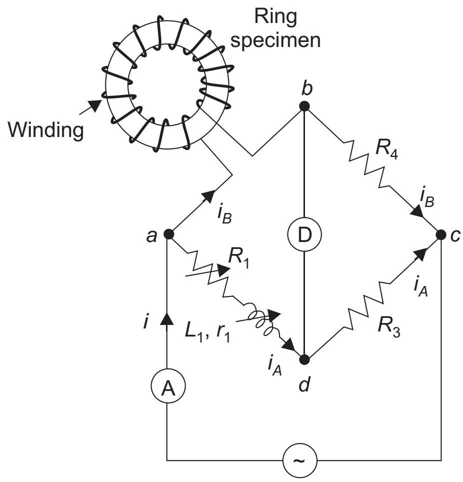


Fig. 8.26 Maxwell's Bridge Method

Let the effective resistance and inductance of arm *ab* be R_s and L_x, respectively. From the result of Maxwell's bridge (refer to Section 7.5.2), we have:

$$R_s = \frac{R_4}{R_3}(R_1 + r_1)$$

The effective resistance R_s of arm *ab* can also be written as:

$$R_s = \frac{\text{Core loss} + \text{Copper loss in winding}}{(\text{Current})^2}$$

Substituting the values, we get:

$$R_s = \frac{P_{\text{core}} + i_B^2 R_c}{i_B^2}$$

where R_c is the resistance of the coil winding. On rearranging, we get core loss P_{core} as:

$$P_{\text{core}} = i_B^2 (R_s - R_c) \qquad ...(27)$$

When the bridge is balanced, the voltage drops in arms *bc* and *DC* are equal. Thus, we have:

$$i_B R_4 = i_A R_3 = (i - i_B) R_3$$

As, we have:

$$i = i_A + i_B \quad \text{or} \quad i_B = \frac{R_3}{R_3 + R_4} i$$

the Equation (27) can be written as:

$$P_{\text{core}} = i^2 \left(\frac{R_3}{R_3 + R_4} \right)^2 (R_s - R_c) \qquad ...(28)$$

The above equation can be used to calculate the core loss of the specimen as the value of R_s is known. The coil resistance R_c and current i can be measured using DC supply. Now, the AC permeability of the specimen is determined by the relation derived below. As we know, the inductance of the ring specimen L_x is given as:

$$L_x = \frac{R_4}{R_3} L_1 \qquad \text{(Refer to Section 7.5.2)} \quad ...(29)$$

In terms of ring specimen parameters, it can be written as:

$$L_x = \frac{N^2}{l_S / \mu_S A_S} \qquad ...(30)$$

where N is the number of turns of magnetizing winding.

From Equation (29) and (30), we get:

$$\mu_S = \frac{L_x l_S}{N^2 A_S} = \frac{R_4 l_S L_1}{R_3 N^2 A_S} \quad ...(31)$$

8.7.3 Separation of Core Losses

In an iron core, the dissipated power can be represented by two components, namely, *eddy current loss* and *hysteresis loss*. Sometimes, it is necessary to consider the effects of these two losses separately in order to analyze the effects of different compositions of the materials and different designs of the apparatus. The hysteresis loss is directly proportional to the frequency of the waveform while the eddy current loss is directly proportional to the square of the waveform frequency.

The iron core power loss equation is represented as:

$$P = A_H f + A_E f^2 \quad ...(32)$$

where A_E and A_H are eddy current and hysteresis constants, respectively. These constants are determined by the core and its flux density.

Rewriting Equation (32) to determine the values of A_E and A_H, we get:

$$\frac{P}{f} = A_H + A_E f \quad ...(33)$$

Now, plotting a graph of P/f against f for different values of f, we obtain a straight line graph as shown in Figure 8.27.

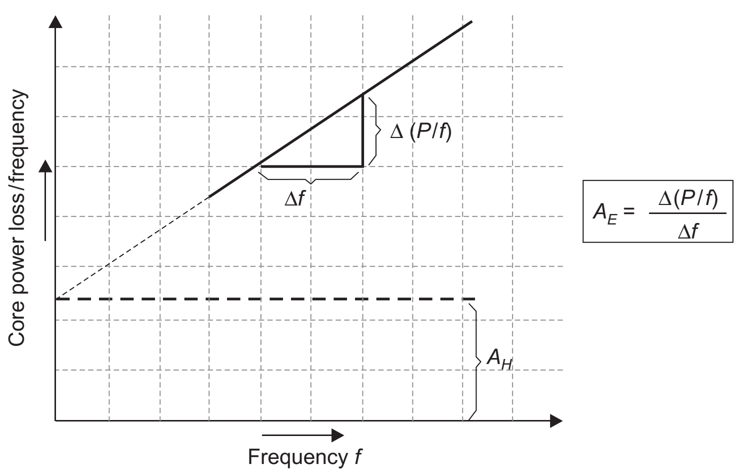


Fig. 8.27 Graph of P/f vs f

Thus, the values of A_E and A_H can be determined by simply extending the line to the value $f = 0$.

Example 5 Calculate the actual core loss in a magnetic core if a 33 W measurement is obtained for an input frequency of 60 Hz and V_2 = 77 V using wattmeter method. The number of turns in primary and secondary windings is given to be 400 and 500, respectively and the resistances of secondary and wattmeter pressure coil windings are 18 Ω and 74 kΩ, respectively.

Solution: Given that: P = 33 W, V_2 = 77 V, N_1 = 400, N_2 = 500, R = 18 Ω, and R_p = 74 kΩ = 74000 Ω

The core loss P_{core} in a magnetic core using wattmeter method is given as:

$$P_{\text{core}} = P\left(1+\frac{R}{R_p}\right)\left(\frac{N_1}{N_2}\right)-\left(\frac{V_2^2}{R+R_p}\right) \qquad \text{[Refer to Eqn.(26)]}$$

Substituting the given values, we obtain:

$$P_{\text{core}} = 33\left(1+\frac{18}{74000}\right)\left(\frac{400}{500}\right)-\left(\frac{(77)^2}{18+74000}\right)$$

$$\Rightarrow \qquad P_{\text{core}} = 26.325 \text{ W}$$

Example 6 Calculate the hysteresis and eddy current losses in a magnetic core at 60 Hz if the power loss is given to be 12 W at 80 Hz and 5.2 W at 40 Hz.

Solution: The power loss in terms of hysteresis and eddy current loss can be expressed as:

$$\frac{P}{f} = A_H + A_E f \qquad \text{[Refer to Eqn. (33)]}$$

At 80 Hz, $\frac{P}{f} = \frac{12}{80} = 0.15$

At 40 Hz, $\frac{P}{f} = \frac{5.2}{40} = 0.13$

Also, the change in frequency Δf = 80 – 40 = 40 Hz

The eddy current coefficient A_E is given as:

$$A_E = \frac{\Delta(P/f)}{\Delta f} = \frac{(0.15-0.13)}{40} = 0.5\times 10^{-3}$$

The hysteresis coefficient A_H is given as:

$$A_H = \left.\frac{P}{f}\right|_{40\text{ Hz}} - \left.A_E\right|_{40\text{ Hz}}$$

Substituting the values, we get:

$$A_H = 0.13 - (0.5 \times 10^{-3} \times 40) = 0.11$$

The power at 60 Hz can be calculated using the relation given as:

$$P = A_H f + A_E f^2 \qquad \text{[Refer to Eqn. (32)]}$$

$$\Rightarrow \qquad P = 0.11 \times 60 + 0.5 \times 10^{-3} (60)^2 = 6.6 + 1.8 \text{ W}$$

Thus, Hysteresis loss = 6.6 W and eddy current loss = 1.8 W

Let us Summarize

1. The flux density of a magnetic field is determined using various methods that include magnetometers, and ballistic galvanometer.
2. An instrument which is used to measure magnetic flux density is known as a magnetometer. This instrument is also known as Gauss meter or Tesla meter since its output can be expressed in Gauss or Tesla.
3. The different types of magnetometers are induction coil magnetometers, fluxgate magnetometers, and Hall-effect magnetometers.
4. The induction coil magnetometer consists of a coil placed in a changing magnetic field which results in an induced voltage at the terminals of the coil.
5. There are two types of fluxgate magnetometers, namely, twin-rod fluxgate and ring-core fluxgate.
6. When a strip of current-carrying conductor is subjected to a transverse magnetic field, a voltage is produced at the edges of the conductor which is perpendicular to the direction of both magnetic field and current. This phenomenon is known as Hall-effect.
7. In addition to different magnetometers, flux density of a magnetic field can also be measured using either a ballistic galvanometer or a flux meter.
8. The curve obtained by plotting the various values of magnetic flux density *B* of a magnetic material corresponding to different values of its magnetic field intensity *H* is known as *B-H* curve.
9. The *B-H* curve can be determined by two methods, namely, method of reversals and step-by-step method.
10. In magnetic materials, the flux density *B* always lags behind the magnetic field strength *H* and it forms a loop when we draw the *B-H* curve. This loop is known as hysteresis loop and its area gives the amount of heat loss in the material.
11. The hysteresis loop can also be determined by two methods, namely, method of reversals and step-by-step method.
12. In permeameters, the effects of self-demagnetization are reduced or even removed completely in some cases. This is achieved by providing a return path having a low reluctance.
13. The different types of permeameters are Hopkinson permeameter, Ewing double bar permeameter, Illiovici permeameter, Burrow's permeameter, Fahy's simplex permeameter, and National physical laboratory form of permeameter.
14. AC testing can be performed using the same search coil technique to calculate the flux density in a magnetic core.
15. The iron loss or core loss in ferromagnetic materials can be measured by two methods, namely, wattmeter method and bridge method.
16. In an iron core, the dissipated power can be represented by two components, namely, eddy current loss and hysteresis loss.

EXERCISES

Fill in the Blanks

1. Induction coil magnetometers work for _________ magnetic field.
2. Self demagnetization occurs in _________ shaped specimen.
3. Core power loss consists of _________ and _________ losses.
4. The flux density *B* always _________ the magnetic field strength *H*.
5. Two methods for measuring iron losses are _________ and _________.

Multiple Choice Questions

1. Magnetometer is also known as
   (a) flow meter (b) Gauss meter
   (c) flux meter (d) field meter
2. 1 Tesla is equal to:
   (a) 10^4 Gauss (b) 10^2 Gauss
   (c) 10^3 Gauss (d) 10^{-4} Gauss
3. Specific core loss is defined as core loss per
   (a) kg (b) km
   (c) square kg (d) square km
4. How many bars are used in Ewing double bar permeameter?
   (a) Five (b) Three
   (c) Two (d) Four
5. The core power loss equation in terms of eddy current and hysteresis loss can be represented as
   (a) $P = A_H f + A_E f^2$ (b) $P = A_H f^2 + A_E f$
   (c) $P = A_H f^2 + A_E f^2$ (d) $P = A_H f + A_E f^3$

State True or False

1. Magnetometers are used to determine *B-H* curve.
2. Hopkinson permeameter is also known as bar and yoke method.
3. The wattmeter used in the wattmeter method for loss measurement is designed for high power factors.
4. Hysteresis loss is directly proportional to the cube of operating frequency.
5. Hall-effect magnetometers are used for applications having very small magnetic field strength.

Descriptive/Numerical Questions

1. List out various types of magnetometers. Explain the working of a fluxgate magnetometer with a proper diagram.
2. Explain the working of a magnetometer that can be used to measure the flux density of a stationary field.
3. List out the methods used for the measurement of iron loss in ferromagnetic materials.
4. What are permeameters? Explain any two types of permeameters in detail.
5. What are the advantages of bridge method over wattmeter method? Explain Maxwell's bridge method in detail.

6. Explain AC testing of magnetic materials along with necessary diagrams.
7. Describe the basic magnetic measurement using *B*-*H* curve.
8. Classify different types of iron loss.
9. Calculate the Hall-effect coefficient of a device that produces an output of 3.2 mV when a magnetic field having 1.0 T flux density is measured using it. The field is applied perpendicular to the device and the current is given to be 4.9 A.
10. The AC testing of a magnetic core having 0.9×10^{-3} m^2 cross-sectional area, magnetic path length of 22 cm, and a relative permeability of 855 is performed. Calculate the maximum flux density and the required primary current if the primary voltage is 98 V, 30 Hz. The number of turns in primary and secondary windings are given to be 400 each.
11. Write short notes on:
    (a) Burrow's permeameter
    (b) Step-by-step method for determining *B*-*H* curve
    (c) Method of reversals for determining hysteresis loop

9

Cathode Ray Oscilloscope

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Understand the need for an oscilloscope
- Learn about the structure and working of an oscilloscope
- Know the function of various circuits of an oscilloscope
- Learn about various probes required while using an oscilloscope
- Learn various measurement techniques for measuring frequency, phase, and time delay
- Explain different types of special oscilloscopes
- Appreciate the concept of Lissajous figures

9.1 INTRODUCTION

The cathode ray oscilloscope (abbreviated as CRO), generally referred to as an oscilloscope, is an instrument that can measure, display, and analyze waveforms and other phenomena in electronic and electrical circuits. It is the most versatile and important tool used for the development of electronic circuits and systems. The amplitude of many electrical signals, such as voltage, current, power, pressure, and acceleration can be displayed on an oscilloscope primarily as a function of time. Basically, oscilloscopes operate on voltages, so other physical quantities are converted into voltages using transducers to present their visual representations.

A beam of electrons, which determines the CRO display, produces a luminous spot by striking on a fluorescent screen. The luminous spot creates a two-dimensional display when the electron beam is deflected in either of the two orthogonal axes, namely, X and Y. The horizontal input voltage is an internally generated ramp voltage called **time base**. The luminous spot periodically moves in horizontal direction from left to right over the display area or screen due to this horizontal voltage. The voltage to be measured is applied as a vertical input to the CRO. The luminous spot moves up and down with respect to the instantaneous value of this vertical voltage. Thus, the waveform of the input voltage is displayed with respect to time. This display appears to be stationary when the input voltage repeats itself at a fast rate. Thus, the time varying voltages are visualized on the CRO. In this chapter, we will study the basic construction and working of a CRO in detail.

In addition to simple CROs, the multi-input CROs, namely *dual beam* and *dual trace* CRO and some special CROs, namely *sampling oscilloscope* and *storage oscilloscope* are also discussed here. Various patterns can be observed on the screen of an oscilloscope for different types of inputs applied to it. These patterns are known as *Lissajous figures* that are also described in this chapter.

9.2 Block Diagram of Cathode Ray Oscilloscope

The basic structure of a cathode ray oscilloscope consists of a vertical amplifier, a horizontal amplifier, a trigger circuit, a time-base generator, a delay line, a power supply block, a cathode ray tube, and a phosphor screen as shown in Figure 9.1.

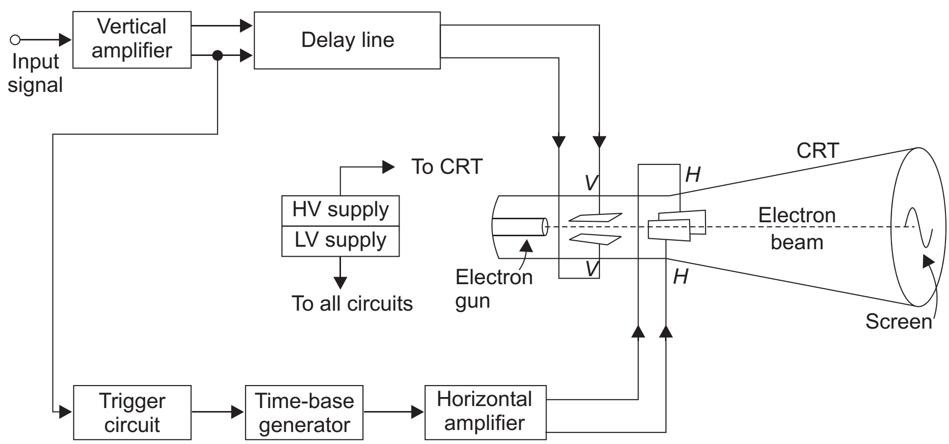


Fig. 9.1 Block Diagram of an Oscilloscope

The cathode ray tube is considered as the heart of the instrument and performs three specific tasks as follows:

- Generates an electron beam
- Accelerates the beam to a high velocity
- Deflects the beam to create an image

A high voltage (of the order of a few thousand volts) is fed to the cathode ray tube from the power supply block for acceleration, while a low voltage is fed to its electron gun heater. All other control circuits are fed with normal voltages, usually of the order of a few hundred volts.

The high velocity electron beam strikes the phosphor screen of CRO and thereby becomes visible creating a visible spot in the horizontal direction. The deflection of this spot is at a constant time-dependent rate, generated by the time-base generator circuit. This circuit provides an adequate voltage to CRT so that the spot deflects in accordance with the constant rate. The output of a time-base generator is a saw-tooth waveform possessing variable amplitude and frequency components. The increased amplitude of this signal causes the spot on the screen to move from the left to right direction. On the other hand, the spot shifts to the extreme left of the screen when the amplitude decreases to zero and gets ready for the next sweep. This is called **fly-back**. The sweep and fly-back occur so swiftly that it apparently seems to be a straight line on the screen.

The vertical amplifier boosts the incoming signal (which is to be measured) to an appropriate level so that it provides a suitable deflection of the electron beam. The beam is deflected by the horizontal and vertical deflection plates that are fitted between the electron gun and the screen which deflects the beam horizontally as well as vertically. The deflection of horizontal amplifier is synchronized with the vertical input signal by means of a trigger circuit. It is necessary so that the horizontal amplifier deflects at the same instant of time as the vertical input signal.

It must be noted here that the frequencies of input signal and time-base generator circuit must be the same or the deflection gets drifted sideways. For this, the input signal is sampled by the trigger circuit to ensure that both time-base generator and input signal possess the same frequency.

9.3 CATHODE RAY TUBE

As stated earlier, a cathode ray tube (abbreviated as CRT) is the heart of an oscilloscope. It is basically a glass tube consisting of an assembly of electrodes. The tube has a phosphor coated viewing screen at one end. The major components of a CRT are a glass envelope, a fluorescent screen, electron gun assembly, deflection plates, and a base. Figure 9.2 depicts the structure of a CRT.

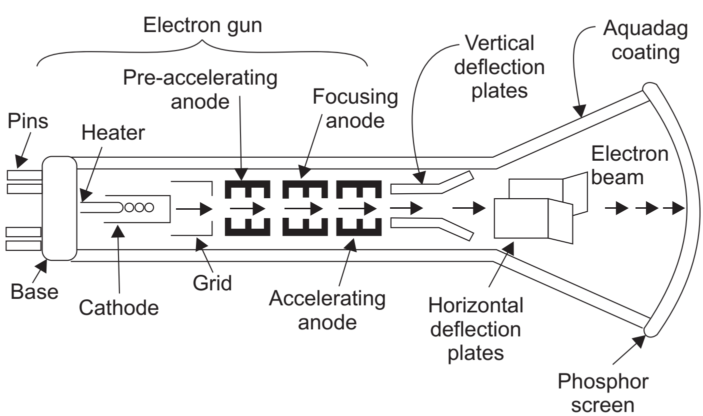


Fig. 9.2 Structure of a Cathode Ray Tube

The electron gun assembly is responsible for generating an electron beam. This beam is sharp, highly focused, and possesses sufficient energy since electrons are accelerated to a high velocity. The beam from the gun assembly passes through the deflection plates which are supplied with an operating voltage to deflect the beam. The beam then hits the phosphor coated screen to deflect a bright spot. The horizontal plates cause the beam to move horizontally while the vertical plates cause it to move vertically. All these components are together enclosed in an evacuated glass tube where the electrons are free to move. Let us discuss each of these components in detail.

9.3.1 Electron Gun Assembly

The electron gun assembly consists of a heater, a cathode, a pre-accelerated anode, a focusing anode, an accelerating anode, and a control grid. The cathode, which requires a 600 mA operating current and 6.3 V operating voltage, is heated indirectly by the heater element to generate electrons. The operating current can be as low as 300 mA for highly efficient systems. A current of 140 mA at 1.5 V voltage is required for specially designed low power systems. The gun assembly emits electrons and forms them into a beam. The emission of electrons can be made high with a coating of strontium oxide and barium at the cathode end. The emitted electrons then pass through the negatively biased control grid, which is a nickel cylinder. At the centre of the control grid, there is a hole of about 0.25 mm so that the electrons can flow through it. This hole is co-axial with the CRT axis. The control grid controls the intensity of the electron beam by controlling the number of electrons emitted by the cathode.

After passing through the control grid, the electrons are accelerated by pre-accelerating and accelerating anodes. These anodes are kept at a high positive potential of about 1500 V. The accelerated electrons are then focused by the focusing anode which is fed with a variable voltage of about 500 V. All these electrodes are cylindrical in shape and have a hole at their centre. These holes are also co-axial with the CRT axis. The focused electrons now pass through two pairs of deflection plates, that is, horizontal and vertical. The electron beam hits the phosphor screen and becomes visible. The deflection of the beam can be seen on the screen.

9.3.2 Deflection Plates Assembly

We know that there are two pairs of deflection plates in the CRT of an oscilloscope. The electron beam after being accelerated and focused, reaches these deflection plates. One of these pairs is mounted horizontally and is known as **vertical deflection plates**. These plates, also called ***Y*-plates**, generate an electric field in the vertical plane. The other pair is vertically mounted and known as **horizontal deflection plates** or ***X*-plates**. The electric field generated by these plates is in the horizontal plane. Both pairs of plates are so located that the electron beam can easily pass through them.

If the deflecting plates are grounded or opened, the electron beam passes through them and hits the tube screen exactly at its centre, creating a luminous bright spot. This is due to no deflecting force experienced by the electrons in such a case. On the contrary, when one plate of a pair of deflecting plates is connected to a positive voltage supply and another one is connected to the negative supply, the electrons hit the screen at a location determined by the applied voltage. This is called **electrostatic deflection**. In such a case, the electrons are attracted towards the positive plate, but since they travel axially, they never hit the plate; rather, they are deflected by the plate such that they strike the fluorescent screen at some specific location in accordance with the applied voltage (see Figure 9.3).

On application of an AC supply voltage, the plates alternatively deflect the beam horizontally and vertically. When the beam is deflected horizontally, a horizontal line is produced while a vertical line is produced when it is deflected vertically. There are two ways of expressing the sensitivity of a CRT to the corresponding voltage applied to the deflection plates. These are *deflection factor of the tube* and *deflection sensitivity*. **Deflection factor**

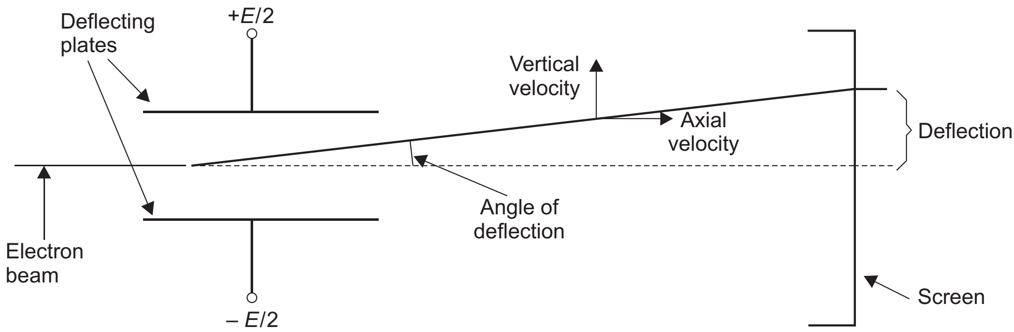


Fig. 9.3 Electrostatic Deflection

can be defined as the voltage that can generate one division of deflection on the screen and is expressed in volts/centimetre (V/cm). On the other hand, the deflection generated on applying 1 V voltage is known as **deflection sensitivity**, expressed in centimetre/volt (cm/V).

The electric fields of one pair of plates (that is, horizontal or vertical) may influence the fields of the other pair. Therefore, they must be isolated from each other. A grounded isolation shield is incorporated between the two pairs of plates for this purpose.

9.3.3 Fluorescent Screen

The CRT screen is a rectangular glass face plate with dimensions 8 cm (vertical) ×10 cm (horizontal). It is marked with graticules on it where each centimetre on the graticule represents one division. The screen is coated with phosphor from inside which can convert electrical energy into light energy. When the electron beam strikes the phosphor crystals, the energy level of the electrons present on the phosphor gets increased. This effect is known as **cathode-luminescence**. When the phosphor gets excited, light is emitted, which is known as **fluorescence**. The phosphor crystals return to their initial state when the electron beam is switched off and release light energy. This light energy may retain for a few seconds or more; it is known as **persistence** or **phosphorescence**. This light or glow may be red, green, blue, or white in colour in correspondence with the phosphor used. However, blue phosphor is usually used.

A negative voltage is developed on the screen after primary emission on account of accumulation of electrons. This negative potential gradually becomes so large that the screen starts repelling the electron beam. Upon striking the screen, electrons release some *secondary emission electrons* which are accumulated by *aquadag*, a graphite coating around the neck of the tube. This ensures that the negative voltage does not accumulate on the screen as well as brings the screen in electrical equilibrium state.

Some other CRTs employ a screen with a thin aluminium film. This film reflects the light emitted by the electrons to the screen. This enhances the brightness of the luminescence. The electron beam hits this film and passes through it. However, the secondary emission electrons are accumulated and grounded by the film so that they may not damage the screen. This function is essentially similar to that of a *heat sink* which conducts the redundant heat away to avoid damage.

9.4 Basic CRO Circuits

The circuit of a CRO consists of a number of sub-circuits. To understand the working of each one of them adequately, a more detailed block diagram of a typical oscilloscope is considered in Figure 9.4. It shows the various sections of CRO.

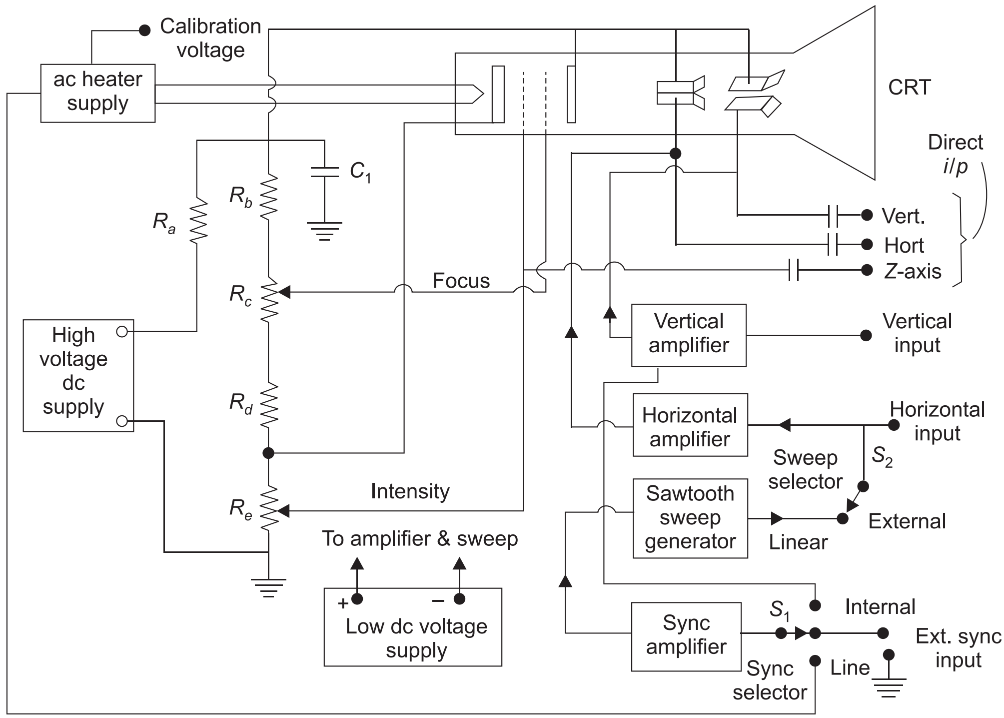


Fig. 9.4 Circuit Diagram of a Typical CRO

An AC filament supplies the required voltage to the cathode heater element in addition to providing AC calibrating voltage. On the other hand, the dc voltage is supplied by high voltage dc supply circuit. Resistances R_a to R_e make a voltage divider circuit through which the dc voltage reach the CRT. Resistances R_c and R_e among these resistances are variable and are referred to as **focus control** and **intensity control**, respectively. When R_c is adjusted, the potential of focusing anode is controlled while the voltage of the control grid is varied by adjusting resistance R_e, thereby determining the number of electrons leaving from cathode. Capacitor C_1 connects the deflection plates as well as the accelerating anode to the ground. Let us discuss the major blocks in detail.

9.4.1 Vertical Deflection System

The vertical deflection system of an oscilloscope mainly consists of a vertical amplifier circuit. The sensitivity S and bandwidth BW of the oscilloscope are largely determined by the vertical amplifier. The sensitivity of an oscilloscope is in proportion to the gain of the vertical amplifier. The vertical sensitivity or gain of an oscilloscope determines the extent to which an input signal can be deflected by the instrument while its bandwidth is a measure of the frequency range which an oscilloscope can reproduce accurately. Vertical sensitivity is the smallest possible deflection factor while bandwidth is the frequency range over which the amplifier gain falls within 3 dB of the mid-band frequency gain.

Another important parameter of a vertical amplifier is its **rise time** t_r, which is defined as the time period the edge of a pulse takes to increase from 10% to 90% of its maximum amplitude. In case of observation of a square or pulse wave, the rise time of the oscilloscope must be faster than that of the pulse under observation to accurately reproduce it. An approximation of the relation between the bandwidth BW and rise time t_r is given as:

$$t_r \times BW = 0.35 \qquad ...(1)$$

Let us now discuss the basic structure of a vertical amplifier as shown in Figure 9.5. The major components of a vertical amplifier are pre-amplifier circuit and main amplifier circuit.

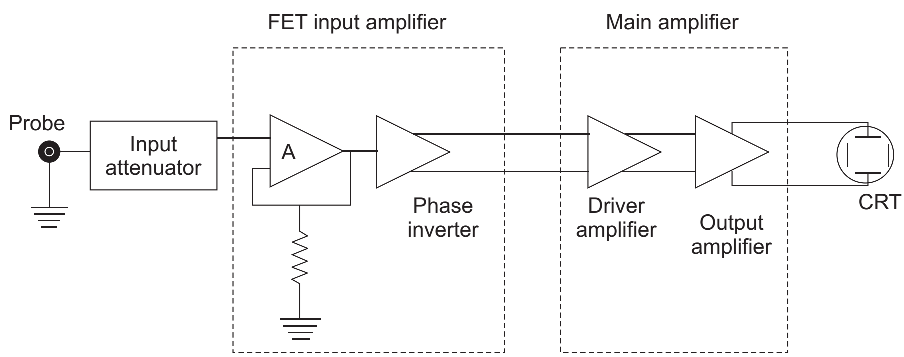


Fig. 9.5 Vertical Amplifier

The gain or sensitivity of vertical amplifier is constant and is expressed in V/div. The fixed gain makes it easier to design the amplifier with desired bandwidth and stability. The input attenuator unit can be switched to an appropriate value in order to keep the amplifier in the range of its signal handling capability. The attenuator unit is isolated from the amplifier section by a high input impedance FET source follower which is usually the first element of the pre-amplifier unit. A BJT is employed between FET and phase inverter circuit in order to match the low input impedance of the phase inverter to the moderate output impedance of FET.

The output of the phase inverter is two out-of-phase signals used to drive the push–pull amplifier at the output stage. The push–pull amplifier in turn provides two equal voltage signals with opposite polarity which are fed to the vertical deflection plates of the CRT. The push amplifier suppresses the even harmonics, thereby provides greater power output, cancels out the hum voltage, and reduces various non-linear and defocusing effects since no plate of CRT is grounded.

9.4.2 Horizontal Deflection System

The major components of the horizontal deflection system of an oscilloscope are a *time-base generator* and a *horizontal amplifier*. The time-base generator generates a sweep signal which is then amplified by the horizontal amplifier. The output of horizontal amplifier is fed to the horizontal deflection plates of the CRT. External signals can also be applied to this system by selecting *Ext* by sweep selector switch on the front panel of instrument.

Time-base generator

The time-base generator consists of a unijunction transistor (also known as UJT) as shown in Figure 9.6. The circuit provides a sweep voltage signal at its output terminals and thus, sometimes it is referred to as **sweep generator**.

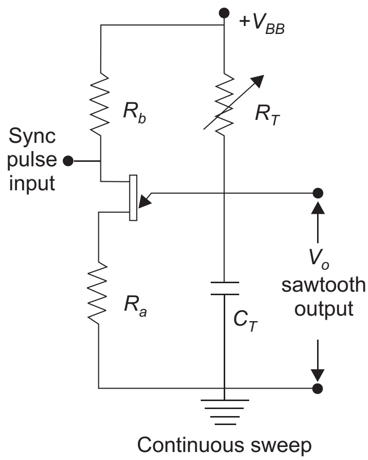


Fig. 9.6 Time-base Generator

In the above circuit, resistor R_T, known as **timing resistor**, is incorporated to keep the operating frequency in a certain usable range while capacitor C_T, known as **timing capacitor**, is adjusted to change this operating range.

Initially, the UJT is off when the power is applied first, while capacitor C_T starts charging exponentially through resistor R_T and emitter voltage V_E tends to attain a voltage equal to V_{BB}. The voltage across C_T during charging V_o can be expressed as:

$$V_o = V_{BB}[1 - \exp(-t / R_T C_T)] \qquad ...(2)$$

where V_{BB} is the supply voltage and t is the time in seconds. The UJT turns on when the emitter voltage V_E becomes equal to peak voltage V_p. Thus, the UJT provides a low resistance path for the capacitor to discharge rapidly. Now, the emitter voltage V_E starts decreasing and attains the minimum value very swiftly, turning the UJT off. The capacitor again starts charging and the process is repeated over and over again. The output waveform of the circuit is shown in Figure 9.7.

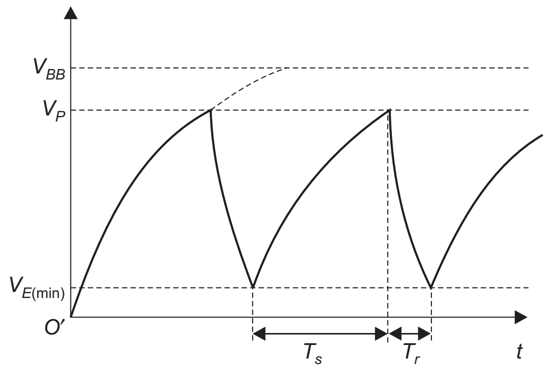


Fig. 9.7 Output Waveform

Here, T_S represents the *sweep time* during which the electron beam travels from left to right on the CRT screen. If the amplitude of the waveform is increased, the beam shifts to the right due to which the positive supply attracts the electrons. On the other hand, T_r represents the *retrace time* in which the beam returns to the left of the screen. From the figure, we can say that T_r is considerably smaller than T_S. Thus, during the retrace time, the movement of the beam is significantly faster.

The linearity of the output sweep signal can be improved if a low voltage is supplied to UJT and a high voltage is applied to the timing resistance-capacitance circuit. The sweep signal must not drift on the screen and hence, must be locked. This can be achieved by a synchronizing pulse which brings the frequency of this sweep signal exactly equal to that of the input signal.

For some applications, such as music or voice, the pattern obtained on the screen goes out of synchronization very frequently. This is because the frequency and amplitude of such signals vary frequently. This causes the displayed waveform to become unstable. This can be avoided using a triggered sweep CRO. In this mode, the input signal generates narrow pulses which trigger the sweep signal so that it always remains synchronized with the driving signal. A triggered sweep can display even short duration pulses. Figure 9.8(a) shows the circuit of a triggered sweep.

It can be seen from the figure that an additional voltage divider circuit, formed by resistances R_c and R_d is incorporated to the circuit shown in Figure 9.6. This voltage divider keeps the cathode voltage of diode D below the peak voltage V_p. As stated earlier, initially, when power is applied to the circuit, the UJT is off. Capacitor C_T starts charging through resistance R_T and charges until the diode starts conducting. The diode voltage V_D in turn, clamps the voltage across C_T at V_D, thereby, preventing it to reach V_p which is required to turn the UJT on. However, this required peak voltage can be made lower by applying a negative pulse at the base of UJT. As a result, the UJT triggers on and the capacitor starts discharging through it. When the capacitor voltage decreases to a level insufficient to keep the UJT conducting, the UJT turns off and the capacitor again tends to charge towards the supply voltage V_{BB}. The voltage across capacitor is again clamped by the diode D and the process continues. The resultant waveforms are shown in Figure 9.8(b).

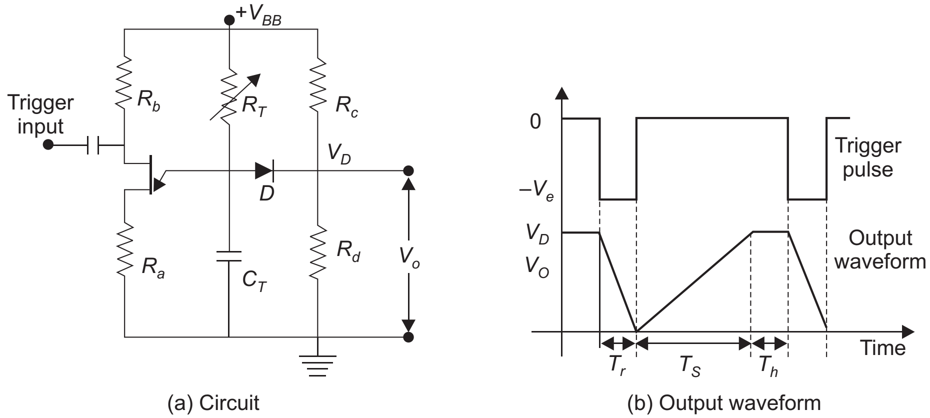


Fig. 9.8 Triggered Sweep

9.4.3 Synchronization Amplifier

Synchronization amplifier is used in an oscilloscope to synchronize the sweep signal to the vertical input signal. It requires that the time-base generator should be driven at a frequency which is a sub-multiple of that of the vertical input signal. The synchronization yields a stationary pattern which otherwise may move randomly over the CRT screen.

A synchronization selector switch S_1 is provided next to the synchronization amplifier as shown in Figure 9.4. This switch can be moved to any of the three positions, namely *internal*, *external*, and *line*. If the switch is at the internal position, the signal from vertical amplifier triggers the sync amplifier while if it is at the external position, the triggering is performed by an external signal. When the switch is connected to line position, the supply voltage through the low amplitude AC supply provides the trigger. The signal after being synchronized is fed to the sweep generator.

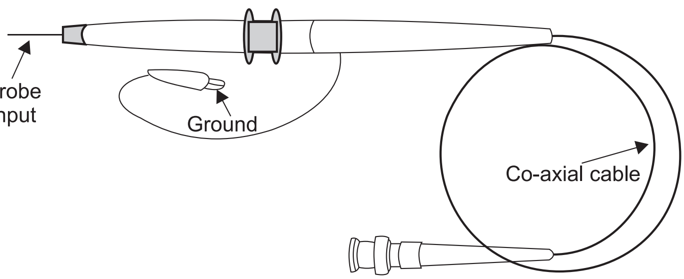
9.5 CRO PROBES

An input signal is fed to an oscilloscope via its probes. We may also say that the probes of a CRO connect it to the circuit under test. There are three types of oscilloscope probes, namely *direct probes*, *attenuator probes*, and *active probes*.

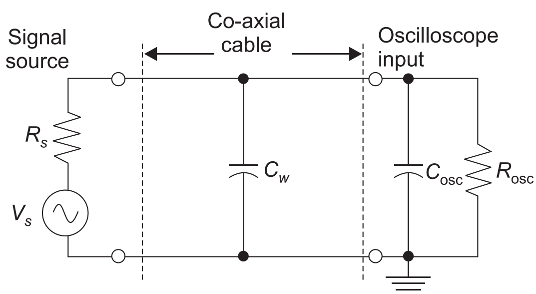
9.5.1 Direct Probes

Direct probes have co-axial cables at one end to carry the input signal from the other end. There is an additional terminal in such probes which is connected to the ground potential. The basic structure of a probe as well as its equivalent circuit can be seen from Figure 9.9.

The probe shown in the figure is referred to as **direct probe** or **1:1 probe** since this probe does not incorporate any resistance to attenuate the signal under test. The central conductor of the co-axial wire contains the input signal to be displayed while its braided outer conductor, which surrounds the central one, is grounded so that any unwanted signal may not get picked up and fed to the oscillator.



(a) Basic Structure



(b) Equivalent Circuit

Fig. 9.9 Direct Probe

Since the input impedance and sensitivity of an oscilloscope are considerably high, the probes must be shielded so that they may not pick up the hum signal. The input impedance is sometimes required to be increased further to avoid overloading or instability of the circuits as the input impedance of the CRO is very high as compared to the test circuit. In addition, the signal loss due to attenuation as well as undesired phase shifts of the signal can be avoided if the input impedance of the oscilloscope in addition with the input impedance of the probe is significantly higher than that of the input signal source.

The probes possess some stray capacitance which may get added to the input capacitance of the oscilloscope and cause undesired oscillations in a sensitive circuit when it is connected to the CRO. An isolation probe is used to eradicate such effects. This probe consists of a carbon resistance and a shielded core connected in series.

9.5.2 Attenuator Probes

Attenuator probes, also known as **passive probes**, are used to attenuate the signal under test and provide significantly larger input impedance than that of direct probes, thereby reducing the loading effects. The attenuation is generally done by a factor of 10 and hence, these are also referred to as **10:1 probes**. Attenuation probes incorporate compensation mechanism for input capacitance of oscilloscope as well as the capacitance of co-axial cables. Figure 9.10 illustrates an attenuator probe with its equivalent circuit.

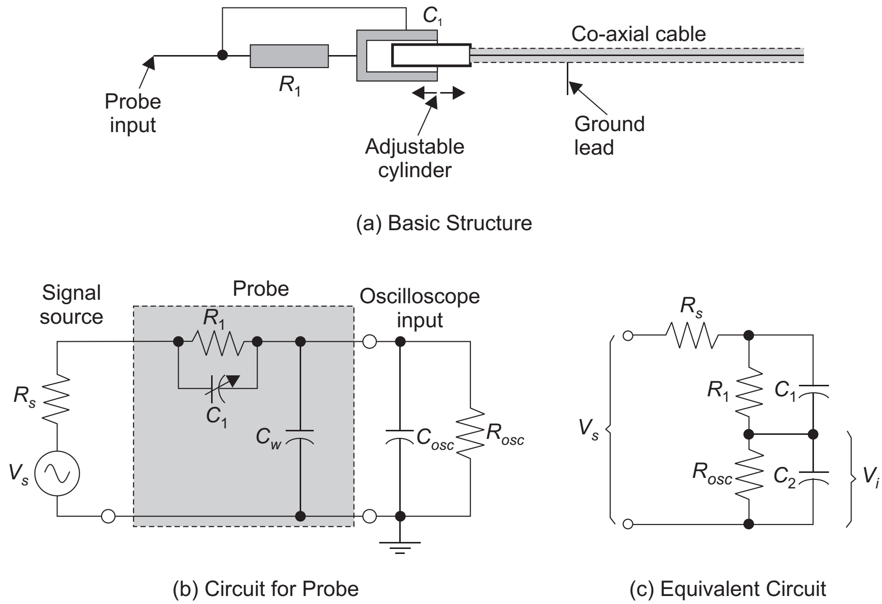


Fig. 9.10 Attenuator Probe

In Figure 9.10 (b), C_{osc}, C_w, R_s, and R_{osc} represent input capacitance of the oscilloscope, input capacitance of co-axial cable, source resistance, and input resistance of the oscilloscope, respectively. The typical value of R_{osc} is about 1 MΩ. A high input resistance R_1 (typically of 9 MΩ) is connected in parallel to a variable capacitance C_1 [see Figure 9.10 (b)]. This circuit can be modified as the equivalent circuit shown in Figure 9.10 (c) in which capacitance C_2 is shown as the sum of capacitances C_{osc} and C_w. The oscilloscope input voltage V_i can be expressed as:

$$V_i = V_s \frac{R_{osc}}{R_1 + R_s + R_{osc}} \qquad ...(3)$$

Equation (3) is valid only at low and medium frequencies where capacitive impedances are large enough to be no longer effective. If the source resistance R_s is much smaller than the oscilloscope resistance R_{osc}, then the above equation is reduced to:

$$V_i \approx V_s \frac{R_{osc}}{R_1 + R_{osc}} \qquad ...(4)$$

Substituting the typical values of resistances, we get:

$$V_i = V_s \frac{1 \times 10^6}{9 \times 10^6 + 1 \times 10^6} = \frac{V_s}{10}$$

Now, if capacitances are assumed to be acting alone, then the signal attenuation due to them can be determined as:

$$V_i = V_s \frac{X_{C2}}{X_{C1} + X_{C2}}$$

Or, $$V_i = V_s \frac{1/(\omega C_2)}{1/(\omega C_1) + 1/(\omega C_2)}$$

Or, $$V_i = V_s \frac{C_1}{C_1 + C_2} \qquad ...(5)$$

If the signal attenuation by capacitive and resistive networks is same, then the input voltage V_i across R_{osc} becomes equal to that across C_2 in amplitude as well as in phase. Thus, from Equations (4) and (5), we may write as:

$$\frac{R_{osc}}{R_1 + R_{osc}} = \frac{C_1}{C_1 + C_2} \qquad ...(6)$$

When this situation is met, the combination of CRO and probe is compensated for all frequencies. The undesired phase shifts and additional attenuation due to capacitor C_2 do not exist anymore since C_1 compensates for C_2. The required value of C_1 which compensates for C_2 can be expressed from Equation (6) as:

$$C_1 = C_2 \frac{R_{osc}}{R_1} \qquad ...(7)$$

Capacitance C_1 must be adjusted to the value obtained by Equation (7) to provide the required compensation as different CROs have different values of input impedance. A square wave, generated within the CRO, is applied to a terminal on its front panel and the capacitance is varied so that an undistorted display of the waveform can be obtained on the screen. The displayed waveform should be perfectly square. This way the probe can be calibrated before use.

9.5.3. Active Probes

Active probes provide a good way to connect high frequency signals with fast rise time to the oscilloscope. These probes incorporate amplifiers, usually FET amplifiers, which increase their input resistance in addition to decreasing their input capacitance overall providing high input impedance. The input amplifier, with unity gain and 1 MΩ input-resistance in parallel with 3.5 pF capacitance, serves as a voltage follower circuit. Moreover, the input impedance can be made as high as 10 MΩ with the aid of FET input amplifier stages, and resistive attenuation can be employed to further minimize the capacitive effect. The driving power for these probes may be generated within the probe by its power supply section or it may be derived from the oscilloscope. Figure 9.11 depicts the configuration of an FET amplifier active probe.

As FET amplifies the input signal, the tip of the voltage probe must be incorporated in the FET in order to eradicate the interconnecting cable capacitive effects. No signal attenuation exists between the probe and the amplifier. The active probe is capable of handling the signals over the range of less than a few volts. This range is determined by the dynamic range of the

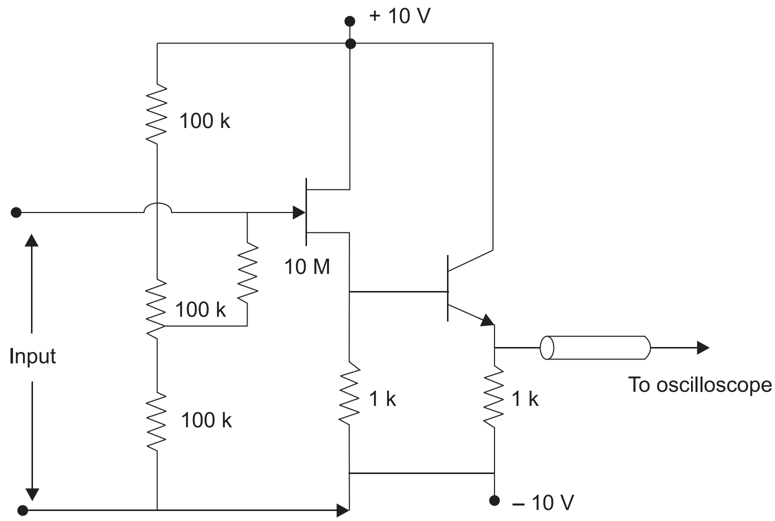


Fig. 9.11 Connections for an Active Probe

FET being employed. However, this range can be further increased by incorporating external attenuators.

In comparison to attenuator probes, these probes provide less attenuation to the signal. Thus, active probes are more suitable for the applications where small signals are to be measured. However, active probes are bulky and expensive.

Example 1 For compensating a 10:1 probe, if the input capacitance of an oscilloscope is 20 pF and the capacitance of the co-axial cable is given as 80 pF, determine:

(a) the required capacitance C_1

(b) the input capacitance of the probe seen from the source end

Solution: Given that: oscilloscope capacitance C_{osc} = 20 pF, co-axial cable capacitance C_w = 80 pF, C_2 = 80 pF + 20 pF = 100 pF

(a) We know that: R_{osc} = 1 MΩ and R_1 = 9 MΩ

Thus, capacitance C_1 can be calculated as:

$$C_1 = C_2 \frac{R_{osc}}{R_1} \qquad \text{[Refer to Eqn. (7)]}$$

Substituting the required values, we get:

$$C_1 = (100 \times 10^{-12}) \left(\frac{1 \times 10^6}{9 \times 10^6} \right) = 11.11 \text{ pF}$$

(b) The input capacitance C_T of the probe can be determined by considering the series combination of C_1 and C_2.

Thus, we may write as:

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

Substituting the required values, we get:

$$\frac{1}{C_T} = \frac{1}{11.11 \times 10^{-12}} + \frac{1}{100 \times 10^{-12}}$$

This gives,

$$C_T = 10 \text{ pF}$$

9.6 MEASUREMENT OF FREQUENCY, TIME DELAY, AND PHASE ANGLE

The signal waveform under test is applied to Y-plates of the CRO while the generated sawtooth signal waveform is applied to its X-plates. The beam under this condition encounters force in horizontal as well as vertical direction. The horizontal force tends to move the beam from left to right while the vertical force moves it up and down. Both the movements are proportional to the voltage applied to the deflection plates. The horizontal deflection is determined by the voltage applied to X-plates and is a straight line due to linearity of the sawtooth sweep signal. The vertical deflection is determined by the input signal applied to Y-plates and is in accordance with the magnitude as well as polarity of the input signal. Figure 9.12 illustrates an obtained waveform of a sinusoidal input signal.

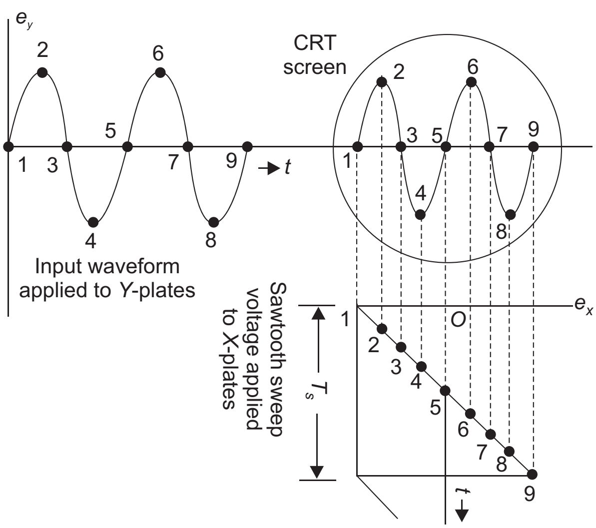


Fig. 9.12 Resultant Waveform from a Sinusoidal Signal

The number of cycles of the output waveform depends upon the frequency of the input and sweep signal. A signal appears when both the frequencies are equal. However, for unequal frequencies, the number of cycles become less than or greater than 1 depending upon whether the sweep signal frequency is higher or lower than the input signal frequency, respectively. Here it is to be noted that to obtain a stationary waveform, the ratio of the two frequencies must be an integer. Here, the applied input signal possesses a frequency twice of that of the sawtooth sweep signal. It is evident from the figure that the sweep voltage drops in an abrupt manner at the end of each cycle, thereby causing the luminous spot to be moved again to its original position. Various parameters of the obtained waveform can be determined using different measurement techniques. Let us discuss the measurement methods for *frequency*, *phase angle*, and *time delay*.

9.6.1 Frequency Measurement

Frequency of a waveform, denoted as f, can be determined using the relation between time period T and frequency f, given as:

$$f = \frac{1}{T}$$

Thus, if the time period of the obtained signal is determined once, it is easier to determine its frequency. The measurement of time period of a sinusoidal waveform can be done by measuring it for one cycle in horizontal divisions. The obtained value is then multiplied by the value indicated or set at the Time/Div control on the front panel of the oscilloscope. It should be noted here that the Time/Div knob must be set at its calibrated value. Mathematically, it can be expressed as:

$$T = (\text{Horizontal divisions/Cycle}) \times \text{Value selected at Time/Div knob} \qquad ...(8)$$

Consider the waveform shown in Figure 9.13 in which a sinusoidal waveform is marked in 8.5 horizontal divisions and 4.4 vertical divisions. A Time/Div control knob is also shown which is set to a calibrated value of 0.5 ms.

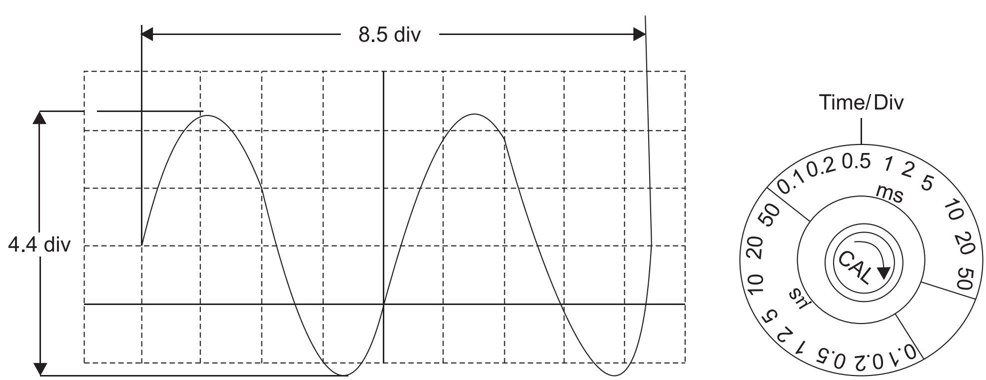


Fig. 9.13 Waveform under Measurement

The time period T of this waveform can be calculated using Equation (8) as:

$$T = \frac{8.5 \times 0.5 \times 10^{-3}}{2} = 2.12 \text{ ms}$$

Therefore, the frequency of the waveform can be calculated as:

$$f = \frac{1}{2.12 \times 10^{-3}} = 471.7 \text{ Hz}$$

9.6.2 Phase Angle Measurement

Measurement of the phase angle of the waveform is another important parameter, displayed on the CRO screen. The phase difference between two waveforms can be obtained by multiplying the number of horizontal divisions in one degree by the number of horizontal divisions between their commencements. Consider the two waveforms illustrated in Figure 9.14 where each of them has 6 divisions in one full cycle.

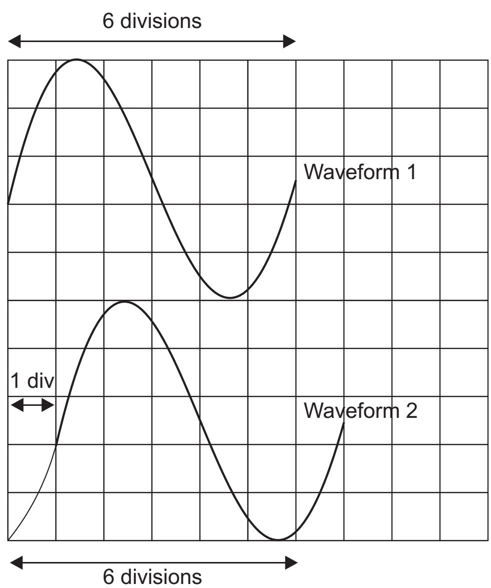


Fig. 9.14 Phase Difference Measurement

It can be seen from the figure that there is time difference of 1 division in the commencements of the two waveforms. The phase difference between these waveforms can be determined as follows.

We know that one full cycle = 360°

$$\Rightarrow \quad 6 \text{ divisions} = 360°$$

$$\Rightarrow \quad 1 \text{ division} = 360°/6 = 60°$$

Now, the phase difference ϕ can be obtained as:

ϕ = Phase difference in division between commencements × Degrees per division

This yields,

$$\phi = 1 \text{ div} \times (60°/\text{div}) = 60°$$

9.6.3 Time Delay Measurement

It is important that the difference between the occurrences of two waveforms is calculated in terms of time. Consider the waveforms shown in Figure 9.15 to understand the concept better.

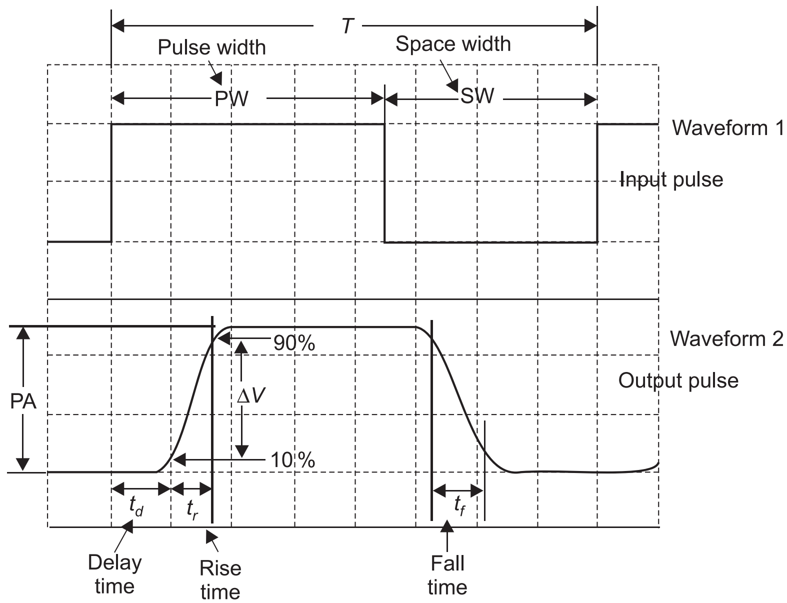


Fig. 9.15 Time Delay Measurement

It is evident from the figure that the two waveforms do not occur at the same instant, rather there is a time delay in their commencements. This time delay t_d can be seen as the difference in the commencements of these waveforms. It can be measured if the waveform 1 is assumed to be the input of a circuit while waveform 2 as its output and can be defined as the time intervened from the occurrence of the input pulse until the amplitude of the output pulse reaches 10% of its final value.

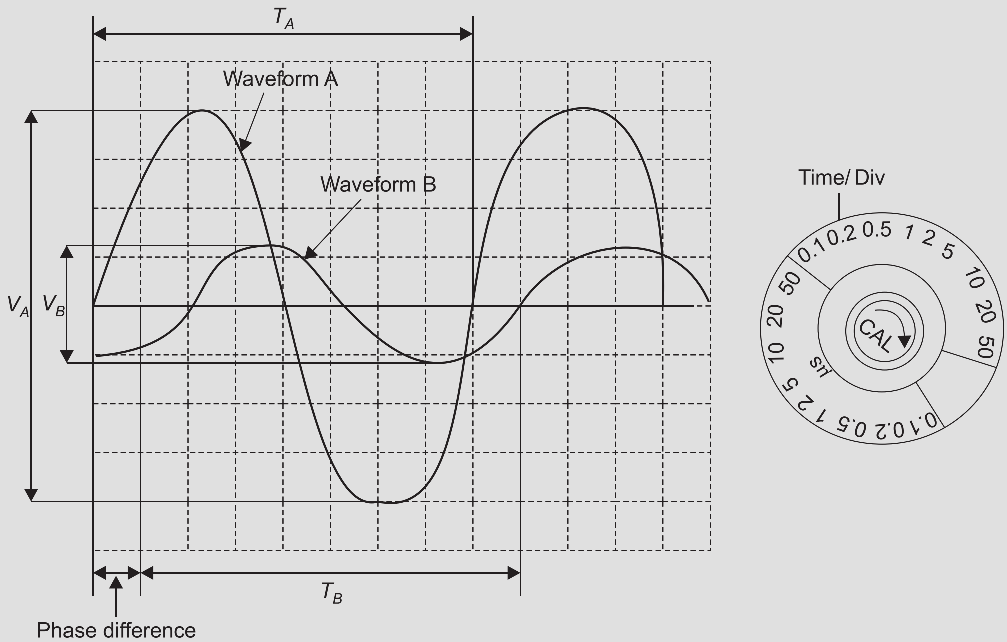
The Time/Div control on the front panel of the oscilloscope determines the value of time delay. If it is set to 0.5 ms, then the time delay of the waveforms shown in the figure comes to 0.5 ms since there is a difference of one horizontal division in the occurrences of the input and output waveforms.

In addition, there are two more parameters, namely rise time and fall time in terms of time specifications of the displayed waveforms. These time periods can be determined on waveforms which do not possess steep leading and lagging edges as waveform 2 in Figure 9.15. The **rise time** t_r can be defined as the time taken by the waveform to rise from 10% to 90% of its maximum amplitude while the **fall time** t_f can be defined as the time taken to fall from 90% to 10% of its final value.

Example 2 Consider the waveforms given in the following figure and calculate:

(a) their frequency

(b) their phase difference.



Solution:

(a) The frequency of the waveforms can be computed as:

$$f = \frac{1}{T}$$

For waveform A, the time period T comes out to be:

T_A = (horizontal divisions/cycle) × value selected at Time/Div knob [Refer to Eqn. (8)]

= 8 × 0.2 ms/div = 1.6 ms

So, frequency comes out as:

$$f_A = \frac{1}{T_A} = \frac{1}{1.6 \times 10^{-3}} = 625\,\text{Hz}$$

Now, for waveform B, the time period is:

$$T_B = 8 \times 0.2 \text{ ms/div} = 1.6 \text{ ms}$$

So, frequency comes out as:

$$f_B = \frac{1}{T_B} = \frac{1}{1.6 \times 10^{-3}} = 625\,\text{Hz}$$

(b) The phase difference between these waveforms can be determined as:

ϕ = Phase difference in division between commencements × Degrees per division

$$\phi = 1 \text{ div} \times [(360°/8)/\text{div}] = 45°$$

9.7 MULTI-INPUT OSCILLOSCOPES

The oscilloscopes studied so far are simple oscilloscopes with only one input signal applied to them. However, there are some other oscilloscopes which are fed with more than one input. These oscilloscopes can be categorized as multiple-trace oscilloscopes and multiple-beam oscilloscopes. Both these oscilloscopes are common in two input configuration; although four and eight input configurations are also possible. Therefore, we confine our discussion in this section to the two input configuration. The same principle can be applied to all higher input configurations.

9.7.1 Dual Beam Oscilloscope

Dual beam oscilloscopes can be used to display two different waveforms simultaneously. As the name suggests, these oscilloscopes incorporate two separate electron beams. Two separate beams require separate vertical deflection systems which imply that the amplitude of the two waves can be independently controlled. However, there can be two separate horizontal deflection systems for the two beams or a single system with a switching mechanism can be used. In single time base or horizontal system, only one beam can be synchronized at a time. Therefore, the frequencies of both the signals must be same so that both the signals can be locked onto the screen to get their simultaneous display. On the other hand, two different horizontal deflection systems provide different sweep rates for two vertical deflection systems. However, the oscilloscopes possessing them become bulky and heavy. Figure 9.16 depicts a dual beam oscilloscope with two different time-base circuits.

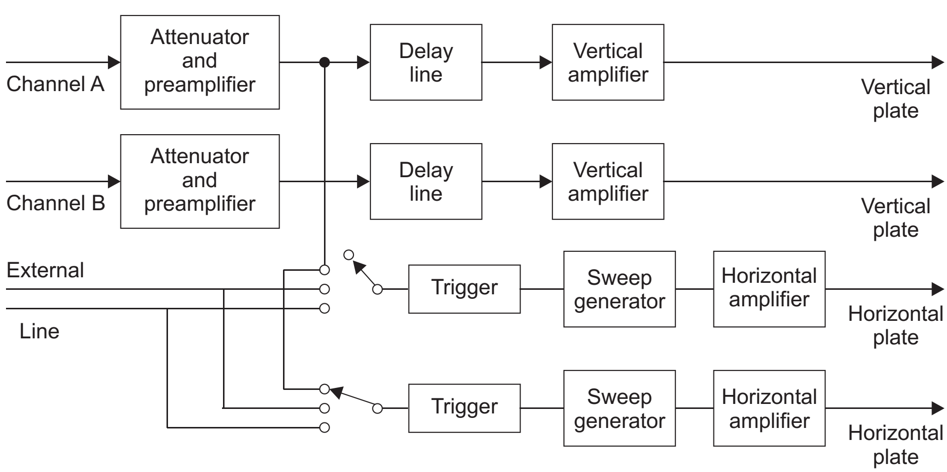


Fig. 9.16 Dual Beam CRO with Separate Horizontal Deflection Systems

The two electron beams can be generated by two methods, namely *double gun tube* and *split beam*. By using the double gun method, the focus and brightness of the beams can be controlled independently. However, the oscilloscope becomes larger in size. On the other hand, the split beam method allows a single electron gun to be used in aid with a horizontal splitter plate kept between the accelerating anode and vertical deflection plates. This splitter plate is used to isolate the two vertical channels or deflection systems. The potential of this plate must be equal to that of the accelerating anode.

The brightness of the beam gets hampered in the split beam method since the split beams have half of the brightness of an electron beam. The brightness is even low in high frequency operations. Thus, to avoid this, two apertures can be made in the accelerating anode so that two beams can pass through it. It must be noted here that for widely spaced speeds of the sweep generators, the brightness of the two beams in the split beam method may significantly differ from each other.

9.7.2 Dual Trace Oscilloscope

Dual trace oscilloscopes have one electron beam split into two and two sets of vertical input channels *A* and *B* are provided with separate input attenuator and positioning control. These are provided to adjust each signal individually. In such oscilloscopes, the electronic switch is responsible for splitting the electron beam into two. This switch can be made to operate in two modes, namely *alternate mode* and *chop mode*. Collectively, these modes are referred to as *switched channel method*. A mode control switch is provided for this on the front panel of the oscilloscope. Figure 9.17 illustrates the configuration of a dual trace oscilloscope.

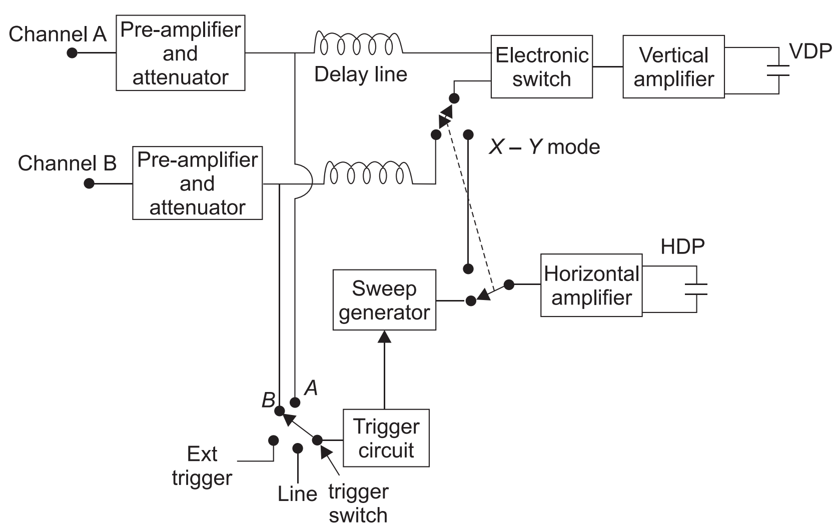


Fig. 9.17 Dual Trace Oscilloscope

It can be seen from the figure that the input signal is provided by two separate channels, namely, channel *A* and channel *B*. In **alternate mode**, the input is applied to the vertical amplifier alternately from channel *A* and channel *B* by means of an electronic switch at the starting of each sweep cycle. Thus, the switching rate of the switch must be synchronized with that of the sweep generator. The different dc offset values are then added to the inputs by this switch so that one waveform can be displayed on the top half of the screen (being biased above ground level) while another one at the bottom half of the screen (being biased below ground level). The two signals are repeatedly displayed on the screen and appear as if they are traced simultaneously due to significantly high frequency of repetition. The resultant waveforms are shown in Figure 9.18 in which waveform 1 from channel *A* is a sinusoidal and waveform 2 from channel *B* is a triangular waveform.

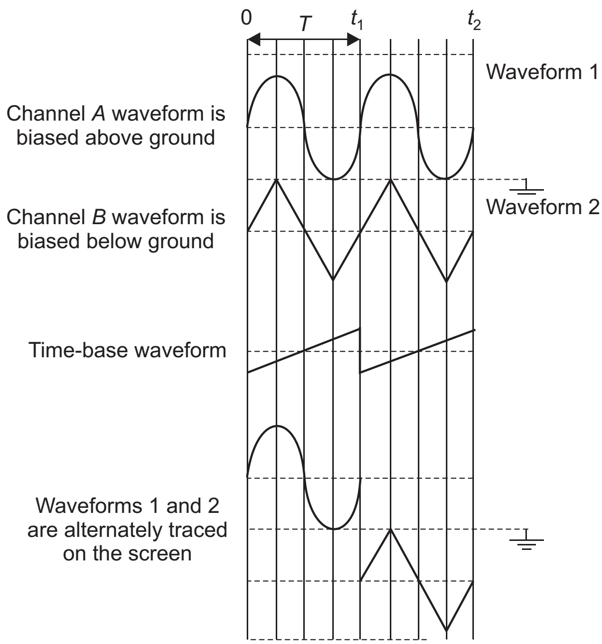


Fig. 9.18 Waveforms of Dual Trace CRO in Alternate Mode

The alternate mode is not suitable for low frequency signals since the two waveforms do not appear simultaneously on the screen and thus, it becomes complicated to compare them.

In **chop mode**, the repetition or switching frequency is much higher than in the alternate mode. In this mode, the electronic switch runs independent of the sweep generator frequency at a high frequency of the order of 100–500 kHz. The waveforms in this mode are traced as dashed lines due to such high frequency. However, the gaps between the dashes are not visible for medium and low frequency waves. The resultant waveforms in chop mode are illustrated in Figure 9.19.

Note: *The alternate mode is used for high frequency signals while the chop mode is used for low frequency signals.*

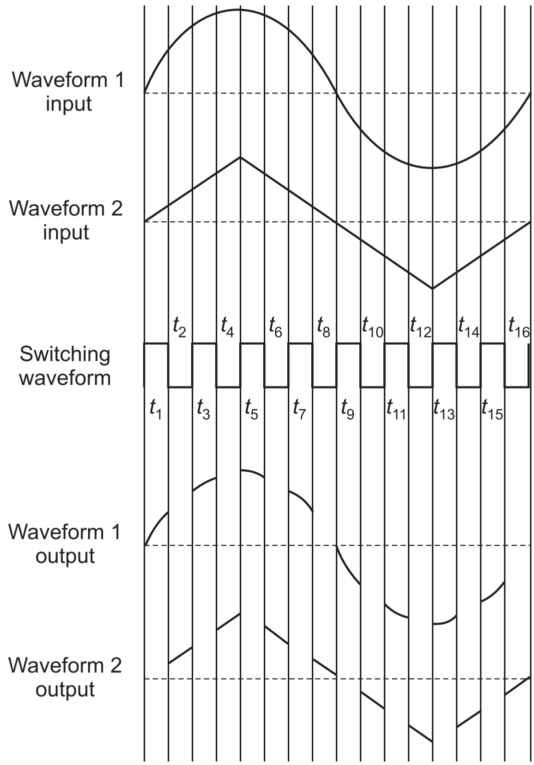


Fig. 9.19 Waveforms of Dual Trace CRO in Chop Mode

9.8 SPECIAL OSCILLOSCOPES

There arise some situations when conventional oscilloscopes cannot be used. For this, some other oscilloscopes have been developed and are together termed as **special oscilloscopes**. These oscilloscopes include *sampling oscilloscopes* and *storage oscilloscopes*. The storage oscilloscopes are then further categorized as *analog storage oscilloscopes* and *digital storage oscilloscopes*.

9.8.1 Sampling Oscilloscope

Sampling oscilloscope is used to analyze waveforms with very high frequency. Analog oscilloscopes (as studied earlier) are not capable of tracing such fast signals since the electron beam, in such a case, moves so swiftly that the produced trace appears highly fainted. To avoid this, sampling oscilloscopes are used which sample the input signal over successive cycles. The original waveform, at a relatively lower frequency, is reproduced on the screen by the series of these samples. These samples are produced from high frequency signal by dot representation which results in low frequency signal as shown in Figure 9.20. The sampling circuit of this oscilloscope must be capable of operating at extremely high frequencies while the CRT and all other circuits may operate at low frequencies.

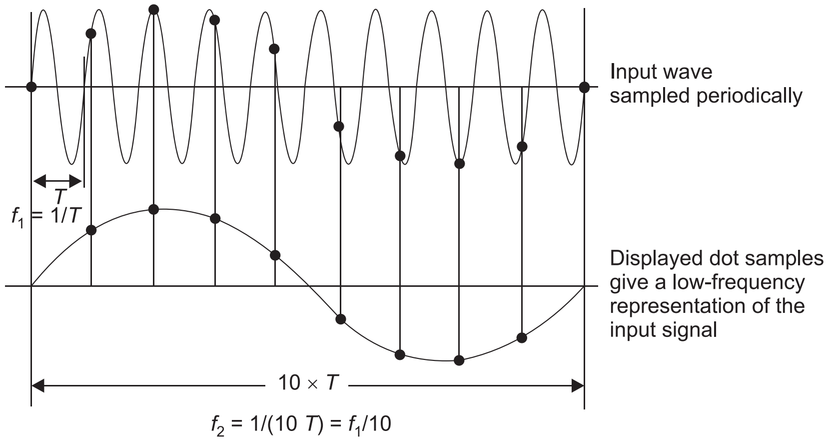


Fig. 9.20 Sampling High Frequency Signal into Low Frequency Signal

The maximum frequency of the signal at which a satisfactory display can be obtained is determined by the rate of the sampling of the oscilloscope. The sampling rate must be kept at least greater than twice the highest frequency component of the signal. This rate is referred to as **Nyquist rate**. If the samples are taken at a rate lower than the Nyquist rate, the consecutive spectrums or waves overlap each other causing an error called **aliasing** due to which variable frequency readings are obtained for a single frequency input signal. It should be noted here that sampling oscilloscopes can only investigate repetitive waveforms—these are not suitable for analyzing transient waveforms. Figure 9.21 shows the block diagram of a sampling oscilloscope which can be operated in two modes, namely *delayed time-base mode* and *expanded mode*.

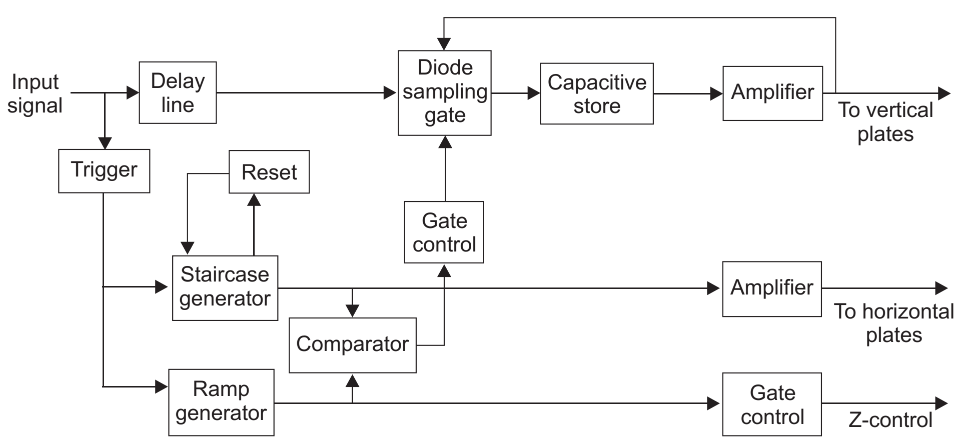


Fig. 9.21 Sampling Oscilloscope

In **delayed time-base mode**, the input signal is applied to the sampling gate via delay line in order to provide sufficient time to the horizontal sweep to be initiated. This biases the diodes in forward direction. The pulses to trigger the signal generators (staircase and ramp) are also obtained from the input. It must be noted here that in addition to a ramp generator, a staircase generator is also incorporated in a sampling oscilloscope. This staircase generator deflects the electron beam in horizontal direction. The signal is sampled at the diode sampling gate stage after which it is fed to a capacitive store unit where it is stored for a predetermined time period. The signal is then amplified by an amplifier stage. A unity gain from this amplifier is fed back to the diode sampling gate unit to ensure that the voltage on the capacitor store stage also increases only by the voltage change between two consecutive samples. The amplified signal is then fed to the vertical deflection plates of the CRT.

For horizontal deflection, the staircase generator upon being triggered resets after every specific number of steps and starts again. The output of this generator is amplified and sent to the horizontal deflection plates of the cathode ray tube. This voltage causes the spot to move from one side of the screen to another in a series of steps. In addition, the staircase output is also fed to the comparator unit where it is compared to the output of the ramp generator. Whenever the output of the staircase generator becomes equal to that of the ramp generator, the comparator produces a short-duration pulse. This pulse is fed to the diode sampling gate via gate control unit. This turns the sampling gate on. The sampling gate then samples the input signal and holds its output at a constant level for the time the next sample is taken. The output of the ramp generator is also provided to another gate control unit for controlling the intensity of the signal. The resultant waveforms at various stages of a sampling oscilloscope are shown in Figure 9.22.

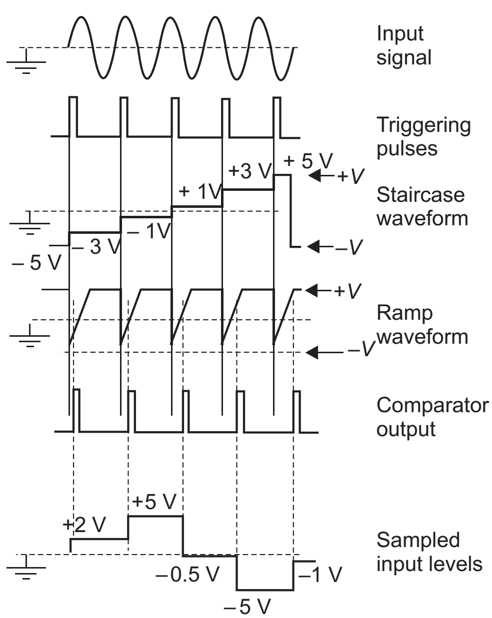


Fig. 9.22 Waveforms of a Sampling Oscilloscope

If the oscilloscope is operating in the **expanded mode**, the resultant waveforms remain similar to those of obtained in the delayed time-base mode. The only difference that in the sampled portion of the waveform is displayed in an expanded form. The horizontal deflection sensitivity of the oscilloscope is increased to obtain a full screen display of the sampled portion. In addition, the bias voltage as well as density of the staircase generator is adjusted to expand waveform.

9.8.2 Storage Oscilloscope

When an electron beam strikes the fluorescent screen of the oscilloscope, it remains there for a few milliseconds. This is due to the *persistence* of the material coated on the screen. This persistence is sufficient for almost all signals; however, it is inadequate when a low frequency signal is to be analyzed. The output of a low frequency signal is a dotted trace instead of the actual waveform. This limitation of an oscilloscope can be avoided if the persistence of the screen is made longer. This can be achieved by means of a storage oscilloscope. In addition, non-repetitive waveforms such as transient waveforms can also be analyzed on a storage oscilloscope. Based on the technique used to store signals, storage oscilloscopes can be categorized into **analog storage oscilloscope** and **digital storage oscilloscope**. Let us discuss each of them in detail.

Analog storage oscilloscope

Analog storage oscilloscopes operate on the principle of secondary emission electrons (refer to Section 9.3.3). Special cathode ray tubes such as **bi-stable storage CRTs** and **variable persistence CRTs** are required for storing the input signal in analog storage oscilloscopes. This section contains detailed description about these CRTs.

Bi-stable storage CRTs

Bi-stable storage CRTs are capable of storing a waveform for considerably long time periods, say in hours. The stored waveform can be displayed only at a single level of brightness. Any variation in the display intensity is impossible with this tube. Figure 9.23 depicts a bi-stable storage CRT.

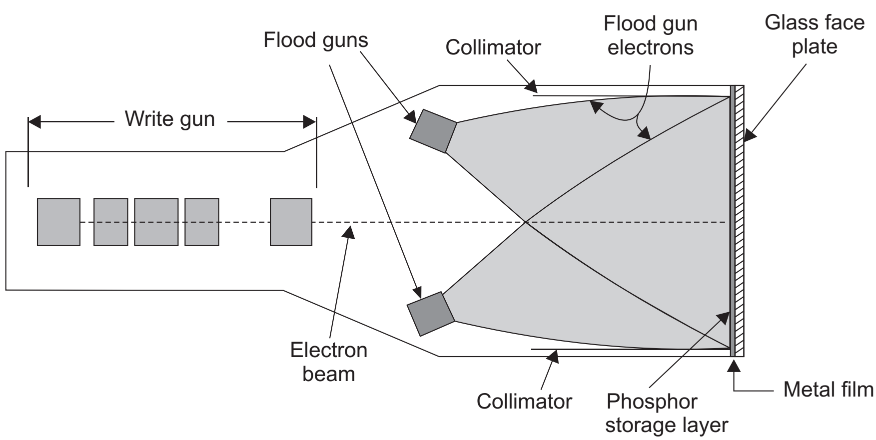


Fig. 9.23 Bi-stable Storage CRT

It can be seen from the figure that the screen of the tube is coated with a phosphor material. This layer possesses good characteristics for secondary emission and is known as **storage layer**. This material provides a discontinuous surface due to the presence of scattered particles in its layer. Thus an extremely high resistance exists between these particles. Between the screen glass and storage layer, there is a thin transparent layer of metal. Besides, there is another metal film known as **collimator** which is coated around the tube neck.

The write gun emits an electron beam; it consists of an arrangement of defection and accelerating electrodes. Low energy electrons are emitted by heating two cathodes, known as **flood guns**. It is to be noted here that the cathode is kept at the ground level potential while the collimator may be kept at ground or slightly positive potential. Transparent metal film is kept at a potential of +1 to +3 V which attracts the electron cloud emitted by the flood guns. These electrons then reach the screen of the tube. In the absence of electron beam, the electrons generated by the flood guns cannot affect the storage layer. These low-energy electrons are collected at the collimator instead of reaching the phosphor screen. Thus, there is no display on the CRT screen.

Now, let an input signal be applied to the oscilloscope for a very short time and the write gun be energized. As a result, the write gun emits a high-velocity electron beam in order to trace the input signal. The electrons in the beam have considerable energy to hit the storage layer thereby producing secondary emission. Wherever the secondary emission occurs on the screen, electrons get lost from those points thereby making the screen positively charged. A path of positive charges in the shape of input waveform gets formed at the storage layer which can be displayed over long time. This positive charge path attracts the electrons emitted by the flood guns. Upon passing through this path, these low-energy electrons reach the metal film which is more positive than the charge path. Further, they pass through the storage layer and cause the phosphor to glow continuously. In this way, the transient waveforms are made to display for hours.

The display can be erased if the metal film is made negative which causes the low-energy electrons to go back to the storage layer and get accumulated. This brings the written area to the same potential as that of the surrounding material.

Variable-persistence storage CRTs

Variable-persistence storage CRTs are similar to bi-stable storage CRTs, except that a *storage mesh* as well as a *collector mesh* is incorporated in it. These meshes are made up of fine wire. On the inner side of the storage mesh, the storage layer is deposited. A 0 V to –10 V potential is applied to the storage mesh while a 100 V potential is applied to the collector mesh. In addition, the flood guns are aided with control grids at their front. Figure 9.24 shows the basic structure of a variable-persistence storage CRT.

As in the case of a bi-stable storage CRT, here also, the electron beam strikes the storage layer and produces secondary emission thereby making a path of positive charge in the shape of the signal applied at the input. However, the low-energy electrons from secondary emission and flood guns, which are not able to pass through the storage layer (due to negative charge of the rest of the layer) attract backwards to the collector mesh, kept at high positive potential. It should be noted here that despite the low-persistence phosphor in this CRT, the low energy electrons, which pass via the positive trace on the storage layer maintain the screen display.

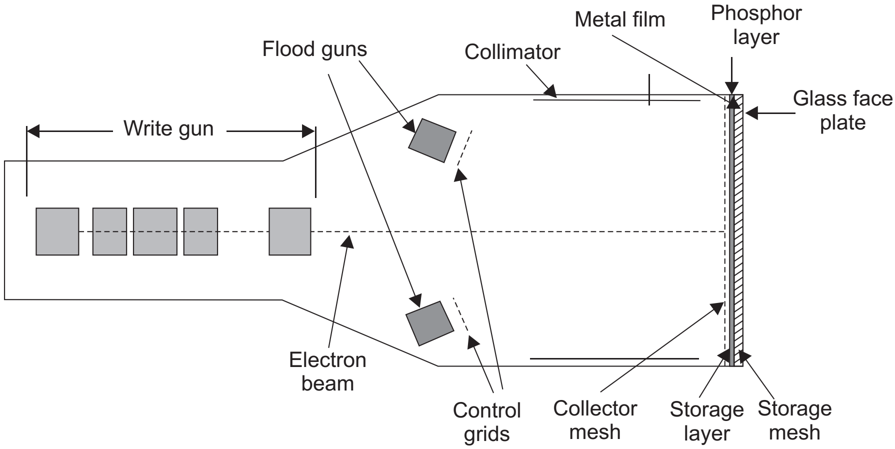


Fig. 9.24 Variable-Persistence Storage CRT

The number of electrons emitting from flood guns and the amount of their energy determines the waveform brightness. These electrons are controlled by the grid voltage of flood guns. If a high negative potential is applied to these grids, the electrons get completely cut off while for maximum electrons to flow, a 0 V potential must be applied to these grids.

The display can be erased if the storage mesh is kept at a high positive potential equal to that of the collector mesh for a brief instant of time. Therefore, the electrons emitted from the flood guns generate the secondary emission all over the storage layer which causes the traced waveform to be erased. After this, the storage mesh returns to its normal potential level. The positively charged areas can be discharged by applying a negative pulse ranging between –4 V and –11 V to the storage mesh repetitively. The persistence of the waveform can be varied if the stored waveform is erased slowly. The width of this repetitive pulse is variable while the frequency is generally kept stable at 1200 Hz. The wider the pulse, the shorter is the persistence and vice-versa. Another method to vary the persistence includes varying the frequency, keeping pulse width constant. The persistence can be increased by decreasing the frequency and vice-versa.

Digital storage oscilloscope

In a digital storage oscilloscope, the analog signal is sampled at regular intervals and converted to its digital equivalent by an analog-to-digital converter (abbreviated as ADC). The block diagram of a digital storage oscilloscope is depicted in Figure 9.25.

An analog input signal is applied to the circuit. This signal is first amplified by an amplifier and then sent to an analog-to-digital converter circuit and a trigger circuit. The trigger circuit gives triggering pulses to the time-base generator which generates a staircase waveform for horizontal amplifier. The horizontal amplifier amplifies the incoming signal and sends its output signal to the horizontal deflection plates of the CRT. Note that the CRT of a DSO is similar to that of a conventional oscilloscope. On the other hand, the ADC circuit converts this analog signal into its digital form by means of sampling. The digital signal so obtained

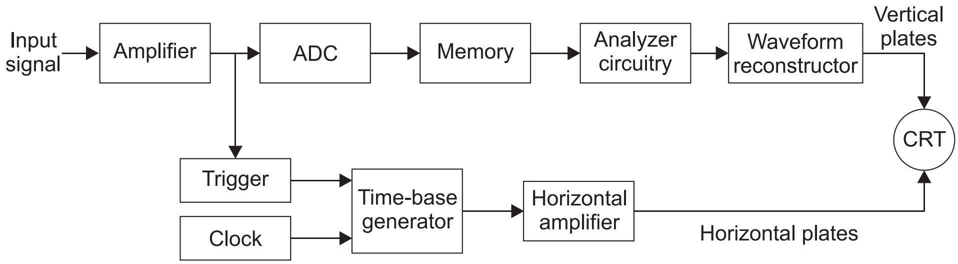


Fig. 9.25 Digital Storage Oscilloscope

is stored in the memory unit, which is generally an arrangement of a number of set-reset flip-flops, for later retrieval. The stored signal is then analyzed to produce a variety of information by the analyzer circuitry. Upon analyzing, the digital signal is converted back to its analog form by the waveform re-constructor unit or digital-to-analog converter (DAC). The analog signal is then sent to the vertical plates of the CRT. In this way, the analog signal applied at the input is represented in the form of a dot waveform on the oscilloscope screen. The waveforms at various stages of a digital storage oscilloscope are illustrated in Figure 9.26.

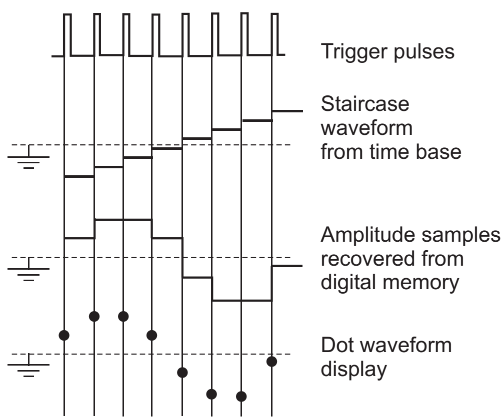
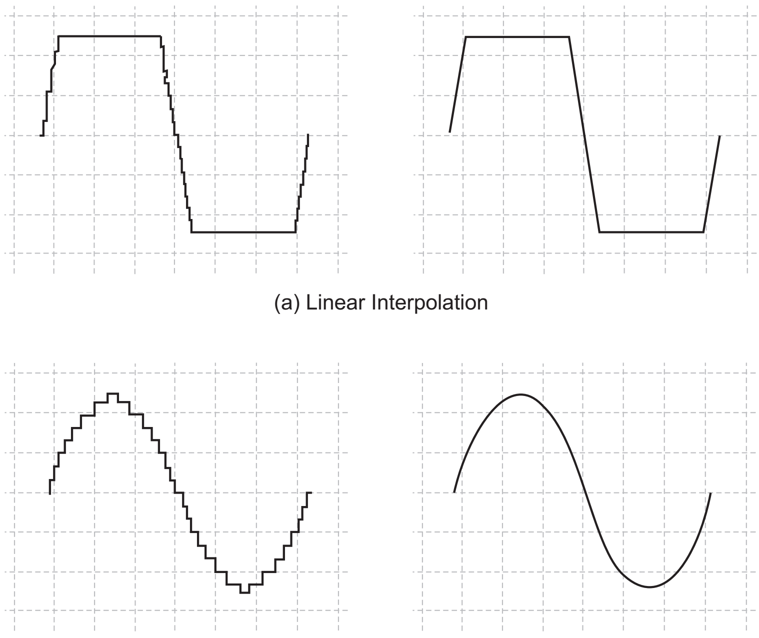


Fig. 9.26 Waveforms at Various Stages of a DSO

Dot or step waveform output of a digital storage oscilloscope is not able to represent the actual smooth waveform. Thus, sometimes it becomes necessary to modify the obtained output waveform with the aid of additional samples in between the actual acquired samples. This process is known as **interpolation** and provides a smooth waveform (closer approximation of the input waveform) which has relatively higher sampling rate. In other words, the effective sampling density of a waveform is increased by interpolation. However, the bandwidth of the signal remains unaltered. This optimal view of the waveform is possible using interpolation which further leads to accurate measurements.

Broadly, there are two types of interpolation, namely, *linear interpolation* and *sine-wave interpolation*. Linear interpolation is done on square or pulse waveforms while sine-wave interpolation is done on sinusoidal waveforms (see Figure 9.27).



(a) Linear Interpolation

(b) Sine-wave Interpolation

Fig. 9.27 Interpolation

The simplest form of interpolation is linear interpolation in which a straight line is used to join two consecutive samples. On the other hand, sine-wave interpolation introduces a sinusoidal function in between the levels of two consecutive samples. If it is used for a pulse waveform, it may distort the signal by introducing overshoots in it.

Features of DSO

Digital storage oscilloscopes possess some features that make them more suitable for waveform analysis. These features are discussed below.

- **Auto-set:** Some digital storage oscilloscopes provide auto-set function which enables them to automatically choose the most suitable time and voltage settings. These settings are displayed on the screen with the waveform. However, manual selection can also be done in such oscilloscopes.
- **Waveform Processing and Measurement:** Considerably accurate measurements can be done using a DSO in comparison to an analog oscilloscope. These oscilloscopes can print the desired measurements on the screen while displaying the waveform. For this, there are two cursors which point to the specific locations of the waveform between which the measurement has to be done. The movement of these cursors can be controlled by the controls on the front panel of the oscilloscope. Some DSOs incorporate special circuitry to process the waveform in the absence of cursors. The measurements of peak-to-peak voltage, rms voltage, frequency, and time-period are done automatically and displayed on the screen.

- **Zoom and Restart:** Some DSOs possess these features which enable them to view some part of a waveform in detail. In zoom option, the desired portion of the waveform can be centred on the screen. The portion to be analyzed is then zoomed in by pushing a zoom button. The required changes in context of time-base and time-delay are automatically introduced. On the other hand, if the restart selection is made, the waveform is re-sampled at higher sampling rate along with expantion in the time domain. This provides the maximum information.
- **Pre-triggering and Post-triggering:** In DSOs, some portions of the stored waveform can be displayed prior to the application of a triggering pulse. This is called **pre-triggering**. The time of pre-triggering depends upon the oscilloscope and may lie from 50% to 100% of the waveform time period. On the other hand, if the starting point of the waveform is displayed after a substantial time delay of the triggering, it is known as **post-triggering** and this time-delay is called **hold-off time**. The hold-off time is variable and much longer (say several times) than the waveform time period. This feature enables a DSO to examine any desired portion of the waveform.
- **Glitch and Runt Catching:** If the sampling rate of the DSO is not sufficiently high, it may miss the glitches, that is, very fast spike-type changes in the waveform. These glitches are invisible to the oscilloscope during sampling time-interval. In digital logic circuits, where the switching of flip-flops is done at high speed, glitches may get introduced in the waveform (see Figure 9.28). Thus, the oscilloscopes with very high sampling rate should be used. Another special type of pulse, known as **runt**, occurs in digital logic circuits. It cannot trigger the circuit; however, by incorporating a maximum/minimum level detector, it can be made to do so. The waveform before and after the glitch or runt, is then displayed (see Figure 9.29).

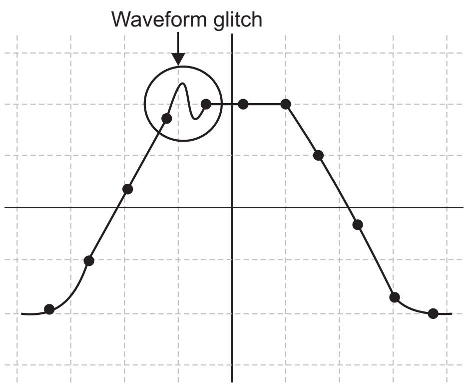


Fig. 9.28 Glitch in the Waveform

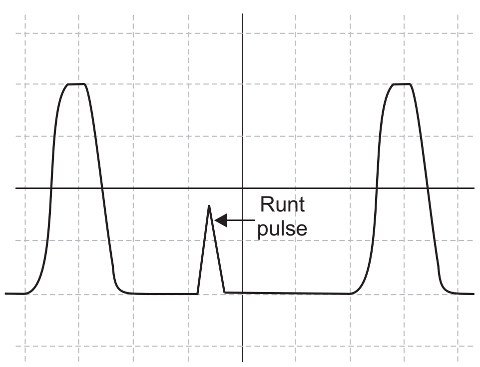


Fig. 9.29 Runt in the Waveform

Modes of operation

A digital storage oscilloscope can be operated in two modes, namely *baby-sitting mode* and *roll mode*.

- **Baby-sitting mode:** Some waveforms possess infrequent anomalies, that is, the transients in them occur only once in several hours. This situation can be dealt with if the oscilloscope is made to operate in baby-sitting mode. In this mode, the waveform is continuously sampled and stored in order to retain several immediately before and after cycles of an anomaly. Thus, when an anomaly occurs, these waveforms are available for analysis as shown in Figure 9.30.

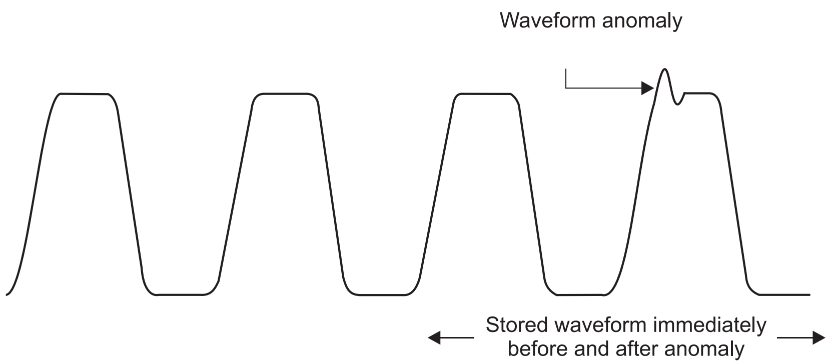


Fig. 9.30 A Waveform in Baby-Sitting Mode

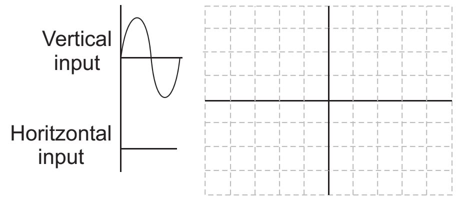
- **Roll mode:** Some waveforms vary in a very slow manner. Such waveforms are stored in a DSO when it is operated in roll mode. The stored waveforms are then displayed at a considerably high speed since the time-base generator of the oscilloscope can be adjusted to provide an accurate sampling time interval.

9.9 LISSAJOUS FIGURES

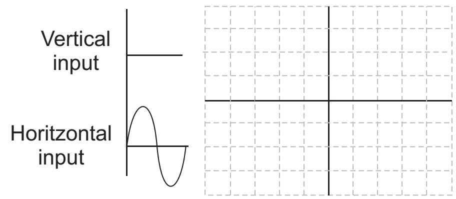
When the time-base generator of the oscilloscope is disconnected and a different sine wave input is applied to the vertical and horizontal deflection system, the resultant waveform is determined by the relationship between these waveforms. If the frequency of these waveforms is equal, the obtained waveform is normal while a very complex figure may be obtained for different frequency waveforms. These figures or patterns are referred to as **Lissajous figures** or **Lissajous patterns**. Let us discuss some of the common lissajous figures.

If a sine wave input signal is applied to the horizontal deflection plates while no signal is applied to the vertical deflection plate, then the output signal is obtained as a straight horizontal line. On the other hand, if the input sine signal is applied only to the vertical deflection system, the output signal is obtained as a straight vertical line as illustrated in Figure 9.31.

When two input sine signals possessing equal amplitude and frequency are applied to both the deflection plates, the obtained signal is a straight line, tilted at an angle of 45° from the horizontal plane. When both the waveforms reach their zero potential point at the same instant, the deflection is neither vertical nor horizontal. Thus, the electron beam strikes at the centre of the screen. This is shown in Figure 9.32 (a), at positions *A* and *C*. On the other



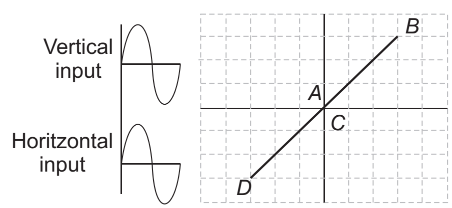
(a) When input signal is applied to only horizontal deflection system



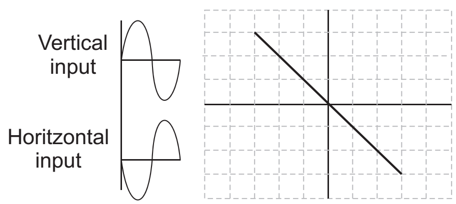
(b) When input signal is applied to only vertical deflection system

Fig. 9.31 Output Waveforms when Input Signal is Applied to only one Deflection System

hand, when both the waveforms attain their maximum positive amplitude at the same instant, the maximum positive deflection occurs vertically as well as horizontally on right-hand side, marked as *B* in the figure. Similarly, when the waveforms reach their maximum negative amplitude, the maximum negative deflection occurs vertically and horizontally on left-hand side, marked as *D* in the figure. If the input waveforms are anti-phase, the resultant waveform is deflected as a straight line tilted at an angle of 135° from the horizontal plane. It is shown in Figure 9.32 (b).



(a) When the waveforms are in-phase

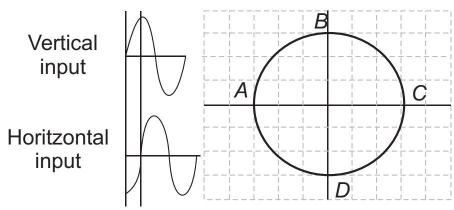


(b) When the waveforms are anti-phase

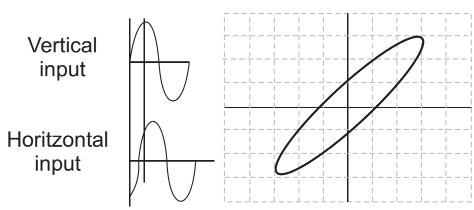
Fig. 9.32 Output Waveforms When Equal Input Signals are Applied to both the Deflection Systems

If input signals with 90° phase difference are applied to the two deflection plates, then the resultant display comes out to be circular. When the vertical input is at zero and horizontal input is at its maximum negative amplitude, there is no vertical deflection while maximum horizontal deflection on the left-hand side occurs, marked as *A* in Figure 9.33(a). Now, when the vertical input attains its maximum positive amplitude and horizontal input reaches its zero level, maximum positive vertical deflection occurs which is marked as point *B* in the figure. Point *C* shows the maximum positive horizontal deflection on the right-hand side when the vertical input is zero and the horizontal input is its maximum level. When the horizontal input is zero and vertical input is at its maximum negative amplitude, the maximum deflection occurs at negative vertical plane, marked as point *D*. In this way, the beam strikes the screen at these four points in a rotating manner making a circular display.

On the other hand, if the phase difference between the applied signals is less than 90° but greater than zero, the resultant signal is an ellipse as shown in Figure 9.33(b). The lesser the phase difference, the narrower is the ellipse. If the phase difference lies between 90° and 180°, the resultant signal is again an ellipse, but tilted in the opposite direction. Measurements are made on the displayed signal to determine the actual phase difference.



(a) When the waveforms are out of phase by 90°

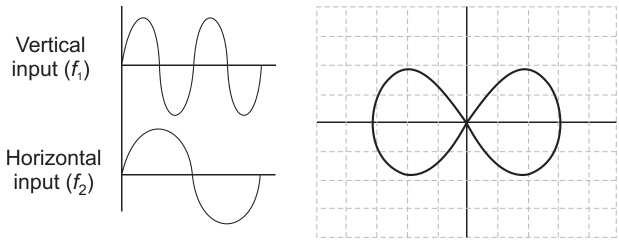


(b) When the waveforms are out of phase by 0 – 90°

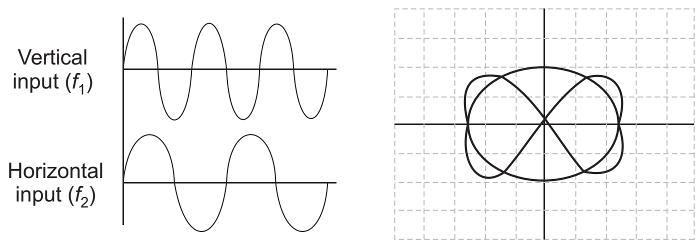
Fig. 9.33 Output Waveforms When Out of Phase Input Signals are Applied to the Deflection Systems

The Lissajous figures studied above are the result of phase relationship between the vertical and horizontal input signals. However, the frequency of the input signals is assumed to be identical in this case. Now, let us consider the figures that result from signals possessing different frequencies. Figure 9.34 illustrates such signals and their resultant waveforms.

In the first case, the ratio of the vertical input frequency is twice that of horizontal input frequency. The electron beam moves in the influence of horizontal input—first from the centre of the screen to the right-hand side and then back from the centre to the left-hand side after which it comes back to the centre again. Meanwhile, the vertical input signal causes the



(a) When $f_1/f_2 = 1/2$



(b) When $f_1/f_2 = 3/2$

Fig. 9.34 Output Waveforms When Different Frequency Signals are Applied to the Deflection Systems

beam to deflect from centre to top and then centre to bottom, and back to the centre. This vertical deflection occurs twice the horizontal deflection since its frequency is twice the horizontal input signal.

In the second case, the ratio of the frequencies is 3:2 and a more complex output signal is obtained in this case. Any other ratio of the frequencies would yield even more complex Lissajous figures. In addition, a continuously varying waveform is obtained if the ratio of the frequencies is not exact. Thus, to obtain a stationary output waveform, this ratio must be exact. This ratio can be easily determined from the output waveform using the given relation:

$$\frac{f_1}{f_2} = \frac{\text{Number of peaks at either top or bottom}}{\text{Number of peaks at either left or right side}} \qquad \text{...(9)}$$

where f_1 is the vertical frequency and f_2 is the horizontal frequency.

Let us Summarize

1. The cathode ray oscilloscope (abbreviated as CRO), generally referred to as an oscilloscope, is an instrument that can measure, display, and analyze waveforms and other phenomena in electronic and electrical circuits.
2. The basic structure of a cathode ray oscilloscope consists of a vertical amplifier, a horizontal amplifier, a trigger circuit, a time-base generator, a delay line, a power supply block, a cathode ray tube, and a phosphor screen.
3. A cathode ray tube (abbreviated as CRT) is the heart of an oscilloscope which is basically a glass tube consisting of an assembly of electrodes. The tube has a phosphor coated viewing screen at one end.

4. The major components of a CRT are glass envelope, fluorescent screen, electron gun assembly, deflection plates, and a base.
5. The basic CRO circuit consists of vertical deflection system, horizontal deflection system, and synchronization amplifier.
6. An input signal is fed to an oscilloscope via its probes. It can also be said that the probes of a CRO connect it to the circuit under test. There are three types of oscilloscope probes, namely direct probes, attenuator probes, and active probes.
7. The signal waveform under test is applied to *Y*-plates of the CRO while the generated sawtooth signal waveform is applied to its *X*-plates. The beam under this condition encounters force in horizontal direction as well as in vertical direction.
8. Various parameters of the obtained waveform can be determined using different measurement techniques such as frequency, phase angle, and time delay.
9. There are some oscilloscopes which are fed with more than one input. These oscilloscopes are categorized as multiple-trace oscilloscopes and multiple-beam oscilloscopes.
10. In place of conventional oscilloscopes, some other oscilloscopes have been developed and are termed as special oscilloscopes. These oscilloscopes include sampling oscilloscopes and storage oscilloscopes. Storage oscilloscopes are then further categorized as analog storage oscilloscopes and digital storage oscilloscopes.
11. A very complex figure may be obtained for different frequency waveforms known as Lissajous figures or Lissajous patterns.

EXERCISES

Fill in the Blanks

1. A sampling CRO is used for _________ signals.
2. The alternate mode is not suitable for _________ frequency signals.
3. A DSO is operated in baby-sitting mode to deal with _________.
4. If the vertical and horizontal inputs are 90° out of phase, but possess equal frequency, then the Lissajous figure is _________.
5. The attenuator probes provide larger _________ and reduce the _________ on the circuit under test.

Multiple Choice Questions

1. When a 50 Hz sinusoidal signal is given to the *Y*-plates and the time-base generator is turned off in the *X*-plates of a CRO, the trace on the screen is a
   (a) horizontal line (b) vertical line
   (c) spot (d) sinusoidal waveform
2. In a CRT, the focusing anode is located
   (a) between pre-accelerating and accelerating anodes
   (b) after accelerating anode
   (c) before accelerating anode
   (d) none of these
3. The time base of a CRO is developed by a
   (a) square waveform (b) sawtooth waveform
   (c) sine waveform (d) none of these

4. An aquadag is used in a CRO to collect
   (a) primary electrons (b) secondary emission electrons
   (c) both of the above (d) none of these
5. The synchronization amplifier is used to
   (a) match the sweep rate of horizontal amplifier to the frequency of vertical input signal
   (b) adjust the intensity of the signal
   (c) control the amplifier gain
   (d) commence the horizontal sweep signal at the same point on the vertical signal

State True or False

1. The vertical amplifier of a CRO can be designed to provide a constant gain-bandwidth product.
2. In horizontal deflection, during retrace time, the electron beam moves from left to right on the screen.
3. The time taken by the waveform to rise from 10% to 90% of its maximum amplitude is known as its fall time.
4. The secondary emission electrons are collected by the aquadag at the CRT neck.
5. The sampling of the signal must be done at Nyquist rate in order to avoid aliasing.

Descriptive/Numerical Questions

1. Explain the basic block diagram of an oscilloscope. Explain the function of each block.
2. Explain the function of a time-base generator in a CRO.
3. Explain the functions of the internal structure of a cathode ray tube with a neat diagram.
4. Explain the working principle of electrostatic deflection system in a CRT.
5. How do we measure phase and frequency of an AC quantity with the help of a CRO?
6. Sketch the basic block diagram of a delayed time-base (DTB) system. Explain the system operation.
7. Explain multi-beam and multi-trace oscilloscopes in detail.
8. What are the differences between dual beam and dual trace CROs?
9. For the waveforms shown in Figure 9.14, determine their frequencies and time delay between them.
10. Describe the following modes of operation available in a dual trace oscilloscope:
    (a) Alternate mode (b) Chop mode
11. If the input capacitance of the oscilloscope is given to be 40 pF while the capacitance of the co-axial cable is given as 90 pF, determine the following to compensate a 10:1 probe:
    (a) The value of capacitance C_1
    (b) Input capacitance of the probe
12. With the help of a neat diagram, explain the working of a sampling oscilloscope.
13. Explain the working of a digital storage oscilloscope.
14. What are the merits and demerits of a digital storage oscilloscope?
15. Define active and passive CRO probes.
16. What are Lissajous patterns or figures? Explain how frequency and phase can be measured using these figures.

10

Display Devices and Recorders

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Describe various display devices such as decade counting assembly, light emitting diode, liquid crystal display, nixie tube, segmental gas discharge display, and dot matrix display
- Appreciate the concept of recorders including *X-Y* recorders, magnetic tape recorders
- Draw with the use of plotlers
- Explain the DC and AC signal conditioning in measurement systems
- Describe the data acquisition system

10.1 INTRODUCTION

Measurement systems, as studied in Chapter 1, have their final stage as the data presentation. This stage is responsible for displaying the result of the measurements and thus it consists of display devices and recorders. The measurement results are displayed in an understandable form and can also be recorded for later observation. Thus, we may say that a time-varying input waveform is instantly displayed by the display devices while it is stored and then displayed at some other time by the recorders. The recorders retain the information or result even after the input signal has ceased to exist.

There are some factors on which the choice of data presentation stage depends. These include the information content of the output, that is, whether it is single-valued or it is a function of time, and the expected use of the output, that is, whether it is to be analyzed instantly or at a later time or it is to be applied as an input to some other system. Depending on these two factors, the choice is made between a display device and a recorder.

In this chapter, we will study several display devices and recorders which include *light emitting diode* (*LED*), *liquid crystal display* (*LCD*), *seven segment display*, *dot matrix display*, *X-Y recorders*, *plotters*, *nixie tube*, and *magnetic tape recorders*.

10.2 DISPLAY DEVICES

As stated earlier, the result of computation of a measurement system or other electronic systems should be displayed on a display device. These digital displays can be broadly classified as planar and non-planar, based on whether they display the character on the same plane or different planes. The planar displays include *segmental displays* like *light emitting diodes* and *liquid crystal displays*, *dot matrix displays*, and *segmented gas discharge displays* whereas the non-planar displays include *nixie tubes* and *gas discharge tubes*.

10.2.1 Decade Counting Assembly

A decade counting assembly (abbreviated as DCA) is a numeric display unit, capable of displaying all numerals from 0 to 9. The DCAs and other display devices require triggering units to calculate or display the count. In order to display numbers from 0 to 9, ten characters are required by a typical decimal indicating system. Therefore, for each DCA, a total of ten trigger circuits are required. The first trigger circuit triggers the circuit of one to initiate the counting, thereby displaying one. The second trigger circuit then triggers the circuit of two in order to display a two. This circuit also feeds a pulse back to the circuit one to turn it off. All successive trigger circuits trigger their respective circuits in the same manner for counting an event. The DCA unit displays a zero after displaying a nine. Usually, a single DCA is not used in practice; therefore, a number of these assemblies are connected together for this purpose. An arrangement of five decade counting assemblies capable of counting up to 99999 is shown in Figure 10.1.

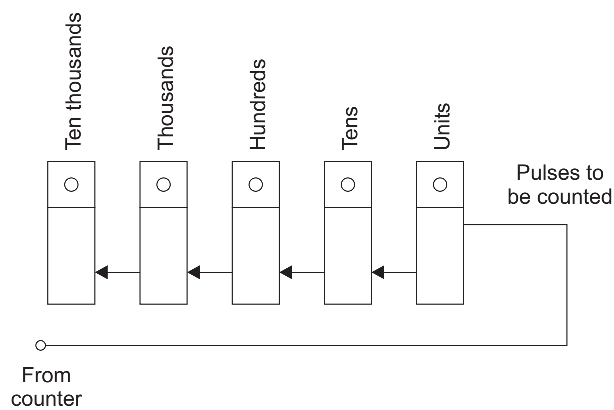


Fig. 10.1 An Arrangement of Decade Counting Assemblies

It can be seen from the figure that five DCAs are cascaded and a number of pulses are fed to them by a counter. These input pulses are to be counted by the arrangement. When a count up to nine is reached by the first DCA unit, it counts a zero for the tenth count and activates the next DCA unit, that is, the one placed at tens position. This causes this DCA to start counting from one to nine. On the tenth count, it also counts a zero and activates the next, that is, the DCA placed at hundreds position. This process continues with successive DCAs also. The arrangement shown in the above figure provides a clear readout.

10.2.2 Light Emitting Diode Display

A light emitting diode, abbreviated as LED, is basically a p-n junction semiconductor device which provides light output when fed with an electric current in its forward biased configuration. On applying an operating voltage, the electron-hole recombination takes place. The energy released due to the recombination process consequently forms photons. This process is known as **electroluminescence**.

A seven segment display can be made using LEDs. The basic configuration of the display is depicted in Figure 10.2 in which the positive terminals of each device are connected to each other and fed with a positive voltage supply while the negative terminal of each device is named as *a*, *b*, *c*, *d*, *e*, *f*, and *g*.

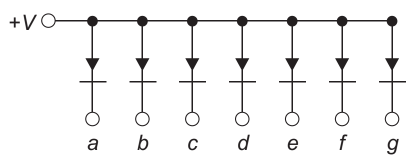


Fig. 10.2 Basic Configuration of an LED Seven Segment Display

This configuration is referred to as **common anode connections**. On the other hand, if the supply voltage is connected at the negative terminals and the positive terminals are left open, the configuration is known as **common cathode connections**. A seven segment numeric display is shown in Figure 10.3 in which all the segments are marked as *a*, *b*, *c*, *d*, *e*, *f* and *g*.

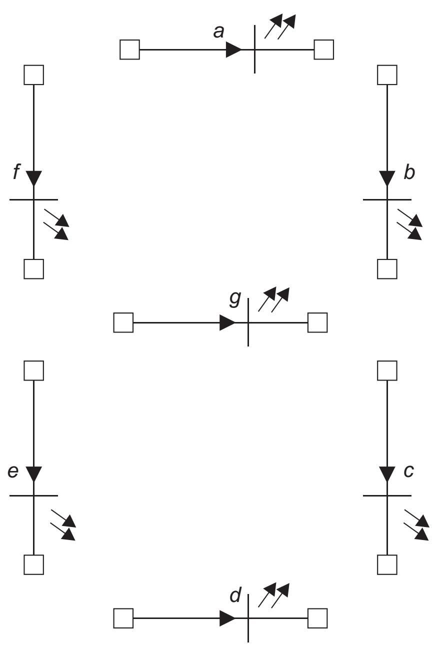


Fig. 10.3 Seven Segment LED Display

Any numeral (from 0 to 9) can be displayed if a current passes through the appropriate segments. The magnitude of the current must lie in the range of 10 mA to 20 mA for sufficient brightness with a typical voltage drop of 1.2 V. The major disadvantage of an LED display is its large current requirements while its long life, ruggedness, and the ability to be operated from a DC supply of low-voltage can be counted as its advantages.

Characteristics of LED

The various characteristics of LEDs are discussed as follows.

- When a current flows in an LED, an output is generated in the form of light.
- LEDs have quick turn ON-OFF time of less than 1ns so they are fast devices.
- LEDs can be stacked together to form numeric and alphanumeric displays due to their small size.
- The semiconductor materials like silicon and germanium cannot be used to make LED as they release most of the energy in the form of heat. On the other hand, the materials such as gallium arsenide (GaAs) which gives red light, gallium arsenide phosphide (GaAsP) giving red or yellow light, or gallium phosphide (GaP) giving red or green light are used extensively in manufacturing LEDs. These materials emit photons having wavelength in the visible region.

10.2.3 Liquid Crystal Display

A liquid crystal display, abbreviated as LCD, works on the principle of a liquid crystal cell which either transmits or reflects the light and can be used for similar applications as those of an LED. The liquid crystal cells do not generate light of their own and thus, require an external source of light to produce an image. The energy consumption of these cells is very low since it is required to activate the liquid crystal. The LCDs can be categorized into two types, namely, *dynamic scattering type* and *field effect type*.

In **dynamic scattering type diodes**, the light is scattered in all directions due to molecular turbulence when the liquid is activated by applying potential across the cell. Figure 10.4 shows a dynamic scattering liquid crystal cell consisting of liquid crystal layered between glass sheets whose inside faces are deposited by transparent electrodes.

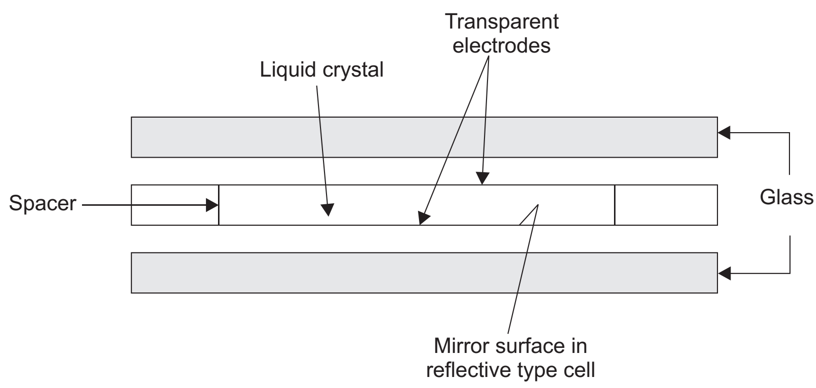


Fig. 10.4 Liquid Crystal Diode Cell

In **field effect type diodes**, the two thin polarizing optical filters are placed at the inside of each glass sheet, rest it is similar in construction to dynamic scattering liquid crystal. The twisted nematic type material is used for its manufacturing which can twist the light passing through the cell when it is not energized and the cell appears bright on light passing through optical filters. On the other hand, the cell appears dull when it is energized and the light does not twist.

The supply voltage of the LCDs should be an AC voltage such as sinusoidal or a square waveform since a direct supply would yield a direct current which may damage the device by producing plating of the cell electrodes. The typical peak-to-peak magnitude of this source voltage should lie between 3 V to 8 V with a frequency of 60 Hz. Similar to the common anode or common cathode connections of LEDs, one terminal of each liquid crystal cell is connected together and this common terminal is known as a **back plane**. The voltage is applied to this back plane anti-phase to the segments to be energized. This implies that twice the peak-to-peak square input voltage is applied to the segments and the back plane.

The use of LCDs is advantageous as they consume low power and are less expensive. On the contrary, they are disadvantageous as they are slow devices as compared to LED with large turn ON-OFF time and occupy larger area.

10.2.4 Nixie Tube

The nixie tube is a device based on the principle of gaseous discharge glow which states that when the gas breaks down, it leaves a glow of discharge. There is a set of electrodes that are stacked behind one another. These electrodes are formed in the shapes of different digits and work as cathodes. On the other hand, a gauze electrode works as an anode. When a digit is selected, it gets surrounded by a gaseous discharge (usually a neon gas) and thereby, glows. Usually, an orange-red glow is produced by the discharge; however for different gases, several other colour glows can be seen. In this way, the digits are displayed at the readout in various planes. The basic structure and schematic symbol of a nixie tube are depicted in Figure 10.5.

As can be seen from the figure, a gas-filled glass envelope with connecting pins at its bottom encases all these electrodes. A positive voltage is applied to the anode while a negative voltage is applied to a particularly selected cathode. This causes a simple gas discharge diode to be formed which glows the chosen digit. The switching between different digits is done with the aid of a transistor gate which is incorporated at each cathode. The circuitry required to drive a nixie tube is simple as compared to other display device circuitries. However, the gas discharge requires a relatively higher voltage, of the order of 150 V to 220 V. In addition, the current requirements of these tubes are of the order of 1 mA to 5 mA. The nixie tubes are larger in size in comparison to seven segment display devices.

10.2.5 Segmental Gas Discharge Display

Segmental gas discharge displays also work on the principle of gaseous discharge glow like the nixie tubes. These displays can incorporate seven or fourteen segments in order to display numeric and alphanumeric characters, respectively. The configuration is depicted in Figure 10.6 in which all the seven segments and decimal points are provided with their separate cathodes.

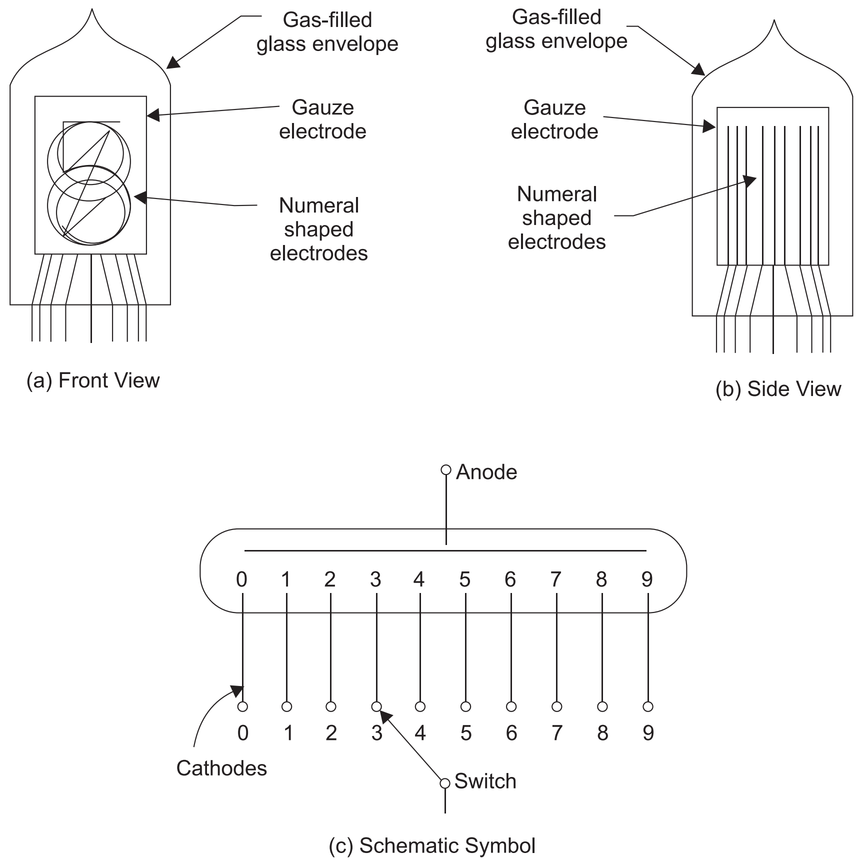


Fig. 10.5 Basic Structure of a Nixie Tube

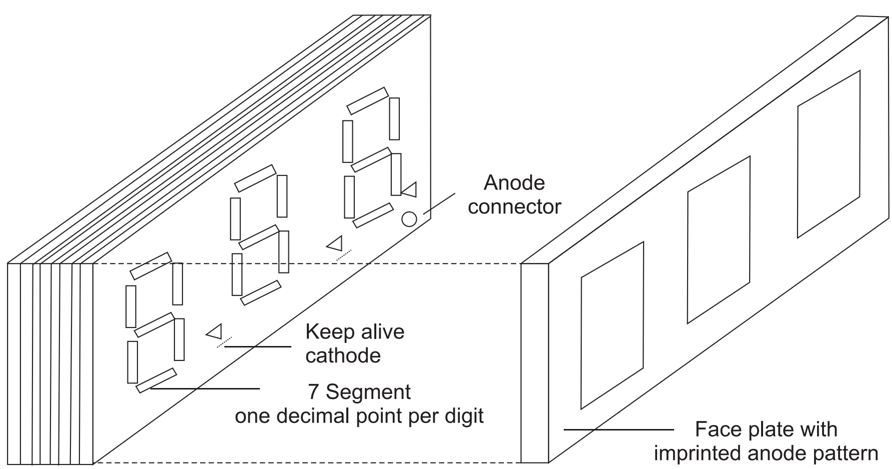


Fig. 10.6 Segmental Gas Discharge Display

On the covering face plate of the display, there is a common anode for each group of seven segments. In addition, each segment group is incorporated with a *keep alive cathode* to ensure fast switching. For this, a small constant current, which generates ions, is passed through these cathodes. The gap between the cathodes and anode is filled with the gas. A voltage of about 150 V to 220 V is required to drive such displays which can be counted as a disadvantage of the segmented gas discharge displays. The switching between various cathodes is done with the aid of high voltage transistors. The low power consumption of these displays is considered as their major advantage. In fact, a current, as low as 200 μA can produce a bright display. The construction of a segmented gas discharge display uses a glass substrate as shown in Figure 10.7.

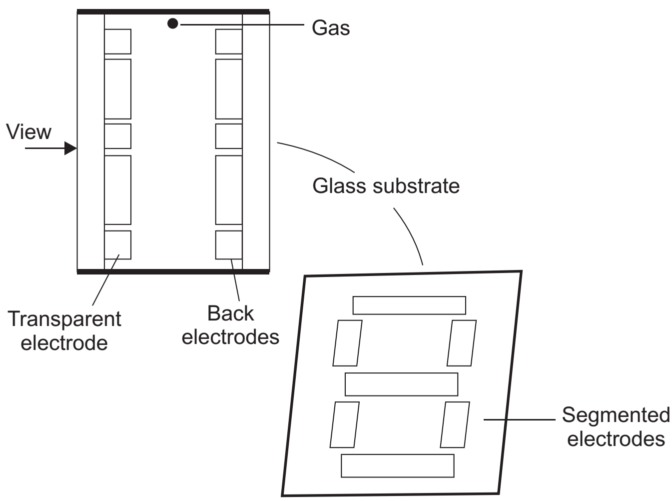


Fig. 10.7 Structure of a Segmental Gas Discharge Display

There are thick-film type and thin-film type electrodes known as **back** and **front electrodes**, respectively. The back electrodes serve as cathode segments, while the front electrodes serve as transparent anodes and a gas, usually neon, is filled between them. The gas gets struck between the selected segment cathode and anode in order to make that cathode glow. In this way, by activating the appropriate segments, any number or character can be displayed.

10.2.6 Dot Matrix Display

The dot matrix displays are used to display alphanumeric characters. A dot matrix made of LEDs which contains an LED at each dot location. The commonly used dot matrices for displaying the characters are 5 × 7, 5 × 8, and 7 × 9. The configuration of *X*-*Y* array connection with 5 × 7 dot matrix display is shown in Figure 10.8.

Any LED in the matrix can be lightened by applying a voltage at its anode and grounding its cathode. For example, if an LED circled in the figure is to be made to glow, it can be achieved by applying a voltage to the fourth row and third column of the matrix. A number of selected LEDs can be made to glow by applying voltage to the appropriate rows and columns simultaneously.

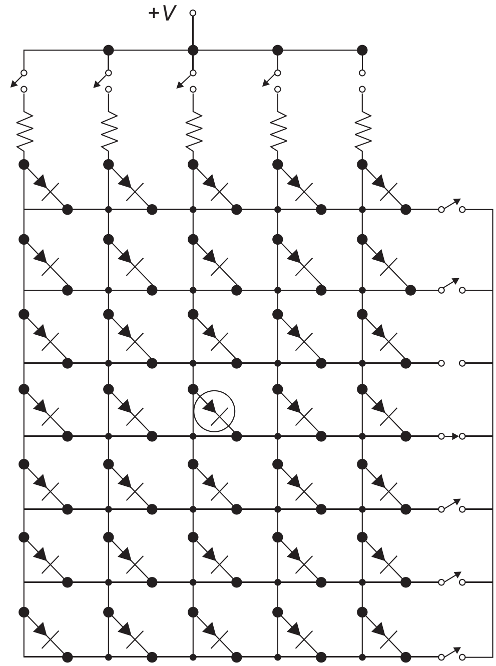


Fig. 10.8 A Dot Matrix Display in X-Y Array Connection

The *X*-*Y* array connection dot matrix display is economical and provides the advantage of being extended in either of vertical or horizontal directions using minimum number of wires. On the other hand, the common anode or common cathode connection of the display proves to be uneconomical.

10.3 RECORDERS

Sometimes, it is required to store the result of a measurement or other processes for future reference. This can be achieved using recorders. The recorders keep the recorded data in printed or written form which can be analyzed and compared at a later instance of time. This provides a better control as well as understanding of the process. The recorders can record both electrical and non-electrical quantities. The electrical quantities like current and voltage can be recorded directly while the non-electrical quantities can be recorded by first converting them into an electrical quantity using transducers or sensors. Let us discuss the *X*-*Y* recorders which possess significance among recorders.

10.3.1 *X-Y* Recorders

An *X*-*Y* recorder is an instrument which provides a graphical record of the relationship between two variables. For this, it uses an arrangement of a stationary sheet of paper (of about 250 × 180 mm) and a moving pen. The pen moves simultaneously in *X* and *Y* directions. These recorders plot the emf as a function of another emf with the aid of two self-balancing potentiometers. One of these potentiometers controls the position of the paper sheet or chart roll and another controls the position of the moving pen. This mechanism is illustrated in Figure 10.9 in which a servomotor and an idler pulley control the position of the pen.

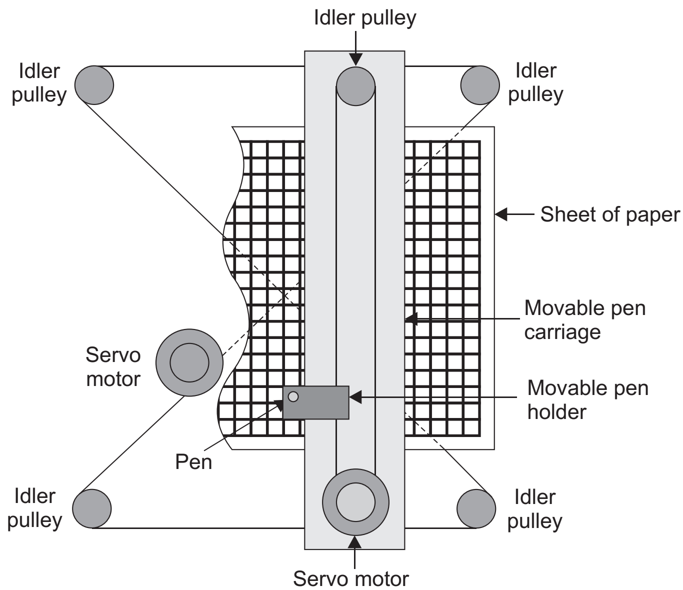


Fig. 10.9 Recording Mechanism of an X-Y Recorder

The figure reveals that the moving or recording pen can move along the pen carriage, which is also movable. This pen carriage as well as the drive system slide across the paper. This movement is controlled by another servomotor. A crossed drive cord is mounted on four idler pulleys and is used by the servomotor controlling the pen carriage position. It is to be noted here that the two self-balancing potentiometer circuits move the pen in right-angled directions, that is, one circuit moves the pen in *X*-direction while another moves it in *Y*-direction. The block diagram of a typical *X-Y* recorder is shown in Figure 10.10 which consists of a separate channel for both *X* and *Y* inputs.

It can be seen from the figure that each channel consists of an input attenuator, a balance circuit, and a reference source. When an input signal enters these channels, it is brought to an acceptable level by the input attenuator. The attenuation is done at the full scale range of the recorder. The signal is then passed through a balance circuit which basically compares the incoming signal with a reference source voltage. The output of this circuit, that is, the difference between these two signals is fed into a chopper circuit. The chopper circuit essentially converts the DC signal to its equivalent AC signal. This AC signal is then passed through an output amplifier which boosts the signal to an appropriate level so that it can drive the corresponding servomotor. The servomotors keep the system balanced even when the quantity under consideration changes. The *X-Y* recorder is used with a suitable transducer which may plot any physical quantity as a function of another physical quantity, such as force, displacement, strain, or pressure. The above procedure takes place simultaneously in both *X* and *Y* directions; hence, one physical quantity can be plotted as a function of another. The specifications of an *X-Y* recorder include:

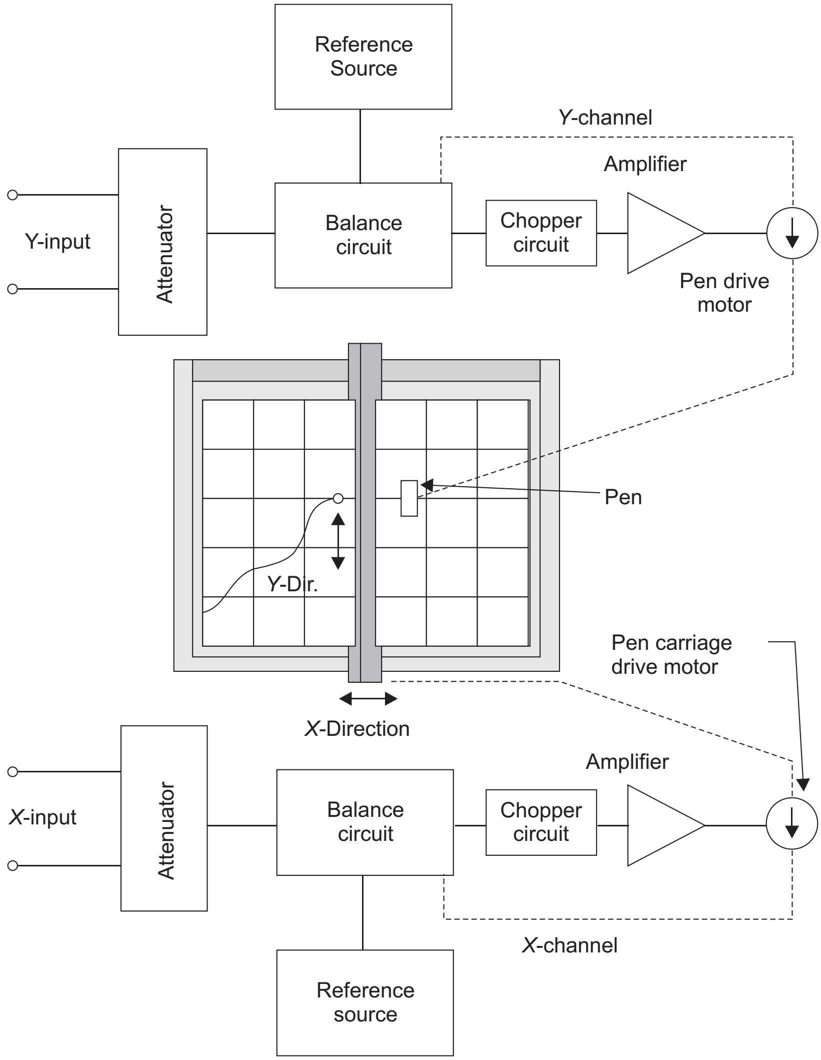


Fig. 10.10 Block Diagram of an *X-Y* Recorder

- Sensitivity = 10 μV/mm
- Frequency response = approximately 6 Hz for both axes
- Accuracy = ±0.3%
- Slewing rate = 1.5 m/s

The *X-Y* recorders find applications in plotting graphs between various physical quantities, such as:

- Characteristic curves of zener diodes, rectifiers, and transistors
- Stress–strain curve and hysteresis curve
- Resistance versus temperature curves of various materials

- Output characteristic graph of electronic calculators and computers
- Speed versus torque curves of various motors
- Regulation curves of power supplies

In addition, the monitoring of power demands or line voltage and recording of an incoming signal from a remote transmitter are accomplished by *X-Y* recorders. These recorders are also used in lie detector circuits, mechanical electrocardiograms, and IR and UV spectrophotometers.

10.3.2 Magnetic Tape Recorders

Magnetic tape recorders are widely used in applications where the recorded data or information is to be retrieved in electrical form again. The *X-Y* recorders studied in the previous section are low frequency recorders whereas the magnetic tape recorders are essentially used for high frequency applications since they possess response characteristics. This enables them to be used in instrumentation systems.

The major components of a magnetic tape recorder are *magnetic tape*, *recording heads*, *reproducing* or *playback heads*, and *tape transport mechanism*. In addition, some other components are required to provide signal conditioning and hence are collectively termed as **conditioning devices**. The tape transport mechanism of a magnetic tape recorder is shown in Figure 10.11.

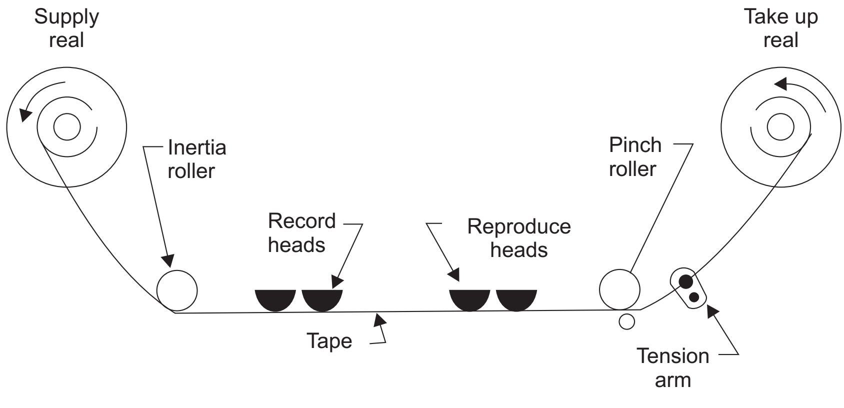


Fig. 10.11 Tape Transport Mechanism

The tape transport mechanism is responsible for moving the magnetic tape along the recording and reproducing heads at a steady speed. During various modes of operation, the tape should be protected against distortion, strain, and wear or tear by this mechanism. For this, there is an arrangement which guides the tape precisely for its directions past the recording and reproducing heads, brings an adequate portion of tape to the contact of heads, and maintains a sufficient tension on it. In addition, this mechanism also incorporates reversing and winding arrangements at a great speed. Some other components, such as pinch roller and inertia roller are used to drive the magnetic tape.

Conditioning devices include some filters and amplifiers which modify or condition the input signal so that it can be recorded onto the tape easily and efficiently.

The magnetic tape seen in the figure is basically a plastic ribbon coated with a layer of fine magnetic iron oxide particles. The typical breadth and thickness of the tape is around 12.7 mm and 25.4 µm, respectively. An electrical signal is fed into the recorder; the recording head produces a corresponding magnetic pattern on the magnetic tape. This pattern is retained by the tape as the magnetic iron oxide particles on the tape conform to it. The structure of a recording head is illustrated in Figure 10.12.

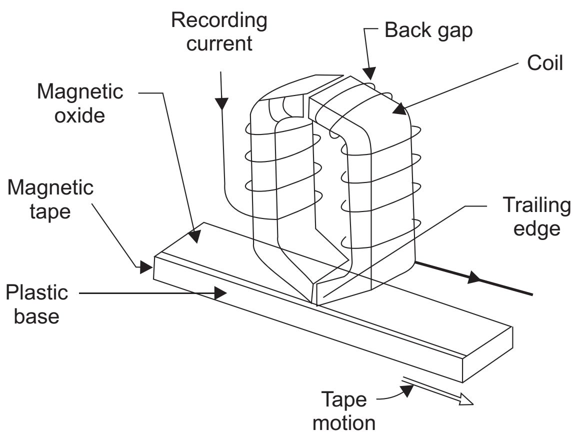


Fig. 10.12 Structure of a Recording Head

When a current passed through the coil, it produces a flux to bridge the air gap and thus, passes through the tape. This causes the iron oxide particles to magnetize when they pass through this air gap. These particles retain the magnetization even after passing through the gap. The recording of the signal starts at the trailing edge of the gap and emerges as a magnetic pattern along the tape as it passes through the gap. This pattern is corresponding to the incoming signal (current) with respect to time.

The reproducing head and recording head are similar in appearance. The reproducing head senses the recorded signal and converts it into its original form, that is, electrical form. The magnetic tape causes variations in the winding reluctance, and consequently, induces a voltage in the winding in accordance with the amplitude and direction of the magnetization on the tape. This voltage is in proportion to the rate of change of flux linkages which further implies that it is proportional to the rate of change of the level of magnetization on the tape. It can be mathematically expressed as:

$$e_{\text{Rep.head}} \; \alpha \; N \frac{d\phi}{dt} \qquad \text{...(1)}$$

where $e_{\text{Rep.head}}$ represents the induced voltage in the reproducing head, N represents the number of turns on the reproducing head winding, and $\frac{d\phi}{dt}$ represents the rate of change in the flux.

Now consider that the incoming signal is represented by $A \sin \omega t$, then we may write as:

$$\phi = K_1 A \sin \omega t \qquad ...(2)$$

where K_1 represents a constant. From Equation (2), we may say that the induced voltage is proportional to the recording head winding current and flux. Now, since the magnetic tape retains the pattern of the flux and generates it again at the output, the induced voltage at the reproducing head can be given as:

$$e_{\text{Rep. head}} = N \frac{d\phi}{dt} = K_1 N A \omega \cos \omega t = K_2 A \omega \cos \omega t \qquad ...(3)$$

From the above expressions, we may conclude that the reproducing head acts as a differentiator and it provides a derivative of the input signal at the output. This implies that the signal at the output is proportional to the flux on the tape as well as to the frequency of the input signal. It can also be concluded from Equation (3) that with every octave rise in the frequency of the input signal, the output signal gets doubled. Thus, the output of the reproducing head experiences a rise of 6 dB octave. To compensate this and to get a flat frequency response curve, the amplifier in the reproducing head circuitry must possess a complementary response of –6 dB. This process is referred to as **equalization** and is shown in Figure 10.13.

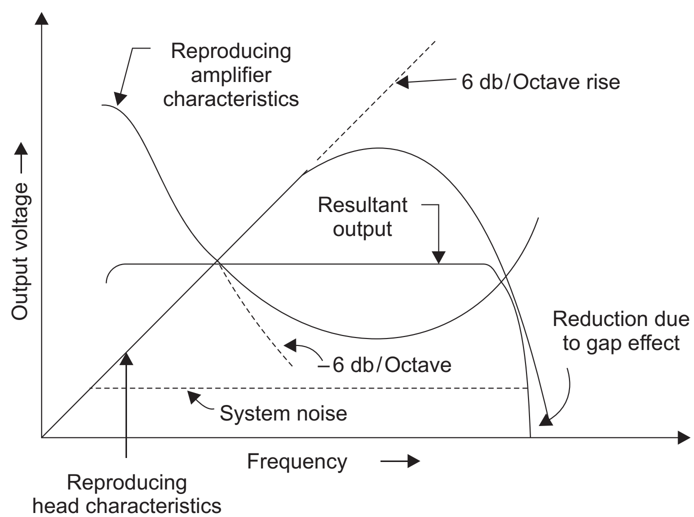


Fig. 10.13 Equalization Technique

There are several advantages of magnetic tape recorders over other recording techniques. Some of them are:

- They can handle large overloads and provide very low distortion.
- They provide automatic reduction of data since the signal is stored in its electrical form.
- They are operative for a wide range of frequencies from DC to more than 4 kHz.

- The dynamic range of operation is in excess of 50 dB, that is, from 100% to 0.3%.
- The signal which is to be recorded is available immediately for processing with no time lost. The recorded signal can be reproduced as many number of times as desired with no signal loss.
- On processing the information, the tape can be erased and a new data can be recorded.

Furthermore, the recording on the magnetic tape can be performed using these three methods:

- Direct recording
- Frequency modulation (FM) recording
- Pulse duration modulation (PDM) recording

Let us now discuss each of these methods in detail.

Direct recording

Direct recording method is the simplest method of recording. This method is employed on music and sound signals. The input signal is first amplified and then mixed with the high frequency bias signal before applying it to the recording head as a varying electric current. It is to be noted here that the amplitude and frequency of the bias signal are much greater than those of the incoming signal. The amplitude should be several times of the amplitude of the input signal while its frequency should be as much as three and a half times of the highest frequency component of the incoming signal. The biased signal is used which eradicates the non-linearity of the magnetizing curve (that is, *B-H* curve). Figure 10.14 depicts the direct recording method.

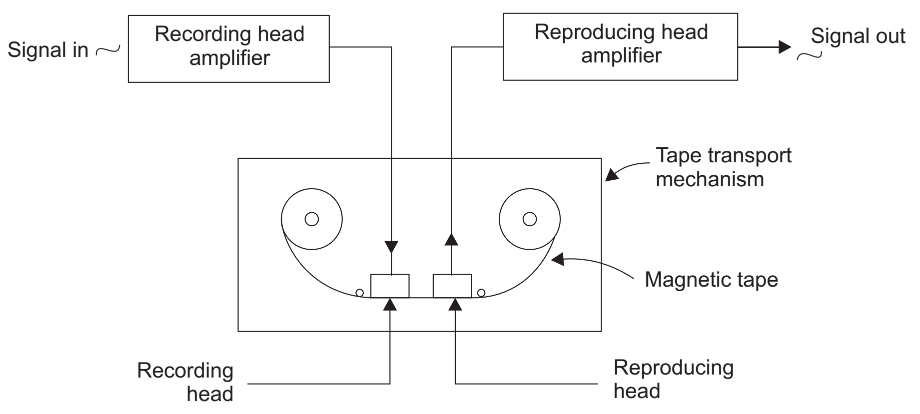


Fig. 10.14 Direct Recording Method

It is evident from the figure that the functioning of the above circuit arrangement is same as that of the tape transport mechanism discussed earlier. The equalization between the recording head amplifier and reproducing head amplifier holds good in direct recording method. The direct recording has a lower frequency range—of the order of 50 Hz which implies that the DC recording cannot be done by this method. Some random surface inhomogenities of the tape coating cause the reduction of the signal and amplitude instability. Such errors are termed as **dropouts**. When transient waveforms are to be recorded, the dropouts are unavoidable. Despite the above limitations, the direct recording method requires the least circuitry for encoding and decoding processes and provides the widest frequency spectrum and a wide dynamic range of operation. The direct method of recording finds application in recording voice commentary. This recording is used for identification and logging purposes and is done on one of the tracks of a multi-track recorder.

Frequency modulated recording

In frequency modulated (FM) method of recording, the signal frequency is varied instead of varying its amplitude. This method eliminates the basic limitations of the direct recording method. The configuration of a frequency modulation system is illustrated in Figure 10.15.

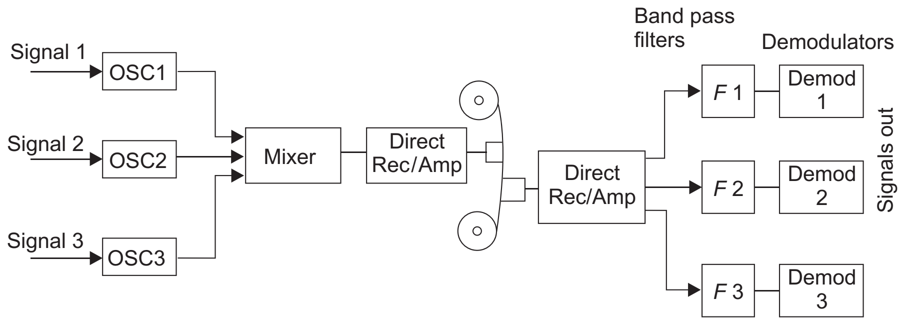


Fig. 10.15 Frequency Modulation System

In this method, a particular frequency is chosen as the centre frequency. When no signal is applied at the input, the modulation has this centre frequency. It can be seen from the figure that a number of signals are fed into the system simultaneously through different oscillators. These signals are modulated with the carrier signals provided by these oscillators. This technique is referred to as **frequency division multiplexing** (FDM). The frequency varies in one direction for a DC signal and in both directions for an AC signal. Furthermore, a positive signal deviates the signal in one direction while a negative signal deviates it in another direction. This variation of the frequency is in accordance with the amplitude of the input signal. The resultant signals are then linearly mixed by a mixer, after which this composite signal is processed for its appropriate amplitude by the amplifier/rectifier stage. The signal is then recorded on the magnetic tape via tape transport mechanism. The modulated signal is then fed to the magnetic tape, where during playback of the recording process, the reproduce head demodulates this signal and feeds it to a low-pass filter. This low-pass filter then removes the unwanted frequencies, the carrier frequency and some other redundant frequencies, which might get added to the signal during its modulation process. This implies that the linearity and wide bandwidth, provided by the direct recording method are utilized here in FM recording method to record a number of signals simultaneously on one track of the tape.

In addition, an increased overall accuracy and a wider dynamic range can be obtained using a *wide deviation frequency modulation technique*. This is because only a single channel of the signal is recorded on one track of the tape in this technique and thus, the entire bandwidth is dedicated to this signal. A 40% deviation of the signal frequency from its centre frequency is possible using this technique. The deviation in the carrier frequency is referred to as **percentage deviation** or **modulation index**, *M* which can be mathematically expressed as:

$$M = \frac{\Delta f}{f_c} \times 100 \qquad ...(4)$$

where f_c represents the carrier or centre frequency and Δf represents deviation of the carrier frequency from its centre frequency. Here, another term, **deviation ratio**, can also be defined as the ratio of carrier deviation to the frequency of the modulating signal. Thus, we may write:

$$\delta = \frac{\Delta f}{f_m} \qquad ...(5)$$

where f_m is the signal frequency input to the modulator.

A high deviation ratio system provides a low noise figure. The deviation in the carrier frequency is limited by the bandwidth of the recorder. However, the frequency of the modulating signal must be kept high so that all data signals can be processed.

It should be noted here that in FM recording method, the speed of the tape movement must be kept constant; otherwise noise may get introduced in the system due to unwanted modulation. This factor puts a limit on the dynamic range of accuracy of the system. Furthermore, the complexity of the circuitry, the constant speed requirement, and inefficient utilization of the magnetic tape may be counted as disadvantages of this method.

The FM recording is applicable where transient phenomena are to be recorded with adequate accuracy, DC and low frequency signals are to be recorded, and data reduction is required by means of large variations in time base circuit.

Pulse modulated recording

Pulse modulated method of recording uses the time-division multiplexing (TDM) technique to simultaneously record a large number of slowly changing variables. In this method, the duration of a pulse is modulated in accordance with the amplitude of the input signal. This requires a large number of channels to be multiplexed. A number of signals from various channels are sampled sequentially in this technique. When the data is sampled at uniformly spaced discrete time intervals, then the time elapsed between two samples can be used to sample another signal. The basic arrangement of a pulse duration modulated recording is depicted in Figure 10.16.

A sine wave input pulse is fed to the system. This sinusoidal input is first converted into an equivalent pulse which has its duration proportional to the amplitude of the sinusoidal wave. By passing these discontinuous readings through appropriate filters, the original signal can be reconstructed on the playback. This signal can be accurately reproduced by this system by using as low as six samples per cycle. The input frequencies of the signal are less than 1.5 Hz.

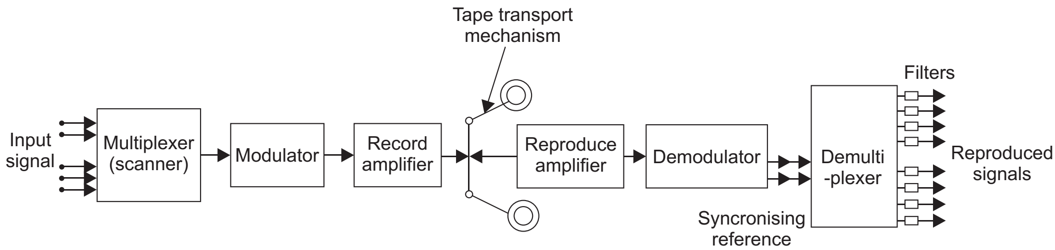


Fig. 10.16 A Pulse Duration Modulation Recording System

The PDM method of recording offers high accuracy and a high signal-to-noise ratio. However, it has a limited frequency response and its highly complex circuitry makes it less reliable.

10.4 PLOTTERS

A plotter is a device which can draw high quality images or drawings on a large piece of paper. Large scale maps, architecture of buildings, diagrams of big machines, and many other high density images are drawn using a plotter. It plots the images in a manner similar to the *X*-*Y* recorder with the exception that the sheet of paper is not stationary in plotters. Figure 10.17 shows a typical plotter.

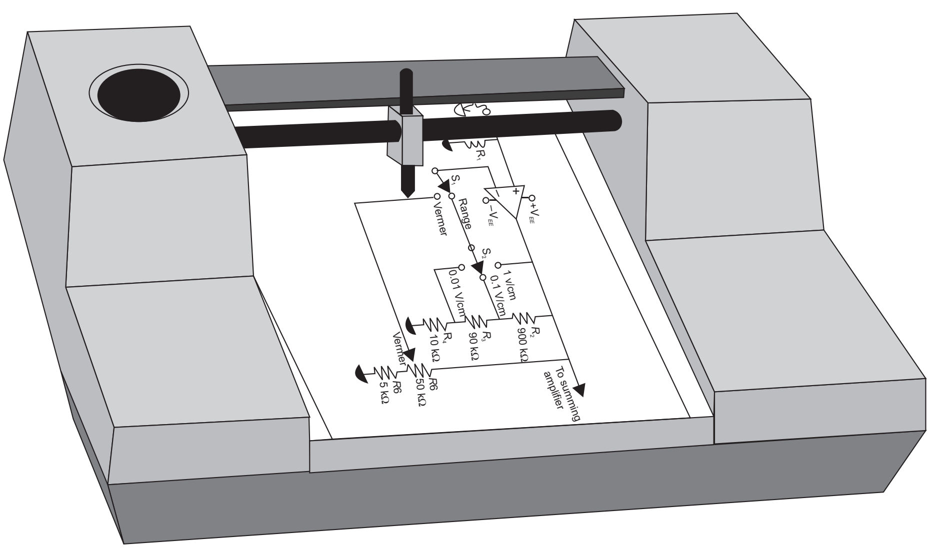


Fig. 10.17 A Typical Plotter

In the plotter shown above, the sheet of paper moves vertically fed by a roller back and forward while the pen moves horizontally. This plotter can perform simultaneous sampling on a number of channels and store these samples in its memory unit. This stored data can then be retrieved from the memory unit and plotted. In contrast, with the aid of analog-to-digital converters, plotting can be done in real time at a speed of 500 points/s. Furthermore, instead of using graph papers, plain sheet of papers can be used since user defined axes and grids can be made using plotters. The plotters can also plot date, time, and other data on the sheet.

Advantages and disadvantages

Some of the advantages of plotters are as follows.

- They offer a typical sensitivity of about 5 mV to 100 V with an accuracy of ±0.15% to ±0.3%. The accuracy depends upon the range selected by the plotter.
- Plotting can be done on large size papers.
- The quality of the drawing is the best as if drawn by an expert.

Some of the disadvantages of plotters are as follows.

- As each line is drawn separately, these devices are slower than printers.
- Text printouts cannot be produced with very high quality.
- They are expensive than printers.

10.5 SIGNAL CONDITIONING SYSTEMS

Signal conditioning is the intermediate stage in the measuring system (as studied in Chapter 1) that gives a faithful representation of dynamic physical quantities in an analog or digital form. This stage basically converts the output data from the first stage to the usable form for the next signal presentation stage. The signal conditioning systems are used to perform various linear and non-linear processes. The linear processes include attenuation, amplification, addition, subtraction, integration, and differentiation; while non-linear processes include filtering, sampling, modulation, demodulation, clipping, clamping, multiplying, and squaring. This stage provides excitation to passive transducers as they do not generate voltage or current on their own. It also provides amplification to both active and passive transducers. The sources that provide excitation may be a DC voltage source or an alternating source.

10.5.1 DC Signal Conditioning Systems

The DC signal conditioning system is shown in Figure 10.18 in which excitation source is DC. The DC amplifier is used due to several advantages as follows.

- It has good thermal and long-term stability.
- It can be calibrated easily at low frequencies.
- It can recover from an overload condition.
- Balanced differential inputs are taken as the input stage.

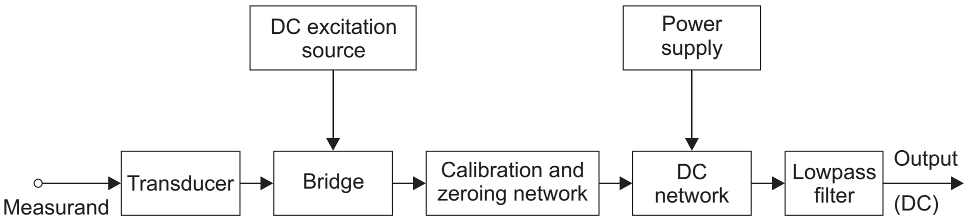


Fig. 10.18 DC Signal Conditioning System

The DC amplifier suffers from a problem of drift and the data available at the output of it is low frequency spurious signals. Hence, special low drift DC amplifiers are used to avoid the low drift. The low-pass filter follows the DC amplifier that eliminates the noise or high frequency components from the data signals. The utility of DC systems lies where common resistance transducers such as strain gauges and potentiometers are employed.

10.5.2 AC Signal Conditioning Systems

The AC signal conditioning system is shown in Figure 10.19 using carrier type AC signal. This system is used to overcome the problem of drift and spurious signals present in the DC systems.

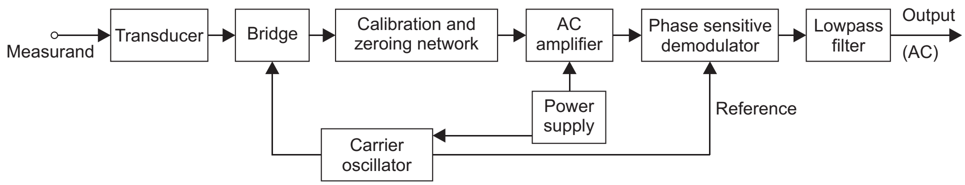


Fig. 10.19 AC Signal Conditioning System

The carrier frequencies employed in AC systems are much higher, that is, 5 to 10 times the signal frequencies lying between 50 kHz and 200 kHz. The variable resistance or inductance type transducers are used whose output is then applied to the bridge circuit giving an amplitude modulated carrier signal that is amplified by an AC amplifier. A phase sensitive demodulator receives this amplified modulated signal and gives a DC output indicating a direction of parameter change in the output of the bridge. It basically filters out the carrier frequency components of the data signal.

The difficulty in this system is in obtaining a stable carrier oscillator than a DC stabilized source. This unstable frequency is rejected by active filters and also the overloading of the AC amplifier is prevented by it. The utility of AC systems lies where the variable reactance transducers are employed and also where signals are transmitted via long cables connecting transducers to signal conditioning equipment.

10.6 DATA ACQUISITION SYSTEMS

After performing conditioning of the signal, it is further processed by electronic circuits. This additional processing is not required in some cases. However, some applications need this processing. Here also linear and non-linear operations are performed on the signal. The block diagram of a typical data acquisition and conversion system is shown in Figure 10.20.

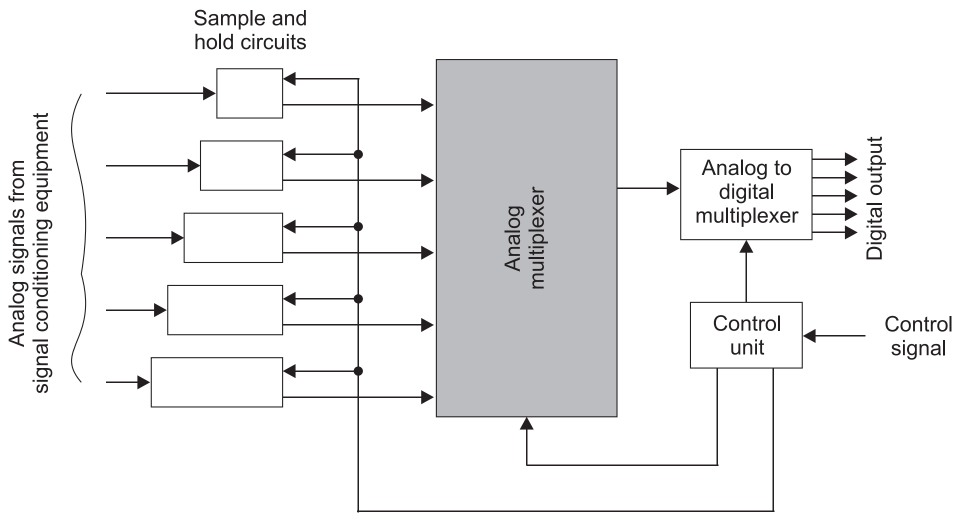


Fig. 10.20 Data Acquisition and Conversion System

The figure depicts a sample and hold circuit, an analog multiplexer, analog to digital converter, and a control unit. The sample and hold circuit, as the name implies, is used to sample different inputs at a specified time and holds the output voltage levels. The analog to digital multiplexer is used to perform the time division multiplexing (abbreviated as TDM) of the inputs. It is performed between different inputs applied to it. Out of many input channels, each channel is connected sequentially to the multiplexer on a time sharing basis. Thus, the input channels are not applied continuously to the multiplexer. The control unit is used to control signals to the analog multiplexer to control its timing for different inputs. It also controls the sample and hold circuit and analog to digital converter. The digital output so obtained can then be applied to any of the digital computer, digital data logger, digital controller, or a digital data transmitter.

Here, it is to be noted that instead of time division multiplexing, frequency division multiplexing (abbreviated as FDM) may be used. In this type of multiplexing, the multiple data analog inputs are not converted into digital form and transmitted at the same time. Also, the received voltage from the signal conditioning block is converted into frequency so that any change in input voltage (which is to be measured) results in a corresponding change in frequency.

CHARACTERISTICS OF A DATA ACQUISITION SYSTEM

The characteristics of a data acquisition system are described as follows.

- The necessary data must be acquired by this system at correct time and correct speed.
- All the data must be used efficiently and the operator must be informed about the state of the system.
- It must be reliable, flexible and capable of expanding for future requirements.
- The data must be collected and stored for diagnosis and recording purpose.

Let us Summarize

1. The choice between a display device and a recorder depends on the information content of the output, that is, whether it is single-valued or it is a function of time, and the expected use of the output, that is, whether it is to be analyzed instantly or at a later time or it is to be applied as an input to some other system.
2. The digital displays can be broadly classified as planar and non-planar, based on whether they display the character on the same plane or different planes.
3. The planar displays include segmental displays like light emitting diodes and liquid crystal displays, dot matrix displays, and segmented gas discharge displays, whereas the non-planar displays include nixie tubes and gas discharge tubes.
4. A decade counting assembly (abbreviated as DCA) is a numeric display unit, capable of displaying all numerals from 0 to 9.
5. A light emitting diode, abbreviated as LED, is basically a p-n junction semiconductor device which provides light output when fed with an electric current in its forward biased configuration.
6. A liquid crystal display, abbreviated as LCD, works on the principle of a liquid crystal cell which either transmits or reflects the light and can be used for the similar applications as those of an LED. They do not generate light of their own and thus, require an external source of light to produce an image.
7. The LCDs can be categorized into two types, namely, dynamic scattering type and field effect type.
8. The nixie tube is a device based on the principle of gaseous discharge glow which states that when the gas breaks down, it leaves a glow discharge.
9. The segmental gas discharge display also works on the principle of gaseous discharge glow like the nixie tubes. Such displays can incorporate seven or fourteen segments in order to display numeric and alphanumeric characters, respectively.
10. The dot matrix displays are used to display alphanumeric characters.
11. The recorders keep the recorded data in printed or written form which can be analyzed and compared at a later instant of time. They record both electrical and non-electrical quantities.
12. An *X*-*Y* recorder is an instrument which provides a graphical record of the relationship between two variables. For this, it uses an arrangement of a stationary sheet of paper (of about 250 × 180 mm) and a moving pen.
13. The magnetic tape recorders are widely used in applications where the recorded data or information is to be retrieved in electrical form again.
14. There are three methods to perform recording on the magnetic tape. They are direct recording, frequency modulated recording, and pulse duration modulation recording.
15. A plotter is a device which can draw high quality images or drawings on a large piece of paper.
16. Signal conditioning is the intermediate stage in the measuring system that gives a faithful representation of dynamic physical quantities in an analog or digital form.
17. The signal conditioning stage provides excitation to passive transducers as they do not generate voltage or current of their own. It also provides amplification to both active and passive transducers. The sources that provide excitation may be a DC voltage source or an alternating source.
18. After performing conditioning of the signal, it is further processed by electronic circuits in data acquisition systems.

EXERCISES

Fill in the Blanks

1. LEDs have _________ turn ON-OFF time.
2. LCDs are of _________ and _________ type.
3. The variable reactance transducers are employed in _________ systems.
4. _________ and _________ devices work on the principle of gaseous discharge glow.
5. Three types of recording techniques used in a magnetic tape recorder are _________, _________, and _________.

Multiple Choice Questions

1. The abbreviation of LCD is
   (a) liquid crystal diode (b) liquid crystal display
   (c) laser crystal diode (d) liquid carrier diode
2. The breadth and thickness of the tape used in a magnetic tape recorder are
   (a) 12.7 mm, 25.4 μm (b) 12.7 mm, 12.4 μm
   (c) 12.4 mm, 25.7 μm (d) 10.7 mm, 25.4 μm
3. In dynamic scattering type diodes, the light is scattered in
   (a) vertical direction (b) horizontal direction
   (c) all directions (d) none of these
4. Slewing rate of *X*-*Y* recorder is
   (a) 1.5 m/s (b) 3.5 m/s
   (c) 2.5 m/s (d) 4.5 m/s
5. Reproducing head in a magnetic tape recorder acts as
   (a) summer (b) integrator
   (c) differentiator (d) none of these

State True or False

1. Dot-matrix displays are used to display alphanumeric characters.
2. *X*-*Y* recorder uses a moving sheet of paper with a pen moving over it.
3. The sheet of paper used in plotters moves horizontally.
4. Magnetic tape recorders are used for low frequency applications.
5. DC signal conditioning systems are unable to recover from an overload condition.

Descriptive/Numerical Questions

1. Explain the principle of working of a magnetic tape recorder. What are its basic components and their functions?
2. What is data acquisition system? Give the block diagram arrangement of a data acquisition system describing the function of each component.
3. Briefly discuss LED and LCD as display devices in instrumentation. Comment on their relative merits and demerits.
4. Why is signal conditioning necessary in an instrumentation system?

5. What is *X-Y* recorder? Explain with a suitable circuit diagram the working of a *X-Y* recorder. Describe its applications.
6. Describe the basic components of a magnetic tape recorder for instrumentation applications using direct recording technique. Describe its advantages and disadvantages.
7. Explain the functions of a plotter.
8. Draw a neat figure and explain briefly each of the building blocks of digital display instrument. Write briefly about seven segment LED display decoder.
9. Describe the principle and working of nixie tubes.
10. Describe the construction and working of LCDs. Mention the difference between light scattering and field effect types of LCDs. Also explain the advantages of LCDs.
11. Discuss in detail various types of recorders.
12. Write two advantages of LED on electronic displays.
13. Draw and explain the block diagram of AC signal conditioning system.
14. Write short notes on:
    (a) Dot matrix display
    (b) Data acquisition system

11

Transducers

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Describe transducers and their advantages
- Explain how to select a transducer for an application
- Discuss different types of transducers in detail along with their working principle and types
- Determine different parameters using transducers such as temperature, pressure

11.1 INTRODUCTION

In Chapter 1 we have studied that a measurement system consists of an input device, a processing or signal conditioning device, and an output device. The quantity to be measured is applied to the input device and an electrical signal proportional to the input is delivered by it to the signal conditioning device. The signal conditioning device converts this signal to a form which can be accepted by the output device. Various operations such as filtering, amplification, or modulation are used for this purpose. The output device is then used for displaying or storing the input data. It may also manipulate the data.

The input quantity is generally non-electrical in nature and must be converted into an electrical quantity. This function is performed by a device known as **transducer**. Therefore, a transducer, also known as **pick up**, is defined as a device that converts one form of energy into another. This energy conversion may be electrical, optical, chemical, mechanical, or thermal. For example, a displacement or a mechanical force or any other physical parameter including intensity of light, liquid level, humidity, heat, flow rate, pH value can be converted into an electrical signal using transducers.

In this chapter, we will study the different types of transducers, the criteria on the basis of which they are classified, and the criteria on the basis of which a transducer is selected for a particular application. The interfacing of transducers in an instrumentation system is also explained.

11.2 ADVANTAGES OF TRANSDUCERS

Conversion of any non-electrical quantity into electrical quantity by a transducer is advantageous due to the reasons given below.

- The attenuation and amplification of the electrical signal can be done easily.
- The friction effects are minimized.
- A very small power level is required to control electronic or electrical systems.
- An electrical signal is easy to use, transmit and process for the purpose of measurement.
- The inertia effects in electrical or electronic signals are due to electrons which have negligible mass. Therefore, mass-inertia effects in such signals are minimized.
- The field of aerospace research and development based on telemetry and remote control, radio monitoring in space research, both require electrical and electronic means instead of mechanical means.
- The electronic devices and components are compact and have led to the miniaturization due to the use of integrated circuits (ICs).

11.3 CLASSIFICATION OF TRANSDUCERS

Transducers are of various types and are classified as *primary* and *secondary* transducers, *analog* and *digital* transducers, *active* and *passive* transducers, and *transducers* and *inverse* transducers. Moreover, transducers can also be classified on the basis of the principle of transduction such as resistive, capacitive, optical, inductive, thermal, and so on. They are *resistive*, *inductive*, *capacitive*, *piezoelectric*, *electromagnetic*, *thermoelectric*, and *photoelectric transducers*.

11.3.1 Primary and Secondary Transducers

Most of the measurement system includes two stages to convert an input signal into an output signal. The input signal is usually mechanical in nature and is detected by a mechanical device, acting as a **primary transducer**. The signal from this stage is then applied to an electrical device, acting as **secondary transducer**. Thus, we can say that a measurement system is a combination of primary and secondary transducers with an intermediate signal, known as **mechanical displacement**. Let us consider an example in which Bourdon tube is used as a primary transducer and a linear variable differential transformer (LVDT) is used as a secondary transducer. The pressure signal is applied to the Bourdon tube which senses it and converts it into displacement of its free end. The core of the LVDT is moved by the displacement of the free end and produces an output voltage proportional to the core movement which in turn is proportional to the free end displacement and hence, the pressure (see Figure 11.1).

11.3.2 Analog and Digital Transducers

The output of a transducer may be a continuous function of time or it may be discrete in nature. Therefore, we have two types of transducers based on the nature of output signal namely, **analog** and **digital** transducers.

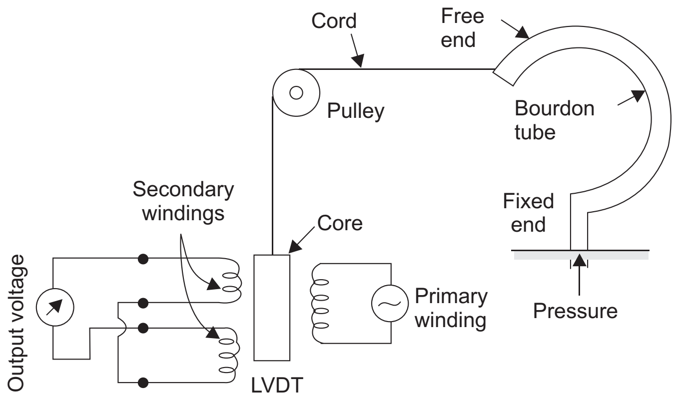


Fig. 11.1 Bourdon Tube and LVDT used to Measure Pressure

The transducers which convert the applied quantity into analog output, that is, a continuous function of time, are known as **analog transducers**. For example, thermocouples or thermistors, LVDT, and strain gauge.

The transducers which convert the applied quantity into pulse output, that is, a discrete function of time, are known as **digital transducers**. For example, shaft encoders, Hall-effect sensors, limit switches, digital resolvers, and digital tachometers.

11.3.3 Active and Passive Transducers

Active transducers are also known as **self-generating type transducers** as they develop their own current or voltage output. These transducers do not require any external power source to produce an output. The physical quantity to be measured provides energy to produce an output signal. The different types of active transducers are *thermocouples*, *piezoelectric crystal*, *photovoltaic cells*, and *tachogenerators*. The physical quantities which can be measured by these transducers are *force*, *temperature*, *light intensity*, and *velocity*. For example, an accelerometer which is used to convert acceleration into electrical voltage does not require any external power source and hence, is an active transducer. In this, a piezoelectric crystal is used which is sandwiched between two metallic electrodes fastened to a base as shown in Figure 11.2. The mass is placed on the top of the sandwich which is fixed. When a certain

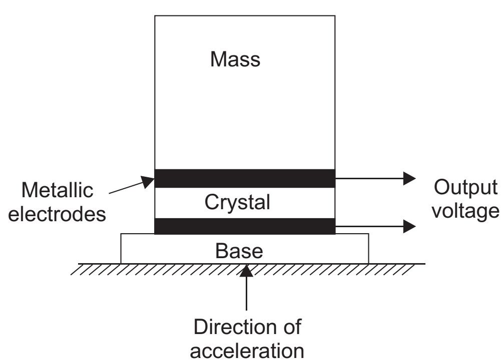


Fig. 11.2 Accelerometer as an Active Transducer

amount of force is applied to the crystal by a mass due to acceleration on the crystal from the base, the output voltage is generated. This output voltage is proportional to the force and hence, the acceleration, when the mass is fixed.

The passive transducers are also known as **externally powered transducers**. In addition to extracting power from the physical quantity being measured, they also require an external power source for transduction. The different types of passive transducers are *capacitive transducers*, *resistive transducers*, and *inductive transducers*. Linear potentiometer is an example of a passive transducer which is used for measuring displacement. It requires an external voltage source for its working. It is a resistive transducer in which a source voltage V_i is applied to measure an input linear displacement x_i as shown in Figure 11.3. The output voltage V_o is mathematically expressed as:

$$V_o = \frac{x_i}{L} V_i$$

Therefore, the linear displacement x_i is given as:

$$x_i = \left(\frac{V_o}{V_i}\right) L$$

where L is the total length of the potentiometer with a total resistance R_t .

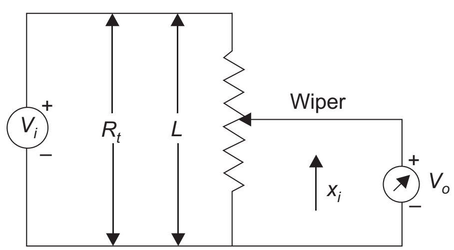


Fig. 11.3 Linear Potentiometer as a Passive Transducer

11.3.4 Transducers and Inverse Transducers

As already discussed, a device which converts a non-electrical quantity into an electrical quantity is known as **transducer**, whereas an **inverse transducer** is the opposite of the transducer which converts an electrical quantity into a non-electrical quantity. For example, when a voltage is applied across the surface of piezoelectric crystal, its dimensions change that cause mechanical displacement. Therefore, this crystal is an inverse transducer. A current-carrying coil moving in a magnetic field, some recording and data indicating devices, oscilloscopes, pen recorders are all examples of inverse transducers. Inverse transducers find application in feedback measuring systems.

The most commonly used feedback measuring system employs the concept of inverse transducer. Figure 11.4 shows the feedback control system that controls the electrical quantities. The output quantity which is electrical in nature is converted into the non-electrical form to compare with the quantity to be measured (that is, non-electrical form) in a measuring system. The resultant error signal generated is transduced into electrical form and amplified to give indication of the output.

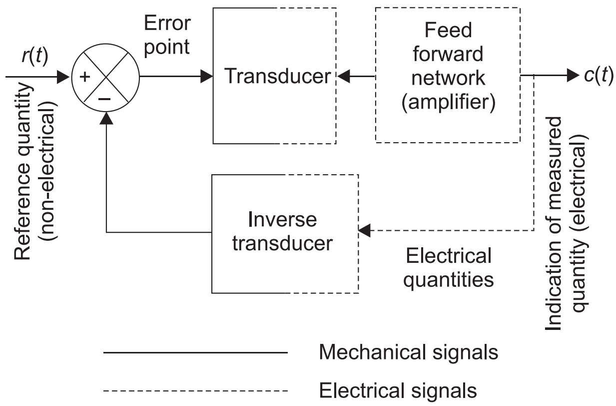


Fig. 11.4 Feedback Control System

A relatively high power level is handled by the control systems through the actuator that determines the output quantity directly. The accuracy of control is largely determined by the transducer and noise at input of the amplifier. The figure above includes inverse transducer that determines the characteristics of the system.

11.4 SELECTION OF A TRANSDUCER

To obtain accurate results, a transducer must be selected appropriately. It must be physically compatible for the required application. Various points regarding the quantity to be measured, principle of the transducer, and required accuracy in the system must be kept in mind while choosing a transducer for a particular application. The points which need to be considered are described as follows.

- **Operating range:** It must be chosen properly to maintain good resolution in the results and also to keep the range within its requirement.
- **Sensitivity:** The sensitivity of the system must be appropriate so as to obtain sufficient output.
- **Minimum sensitivity:** The transducer must respond to the parameter which needs to be measured. It must not respond to any other parameter.
- **Accuracy:** The errors occurring due to repeatability and improper calibration must be small. Also, the errors occurring due to response of transducers to extra quantities must be small. Hysteresis effects must also be considered.
- **Resonant frequency and frequency response:** The response of the transducer must be flat over the entire frequency range.
- **Usage and ruggedness:** The transducer must be rugged in terms of both mechanical and electrical intensities.

- **Environmental capabilities:** It must be resistant to various environmental parameters, such as, pressure, shocks, vibrations, corrosive fluids, and temperature. Also, it must meet the required size and mounting restrictions.
- **Electrical parameters:** Various electrical parameters that need to be kept in mind include signal to noise ratio while using transducers with amplifiers, frequency response of the transducers, and type and length of the required cable.

However, if some error occurs, the following techniques can be used to bring the data in the required accuracy range.

- To reduce the amount of errors, the environment can be controlled artificially. It can be achieved by either physically moving the system or by providing isolation. Heat enclosures and, vibration isolators may be used to provide isolation from the environment. Adopting these methods results in zero error due to environmental factors.
- Environment can be monitored simultaneously for the measurement and thus, the recorded data can be corrected consequently. This technique provides a significant increase in the accuracy of the system.
- Inplace system calibration can be used and accordingly corrections can be done in the data. This is done for errors which are predictable and can be calibrated out of the system. This calibrated data is used to correct the recorded data after calibration of the whole system.

11.5 RESISTIVE TRANSDUCERS

A transducer which works on the concept of change in resistance due to any phenomenon is known as **resistive transducer**. We know that the resistance R of a conductor is given by the relation:

$$R = \rho \frac{l}{A}$$

where ρ is the resistivity, l is the length and A is the cross-sectional area of the conductor. Thus, to form the basis of resistive transducers, any of the above quantities can be varied thereby causing change in resistance. For example, when a metal or semiconductor is strained, its resistance changes which can then be recorded to measure force, pressure, and displacement. This is the basic principle of strain gauges. As resistance measurements can be performed for both DC as well as AC currents, these types of transducers are preferable to the other ones.

11.5.1 Strain Gauges

Strain gauge is a thin, wafer-like device used for measuring the applied strain. It is basically a passive transducer which converts mechanical displacement into change of resistance. It can be bonded or attached to a variety of materials for strain measurement. The metallic strain gauges are made from small diameter resistance wire like constantan, or are etched from thin foil sheets. This device is used for measuring strain and stress in an experiment of stress analysis. When a material to which gauge is attached undergoes compression or tension, the resistance of metal foil or wire changes with length. This property of resistance change is known as **piezoresistive effect**. Such resistance strain gauges are known as **piezoresistive**

gauges. A Wheatstone bridge is used to measure this resistance change which is proportional to the applied strain. The strain gauges are used as secondary transducers in temperature sensors, accelerometers, flow meters, and diaphragm type pressure gauges.

The characteristic that defines the sensitivity of a strain gauge is known as **gauge factor** G_f. It is expressed as the unit change in resistance per unit change in length. Mathematically, it is given as:

$$G_f = \frac{\Delta R/R}{\Delta l/l} \qquad ...(1)$$

where ΔR is change in gauge resistance, R is the nominal gauge resistance, Δl is change in specimen length, l is normal specimen length in unstressed condition. The strain σ is equal to $\Delta l/l$ in lateral direction, then the Equation (1), becomes:

$$G_f = \frac{\Delta R/R}{\sigma} \qquad ...(2)$$

The resistance R of a conductor of uniform cross section is given as:

$$R = \rho \frac{\text{Length}}{\text{Area}} = \rho \frac{l}{(\pi/4)d^2} \qquad ...(3)$$

where l is the length of the conductor, d is the diameter of the conductor, and ρ is specific resistance of the conductor.

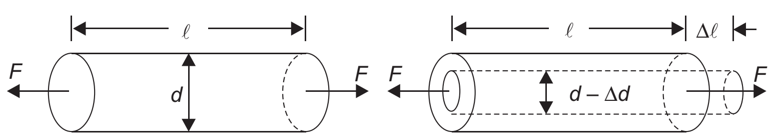


Fig. 11.5 Change in Dimensions of Strain Gauge under Tension

When a conductor of elastic material is subjected to tension, its length increases by Δl while the diameter decreases by Δd as shown in Figure 11.5. The resistance in Equation (3) changes to a value R_s which is given as:

$$R_s = \rho \frac{(l + \Delta l)}{(\pi/4)\,(d - \Delta d)^2} = \rho \frac{l(1 + \Delta l/l)}{(\pi/4)\,d^2(1 - 2\Delta d/d)} \qquad ...(4)$$

Equation (4) can be written using Poisson's ratio μ as:

$$R_s = \rho \frac{l}{(\pi/4)\,d^2} \frac{(1 + \Delta l/l)}{(1 - 2\mu\,\Delta l/\text{l})}$$

where $\mu = (\Delta d/d)/(\Delta l/l)$ is the ratio of strain in lateral direction to strain in the axial direction. The above equation can be simplified as:

$$R_s = R + \Delta R = R\left[1 + (1 + 2\mu)\frac{\Delta l}{l}\right]$$

The gauge factor G_f can now be given in terms of Poisson ratio μ as:

$$G_f = \frac{\Delta R/R}{\Delta l/l} = 1 + 2\mu \qquad ...(5)$$

The gauge factor for most metals is of the order of 1.5 to 1.7 when Poisson's ratio is in the range of 0.25 to 0.35.

Note: *The sensitivity of strain gauge is desired to be high which means a large resistance change that can be easily measured.*

Types of strain gauges

Strain gauges are used according to the type of applications that include experimental stress analysis of machines and structures, and construction of transducers based on torque, force, flow, pressure, and acceleration. For this, there are different types of strain gauges available that include, *unbonded metal strain gauges*, *bonded metal wire strain gauges*, *bonded metal foil strain gauges*, *evaporation deposited thin metal film strain gauges, bonded semiconductor strain gauges, and diffused metal strain gauges*.

Unbonded metal strain gauge

The unbonded metal strain gauge is made up of a wire stretched between two points in an insulating medium such as air. The wire is made of a metal alloy, such as chrome nickel, nickel iron or copper nickel alloys, about 0.003 mm in diameter, having gauge factor 2 to 4 and can sustain a force of 2 mN. The wire length is less than or equal to 25 mm. The pressure is sensed by the flexture element which is connected to a diaphragm via a rod. When a compressive force is applied to them, the wires are tensioned to avoid buckling (see Figure 11.6).

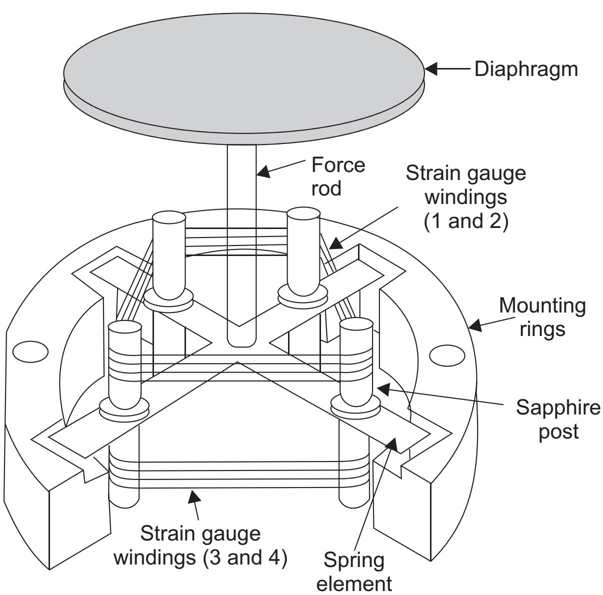


Fig. 11.6 Unbonded Metal Strain Gauge

This type of strain gauge has preloaded resistance wires connected in a Wheatstone bridge (see Figure 11.7). Initially, the resistances and strains of the four arms are equal due to which the output voltage V_o of the bridge is equal to zero. When a pressure is applied, a small displacement is produced which in turn increases tension in two wires and decreases in the other two wires. As a result, the resistance of the two wires under tension increases while the resistance of the remaining two wires decreases. Therefore, the bridge is unbalanced and an output voltage is produced proportional to the input displacement and the applied pressure. The full scale output of the bridge is about 20 mV to 50 mV with each arm resistance of 120 Ω to 1000 Ω and an input voltage applied to the bridge is 5 to 10 V.

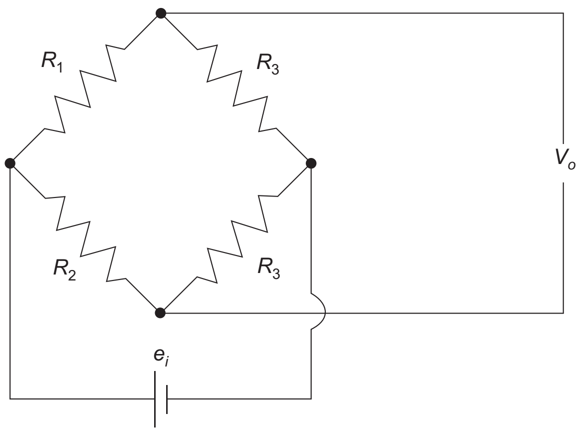


Fig. 11.7 Measurement by Wheatstone Bridge

Bonded metal wire strain gauge

A bonded metal wire strain gauge consists of a base on which a grid made of fine resistance wire is cemented. The base or carrier is made of thin sheet of bakelite, or paper, or teflon and the wire has a diameter less than or equal to 0.025 mm. An adhesive is used to bond the carrier with the specimen under study which permits a good transfer of strain from the carrier to the grid of wires. The wires are embedded in a matrix of cement so they cannot change their shape and follow tensile and compressive strains of the specimen. A thin sheet of material is used to cover the top of the wire to prevent it from any mechanical damage. The spreading of wire enables the stress to be uniformly distributed over the grid. The strain gauges normally used have large dimensions as 25 mm long and 12.5 mm wide. Some of the characteristics of resistance wire strain gauge for excellent results are as follows.

- A large change in resistance for a particular strain results in high sensitivity so a strain gauge should have high value of gauge factor.
- The high resistance of strain gauge minimizes the undesirable variations of resistance in the measurement circuit. The resistances lie in the range of 120 Ω, 350 Ω, and 1000 Ω.
- To achieve high sensitivity, high bridge voltage must be used. The maximum current-carrying capacity of the wires which is around 30 mA limits the bridge voltage.

- The strain gauge should have linear characteristics, that is, the variations in resistance must be a linear function of the strain. This maintains the constancy of calibration over the entire range of the strain gauge.
- The accuracy of measurements may get affected by temperature variations. Thus, it should have low resistance temperature coefficient to minimize these errors.
- It should not have any hysteresis effect in its response.
- It should have a good frequency response and linearity should be maintained within the accuracy limits over the entire frequency range as it is used for dynamic measurements.

The different configurations for bonded wire strain gauges are shown in Figure 11.8. Figures 11.8 (a), (b), and (c) show all single element strain gauges. Figure 11.8(d) represents a combination of strain gauges, known as **rosette**.

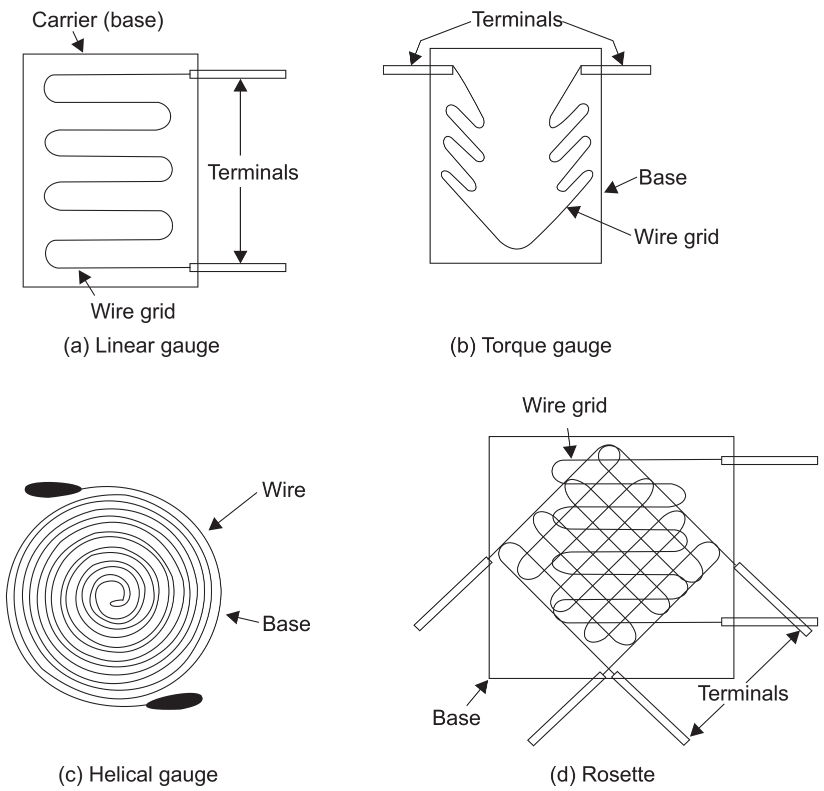


Fig. 11.8 Different Configurations of Bonded Metal Wire Gauge

Bonded metal foil strain gauge

The bonded metal foil strain gauge supersedes the bonded metal wire strain gauge. It is made up of the same material as bonded and unbonded wire strain gauges. The typical bonded metal foil gauge shown in Figure 11.9 is mounted on an insulating carrier film of about 0.025 mm thick made of polymide, or glass phenolic. The bonded foil gauges can be used for higher operating temperature range as they have much greater heat dissipation capacity as compared to wire wound strain gauges due to greater surface area for the same volume. Also, the larger surface area leads to better bonding. The heat dissipation capacity determines the gauge current to be 5 to 40 mA with gauge factor ranging from 2 to 4 and gauge resistances of 120, 350, and 1000 Ω.

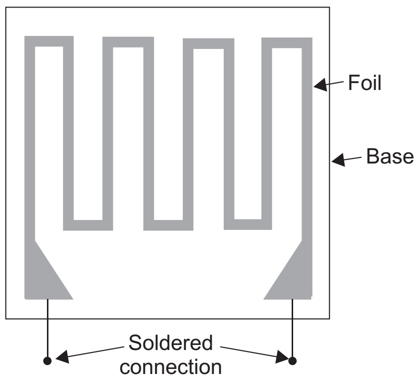


Fig. 11.9 Configuration of Bonded Metal Foil Gauge

The sensing element of foil gauges are made by photo-etching processes using sheets having thickness less than 0.005 mm to provide greater flexibility. These strain gauges are used for many transducers and stress analysis applications.

Evaporation deposited thin metal strain gauge

Evaporation deposited thin film metal strain gauge consists of a suitable elastic metal element which converts the physical quantity into strain. This evaporation process is mostly used for the fabrication of transducers. To form this type of strain gauge, a diaphragm with some insulating material is placed in a vacuum chamber and heat is applied to it till the insulating material vaporizes and then condenses to form a thin dielectric film on the diaphragm. A desired strain gauge pattern can be formed on top of the insulating substrate by placing suitably-shaped templates over the diaphragm, and evaporation and condensation processes are repeated. These thin film strain gauges offer time and temperature stability.

Semiconductor strain gauge

Semiconductor strain gauges are employed where there is a need of very high gauge factor. A higher change in resistance relates to higher gauge factor and hence a higher sensitivity. This gives a good degree of accuracy. In semiconductors, resistance changes with change in applied strain, whereas in case of metallic gauges, the resistance changes due to change in dimensions on applying strain.

This type of strain gauge is made up of some strain sensitive crystal material with leads sandwiched in a protective matrix as shown in Figure 11.10. The leads are generally made of gold, whereas silicon and germanium are used as resistive materials for such strain gauges. These are produced with semiconductor technology using semiconductor wafers or filaments of thickness 0.05 mm and bonding them on an insulating substrate of teflon.

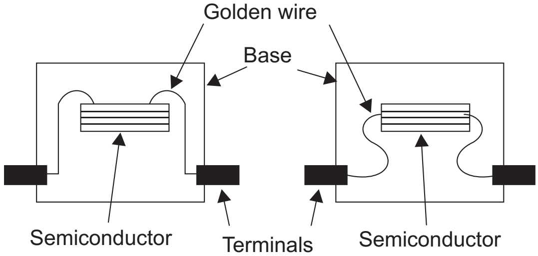


Fig. 11.10 Configuration of Semiconductor Strain Gauge

Some of the advantages of semiconductor strain gauge are as follows.

- They have high gauge factor of about ±130 which permits measurement of very small strains of the order of 0.01 microstrain.
- The frequency response is up to 10^{12} Hz and fatigue life is in excess of 10×10^6 operations.
- These gauges are useful for the measurement of local strains. They are very small with lengths ranging from 0.7 to 7 mm.
- These gauges maintain excellent hysteresis characteristics with less than 0.05%.

Some of the disadvantages of semiconductor strain gauge are as follows.

- They are more expensive and difficult to attach to the object under study.
- Such gauges are very sensitive to temperature variations.
- These gauges have poor linearity and require proper doping to have linear characteristics.

Diffused strain gauge

The diffusion process used in diffused strain gauges is the one which is used in IC manufacture. This results in low manufacturing costs and small size as a single silicon wafer can be used to construct many diaphragms. These types of strain gauges are commonly used in pressure transducers as sensing elements. Also errors due to creep and hysteresis do not occur in such strain gauges.

Example 1 A strain gauge with 30 cm wire length, 33 μm wire diameter and a resistance of 280 Ω is strained positively over the entire length. If the change in resistance observed is 0.7 Ω, calculate:

(a) change in wire length

(b) diameter

Assume the gauge factor to be 2.5.

Solution: Given that: $l = 30 \times 10^{-2}$ m, $d = 33 \times 10^{-6}$ m, $R = 280\ \Omega$, $\Delta R = 0.7\ \Omega$, and $G_f = 2.5$.

(a) From Equation (1), we have the gauge factor as:

$$G_f = \frac{\Delta R / R}{\Delta l / l}$$

Substituting the given values, we get:

$$\frac{\Delta l}{l} = \frac{0.7/280}{2.5} = 0.001$$

$$\Rightarrow \qquad \Delta l = l \times 0.001 = 30 \times 10^{-2} \times 0.001$$

$$\Rightarrow \qquad \Delta l = 0.0003 \text{ m} = 0.3 \text{ mm}$$

Now, we have the relation:

$$G_f = 1 + 2\mu \qquad \text{[Refer to Eqn.(5)]}$$

Substituting the given values, we get:

$$\mu = \frac{2.5 - 1}{2} = 0.75$$

(b) To determine the diameter of the wire, we have:

$$\mu = (\Delta d / d) / (\Delta l / l)$$

$$\Rightarrow \qquad \frac{\Delta d}{d} = \mu \frac{\Delta l}{l}$$

Substituting the given values, we get:

$$\frac{\Delta d}{d} = 0.75 \times 0.001 = 0.00075$$

$$\Rightarrow \qquad \Delta d = d \times 0.00075 = 33 \times 10^{-6} \times 0.00075 = 0.02475 \times 10^{-6}$$

$$\Rightarrow \qquad \Delta d = 0.02475 \times 10^{-6} \text{ m} = 0.024\ \mu\text{m}$$

11.6 THERMOELECTRIC TRANSDUCERS

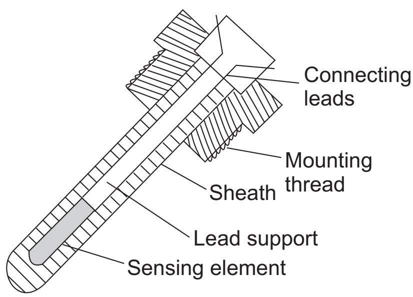
A voltage can be generated in a circuit by joining two dissimilar metals at the ends and keeping both the junctions at different temperatures. The voltage so generated is temperature dependent and also depends on the material of the metals used. This is known as **thermoelectric effect**. It forms the basis of thermocouples. Thermocouples are already discussed in Chapter 4 (refer to Section 4.3). There are two types of thermoelectric transducers, namely, *resistance thermometer* and *thermistor*.

11.6.1 Resistance Thermometer

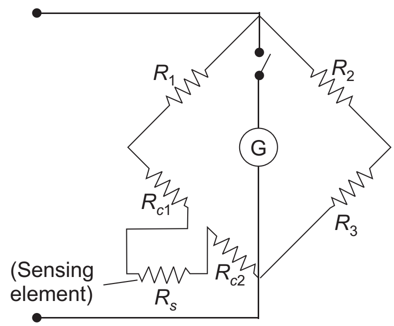
Resistance thermometer, also known as **resistance temperature detector** (abbreviated as RTD), uses a wire whose resistance depends upon temperature. Usually, a wire of pure copper, platinum, or nickel is used. The temperature dependence of resistance can be expressed as:

$$R_{t2} = R_{t1}(1 + \alpha\Delta t) \quad ...(6)$$

where R_{t2} represents the resistance at temperature t_2°C and R_{t1} is the resistance at reference temperature t_1°C which is generally 0°C. Δt represents the change in temperature and α is known as **temperature coefficient of resistance**. Based on the value of α, resistance of the materials may increase or decrease with an increase in temperature. The materials whose temperature coefficient is positive, the resistance tends to increase with an increase in temperature, such as pure metals. However, the materials whose temperature coefficient is negative, resistance tends to decrease with an increase in temperature, such as semiconductors.



(a) Basic construction



(b) Equivalent bridge circuit

Fig. 11.11 Resistance Thermometer

The basic construction of an industrial resistance thermometer along with its equivalent Wheatstone bridge circuit is shown in Figure 11.11. The sensing element, mostly pure platinum, is basically a coiled wire protected by a glass or stainless steel sheath. The wire is kept in place using ceramic or high temperature insulated enamel spacers. Spacers also help in avoiding short circuits. Connecting leads are also provided, having very less resistance as compared to sensing element resistance.

When the bridge is balanced, we have the relation:

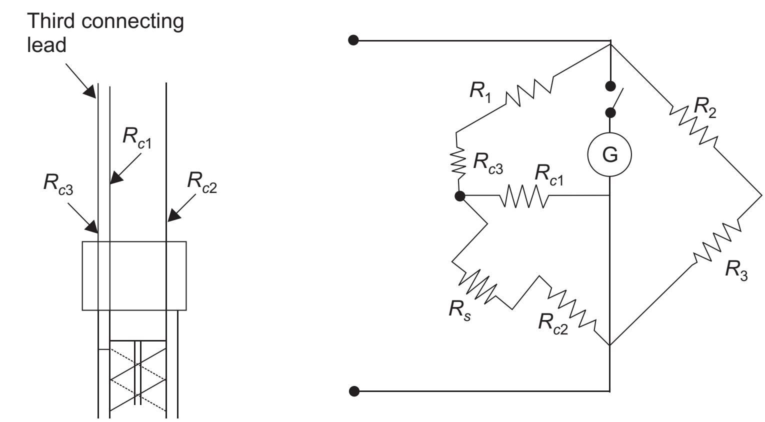
$$\frac{R_1}{R_2} = \frac{R_s}{R_3} \quad \text{(Refer to Section 7.2.1)} \quad ...(7)$$

However, the resistance of connecting leads (R_{c1} and R_{c2}) may get added to the resistance of sensing element as it is far away from the indicator. Thus, Equation (7) can be modified as:

$$\frac{R_1}{R_2} = \frac{R_s + R_{c1} + R_{c2}}{R_3} \qquad ...(8)$$

Now, whenever resistance R_s changes, the balance of the bridge is disturbed, resulting in a galvanometer deflection. The throw of galvanometer can be calibrated with a temperature scale.

The connecting lead resistances, being connected in series with the sensing resistance may introduce some errors. On changing the temperature, if any change occurs in these resistances, it gets incorporated into the measured value of the change in sensing resistance. Thus, to deal with this problem a three-wire connection arrangement is used [see Figure 11.12(a)]. In this arrangement, a connecting lead resistance R_{c3}, similar to other lead resistances is connected in the circuit. R_{c3} is connected serially with R_1 to balance the series combination of R_s and R_{c2}. Also, the resistance R_{c1} becomes part of the voltage measuring circuit and hence, cannot affect the bridge balance [see Figure 11.12(b)].



(a) Third connecting lead (b) Modified bridge circuit

Fig. 11.12 Error Correction using Three-wire Connection

Advantages and disadvantages

Some of the advantages of resistance thermometer are as follows.

- RTDs can be used for wide temperature range from –200°C to 650°C.
- They are extremely accurate transducers.
- There is no need of temperature compensation.
- They are highly tough instruments.

Some of the disadvantages of resistance thermometer are as follows.

- They are expensive.
- They require an external power source.
- A bridge circuit is required to measure the temperature.

Example 2 Calculate the resistance of a platinum resistance thermometer at 75°C, if its resistance at 20°C is given to be 180 Ω. The coefficient of temperature at 20°C is given to be 0.00392.

Solution: Given that: $t_1 = 20°C$, $t_2 = 75°C$, $R_{t1} = 180\ \Omega$, and $\alpha = 0.00392$

Change in temperature Δt is given as:

$$\Delta t = t_2 - t_1 = 75 - 20 = 55°C$$

Now, we have the relation:

$$R_{t2} = R_{t1}(1 + \alpha\Delta t) \qquad \text{[Refer to Eqn. (6)]}$$

Substituting the given values, we get:

$$R_{t2} = 180\,(1 + 0.00392 \times 55) = 218.808\ \ \Omega$$

11.6.2 Thermistor

Thermistors, also known as **thermal resistors**, are semiconductor devices acting as resistors having negative coefficient of temperature. They are manufactured by sintering (baking) various mixtures of metallic oxides such as manganese, cobalt, copper, uranium, iron, and nickel at high temperature. After sintering metallic film or lead contacts are deposited into the thermistors and are pressed into various shapes. Figure 11.13 shows the basic construction of a thermistor.

Here, it is to be noted that most thermistors are based on resistors having negative temperature coefficient, However, thermistors having positive temperature coefficient are also available. Also, the thermistors are available into many shapes including, beads, discs, and probes (see Figure 11.14).

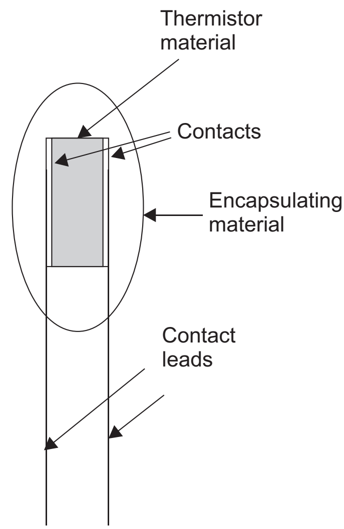


Fig. 11.13 Basic Construction of a Thermistor

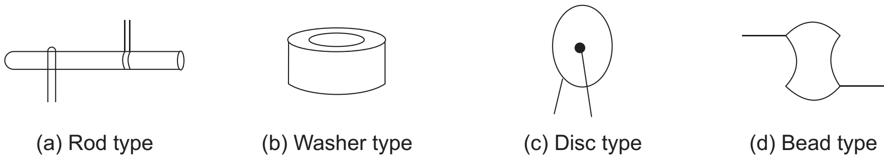


Fig. 11.14 Configurations of Thermistors

The temperature versus resistance graph of a thermistor is shown in Figure 11.15. It can be seen that the resistance decreases with increase in temperature (for negative temperature coefficient). In some cases, this decrease may be 5% for a 1°C increase in temperature implying high sensitivity. The relation between resistance and temperature of a thermistor can be expressed mathematically as:

$$R_{t1} = R_{t2} e^{\beta\left(\frac{1}{t_1} - \frac{1}{t_2}\right)} \quad \text{...(9)}$$

where R_{t1} and R_{t2} are the resistances of the thermistor at absolute temperatures t_1 and t_2 in °K, respectively and β is the material-dependent constant whose value lies in the range of 3500 to 4500°K.

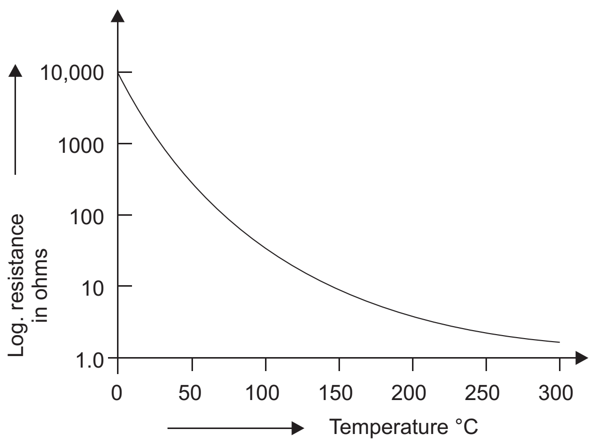


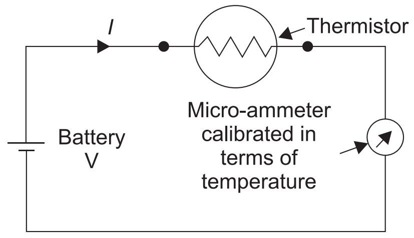
Fig. 11.15 Temperature vs Resistance Graph of a Thermistor

Note: *For applications which require greater power handling capability, parallel or series combinations of thermistors can be used.*

Measurement of temperature

A thermistor can be used to measure temperature by connecting it in a circuit as shown in Figure 11.16(a). It can be seen from the figure that the current flowing in the circuit depends on the resistance of the thermistor which further changes with temperature. So, whenever the temperature changes a change in circuit current is observed by the micro-ammeter. This micro-ammeter can be calibrated to indicate the temperature directly and is able to give a resolution of 0.1°C.

Now, consider the circuit shown in Figure 11.16(b) which represents another circuit to measure temperature using a thermistor. Here, a 4 kΩ thermistor is connected in a bridge circuit to give much higher sensitivity than the previous measurement circuit. Using this circuit, very small changes in temperature (of the magnitude 0.005°C) can be indicated. The thermistor with very high resistance of about 100 kΩ can be used. Therefore, this circuit provides very high sensitivity. A high resistance thermistor is used which is perfect for control and remote measurement.



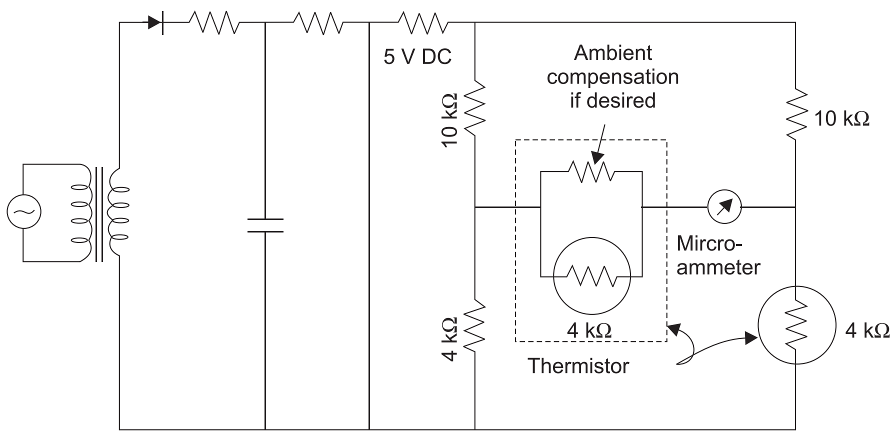


Fig. 11.16 Measurement of Temperature

Advantages and disadvantages

Some of the advantages of thermistors are as follows.

- They have high sensitivity which makes them suitable for precision temperature control, measurement, and compensation. They can also be used for measurement and control of liquid level.
- They have wide applications in lower range of temperatures (–100°C to 300°C).
- The problems due to contact and lead resistances do not occur.
- They are less expensive and smaller in size.

Some of the disadvantages of thermistor are as follows.

- Thermistors are not suitable for wide temperature ranges.
- The current must be limited to avoid self heating.

- High resistance requires filters and shielded power lines.
- The resistance vs temperature curve is not linear.

Example 3 Calculate the resistance of a 44002 A thermistor at 35°C if the resistance at 25°C is given to be 300 Ω. Assume β to be 3118 in the range of 0°C to 50°C.

Solution: Given that: $t_1 = 25°C$, $t_2 = 35°C$, $R_t = 300\ \Omega$, and $\beta = 3118$

Here, it is to be noted that the temperature must be in Kelvin. Thus, we have:

$$t_1 = 25°C + 273 = 298°K$$

$$t_2 = 35°C + 273 = 308°K$$

Now, we have the relation:

$$R_{t1} = R_{t2} e^{\beta\left(\frac{1}{t_1} - \frac{1}{t_2}\right)} \qquad \text{[Refer to Eqn. (9)]}$$

Substituting the given values, we get:

$$R_{t2} = \frac{300}{e^{3118\left(\frac{1}{298} - \frac{1}{308}\right)}}$$

$$\Rightarrow \qquad R_{t2} = 213.67\ \Omega$$

11.7 INDUCTIVE TRANSDUCERS

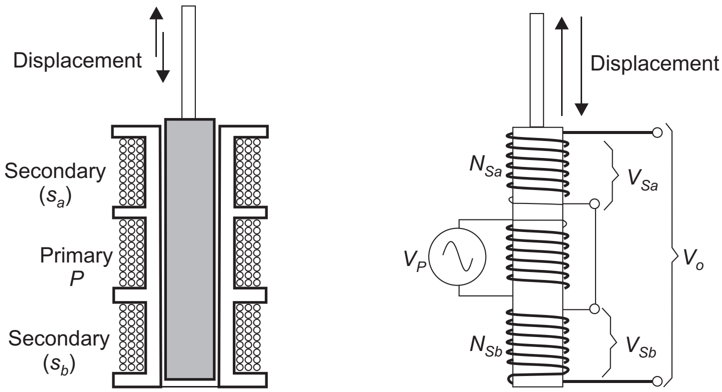
Inductance is a coil-related electrical circuit parameter. It is expressed mathematically as:

$$L = \mu_o \mu_r n^2 G$$

where μ_o is the permeability of free space ($4\pi \times 10^{-7}$), μ_r is the relative permeability, n is the number of turns in the coil, and G denotes the coil geometry. Thus, if any variable is changed other than μ_o (which is constant), the inductance of the coil changes. This change in induction forms the basis of inductive transducers. The various inductive transducers include *linear variable differential transformer*, *rotary variable differential transformer*, and *synchros*.

11.7.1 Linear Variable Differential Transformer

Linear variable differential transformer, abbreviated as LVDT, is the most widely used transducer for converting linear motion (displacement) into electrical signals. It basically consists of one primary and two secondary windings along with an iron core which can be adjusted. The secondary windings S_a and S_b are connected symmetrically on either side of the primary winding P [see Figure 11.17(a)]. The secondary windings, having equal number of turns, are connected in series. The primary winding is supplied with an alternating voltage source [see Figure 11.17(b)].

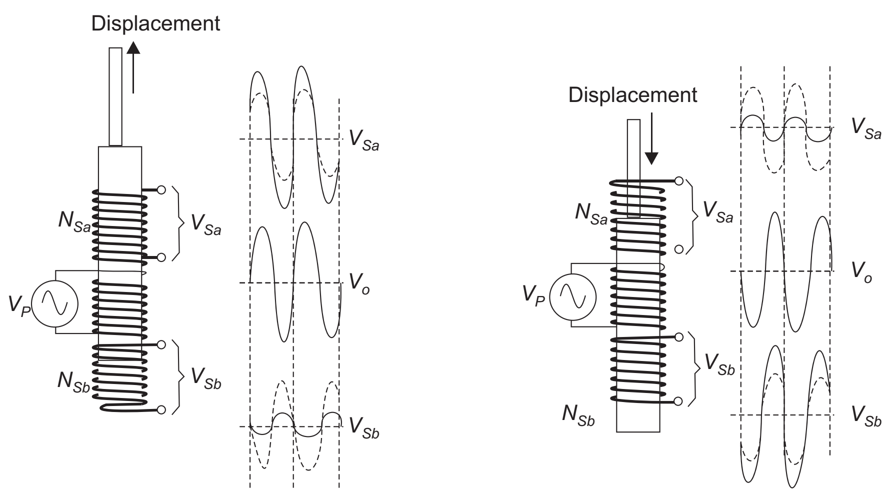


(a) Cross section (b) Coil connections

Fig. 11.17 A Linear Variable Differential Transducer

When the core is at its null position, that is, under zero displacement, both the secondary voltages V_{Sa} and V_{Sb} have equal magnitude. The voltage V_{Sa} is in phase with primary input voltage V_P while the voltage V_{Sb} is opposite in phase with the primary input voltage V_P. In this case, the output voltage V_o is equal to zero as both the secondary voltages cancel each other out. Now, when the core is displaced in upward direction [see Figure 11.18(a)], the voltage V_{Sa} increases while the voltage V_{Sb} decreases. This is due to an increase in the flux from the primary winding linking N_{Sa} to secondary winding while there is a decrease in flux linking to secondary winding N_{Sb}. The output voltage V_o is in phase with the input voltage and its magnitude is given as:

$$V_o = V_{Sa} - V_{Sb}$$



(a) Core displaced upwards (b) Core displaced downwards

Fig. 11.18 Displacement of Core

Now, when the core is displaced in downward direction [see Figure 11.18(b)], the voltage V_{Sb} increases while the voltage V_{Sa} decreases. Here, the output voltage is 180° out of phase with the input voltage and its magnitude is given as:

$$V_o = V_{Sa} - V_{Sb}$$

Thus, the amount of core displaced and its direction can be determined by the amplitude and phase of output voltage.

The response of LVDT is linear. However, it becomes non-linear for very large displacements at which it is not useful. This is shown in Figure 11.19. From the graph it can be seen that output voltage cannot be reduced to zero at null position.

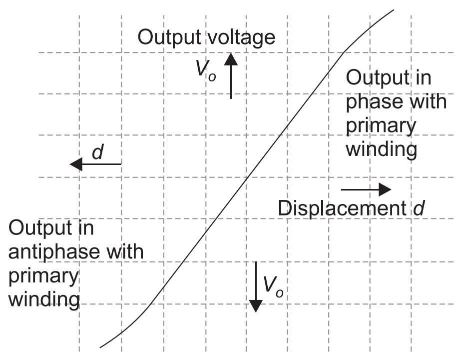


Fig. 11.19 Characteristics of an LVDT

Advantages and disadvantages

Some of the advantages of LVDT are as follows.

- LVDTs have simple construction and lightweight instruments.
- They are very rugged and can tolerate high vibrations and shock.
- Sensitivity of LVDTs is very high, usually around 40 V/mm.
- Power consumption for LVDTs is very small, even less than 1 W.
- The response of LVDTs is linear in nature and they have infinite resolution.
- The output of LVDTs is high and does not require any amplification.

Some of the disadvantages of LVDT are as follows.

- To produce a measurable differential output voltage, relatively large core displacements are required.
- Dynamic response of an LVDT is limited by its core mass and frequency of the applied voltage.
- Performance of the transducer gets affected by the temperature.
- Shielding must be done to avoid the effects of stray magnetic fields.

11.7.2 Rotary Variable Differential Transformer

Rotary variable differential transformer, abbreviated as RVDT, is used to measure angular displacement. The construction and working of RVDT are similar to that of LVDT. However, the core of an RVDT is cam-shaped and is rotated using a shaft. An RVDT is shown in Figure 11.20.

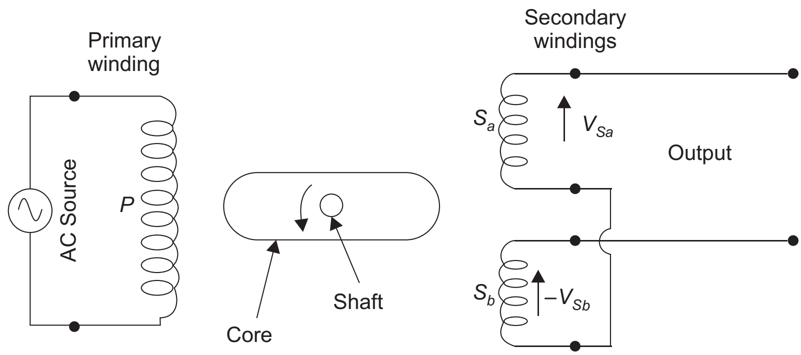


Fig. 11.20 A Rotary Variable Differential Transformer

Initially, when the core is at null position, the output voltages of both secondary windings S_a and S_b are equal and opposite resulting in a net zero output voltage, that is,

$$V_o = V_{Sa} - V_{Sb} = 0$$

However, when the core is at some angular displacement from the null position, some output voltage V_o will be obtained. This output voltage is directly proportional to the angular displacement, that is, RVDT has a linear response. Also, to determine the amount of angular displacement, the magnitude and phase of transducer output voltage is monitored.

11.7.3 Synchros

A synchro has many trade names, such as Selsyn, Microsyn, and Autosyn. It is basically used for shaft angle measurement by converting it into an electrical signal. It works on the inductive principle in which the coupling between primary and secondary windings is varied by changing their relative orientation.

The construction of a synchro is shown in Figure 11.21. It consists of a rotor and a stator, similar to an AC motor. The rotor is capable of revolving around the fixed stator and can have one or three windings. A concentric coil is wound on the dumb-bell shaped rotor which acts as the primary winding. Two precision slip rings are used to connect AC supply to the rotor.

The stator is made up of laminated steel to minimize losses in the core. It consists of a balanced three-phase winding whose windings are displaced by 120° and are star (Y) connected. The windings of the stator act as the secondary windings. An AC voltage is applied to the rotor due to which it gets energized and the coupling between the rotor and stator is a linear or trigonometric function of the rotor position. Usually, a synchro system consists of a group of interconnected synchros according to the application used for.

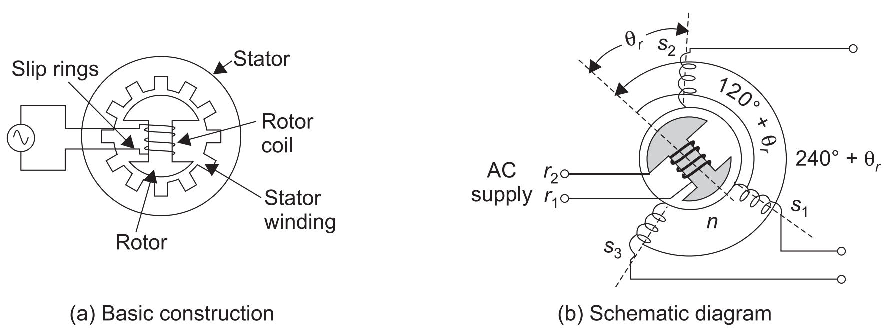


Fig. 11.21 A Basic Synchro

There are two types of synchro systems, namely, *control type* and *torque transmission type*.

Control type

Control type system is formed by the combination of **synchro transmitter** and **synchro control transformer** and is widely used as an error detector in feedback control systems. Consider a synchro device shown in Figure 11.21(b), acting as a synchro transmitter, supplied by an AC voltage given as:

$$v_r = \sqrt{2}\, V_{\text{rms}} \sin \omega t$$

where V_{rms} is the rms value of the rotor voltage and ω is the carrier frequency.

For the position shown in the figure, that is, when the rotor is at an angle of θ_r with the secondary winding of stator s_2, we have:

$$V_{s1n} = kV_r \sin \omega t \cos(\theta_r + 120°), \qquad \text{...(10)}$$

$$V_{s2n} = kV_r \sin \omega t \cos \theta_r \qquad \text{...(11)}$$

and

$$V_{s3n} = kV_r \sin \omega t \cos(\theta_r + 240°) \qquad \text{...(12)}$$

where V_{s1n}, V_{s2n}, and V_{s3n} are the voltages induced in the windings of the stator (secondary windings) s_1, s_2, and s_3, respectively with respect to the neutral n.

The voltages at the terminal of the stator windings are given as:

$$V(s_1 s_2) = V_{s1n} - V_{s2n} = \sqrt{3} kV_r \sin(\theta_r + 240°) \sin \omega t, \qquad \text{...(13)}$$

$$V(s_2 s_3) = V_{s2n} - V_{s3n} = \sqrt{3} kV_r \sin(\theta_r + 120°) \sin \omega t \qquad \text{...(14)}$$

and

$$V(s_3 s_1) = V_{s3n} - V_{s1n} = \sqrt{3} kV_r \sin \theta_r \sin \omega t \qquad \text{...(15)}$$

Equations (13), (14), and (15) can be used to describe the relation between the angle θ_r and the stator terminal voltages as depicted in Figure 11.22.

Let the angle θ_r be equal to 0, then from Equation (11), we get:

$$V_{s2n} = kV_r \sin \omega \text{t}$$

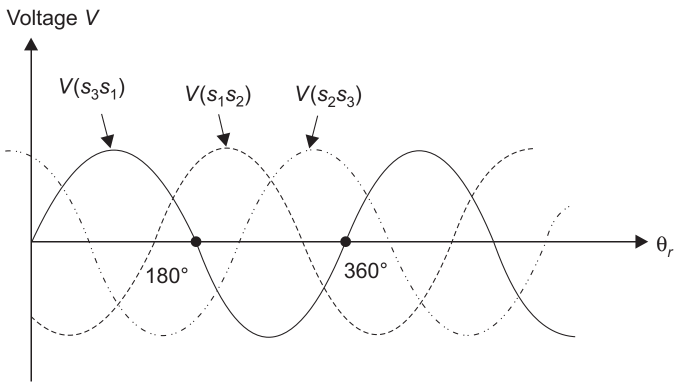


Fig. 11.22 Variation of Terminal Voltages of the Stator with Respect to θ_r

and from Equation (15), we get:

$$V(s_3 s_1) = V_{s3n} - V_{s1n} = 0$$

which implies that at this angle voltage V_{s2n} is maximum and $V(s_3 s_1)$ is equal to zero. This position of the rotor is known as **electrical zero** of the synchro transmitter and is considered as a reference while specifying rotor angular position.

The output of the synchro transmitter is applied to a synchro control transformer. The construction of synchro control transformer is similar to that of synchro transmitter except that a cylindrical shaped rotor is used as shown in Figure 11.23. This circuit acts as an error detector.

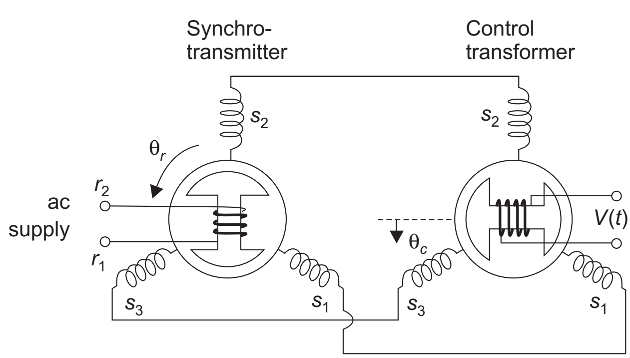


Fig. 11.23 Error Detector using Synchro

In this circuit, currents having same phase but different magnitudes flow through the windings of the stator of the transmitter and control transformer. As a result, an identical flux pattern gets established in the air gap of the control transformer, aligned with the flux of the transmitter. The voltage induced in the control transformer rotor $V(t)$ is given by:

$$V(t) = k'V_r \cos\ \phi\ \sin\ \omega t \qquad ...(16)$$

where ϕ is the angle between the axis of rotors of transmitter and control transformer. When $\phi = 90°$, we get:

$$V(t) = k'V_r \cos\ 90° \sin \omega t = 0$$

This position of the rotor of control transformer at which the voltage induced in it is zero is known as **electrical zero** of the control transformer. It is to be noted here that in Figure 11.23, the rotors of both the transmitter and the control transformer are in their electrical zero positions.

Let the rotor of transmitter be rotated by an angle θ_r and that of the control transformer by an angle θ_c in the same direction, as shown in Figure 11.23. Thus, we have:

$$\phi = 90° - \theta_r + \theta_c \quad ...(17)$$

Substituting Equation (17) into (16), we get:

$$V(t) = k'V_r \cos(90° - \theta_r + \theta_c) \sin \omega t$$

$$= k'V_r \sin(\theta_r - \theta_c) \sin \omega t \quad ...(18)$$

For small values of displacements in angles, we have:

$$\sin(\theta_r - \theta_c) \simeq \theta_r - \theta_c$$

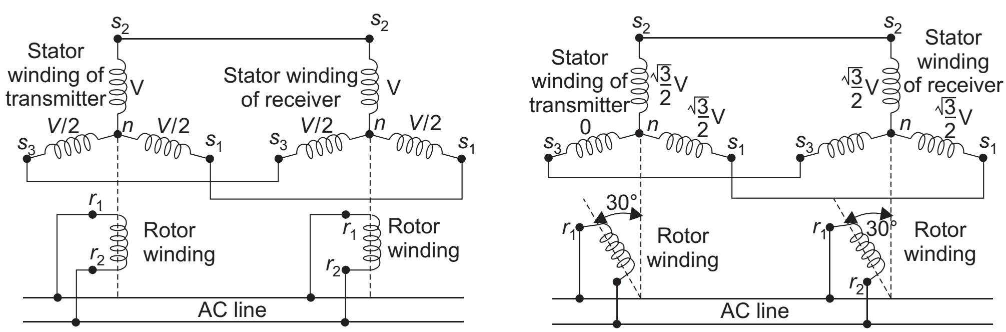
Substituting in Equation (18), we get:

$$V(t) = k'V_r\ (\theta_r - \theta_c) \sin \omega t \quad ...(19)$$

Thus, whenever there is a difference between transmitter and control transformer shaft positions, a voltage will be generated at the rotor terminals of control transformer. This voltage is proportional to the difference in angular positions as can be seen from Equation (19). Hence, this circuit acts as an error detector.

Torque transmission type

Torque transmission type synchros have very less output torque and hence, are used for very light loads, such as pointers. However, in instrumentation systems, this mode is generally used. A torque transmission system is shown in Figure 11.24.



(a) Maximum coupling in rotor and stator s_2

(b) Rotor of transmitter shifted at 30°

Fig. 11.24 Torque Transmission using Synchro

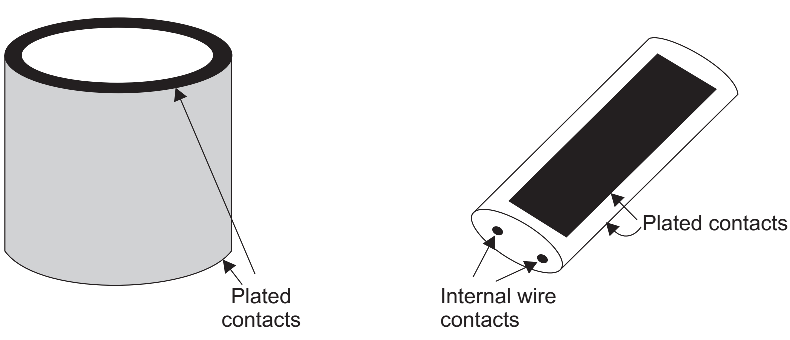
Initially, as shown in Figure 11.24(a), let the stator winding s_2 of the transmitter be placed such that the coupling between it and the rotor winding is maximum. Let the voltage be denoted by V. From Equations (10), (11), and (12), we know that the coupling between rotor (primary) winding and s_1 and s_2 of stator (secondary) windings is a cosine function. Thus, the voltages in these windings at this position are given by $V/2$ each (proportional to cos 60°) and owing to voltage balance, no current will flow through them.

Now consider Figure 11.24(b) in which the voltage balance is disturbed by moving the rotor of the transformer to a new position. As can be seen from the figure, the rotor is moved through an angle of 30°. Due to this, the voltages of stator windings s_1, s_2 and s_3 of the transmitter become $\sqrt{3}/2$ V, $\sqrt{3}/2$ V, and 0, respectively. This results in a voltage imbalance in transmitter and receiver stator windings and thus, currents will flow through them. These currents generate a torque to restore the balance by moving the receiver rotor to a suitable position. The balance is again obtained when the rotor is rotated through the same angle and in the same direction (here, it is through 30°). Hence, a synchro is used for measuring the magnitude and direction of angular displacement.

11.8 PIEZOELECTRIC TRANSDUCERS

When a mechanical stress is applied to some asymmetrical crystalline material such as quartz, barium titanite, and Rochelle salt, a voltage is generated at their surfaces. This voltage is in proportion to the applied stress and becomes zero when the stress is removed. This effect is known as **piezoelectric effect** and the materials having this property are known as **piezoelectric**. The piezoelectric effect is reversible in nature, that is the dimensions of the crystal will change if an alternating voltage is applied to it. This property of crystals is used in piezoelectric transducers.

The materials that show piezoelectric effect are known as **electro-resistive elements** and can be classified as: *natural elements* and *synthetic elements*. Natural elements include quartz and Rochelle salt that provide limited applications due to limited shape in which they can be cut. Synthetic elements include ethylene diamine tartrate, lithium sulphate, and many more substances which can be manufactured in any shape. Figure 11.25 shows two kinds of synthetic piezoelectric transducer. One is cylinder shaped transducer shown in Figure 11.25(a)



(a) Cylinder shaped transducer (b) Bimorph

Fig. 11.25 Synthetic Piezoelectric Transducers that has electrical contact plates on both ends and can be used as pressure transducer to hear sea noises. The other transducer shown in Figure 11.25(b) is called **bimorph** which is basically a ceramic device and forms the basis of cartridge of a record player.

A layer of crystal is placed between two metal plates to construct a piezoelectric transducer. Due to dielectric nature of crystal, its dimensions and relative permittivity are used to calculate the capacitance of the transducer.

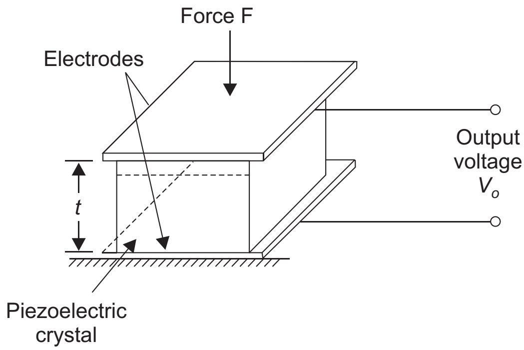


Fig. 11.26 A Piezoelectric Crystal

Consider the piezoelectric crystal shown in Figure 11.26. The charge Q accumulated on the electrodes, when a force F is applied on the crystal, is given as:

$$Q = Fd \qquad ...(20)$$

where d is the charge sensitivity and its unit is Coulombs per Newton (C/N). Charge Q can also be expressed in terms of transducer capacitance C_T as:

$$Q = C_T V_o \qquad ...(21)$$

where V_o is the voltage generated at the output of the transducer.

We know that the capacitance is expressed as:

$$C_T = \frac{\varepsilon_r \varepsilon_o A}{t} \qquad ...(22)$$

where ε_o is the permittivity of free space, ε_r is the relative permittivity of the dielectric, A is the area of the plates, and t is the thickness of dielectric.

Combining Equations (20), (21), and (22), we get:

$$V_o = \frac{Q}{C_T} = \frac{Fdt}{\varepsilon_r \varepsilon_o A}$$

It can also be written as:

$$V_o = gtP \qquad ...(23)$$

where P is the pressure applied and is equal to F/A, and g the voltage sensitivity, is given as:

$$g = \frac{d}{\varepsilon_r \varepsilon_o} \qquad ...(24)$$

The voltage sensitivity is expressed in Vm / N. For any given transducer, it is constant. Here, it is to be noted that the piezoelectric effect is direction sensitive, that is, the voltages produced by tensile and compressive forces are of opposite polarity.

Advantages and disadvantages

Some of the advantages of a piezoelectric transducer are as follows.

- It does not require any external power source, that is, it is self generating.
- It is small in size and very rugged.
- Quartz crystal can be used over a wide range of temperatures.

Some of the disadvantages of a piezoelectric transducer are as follows.

- The output voltage of piezoelectric transducer varies with crystal temperature.
- It cannot be used for measurement of static conditions.
- Synthetic transducers cannot be used for stabilizing the frequency of oscillators.

Example 4 A force of 10 N is applied on a piezoelectric transducer having dimensions 4.5 mm × 3.5 mm and a thickness of 3 mm. If the relative permittivity and voltage sensitivity are given to be 830 and 0.05 Vm/N, calculate the following:

(a) charge sensitivity

(b) charge on transducer

(c) output voltage

Solution: Given that: $F = 10$ N, $t = 3$ mm, $g = 0.05$, and $\varepsilon_r = 830$

Area of the transducer can be calculated as:

$$A = 4.5 \text{ mm} \times 3.5 \text{ mm} = 15.75 \text{ mm}^2$$

(a) The charge sensitivity d of a piezoelectric transducer is computed from Equation (24) as:

$$g = \frac{d}{\varepsilon_r \varepsilon_o}$$

Substituting the given values, we get:

$$d = g\varepsilon_r \varepsilon_o = 0.05 \times 830 \times 8.854 \times 10^{-12} = 367.44 \text{ pC / N}$$

(b) The charge Q is given by the Equation (20) as:

$$Q = Fd$$

Substituting the given values, we get:

$$Q = 10 \times 367.44 = 3.674 \text{ nC}$$

(c) The output voltage V_o is given by the relation:

$$V_o = gtP \qquad \text{[Refer to Eqn. (23)]}$$

where $$P = \frac{F}{A} = \frac{10}{15.75 \times 10^{-6}}$$

$\Rightarrow$ $$P = 0.63 \times 10^6 \text{ N/m}^2$$

Substituting the values in the expression of output voltage, we get:

$$V_o = 0.05 \times 3 \times 10^{-3} \times 0.63 \times 10^6$$

$\Rightarrow$ $$V_o = 94.5 \text{ V}$$

11.9 PHOTOELECTRIC TRANSDUCERS

In photoelectric transducers, when light strikes on a special combination of materials, it may result in a flow of electrons, generation of a voltage, and a change of resistance. This property of light forms the basis of photoelectric transducers. They are classified into three types namely, *photoemissive*, *photoconductive* and *photovoltaic cells*. However, we will discuss only the last two. Photoelectric transducers find applications in many fields such as control engineering, space research satellites, precision measuring devices, electric power source through solar batteries, and exposure meters.

11.9.1 Photoconductive Cell

Photoconductive cell, also known as **photo cell** works on the property of light that can change the resistance of a material upon striking it. When light is incident on a surface, it provides some energy to it which results in the breakdown of electrons. Due to this, free electrons and holes are generated which reduce the resistance of the material. Semiconductor materials, such as cadmium sulphide, lead sulphide, selenium, thalium sulphide, can be used to construct photoconductive cells as their resistance decreases on irradiation. However, generally, cadmium selenide and cadmium sulphide are used.

A photoconductive cell is shown in Figure 11.27. It consists of a disc-shaped base in which a long strip of a light-sensitive material is placed in a zigzag pattern. On each side of the strip, connecting terminals are fitted. It is to be noted here that the connecting terminals

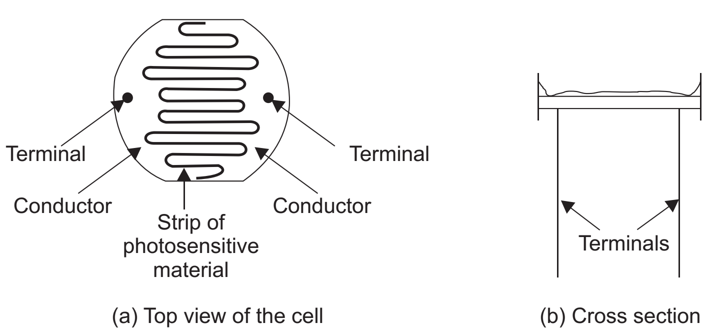


Fig. 11.27 A Photoconductive Cell

are not connected to the ends of the strip, rather they are fitted in the conductor on each side of the strip. This makes the light-sensitive material appear to be a short and wide strip with conductors on both sides [see Figure 11.27 (b)].

The resistance versus illumination graph of a photoconductive cell representing its illumination characteristics is shown in Figure 11.28. Here, the scales used are logarithmic.

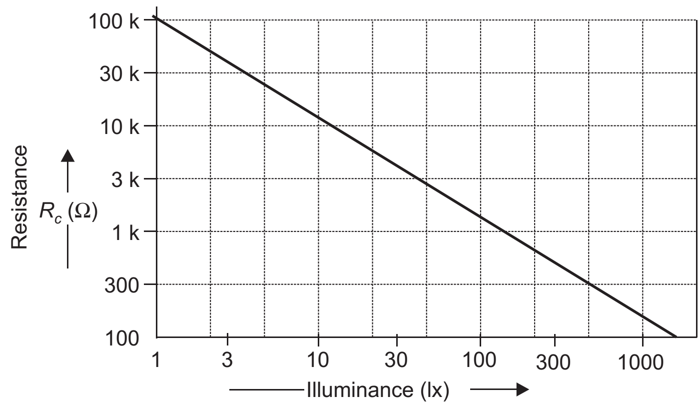


Fig. 11.28 Illumination Characteristics of a Photoconductive Cell

The resistance of the cell when it is not illuminated is known as **dark resistance**. From the figure, it can be seen that it can be greater than 100 kΩ. However, under illumination, the resistance of the cell is reduced to a few hundred ohms. Photoconductive cells are used for telephony (using modulated infrared signals), for detecting aircraft and ships, in burglar alarm circuits, and for counting packages moving on a conveyor belt. They can also be used for relay control as they can carry moderate currents as illustrated in Figure 11.29. In the circuit shown, the relay is energized when the cell is illuminated. When the cell is not illuminated, the current through the circuit is very low due to high resistance of the cell. Thus, the relay is not energized.

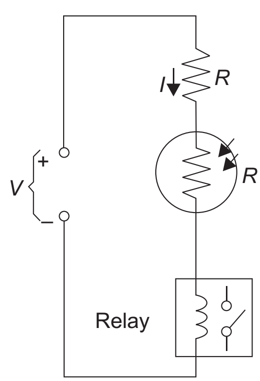


Fig. 11.29 Relay Control using Photoconductive Cell

Resistance R acts as a limiting resistor to limit the current flowing through the relay under illuminated condition.

11.9.2 Photovoltaic Cell

Photovoltaic cell, also known as **solar cell**, works on the principle of generation of voltage in a material when a light is incident on it. The sensitive element used is a semiconductor including silicon and selenium. A photovoltaic cell along with its characteristics curve (current versus luminous flux) is shown in Figure 11.30. It consists of four layers—the topmost layer is made of gold, followed by a barrier layer, and the third layer is made of selenium deposited on a metal base. The gold layer is translucent and acts as the top electrode whereas the metal base acts as the bottom electrode. When light is incident on the top layer, a negative charge builds upon it and a positive charge builds on the bottom electrode. Due to this generation of charges, a voltage is generated in proportion to the incident light.

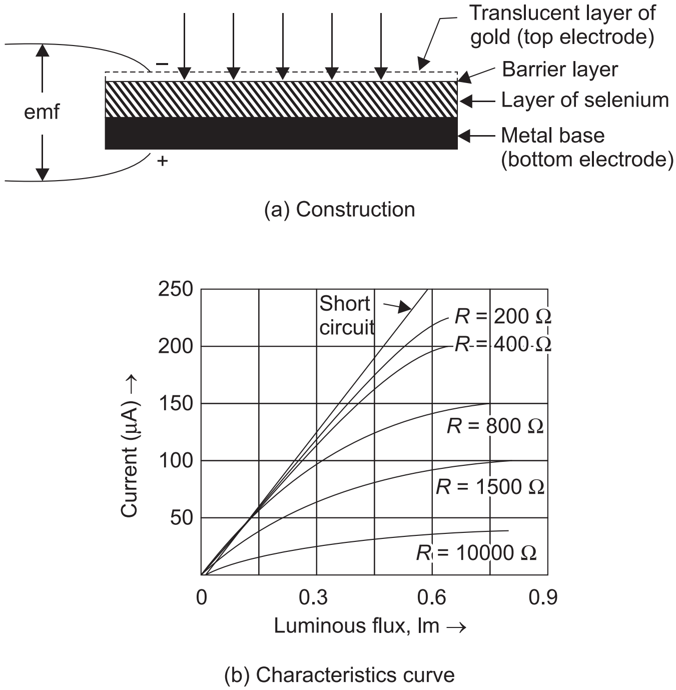


Fig. 11.30 A Photovoltaic Cell

Photovoltaic cells have a temperature range of –100°C to 125°C through which they can work satisfactorily. The short circuit current is not affected by the temperature whereas a few mV/°C change may be observed in the open circuit voltage. They have a very fast response time and do not require any external power source to generate the voltage. They are used in television circuits, sound motion pictures and reproducing tools, and automatic control systems. Gold doped germanium cells may be used as infrared detectors and multiple unit silicon cells may be used as punch card readers.

Example 5 Consider the circuit shown in Figure 11.29 with the characteristics curve of the photoconductive cell shown in Figure 11.28. When the cell is illuminated with 100 lm/m^2, the relay is supplied with 11.5 mA current from the 32 V supply voltage. Calculate the value of

(a) series resistance R

(b) dark current I, if the resistance of relay is negligible in comparison to R and R_C.

Solution:

(a) The value of resistance of photoconductive cell R_C, when it is illuminated with 100 lm/m^2 can be noted down from the characteristics curve as:

$$R_C \approx 2.5 \text{ k}\Omega$$

Applying Kirchoff's Voltage Law in the circuit, we get:

$$V = I\,(R + R_C)$$

Given that: $V = 32$ V and $I = 11.5$ mA. Substituting the given values, we get:

$$R = \frac{V}{I} - R_C = \frac{32}{11.5} - 2.5 = 0.28 \text{ k}\Omega$$

(b) When there is no illumination, the value of resistance of photoconductive cell R_C comes out to be:

$$R_C \approx 100 \text{ k}\Omega$$

Applying KVL in the circuit, we get:

$$V = I\,(R + R_C)$$

Given that: $V = 32$ V, $R_C \approx 100$ kΩ, and $R = 0.28$ kΩ. Substituting the given values, we get:

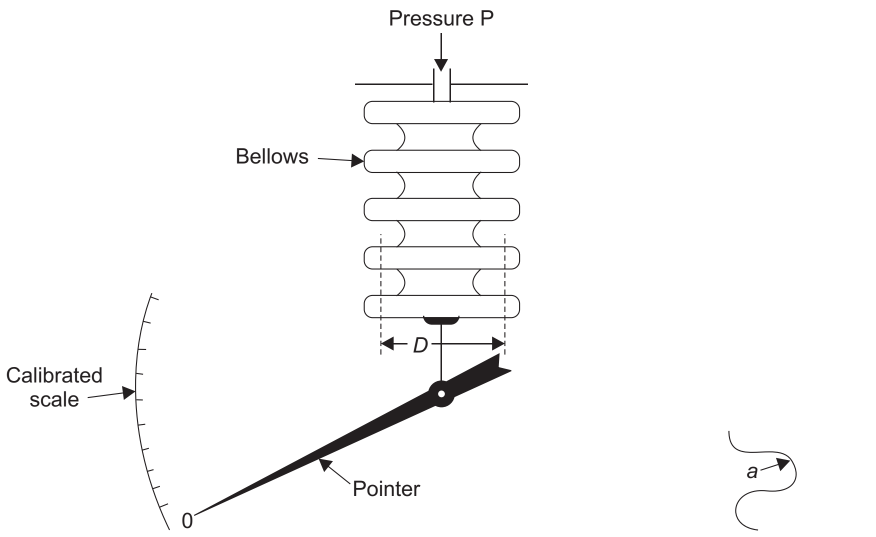
$$I = \frac{32}{0.28 + 100} = 0.32 \text{ mA}$$

11.10 PRESSURE TRANSDUCERS

Mechanical elements used to convert the applied pressure into displacement are known as **force-summing devices**. Elastic members are generally used for this purpose. There are many types of force-summing devices, namely, bellows, diaphragms, bourdon tubes, pivot torque, straight tube, and mass cantilever. Out of these, *diaphragms, bellows*, and *bourdon tubes* are used as primary sensing element in pressure transducers. The working principle of such devices is that the pressure sensitive element is pressed by the fluid whose pressure is to be measured. This results in a mechanical displacement to be measured by electrical transducers. Here, it is to be noted that these devices act as a primary transducer in the circuit, and a secondary transducer is needed to convert the displacement into electrical form.

11.10.1 Bellows

Bellows consist of a series of folded circular parts which can be contracted or expanded axially on application of pressure [see Figure 11.31(a)]. Many methods are adopted for the manufacturing of bellows such as drawn tubing by hydraulic, rolling, soldering and welding of annular sections, spinning, and turning from solid stock. The materials used for the construction of the bellows are bronze, brass, alloys of nickel and copper, steel, beryllium copper, and monel. These materials have the right properties required for the bellows of being flexible, ductile, and have high resistance.



(a) Bellows element

(b) Depiction of radius *a*

Fig. 11.31 Bellows

The displacement d of the bellows element is expressed mathematically as:

$$d = \frac{0.453\, aPND^2 \sqrt{1-\nu^2}}{Et^3} \qquad ...(25)$$

where a is the radius of each corrugation [see Figure 11.31(b)], P is the pressure, N is the number of semi-circular corrugations, D is the mean diameter, V is the Poisson's ratio, E is the modulus of elasticity, and t is the thickness of wall.

A **spring loaded bellows** is generally used to have better accuracy in the measurements. It consists of a spring to oppose the movement of the bellows (see Figure 11.32). Due to this, only a small portion of the maximum stroke is used. This device also increases the life of bellows by overcoming their tendency to move over a greater distance than required when pressure is applied to it.

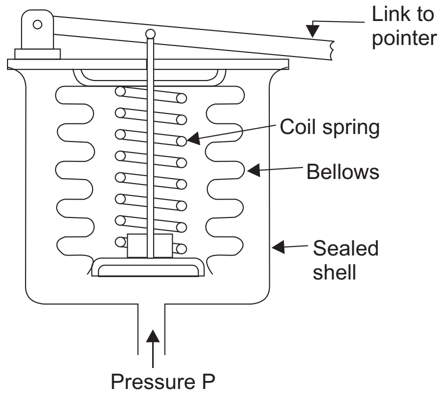


Fig. 11.32 Spring Loaded Bellows

The deflection d of the spring loaded bellow is given by the relation:

$$d = P\frac{A_b}{K_b + K_s} \quad \text{...(26)}$$

where A_b is the effective area of bellows, K_b and K_s are the stiffness constants of bellows and the spring, respectively measured in newtons per square metre (N/m^2).

On rearranging Equation (26), the relation for pressure can be written as:

$$P = d\frac{K_b + K_s}{A_b} \quad \text{...(27)}$$

If the assembly of bellows needs to operate some mechanism or an electric switch, then the pressure is given as:

$$P = \frac{F + d_s(K_b + K_s)}{A_b}$$

where F is the force and d_s is the deflection required to operate that mechanism or switch.

Advantages and disadvantages

Some of the advantages of bellows are as follows.

- They are simple, rugged, and moderate-price devices.
- They can be used to measure low, medium, and high pressure.
- They can determine absolute, gauge, and differential pressures.
- They can provide an accuracy of ±0.5%.

Some of the disadvantages of bellows are as follows.

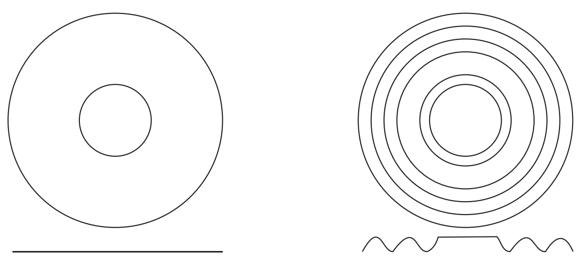
- They are not suitable for dynamic measurements due to their longer relative movement and greater mass.
- They require a temperature compensating device to compensate for changes in ambient temperature.

11.10.2 Diaphragms

Diaphragms work on the same principle as that of bellows. Here also, a pressure is applied which results in a deflection which is then measured. The edges of the diaphragm are fixed, and on application of pressure, a deflection occurs. The measured deflection is directly proportional to the pressure applied thereby giving a measure of the applied pressure. However, the diaphragms have smaller movement and hence they do not require springs.

Basically diaphragms consist of a very thin membrane and hence, they can be used to measure low value of pressures only. Inductive or capacitive transducers are used as secondary transducers to convert the displacement into electrical output. However, when resistive and piezoelectric transducers are used as secondary transducers, membrane type diaphragms cannot be used since they require much greater displacements. Thus, thin circular plates are used in place of membranes. These plates are either machined from a solid piece of metal or they can be clamped between two solid rings around their circumference.

Many materials can be used for the construction of diaphragms including stainless steel, Ni-span *C*, beryllium copper, phosphor bronze, Monel, nickel, Iconel, nylon, Buna *N* rubber, and Teflon. They mainly find applications in measuring low pressures and have a range of $0 - 50\ \text{N/m}^2$ to $0 - 200\ \text{kN/m}^2$. The accuracies provided by a diaphragm lies in the range of $\pm 0.5\%$ to $\pm 1.25\%$ of full span.



(a) Flat type (b) Corrugated type

Fig. 11.33 Diaphragms

The diaphragms are of two types, namely, *flat type* and *corrugated type* as shown in Figure 11.33. Corrugated diaphragms have higher surface area as compared to flat type diaphragms and thus, they give greater deflections. The schematic diagram of a flat type diaphragm is shown in Figure 11.34.

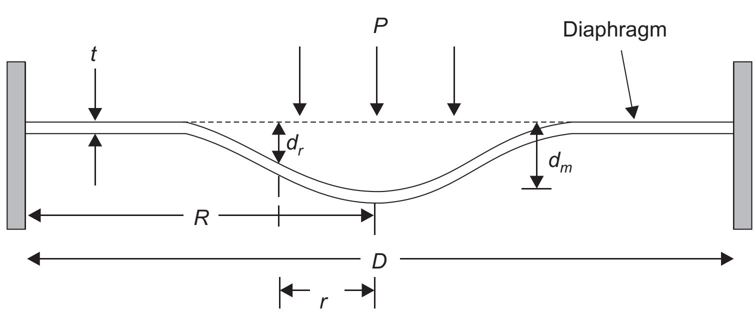


Fig. 11.34 Schematic Diagram of a Flat Diaphragm

The mathematical expression for the applied pressure P is given as:

$$P = \frac{256\, Et^3\, d_m}{3(1-\nu^2)\, D^4} \qquad ...(28)$$

where E is the Young's modulus, t is the thickness of the diaphragm, d_m is the deflection at diaphragm center, v is the Poisson's ratio, and D is the diameter of the diaphragm.

From Equation (28), it can be seen that the relation between pressure P applied to the diaphragm and the deflection at the centre (d_m) are related linearly. However, for $d_m > 0.5\,t$, the linear relation does not hold. Thus, the deflection at the centre of the diaphragm must be less than or equal to half of its thickness. Corrugated diaphragms are used to have a linear response for this condition. On rearranging Equation (28), the deflection at the centre (d_m) can be given as:

$$d_m = \frac{3P(1-\nu^2)\, D^4}{256\, Et^3}$$

$$\Rightarrow \qquad d_m = \frac{3P(1-\nu^2)\, R^4}{16 Et^3} \qquad ...(29)$$

where R is the radius of the diaphragm. From the above relation, it can be seen that the movement of a diaphragm depends on its diameter and thickness.

The deflection at a distance r from the centre of the diaphragm can be expressed as:

$$d_r = \frac{3P(1-\nu^2)\,(R^2 - r^2)^2}{16 Et^3} \qquad ...(30)$$

The maximum stress, denoted by S_m is given by the expression:

$$S_m = \frac{3D^2 P}{16t^2}\ \text{N/m}^2 \qquad ...(31)$$

The lowest natural frequency ω_n for gas or air medium is given by:

$$\omega_n = \frac{20t}{D^2}\sqrt{\frac{E}{3\rho(1-\nu^2)}}\ \text{rad/s} \qquad ...(32)$$

where ρ is the density of the material of the diaphragm. These relations hold when a uniform pressure is applied over the whole surface of the disc.

Note: *Whenever a mechanical device is used to measure deflection, corrugated type diaphragms are preferred as compared to flat type diaphragms.*

11.10.3 Bourdon Tubes

Bourdon tubes are made from an elliptical flattened tube which is bent to produce the desired shape. They are available in four configurations, namely, *C-type*, *helical*, *twisted*, and *spiral* (see Figure 11.35).

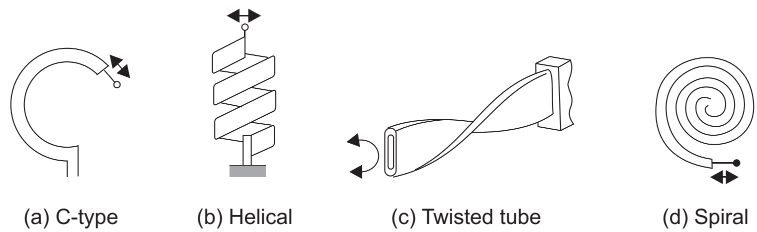


Fig. 11.35 Bourdon Tubes

Bourdon tubes consist of two ends out of which one is sealed and the other one is left open for the fluid to enter. When the fluid enters the tube, it applies pressure on it and as a result the tube tends to straighten out. This causes the closed end of the tube to move to some distance. This displacement after amplifying through mechanical linkages may be used directly to move a pointer on a pressure calibrated scale or it can be applied to a secondary transducer (that is, electrical displacement transducer). The output of the secondary transducer is then used to move the pointer on a pressure calibrated scale.

Advantages and disadvantages

Some of the advantages of bourdon tubes are as follows.

- They are simple and low price device.
- They have good accuracy at high pressure ranges.
- Improved designs are used to provide maximum safety.
- They can be easily adapted in accordance with the secondary transducer used.

Some of the disadvantages of bourdon tubes are as follows.

- They are subjected to hysteresis.
- Shocks and vibrations may affect the measurement.
- They can be used for precision measurements up to a pressure of 3 MN/m^2 due to their low spring gradient.

Example 6 A flat circular diaphragm made up of mild steel has a diameter of 17 mm, Young's modulus of 200 GN/m^2, Poisson's ratio of 0.28, and maximum stress of 310 MN/m^2 is applied with a pressure of 305 kN/m^2, calculate:

(a) its thickness

(b) deflection at the centre of the diaphragm if a pressure of 175 kN/m^2 is applied.

(c) natural frequency if the density of mild steel is given to be 7800 kg/m^3.

Solution: Given that: $D = 17 \times 10^{-3}$ m, $E = 200 \times 10^9$ N/m^2, $v = 0.28$, $S_m = 310 \times 10^6$ N/m^2, and $P = 305 \times 10^3$ N/m^2

(a) The maximum stress S_m is given as:

$$S_m = \frac{3D^2 P}{16t^2} \qquad \text{[Refer to Eqn. (31)]}$$

Substituting the given values, we obtain:

$$t = \sqrt{\frac{3(17 \times 10^{-3})^2 \times 305 \times 10^3}{16 \times 310 \times 10^6}}$$

$$\Rightarrow \qquad t = 2.30 \times 10^{-4} \text{ m} = 0.230 \text{ mm}$$

(b) The deflection d_m at the centre of the diaphragm is given by the relation:

$$d_m = \frac{3P(1 - \nu^2)\, D^4}{256 E t^3} \qquad \text{[Refer to Eqn. (29)]}$$

Substituting the given values, we obtain:

$$d_m = \frac{3 \times 175 \times 10^3 \,\{1 - (0.28)^2\}\, (17 \times 10^{-3})^4}{256 \times 200 \times 10^9 \times (2.30 \times 10^{-4})^3}$$

$$\Rightarrow \qquad d_m = 6.48 \times 10^{-5} \text{ m} = 0.0648 \text{ mm}$$

(c) The natural frequency ω_n of the diaphragm is computed as:

$$\omega_n = \frac{20t}{D^2} \sqrt{\frac{E}{3\rho(1 - \nu^2)}} \qquad \text{[Refer to Eqn. (32)]}$$

Substituting the given values, we obtain:

$$\omega_n = \frac{20 \times 2.30 \times 10^{-4}}{(17 \times 10^{-3})^2} \sqrt{\frac{200 \times 10^9}{3 \times 7800\,(1 - (0.28)^2)}}$$

$$\Rightarrow \qquad \omega_n = 48472.5 \text{ rad/s}$$

11.11 CAPACITIVE TRANSDUCERS

The capacitance C of a parallel plate capacitor is given as:

$$C = \frac{\varepsilon A}{d} \qquad \text{...(33)}$$

where A is the overlapping area of the plates, d is the distance between the plates, and ω is the permittivity of the dielectric medium. ε can be written as:

$$\varepsilon = \varepsilon_r \varepsilon_o$$

where ε_r is the relative permittivity and ε_o is the permittivity of free space (8.854×10^{-12} F/m). This equation forms the basis of capacitive transducers and any of the above mentioned parameter can be changed to change the capacitance. Physical variables like force, displacement, and pressure may induce these changes and hence, can be measured as described in this section.

11.11.1 Capacitive Displacement Transducers

While using a capacitive transducer as a displacement transducer, various configurations can be used. They involve variation of different parameters for variation of capacitance. These configurations are shown in Figure 11.36.

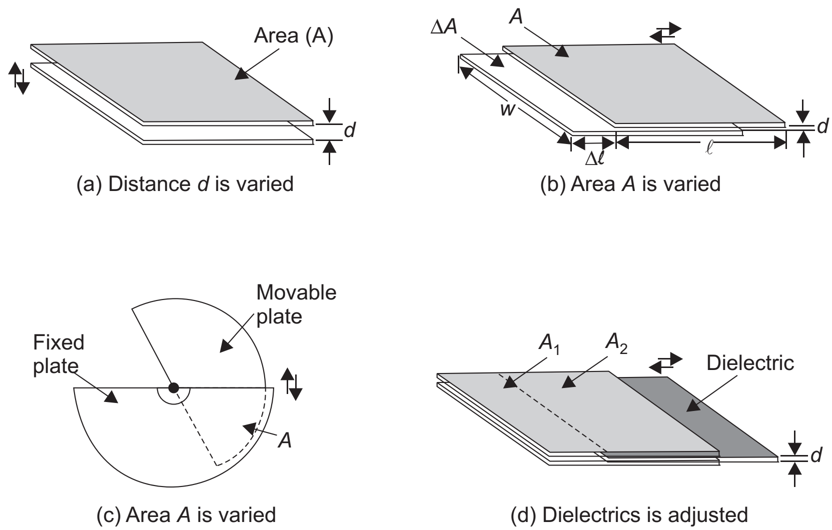


Fig. 11.36 Capacitive Displacement Transducers

Figure 11.36(a) represents a parallel plate capacitor in which the distance between the plates d is varied to vary the capacitance. Here, the change in capacitance ΔC is in inverse proportion to the change in distance Δd and the sensitivity is given by their ratio. Thus, we can write sensitivity as:

$$\text{Sensitivity} = \frac{\Delta C}{\Delta d} \quad \text{...(34)}$$

It is generally expressed in pF/μm. Figure 11.36(b) represents a parallel plate capacitor in which the facing area A of the plates is varied to vary the capacitance. Here, the change in capacitance ΔC is in direct proportion to the change in area ΔA. However, from the figure, it can be seen that the width w remains constant and only length l is varied. Thus, sensitivity can be written as:

$$\text{Sensitivity} = \frac{\Delta C}{\Delta A} = \frac{\Delta C}{\Delta l}$$

Figure 11.36(c) shows one more configuration of varying capacitance of the transducer by varying its area. It is made up of half-disc shaped plates with air acting as the dielectric. The fixed plate is known as **stator** and the movable plate is known as **rotor**. The member under test is mechanically coupled to the rotor. Whenever a displacement is made by the member under test, there is change in the relative position of the rotor with respect to stator. This change causes a variation in the facing plate area thereby a variation in capacitance. Thus, here also the change in capacitance occurs due to change in area and the sensitivity is given as:

$$\text{Sensitivity} = \frac{\Delta C}{\Delta A}$$

This type of capacitive transducer is used in an aircraft to determine the amount of roll. Now consider Figure 11.36(d) in which a movable dielectric is incorporated between the plates. Due to the movement of dielectric, we get two different areas, one in which the air acts as dielectric A_1 and the other A_2 with solid dielectric. This gives two capacitors, connected in parallel, with their areas in fixed proportions, that is, the area of one capacitor can be reduced by increasing the area of the other. Thus, we can say that here also the capacitance varies due to variation in the area and the sensitivity is expressed as:

$$\text{Sensitivity} = \frac{\Delta C}{\Delta A}$$

11.11.2 Capacitive Pressure Transducers

The construction of a capacitive pressure transducer is shown in Figure 11.37. In this transducer, the change in spacing between the plates results in a change in capacitance. It is used to measure pressure in vacuum.

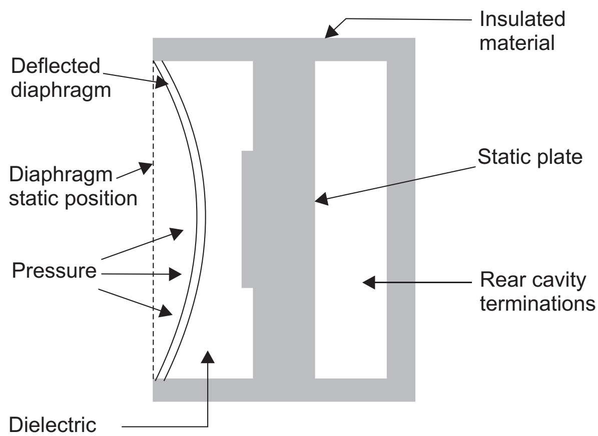


Fig. 11.37 A Capacitive Pressure Transducer

As can be seen from the figure, the transducer consists of a diaphragm and a fixed plate acting as a capacitor. When a pressure is applied on the diaphragm, it moves to the left. Due to this, the gap between the plates changes, thereby changing the capacitance of the transducer. The whole set-up is enclosed in an airtight container. This transducer can be used to receive sound waves, thus acting as a capacitive microphone. Its production is simple and inexpensive. It is a non-linear device and has a frequency range of 40 Hz to 15 kHz. An electrical circuit utilizing capacitive pressure transducer is shown in Figure 11.38.

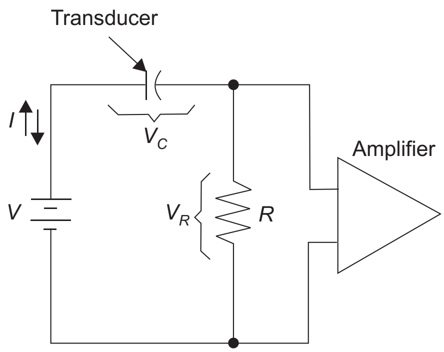


Fig. 11.38 Circuit Utilizing Capacitive Pressure Transducer

When no pressure is applied on the diaphragm, the voltage across transducer is equal to the supply voltage, that is,

$$V_C = V$$

This results in voltage drop across the resistor as there is no current flowing through the circuit. The charge on the capacitor (transducer) at this point is given as:

$$Q = CV$$

Now, when a pressure is applied to the diaphragm, the capacitance C varies, resulting in a variation in Q. Due to this variation in the charge of the capacitor, a current I flows through the circuit and the voltage across the resistor R drops. The voltage across R varies in accordance with the applied signal and thus, has the same waveform. This waveform is amplified using an amplifier for further processing.

11.11.3 Advantages and Disadvantages

Some of the advantages of capacitive transducer are as follows.

- They can be used in small systems as they require very small forces for their operation.
- They have very high sensitivity.
- Capacitive transducers can be used for studying dynamic behaviour as their frequency response is very good—as high as 50 kHz.
- Loading effects are very less due to high input impedance.

Some of the disadvantages of capacitive transducer are as follows.

- Measurement may get affected by stray capacitances and magnetic fields. Thus, error may occur.
- Temperature variation may reduce the accuracy in measurements due to temperature dependence of some dielectrics.
- They require a very complex circuit for their operation.
- Guard rings must be incorporated to remove non-linearity occurring due to edge effects. For low value capacitors, the stray electric field effects must be removed. This is also achieved using guard rings.

Example 7 A capacitive transducer has dimensions 45 mm length, 40 mm width, and a spacing of 0.45 mm between the plates. If a change of 4 pF is observed in the transducer capacitance due to displacement of the member under test, calculate the:

(a) change in displacement
(b) sensitivity

Solution: Given that: l = 45 mm, w = 40 mm, d = 0.45 mm, and ΔC = 4 pF

The capacitance of the transducer is given as:

$$C = \frac{\varepsilon A}{d} \qquad \text{[Refer to Eqn. (33)]}$$

where $A = l \times w = 45 \times 40 = 1800 \text{ mm}^2 = 18 \times 10^{-4} \text{ m}^2$

Substituting the values, we get:

$$C = \frac{8.854 \times 10^{-12} \times 18 \times 10^{-4}}{0.45 \times 10^{-3}} = 35.42 \text{ pF}$$

(a) When a displacement occurs, we have:

$$C - \Delta C = \frac{\varepsilon A}{d + \Delta d}$$

Substituting the values, we get:

$$d + \Delta d = \frac{8.854 \times 10^{-12} \times 18 \times 10^{-4}}{(35.42 - 4) \times 10^{-12}} = 0.50 \text{ mm}$$

Thus, the change in displacement Δd comes out to be:

$$\Delta d = (d + \Delta d) - d$$

$$\Rightarrow \qquad \Delta d = 0.50 - 0.45 = 0.05 \text{ mm}$$

(b) Sensitivity of the transducer is given by:

$$\text{Sensitivity} = \frac{\Delta C}{\Delta d} \qquad \text{[Refer to Eqn. (34)]}$$

$$\Rightarrow \qquad \text{Sensitivity} = \frac{4}{0.05} = 80 \text{ pF/mm}$$

11.12 MISCELLANEOUS TRANSDUCERS

Some of the transducers having their own features are described here. They include *load cells*, *hotwire anemometer*, *ultrasonic flow meters*, *seismic transducers*, *tachogenerators*, and *magneto-strictive transducers*.

11.12.1 Load Cells

A load cell is a combination of elastic member and strain gauge, used for weighing extremely heavy loads. Here, elastic members act as primary transducers while the strain gauges act as secondary transducers. The weight to be measured applies a stress on the elastic member which then produces a strain. This strain is measured to determine the weight. A load cell is shown in Figure 11.39.

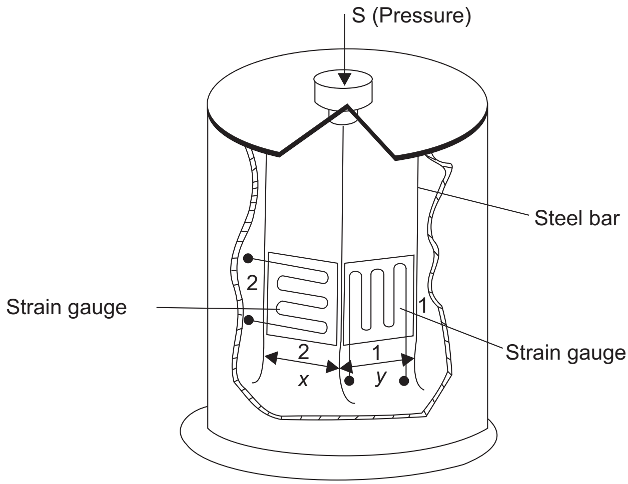


Fig. 11.39 A Load Cell

It consists of a steel bar, acting as an elastic member. When a stress S is applied in the direction shown the bar undergoes compression in the direction of applied stress. However, it experiences an expansion along x and y directions. This results in an increase in the resistance of gauge 2 while a decrease in the resistance of gauge 1. These two gauges along with the other two gauges on the remaining sides of the bar are connected in a bridge circuit and the sensitivity obtained is equal to four times the sensitivity of a single gauge bridge circuit. Due to this reason, a load cell can also measure very small values of the applied stress.

11.12.2 Hotwire Anemometer

Hotwire anemometers are used to study varying flow conditions and hence, are frequently used in research industry. A hotwire anemometer works on the principle of transfer of heat from the surface to the fluid flowing over it. Due to this transfer, the surface temperature reduces. This reduction is in relation with the rate of flow of fluid.

A hotwire anemometer consists of a fine wire placed in a flow stream to which heat is supplied electrically. Then the resistance of the wire is measured to determine its temperature. The measurement of temperature using a Wheatstone bridge is illustrated in Figure 11.40.

In this method, the current flowing through the wire is adjusted so as to keep the temperature constant. This ensures that the bridge is always balanced. Now, this current is measured by determining the voltage drop across the standard resistor connected in series with the wire. A potentiometer is used to measure this voltage drop (see Figure 11.40).

The loss of heat can be mathematically described as:

$$\text{Loss fo heat} = A(u\rho + B)^{1/2} \text{ J/s} \qquad \text{...(35)}$$

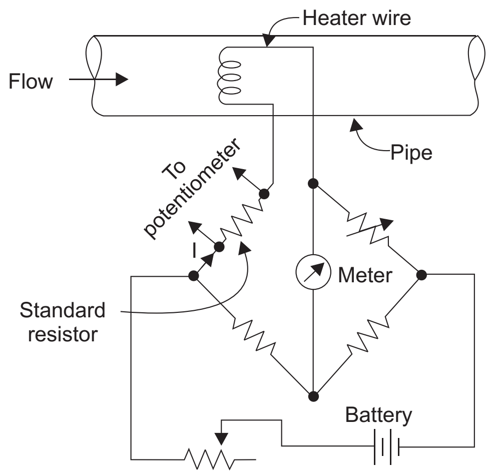


Fig. 11.40 A Hotwire Anemometer Connected in Wheatstone Bridge

where A and B are constants which depend upon physical properties and dimensions of the fluid and wire, u is the velocity of the flow of the heat, and ρ is the density of the fluid.

Let the current flowing through the wire be I. Thus, under equilibrium conditions, the heat generated must be equal to the heat lost. Heat generated in the wire is given as:

$$\text{Heat generated} = I^2R \qquad \text{...(36)}$$

where R is the resistance of the wire. Thus, from Equations (35) and (36), we get the equilibrium condition as:

$$A(u\rho + B)^{1/2} = I^2R$$

On rearranging the above equation, we get:

$$u = \frac{(I^4 R^2 / A^2) - B}{\rho} \qquad \text{...(37)}$$

Thus, the fluid flow rate can be determined by measuring the current flowing through the wire provided its temperature and resistance are kept constant.

11.12.3 Ultrasonic Flow Meters

Ultrasonic flow meters are used for the measurement of flow rate. The construction of an ultrasonic flow meter is shown in Figure 11.41 that consists of two piezoelectric crystals, one

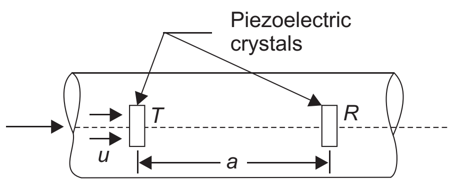


Fig. 11.41 Ultrasonic Flow meter

acting as transmitter while the other acting as receiver. The piezoelectric crystals are kept in a liquid or a gas at a distance a apart. An oscillator is used to provide a sinusoidal signal of 100 kHz to the transmitter which is then received by the receiver.

Let the time taken by the pulse to reach from transmitter to receiver be ΔT. When the signal is travelling in the direction of the flow the transition time ΔT_1 is given as:

$$\Delta T_1 = \frac{a}{u+v} \qquad \text{...(38)}$$

where v is the velocity of sound wave in the medium and u is the linear velocity of the flow. A commutating switch is used to periodically interchange the functions of the transmitter and receiver. When the signal is travelling in the opposite direction of flow the transition time ΔT_2, is given as:

$$\Delta T_2 = \frac{a}{v-u} \qquad \text{...(39)}$$

Subtracting Equation (38) from (39), we get:

$$\Delta T = \Delta T_2 - \Delta T_1 = \frac{2au}{v^2 - u^2} \qquad \text{...(40)}$$

The difference in these transition times is measured using a phase sensitive detector which is in synchronization with the commutator. Generally, we have:

$$v >> u$$

Thus, Equation (40) can be written as:

$$\Delta T \simeq \frac{2au}{v^2} \qquad \text{...(41)}$$

Hence, the difference in transition times ΔT is directly proportional to the flow velocity. However, due to uncertainty in the value of v, some error may occur in the above linear relationship.

Here, it is to be noted that the concept of phase shift can also be used to determine the flow. When the wave is travelling in the direction of flow the phase shift $\Delta\phi_1$, is given as:

$$\Delta\phi_1 = \frac{2\pi fa}{v+u}$$

where f is the frequency of the applied sinusoidal signal. When the wave is travelling in the opposite direction of the flow the phase shift $\Delta\phi_2$ is given as:

$$\Delta\phi_2 = \frac{2\pi fa}{v-u}$$

As we have stated earlier, due to the presence of the term v in Equation (41), some error may get introduced in the measurement. To remove this error, consider a frequency based system whose construction is shown in Figure 11.42. This arrangement consists of two oscillating systems which are self-excited due to feed back. The received pulses are used to trigger the transmitted pulses.

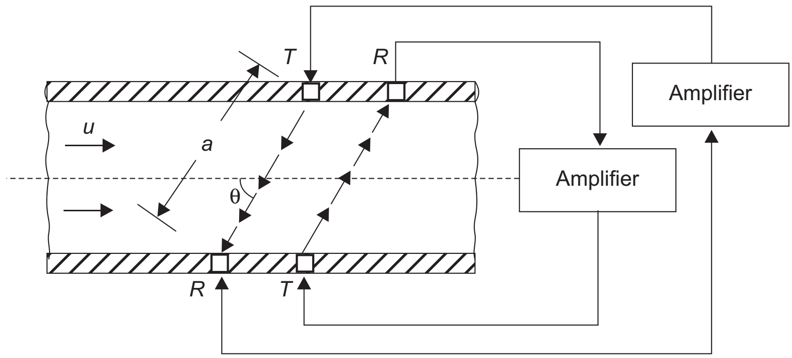


Fig. 11.42 Ultrasonic Flow Meter using Frequency Concept

The pulse repetition frequency in forward loop f_1 is given as:

$$f_1 = \frac{v + u\cos\theta}{a} \qquad ...(42)$$

The pulse repetition frequency in backward propagating loop f_2 is given as:

$$f_2 = \frac{v - u\cos\theta}{a} \qquad ...(43)$$

Subtracting Equation (43) from (42), we get:

$$\Delta f = f_1 - f_2 = \frac{2u\cos\theta}{a} \qquad ...(44)$$

From Equation (44), it can be seen that the output term is independent of v. Hence, no error occurs on account of it.

Advantages and disadvantages

Some of the advantages of ultrasonic flow meters are as follows.

- They have linear relationship between input and output.
- Temperature, density, and viscosity variations do not affect the measurement.
- No moving parts are present in them.
- They have excellent dynamic response.
- Bi-directional flow is possible.

Some of the disadvantages of ultrasonic flow meter are as follows.

- They have high cost.
- High complexity.

11.12.4 Seismic Transducers

Seismic transducers are used to measure the nature of vibrations. A vibration may be expressed as a sinusoidal displacement of the vibrating element. This type of vibration is defined by its frequency and amplitude. The displacement of a sinusoidal vibration can be given as:

$$x = x_m \sin \omega t$$

where x_m is the amplitude and ω is the angular frequency of the vibration.

The velocity u is given as:

$$u = \frac{dx}{dt} = x_m \omega \cos \omega t$$

and the maximum velocity u_o is equal to:

$$u_o = x_m \omega$$

The acceleration a of the vibration is specified as:

$$a = \frac{d^2x}{dt^2} = -x_m \omega^2 \sin \omega t$$

and the maximum acceleration a_o is equal to:

$$a_o = -x_m \omega^2$$

Thus, the vibrating nature of an element can be determined by measuring its displacement, velocity, or acceleration.

Figure 11.43 represents the constructional diagram of a seismic transducer. It consists of a spring and a damper arrangement which are used to connect a mass M to a housing frame. This housing frame is situated on the source of vibration. The characteristics of this source are determined by measuring the displacement between mass and housing frame as the mass remains fixed in its spatial position.

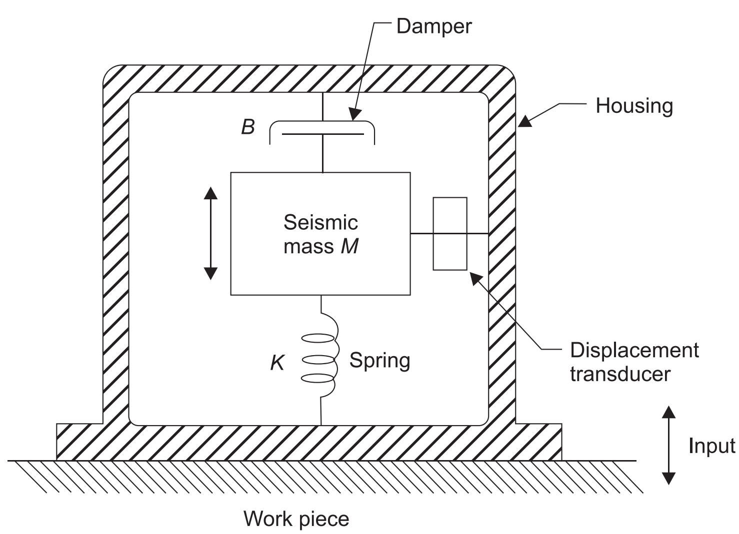


Fig. 11.43 Constructional Diagram of Seismic Transducer

Seismic transducers may be used in two different modes that are *displacement mode* and *acceleration mode*. The mode of operation to be selected is determined by the mass, spring and damper combinations. Usually, when seismic transducers are used in displacement mode, their input frequency must be greater than their natural frequency. This condition requires a soft spring (low value of spring constant) and large mass. Thus, displacement mode is suited for such systems. However, in acceleration mode, the input frequency must be lower than the natural frequency, requiring stiff spring (high value of spring constant) and small mass. Thus, acceleration mode is used for systems having small mass and a stiff spring. When seismic transducers are used in acceleration mode, they are also known as **accelerometers**. Generally, accelerometers are used for general purpose absolute motion, vibrations and shock measurement.

11.12.5 Tachogenerators

Tachometers are used for the measurement of angular velocity. They are of two types, namely, *mechanical tachometers* and *electrical tachometers*. Electrical tachometer generators, often called **tachogenerators**, provide electrical output and thus, have many advantages as compared to mechanical tachometers. Hence, electrical tachometer generators are preferable. They can be of two types—*DC tachometer generators* and *AC tachometer generators*.

DC tachometer generators

A DC tachometer generator is shown in Figure 11.44. The machine whose speed is to be measured is connected to an armature of the generator. This armature is made to revolve in the permanent magnet field to generate an emf. This emf is proportional to the product of speed and flux and as the flux is a constant quantity, the emf becomes directly proportional to the speed. A moving coil voltmeter, directly calibrated in terms of speed, is used to measure this generated voltage.

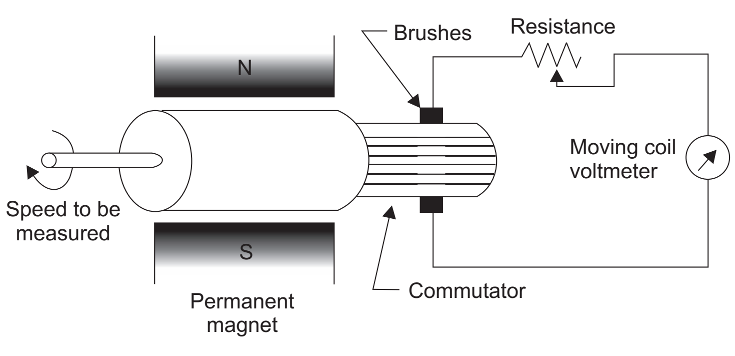


Fig. 11.44 DC Tachometer Generator

To determine the direction of rotation, polarity of the output voltage is considered. A series resistance is also connected in the circuit. This resistor is connected to limit the current through the device in case of a short circuit.

Advantages and disadvantages

Some of the advantages of DC tachometer generator are as follows.

- By noting down the polarity of the output voltage, the direction of rotation can be determined.
- Conventional DC voltmeters can be used to measure output voltage which is around 10 mV/rpm.

Some of the disadvantages of DC tachometer generator are as follows.

- Commutator and brushes may require periodic maintenance at regular intervals as contact resistance of brushes may vary and result in error.
- The meter should have a high resistance in comparison to the output resistance of generator to keep the current small, because a high current may distort the permanent magnetic field resulting in non-linearity.

AC tachometer generators

AC tachometer generators are used to overcome the limitation of DC tachometer generators. They employ a rotating magnet which can be either a permanent magnet or an electromagnet and the coil is wound on the stator. Thus, the problems caused by the commutator do not exist in these tachometer generators.

An emf is induced in the stator coil when the magnet is rotated whose magnitude and frequency both are in direct proportion to the rotational speed. Thus, to measure the speed of rotation, any one of the magnitude or frequency can be monitored. Figure 11.45 shows the circuit required when the magnitude is monitored to measure the speed of rotation.

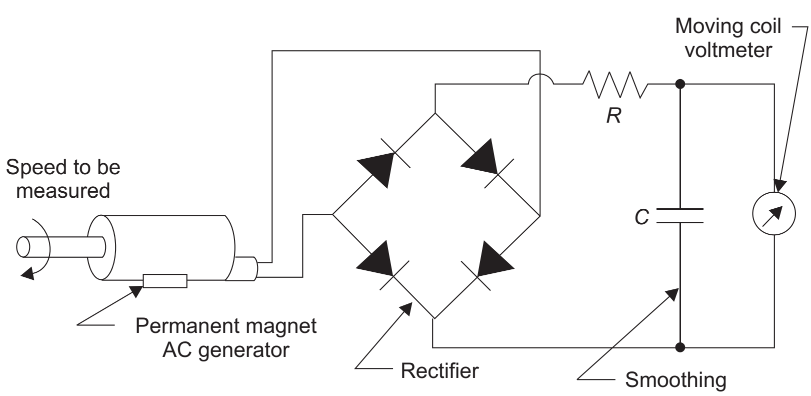


Fig. 11.45 AC Tachometer Generator using Amplitude of Voltage

A permanent magnet moving coil instrument is used to measure the output voltage and hence must be rectified prior to the measurement. Thus, a smoothing circuit is used as shown in the figure. Although ac tachometer generators are used to overcome the limitations provided by DC tachometer generators, they also have some limitations as described below.

- These devices may encounter some problem at low speeds. At low speed the output voltage has a low frequency which presents a problem while smoothening the ripples. Thus, they must be designed so as to have high frequency output voltage even at low speeds.
- The impedance of the coil of the tachometer increases with an increase in the frequency of the output voltage. Thus, high speed also causes some problems in the measurement. The input impedance of the device must be maintained larger than the impedance of coils to maintain linearity in the output.

11.12.6 Magnetostrictive Transducers

Since *B*-*H* curve of a ferromagnetic material is non-linear, its permeability changes when it is subjected to a changing magnetic field. This change in the permeability of the material causes a change in its dimensions. This effect is known as **magnetostriction** and is the basis of magnetostrictive transducers. This effect is strong in nickel-iron alloys, however, the change observed in dimensions, when magnetization is changed, is very small.

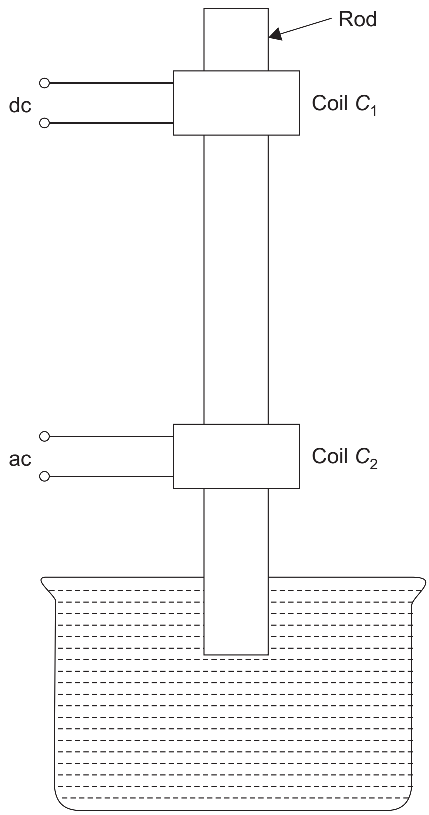


Fig. 11.46 A Magnetostrictive Transducer

Consider Figure 11.46, showing a magnetostrictive transducer used for generation of ultrasound. It consists of a rod and two coils C_1 and C_2. Coil C_1 is used to provide DC field while coil C_2 provides ac field. Let the rod be premagnetized using coil C_1. Now when the field produced by coil C_2 is imposed on it, there will be a change in its magnetization which further results in the change in the length of the rod at regular intervals.

The frequency of lengthwise oscillation of the rod generally lies in ultrasonic range or the high end of the audio frequency range. Thus, this arrangement can be used for the generation of ultrasonic oscillations. The ultrasonic oscillators may be further used for producing cavitation in liquids, formation of stable emulsions of mercury and water or of oil and water, and for breaking particles like bacterial cells.

Let us Summarize

1. The input quantity is generally non-electrical in nature and must be converted into an electrical quantity by a device known as **transducer**. Therefore, a transducer, also known as **pick up**, is defined as a device that converts one form of energy into another.
2. The conversion of any non-electrical quantity into electrical quantity by a transducer proves to be advantageous due to many reasons.
3. Transducers are of various types and are classified as primary and secondary transducers, analog and digital transducers, active and passive transducers, and transducers and inverse transducers.
4. Transducers can also be classified on the basis of the principle of transduction such as resistive, capacitive, optical, inductive, thermal, and so on. They are resistive, inductive, capacitive, piezoelectric, electromagnetic, thermoelectric, and photoelectric transducers.
5. Various points regarding the quantity to be measured, principle of the transducer, and required accuracy in the system must be kept in mind while choosing a transducer for a particular application.
6. A transducer which works on the concept of change in resistance due to any phenomenon is known as resistive transducer.
7. Strain gauge is a thin, wafer-like device used for measuring applied strain. It is basically a passive transducer which converts mechanical displacement into change of resistance.
8. Different types of strain gauges are available that include unbonded metal strain gauges, bonded metal wire strain gauges, bonded metal foil strain gauges, vacuum deposited thin metal film strain gauges, bonded semiconductor strain gauges, and diffused metal strain gauges.
9. A voltage can be generated in a circuit by joining two dissimilar metals at the ends and keeping both the junctions at different temperatures. This voltage so generated is temperature dependent and also depends on the material of the metals used which is known as thermoelectric effect.
10. The two types of thermoelectric transducers are resistance thermometer and thermistor.
11. The change in induction of the coil forms the basis of inductive transducers. The various inductive transducers include linear variable differential transformer, rotary variable differential transformer, and synchros.
12. When a mechanical stress is applied to some asymmetrical crystalline materials, a voltage is generated at their surfaces. This voltage is in proportion to the applied stress and becomes zero when the stress is removed. This effect is known as piezoelectric effect and the materials having this effect are known as piezoelectric.
13. In photoelectric transducers, when light strikes a special combination of materials, it may result in the flow of electrons, generation of a voltage, and a change of resistance. This property of light forms the basis of photoelectric transducers. They are classified into three types, namely, photoemissive, photoconductive and photovoltaic cells.

14. Mechanical elements used to convert the applied pressure into displacement, are known as force-summing devices. Elastic members are generally used for this purpose, such as diaphragms, bellows, and bourdon tubes, used as primary sensing elements in pressure transducers. The working principle of such devices is that the pressure sensitive element is pressed by the fluid whose pressure is to be measured.
15. The capacitive transducer depends on the parameters of parallel plate capacitor such as area of the plates, distance between the plates, and permittivity of the dielectric medium between the plates. Any change in these parameters, changes the capacitance.
16. Some of the transducers having their own features are load cells, hotwire anemometer, ultrasonic flow meters, seismic transducers, tachogenerators, and magnetostrictive transducers.

EXERCISES

Fill in the Blanks

1. RVDT is used for measuring _________ displacement.
2. Seismic transducers can be used in _________ and _________ mode.
3. An error detector made using synchro consists of synchro _________ and _________.
4. A load cell is made up of _________ and _________.
5. Tachometers are used for the measurement of _________.

Multiple Choice Questions

1. A transducer converts
   (a) mechanical energy into electrical energy
   (b) mechanical displacement into electrical signal
   (c) one form of energy into another form of energy
   (d) electrical energy into mechanical form
2. LVDT is a:
   (a) capacitive transducer (b) resistive transducer
   (c) inductive transducer (d) none of these
3. Thermistor is used for the measurement of
   (a) temperature (b) pressure
   (c) flow (d) displacement
4. Piezoelectric transducers are
   (a) passive transducer (b) active transducer
   (c) inverse transducer (d) both (b) and (c)
5. The gauge factor is defined as
   (a) $\dfrac{\Delta l}{l} \Big/ \dfrac{\Delta R}{R}$ (b) $\dfrac{\Delta R}{R} \Big/ \dfrac{\Delta l}{l}$
   (c) $\dfrac{\Delta R}{R} \Big/ \dfrac{\Delta d}{d}$ (d) $\dfrac{\Delta R}{R} \Big/ \dfrac{\Delta \sigma}{\sigma}$

State True or False

1. LVDT is used to measure pH values.
2. The relationship between the output of ultrasonic flow meter and flow velocity is non-linear.
3. Synchro is also known as Selsyn.
4. Spring loaded bellows reduce the life of bellows.
5. Tachometers are used to measure vibrations.

Descriptive/Numerical Questions

1. What is the basic principle of strain gauge based measurements? Write down the expression for the gauge factor of a strain gauge in terms of poisson's ratio (μ).
2. What is the difference between:
   (a) Active and passive transducers
   (b) Primary and secondary transducers
   (c) Transducer and inverse transducers
   (d) Analog and digital transducers
3. Explain how to use a bonded resistance wire strain gauge.
4. Show the construction of LVDT. Explain its operation and list any three advantages.
5. Describe the method for measurement of temperature with the use of RTD and thermistor.
6. With the help of a neat diagram explain the working of:
   (a) Hotwire anemometer
   (b) Ultrasonic flow meter
7. A resistance strain gauge with a gauge factor of 4 is connected to a steel member which is subjected to a strain of 1×10^{-6}. If the original gauge resistance is 150 Ω, calculate the change in resistance.
8. Describe the working and construction of resistance thermometers.
9. Describe the various factors influencing the type of transducer for a particular application.
10. Explain the photoconductive and photovoltaic cells.
11. Explain in detail the working of tachometers and magnetostrictive transducers.
12. What are the two modes in which a seismic transducer can work? Explain in detail.
13. What do you understand by piezoelectric effect? How can it be used to measure pressure?
14. Write short notes on:
    (a) Piezoelectric transducer
    (b) Photoelectric transducer
    (c) Thermistor
    (d) Capacitive transducer

12

Generation of Signals

Chapter Objectives

After reading this chapter, you will be able to:

- Appreciate the concept of generation of signals
- Discuss the pulse- and square-wave generators along with their waveforms
- Explain the working of function generator that produces various waveforms such as triangular, pulse, sinusoidal, square, and ramp wave
- Describe the arbitrary waveform generator that produces virtually any desired waveform
- Describe audio frequency signal generator with its components such as oscillator circuit, sine-to-square wave converter, and attenuator circuit
- Describe radio frequency signal generator with its components such as oscillator circuit, amplifier circuit, and attenuator circuit
- Explain the working of frequency synthesizer signal generator
- Discuss the generation of frequency over a wide range in a sweep frequency generator
- Explain the generation of video signals in a video signal generator

12.1 INTRODUCTION

Signals are generated by signal generators in various forms, such as sine wave, square wave, pulse wave, and sweep wave. These signals may lie in low-frequency range to radio-frequency range. Based on the type of signal generated, the signal generators can be categorized as *sine-wave generators*, *pulse generators*, *sweep frequency generators*, *oscillators*, and *function generators*. Among these, the low-frequency signal generators can provide a maximum output voltage adjustable in the range of 0 to 10 V with a frequency of 100 kHz. The function generators are generally low-frequency generators and are capable of generating three types of waveforms, namely, *sine waveform*, *square waveform*, and *triangular waveform*. On the other hand, the radio-frequency signal generators have different circuit requirements, such as an output level meter, a calibrated attenuator, and a radio frequency screening. In some high-frequency signal generators, like frequency synthesizer, the output signal is stabilized with the aid of a piezoelectric crystal. However, the frequency can be adjusted by employing a phase-locked-loop technique. It is to be noted here that no energy is created by an oscillator, it only converts a DC source into an AC source at a particular frequency.

Pulse generators are used to generate pulse waveforms which can be adjusted in their amplitude, width, and repetition frequency. Some special pulse generators may also incorporate the controlling mechanism for controlling the rise time, delay time, fall time, and DC bias level of the waveform. On the contrary, a sweep signal generator generates a sine waveform in which the frequency increases from a minimum value to the maximum value. This increment occurs over a specific time period. In addition, a ramp voltage is also generated with an amplitude proportional to the instantaneous frequency. The sweep signal generators find application in analyzing the frequency response of various electronic instruments.

Some characteristics to be followed for all types of signal generators are: (i) the frequency of the signal should be known and stable, (ii) the output signal should be free from distortion, (iii) and the output signal amplitude should be controllable from very small to large values.

12.2 PULSE- AND SQUARE-WAVE GENERATORS

Pulse- and square-wave generators are generally used with an oscilloscope for measurement purposes. The generated waveforms are displayed on the oscilloscope screen; they provide sufficient information about the system under test. The major difference between both of these generators lies in their duty cycle which can be defined as the ratio of the width of the pulse and the time period or **pulse repetition time** it takes. Mathematically, it can be given as:

$$\text{Duty cycle} = \frac{\text{Width of the pulse}}{\text{Time period of the pulse}} \quad \text{...(1)}$$

It can also be defined as the ratio of the average value of the generated pulse over one cycle and the peak value. It is to be noted here that the average as well as peak value of a pulse are inversely related to its time duration. Thus, the duty cycle can also be expressed as:

$$\text{Duty cycle} = \frac{\text{Average value of the pulse}}{\text{Peak value of the pulse}} \quad \text{...(2)}$$

It must be noted here that the duty cycle of a square wave is always equal to 0.5 since a square-wave generator produces a waveform in which there are equal time periods for both *on* and *off* portions (that is, equal positive and negative half cycles). Irrespective of the operating frequency, the duty cycle of a square wave always remains 0.5 or 50% as depicted in Figure 12.1.

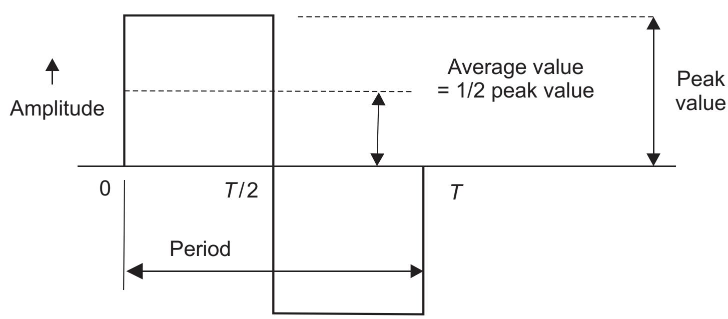


Fig. 12.1 A Square Waveform

In case of a pulse wave, these time periods may differ on the basis of its *on* and *off* periods and it can be expressed as:

$$\text{Duty cycle} = \frac{\text{Average value of the pulse}}{\text{Peak value of the pulse}} = \frac{t}{T}$$

In such a case, the average value is given as $\frac{t}{T} \times \text{peak value of the wave,}$ where t and T represent the time periods of the *on* portion of the pulse width and of the entire one cycle of the wave, respectively. A pulse waveform is shown in Figure 12.2.

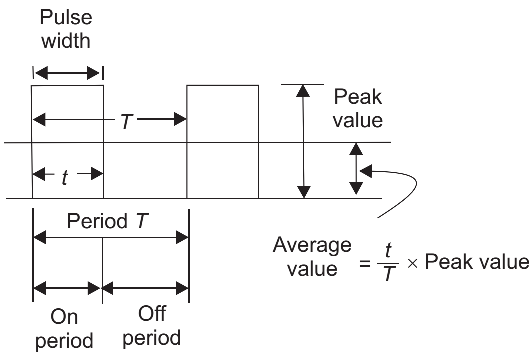


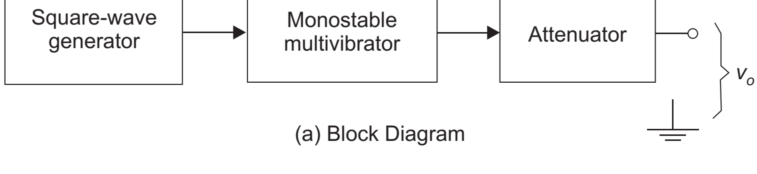
Fig. 12.2 A Pulse Waveform

The variation in the value of the duty cycle of a pulse wave depends upon its *on* time period and generally lies in the range of 50% to 95%. The duty cycle of a short duration pulse is ganerally low and vice versa. Furthermore, the power dissipated by the components under test can be lowered with these short duration pulses. The pulse generator supplies more power during its *on* period in comparison to a square-wave generator.

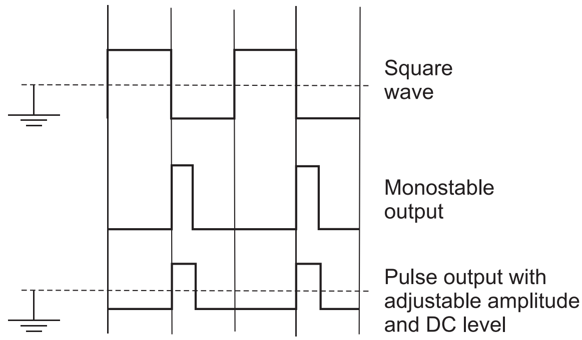
12.2.1 Block Diagram of Pulse-Wave Generator

A pulse-wave generator basically consists of a square-wave generator, a monostable multivibrator, and an output attenuator stage. This is shown in Figure 12.3 with the corresponding output waveforms at each stage.

It can be seen from the figure that the negative-going edge of the output of the square-wave generator triggers the monostable multivibrator, which in turn produces a pulse waveform of constant width. The frequency of this waveform is determined by the square-wave frequency. In addition, the amplitude of this pulse waveform can be adjusted by the output attenuator. Furthermore, the output attenuator can also adjust the DC level of the pulse.



(a) Block Diagram



(b) Corresponding Waveforms

Fig. 12.3 Basic Pulse Generator

The detailed description about each block of Figure 12.3(a) is discussed here

Square-wave generator or astable multivibrator

An astable multivibrator, also known as **free-running multivibrator** is an instrument which provides continuous oscillations between high and low output states and thus, there is no stable state. This is considered as the simplest square-wave generator. The configuration of a basic op-amp astable multivibrator as square-wave generator circuit is shown in Figure 12.4.

The op-amp charges the capacitor C_a through resistor R_a as shown in the figure. With the aid of resistances R_b and R_c, the op-amp works like an inverting type Schmitt trigger circuit. It is evident from Figure 12.4(b) that with the positive maximum level (marked as $V_{o,\text{sat}}$) of the op-amp output, C_a charges positively till the voltage across it, v_{Ca}, gets charged up to the *upper trigger point* or UTP of the Schmitt trigger. At this instant, the output signal of the op-amp attains its lowest possible level (marked as $-V_{o,\text{sat}}$). The low op-amp output causes the charging current I_a of capacitor C_a to flow in the opposite direction such that C_a may discharge and again get charged with a negative voltage. Now, the instant when voltage v_{Ca} across capacitor C_a reaches the *lower trigger point* or LTP of the Schmitt trigger, the output of the op-amp again attains its highest level. This process continues and provides a square waveform at the Schmitt trigger output. In addition, an exponential waveform also gets produced across capacitor C_a. It must be noted here that the time period t, the capacitor takes to charge from LTP to UTP determines the frequency f of the square waveform and it is given as:

$$f = \frac{1}{2t} \qquad \text{...(3)}$$

By adjusting the value of resistance R_a, this time period t can be varied, thereby controlling the frequency f. The calculation of t can be done using the following equation:

$$e_C = E - (E - E_o)\varepsilon^{t/RC}$$

which can be re-arranged as:

$$t = RC \ln\left(\frac{V_o - \text{LTP}}{V_o - \text{UTP}}\right) \qquad \text{...(4)}$$

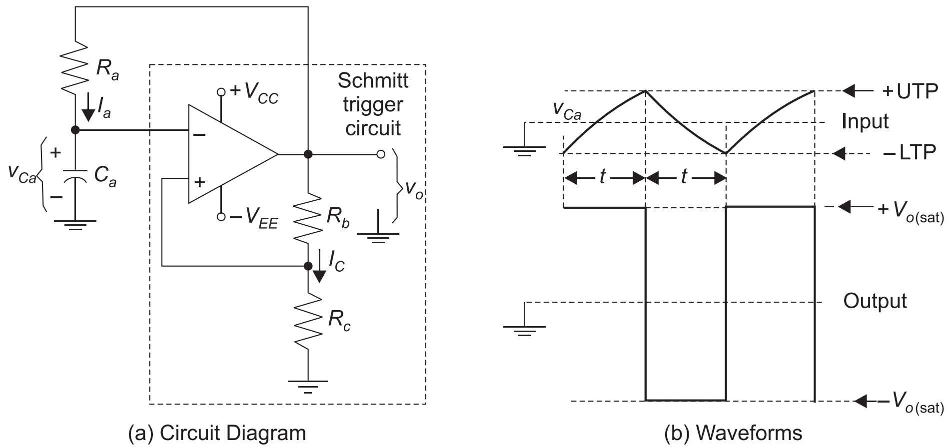


Fig. 12.4 Astable Multivibrator

The upper and lower voltage levels of the Schmitt trigger are obtained using the relation:

$$|\mathrm{UTP}| = |\mathrm{LTP}| = V_o\left(\frac{R_c}{R_b + R_c}\right) \quad ...(5)$$

If these voltage levels are smaller than the output voltage, then the charging current I_a of the capacitor C_a is considered to be approximately constant, which, in turn, charges the capacitor at a constant rate expressed as:

$$t = \frac{C\Delta V}{I} \quad ...(6)$$

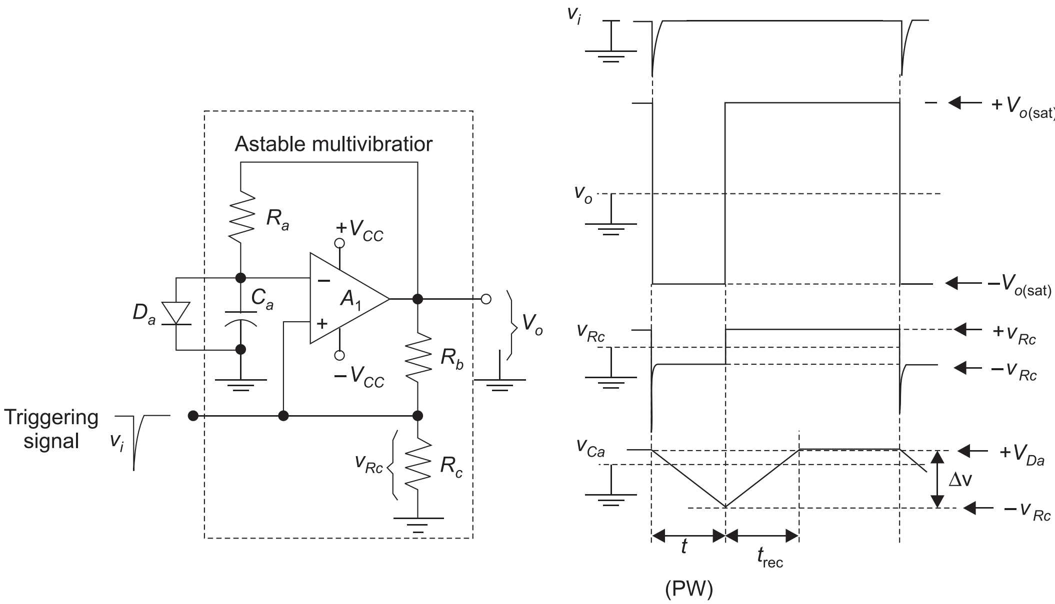
where ΔV represents the difference between UTP and LTP, and I is equal to the charging current I_a and can be given as:

$$I = I_a \approx \frac{V_{o,\mathrm{sat}}}{R_a}$$

Equation (6) can be used in place of Equation (4) to calculate the time period the capacitor takes to charge between the LTP and UTP.

Monostable multivibrator

Monostable multivibrator, unlike an astable multivibrator, is one which provides one stable state. The construction of its circuit is similar to that of the astable multivibrator, except that an additional diode is incorporated in the circuit and a triggering pulse is to be applied to it. The circuit is depicted in Figure 12.5(a) with its corresponding waveforms shown in Figure 12.5(b).



(a) Circuit Diagram (b) Corresponding Waveforms

Fig. 12.5 Monostable Multivibrator

When a triggering pulse is applied to the circuit, the state of its output signal gets changed and remains in that state for a specific time period. After this, the signal gets back to its initial state and thus, a constant-width pulse waveform is generated at the output whenever a triggering pulse is applied.

When the output signal of the op-amp is at its positive maximum level $V_{o,\text{sat}}$, the capacitor C_a charges positively. However, the presence of diode D_a does not allow the capacitor to charge higher than the voltage drop of D_a, represented as V_{Da}. At this instant, the voltage across resistance R_c represents the UTP of the Schmitt trigger circuit. It can be seen from the figure that voltage V_{Da} is applied to the op-amp at its inverting input terminal while voltage v_{Rc} is applied at its non-inverting terminal. Now, when v_{Rc} is greater than V_{Da}, the output of the op-amp remains at its positive saturation level, that is, $V_{o,\text{sat}}$.

On the other hand, when a negatively-going triggering pulse is applied to the non-inverting terminal of the op-amp, its output signal instantly switches to its negative saturation level, that is, $-V_{o,\text{sat}}$. This is because on applying a negative pulse makes the non-inverting terminal voltage below the level of voltage V_{Da} at its inverting terminal. In addition, the change in the output level (from $+V_{o,\text{sat}}$ to $-V_{o,\text{sat}}$) in turn changes the state of voltage v_{Rc} as well from $+v_{Rc}$ to $-v_{Rc}$. This provides a bias voltage at the non-inverting input terminal so that the output remains at $-V_{o,\text{sat}}$.

The negative output causes the capacitor to discharge through resistance R_a thereby charging again with this negative voltage. This causes the inverting input terminal of the op-amp to be more negative. At the instant, when the capacitor voltage becomes greater than the voltage across resistance R_c, the inverting terminal voltage of the op-amp becomes more negative as compared to that of the non-inverting terminal which in turn switches the op-amp output signal from $-V_{o,\text{sat}}$ to $+V_{o,\text{sat}}$ again. The change in the output signal again changes the state of voltage across resistance R_c from $-v_{Rc}$ to $+v_{Rc}$ so that the op-amp output signal level can be held at its state, that is, at $+V_{o,\text{sat}}$.

Figure 12.5(b) reveals that whenever the circuit is triggered, a negatively-going pulse gets produced at the output. The width of this pulse is determined by the value of resistance R_a, capacitance C_a, and voltage v_{Rc}. Here, note that the time period between two consecutive trigger pulses is known as **recovery time** and denoted by t_{rec}. If the circuit is triggered before this time period, the width of the output pulse may get affected. However, this pulse width can be controlled if resistance R_a is variable. Also, by using different capacitors in place of C_a, the range of the pulse width can be changed. The width of the output pulse can be determined using equation:

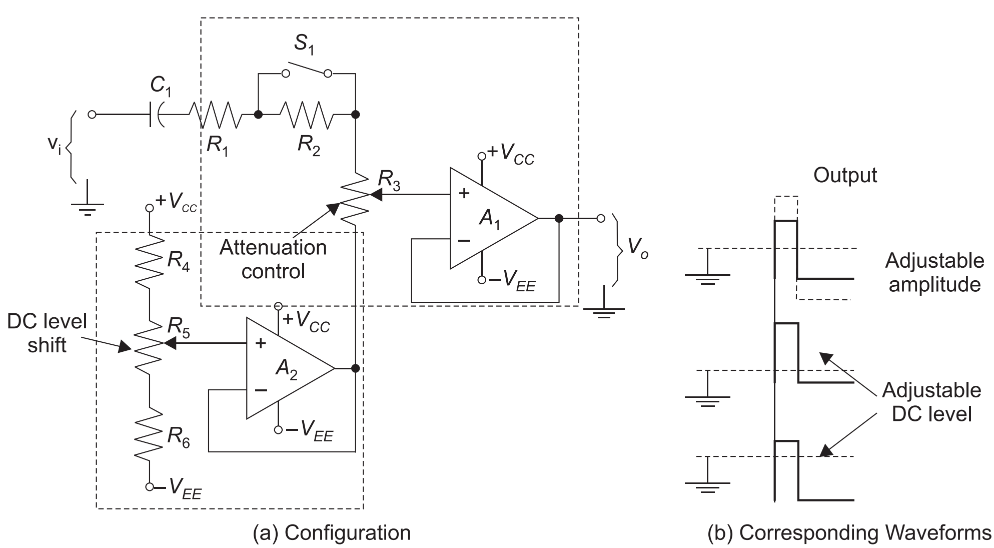
$$\text{PW} = RC \ln\left(\frac{v_o - \text{LTP}}{v_o - V_{Da}}\right) \qquad ...(7)$$

The capacitor charging equation given by Equation (6) can be used here if the saturation voltage $V_{o,\text{sat}}$ is considerably greater than LTP and V_{Da}.

Output attenuator

The output attenuator offers low impedance at the output and allows the amplitude of the output signal of the pulse generator to be adjusted. The configuration of an output attenuator and DC offset control is shown in Figure 12.6 in which op-amp A_1 together with resistances R_1, R_2, and R_3 make an output attenuator while op-amp A_2 with resistances R_4, R_5, and R_6 constitute an arrangement for DC offset control.

The DC offset control circuit in the above figure is responsible for DC level shifting and thus for providing control on the DC offset. Capacitor C_1 is meant to pass the pulses generated by the pulse generator to the output attenuator circuit. Here, resistance R_3 is **attenuation control** which sets the DC output level of the attenuator to any desired level.



(a) Configuration (b) Corresponding Waveforms

Fig. 12.6 Output Attenuator and DC Offset Control

Op-amp A_2 is essentially a voltage follower circuit, the DC output voltage level of which can be set by the variable resistance R_5, known as **DC level shift control**. If the moving contact of R_5 is set to the ground level, the output of amplifier A_2 also comes up at the same level. This in turn provides a pulse at the output of amplifier A_1 which is symmetrical above and below the ground level. On the contrary, if the moving contact of R_5 is set to +5 V, then the pulse output is obtained as a symmetrical pulse above and below +5 V voltage level. Also, if the contact is set to a position at –5 V, the output signal so obtained is symmetrical above and below the –5 V level.

A pulse generator provides an output signal with a frequency in the range of 0.0001 Hz to 20 MHz. An arrangement is provided to vary this range in decade steps. The peak-to-peak amplitude of the output signal can be varied in the range of 3 mV to 30 V. Also, the waveform produced may be normal or inverted with aid of a DC offset of +15 V applicable to the output signal. In addition, the width of the output pulse wave can be adjusted between 25 ns to 1 ms. This range can also be varied in decade steps. It must be noted here that the triggering can be done manually or in a continuous manner or the output pulse can even be gated by an external source. The circuit may also include some mechanism to provide the pulse delay facility.

Example 1 Determine the frequency of the output square-waveform of an astable multivibrator which is given with following specifications:

$$R_a = 25\ \text{k}\Omega,\quad R_b = 6.5\ \text{k}\Omega,\quad R_c = 5.5\ \text{k}\Omega,\quad C_a = 0.3\ \mu\text{F},\quad \text{and}\quad V_{CC} = 12\ \text{V}$$

Solution: The output voltage V_o can be obtained as:

$$V = V_{o,\text{sat}} \approx \pm(V_{CC} - 1\ \text{V}) = \pm(12 - 1)\ \text{V} = \pm 11\ \text{V}$$

The trigger points UTP and LTP can be determined as:

$$|\text{UTP}| = |\text{LTP}| = V_o\left(\frac{R_c}{R_b + R_c}\right) \qquad \text{[Refer to Eqn. (5)]}$$

Substituting the required values, we get:

$$|\text{UTP}| = |\text{LTP}| = \pm 11\left(\frac{5.5\times 10^3}{6.5\times 10^3 + 5.5\times 10^3}\right) = \pm 5.04\ \text{V}$$

Now the time period t of the waveform can be obtained as:

$$t = R_a C_a \ln\left(\frac{V_o - \text{LTP}}{V_o - \text{UTP}}\right) \qquad \text{[Refer to Eqn. (4)]}$$

Substituting the required values, it comes out as:

$$t = 25\times 10^3 \times 0.3\times 10^{-6} \ln\left(\frac{11-(-5.04)}{11-5.04}\right) = 7.425\ \text{ms}$$

The frequency of the waveform can be determined as:

$$f = \frac{1}{2t} \qquad \text{[Refer to Eqn. (3)]}$$

Substituting the value of t in the above relation, we get:

$$f = \frac{1}{2 \times 7.425 \times 10^{-3}} = 67.3 \text{ Hz}$$

12.3 FUNCTION GENERATOR

Function generator is an instrument capable of producing various waveforms such as triangular wave, pulse wave, sinusoidal wave, square wave, and also ramp wave. Any of these waveforms can be generated with the desired frequency and amplitude ranges. The DC offset adjustment feature is also incorporated. A number of outputs of the generator can be obtained at the same time. The basic block diagram of a function generator is illustrated in Figure 12.7.

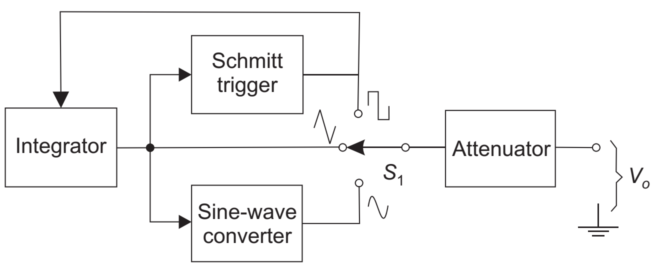


Fig. 12.7 Block Diagram of a Function Generator

The function generator consists of an integrator, Schmitt trigger, sine-wave converter, and attenuator. The integrator circuit produces a triangular waveform at its output. This waveform is basically a negative-going ramp signal. The output signal of the integrator circuit is fed to the Schmitt trigger circuit as well as to the sine-wave converter circuit. The Schmitt trigger converts this triangular signal into a square wave while the sine-wave converter converts it into sinusoidal waveform. Let us discuss each of these blocks in detail.

12.3.1 Integrator Circuit

The circuit of a basic integrator is depicted in Figure 12.8. It can be seen from the figure that the non-inverting terminal of the op-amp is held at ground potential which ensures that the inverting terminal voltage is also at the same potential always. The inverting input terminal is connected to op-amp output through a capacitor C_1. Thus, if initially, there is no charge in C_1, the voltage at the output becomes zero. Now consider that C_1 gets charged with positive charge on the right plate and negative charge on the left plate with the terminal voltage of 1 V, the output voltage also comes out to be +1 V with respect to the ground, while the inverting terminal still remains at ground potential. Till the capacitor terminal voltage remains constant, all circuit voltages retain their stable values. On the other hand, if C_1 is charged

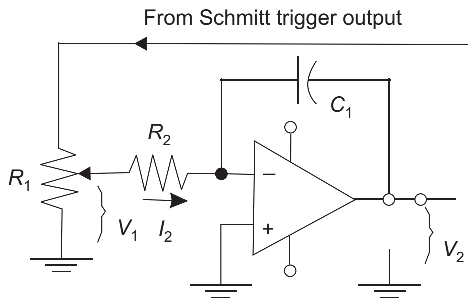


Fig. 12.8 Integrator Circuit

with the opposite polarity (that is, with positive charge on the left plate and negative charge on the right plate), the output voltage comes out to be – 1 V with respect to ground; while the inverting terminal is still at the ground potential.

Now suppose that a positive input voltage $+v_1$ is fed to the circuit via resistance R_2. This causes a constant current I_2 to flow through this resistance since the left terminal of R_2 is at $+v_1$ while its right terminal is at ground level (as non-inverting terminal is at ground potential). Therefore, current $I_2 = v_1/R_2$ is much larger than the op-amp input bias current. Now I_2 flows through C_1 and charges it with positive charge on the left plate and negative charge on the right plate. On charging, the voltage across C_1 increases linearly while the op-amp output voltage decreases linearly since the left plate of the capacitor is held at ground potential. On the other hand, if a negative input voltage is applied, that is, $-v_1$, current I_2 is reversed and capacitor C_1 charges with opposite polarity, the integrator output voltage also gets reversed. It is to be noted here that the input voltage fed to the integrator is obtained from the Schmitt trigger output connected next to it. The voltage v_1 as well as current I_2 are controlled by potentiometer R_1.

12.3.2 Schmitt Trigger Circuit

The working of a Schmitt trigger circuit is shown in Figure 12.9. The inverting input terminal is at ground potential.

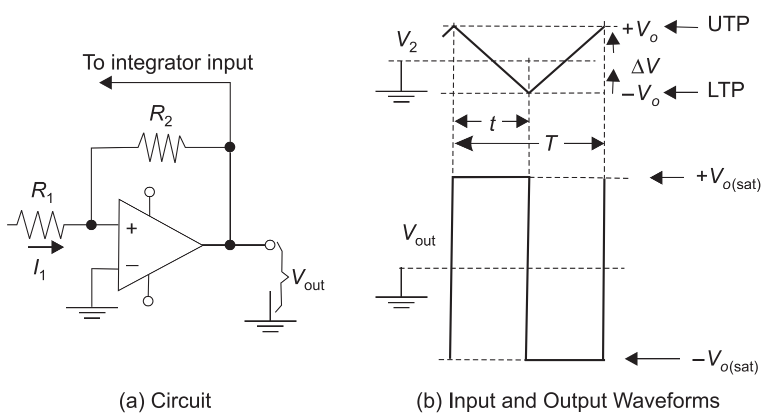


Fig. 12.9 Schmitt Trigger Circuit

As stated earlier, the output of the integrator circuit is fed to the Schmitt trigger input. When this input voltage v_2 increases to the LTP of the Schmitt trigger, it causes the polarity of the output voltage v_{out} to drop rapidly from most positive level to its most negative level [see Figure 12.9(b)]. Also, when this input voltage v_2 reaches the UTP of the Schmitt trigger, the polarity of the output voltage v_{out} rises from its most negative level to its most positive level. Note that to drive the output of the op-amp to saturation (either in positive or negative direction), a very small voltage difference is required between inverting and non-inverting terminals.

The combined operation of integrator and Schmitt trigger circuits can be explained as when the output voltage v_{out} of Schmitt trigger is positive, the integrator input voltage v_1 is positive while its output is at ground level. Thus, capacitor C_1 charges through current I_2 with positive charge on left plate and negative on right plate. The integrator output voltage v_2 decreases linearly reaching the LTP of the Schmitt trigger resulting in negative output voltage of the Schmitt trigger, $-v_{\text{out}}$. Now, the polarity of v_1 is reversed resulting in I_2 in reverse direction and C_1 charges with opposite polarity. Therefore, the output voltage of integrator v_2 increases to attain a value equal to UTP of the Schmitt trigger. Now, when v_2 becomes equal to the Schmitt trigger UTP, its output rapidly switches the polarity and again becomes positive. Thus, the input voltage to the integrator also becomes positive. In addition, current I_2 charges the capacitor C_1 positively which causes the voltage v_2 to become negative. This process repeats itself again and again producing a triangular waveform at the integrator output and a square waveform at the Schmitt trigger output. Figure 12.9(b) reveals that the square wave becomes positive when the triangular wave is negative and vice-versa. It must be noted here that the time period, which the capacitor C_1 needs to charge from the UTP to LTP and vice-versa determines the frequency of the output waveforms. Here, again the capacitor charges linearly according to the following equation

$$C_1 = \frac{I_2 t}{\Delta V} \qquad \text{[Refer to Eqn. (6)]} \quad ...(8)$$

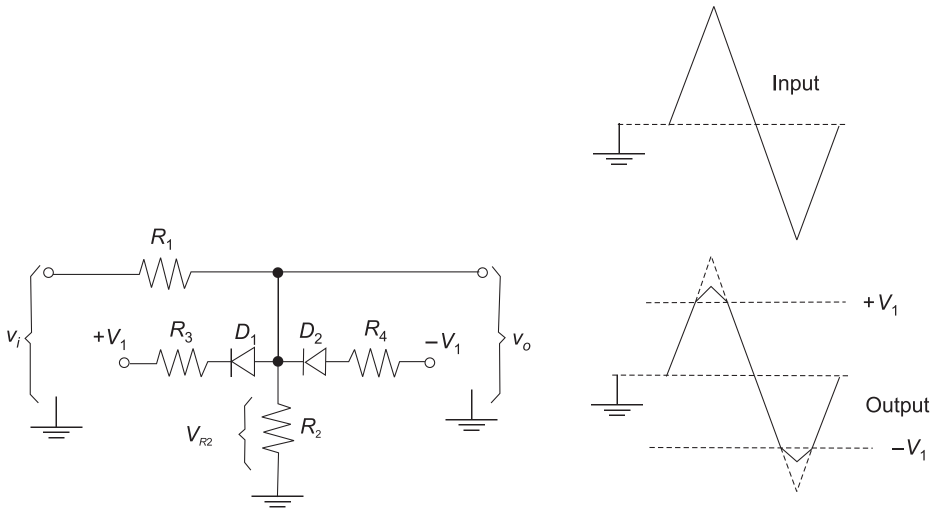
where ΔV represents the difference between UTP and LTP. Note that t is equal to half of the time period of the square waveform and its value gets changed when I_2 is adjusted through R_1. Thus, we may say that the potentiometer R_1 also controls the frequency of the waveforms. Furthermore, different frequency ranges can be selected by connecting different capacitors in the circuit.

12.3.3 Sine-Wave Converter

The triangular waveform from the integrator circuit is fed into the sine-wave converter circuit. This circuit is responsible for converting the triangular waveform into a sinusoidal wave. This can be done using diodes and resistors shown in Figure 12.10.

To understand the concept of this conversion, let us first consider that diodes D_1, D_2, and resistors R_3 and R_4 are not present in the circuit. In that case, the circuit would work as a simple voltage divider, producing an output as an attenuated version of the input signal. The output signal can then be expressed as:

$$v_o = v_i \frac{R_2}{R_1 + R_2} \quad ...(9)$$



(a) Circuit

(b) Input and Output Waveforms

Fig. 12.10 Sine-Wave Conversion

Now assume that diode D_1 and resistor R_3 are connected in the circuit. The circuit would still act as a voltage divider circuit till the moment voltage V_{R2} surpasses voltage $+V_1$, thereby making diode D_1 forward biased. As a result, resistors R_2 and R_3 become parallel to each other. In this case, the output voltage can be expressed as:

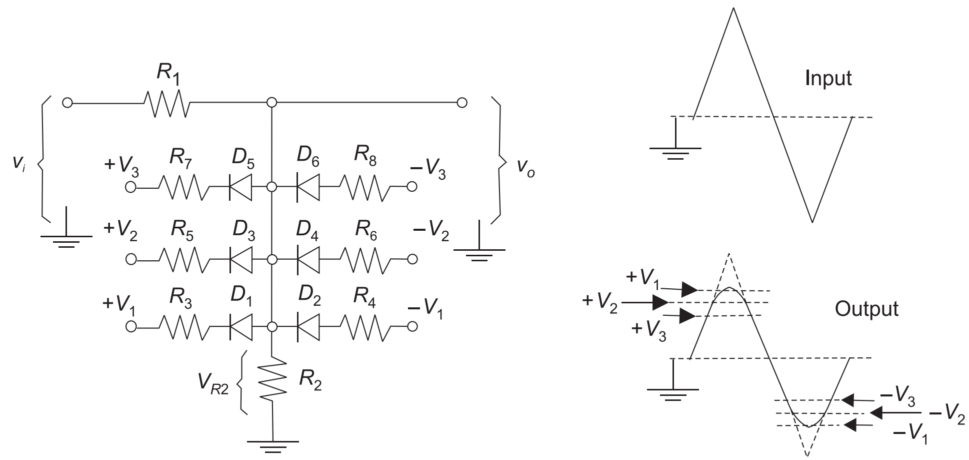
$$v_o = v_i \frac{R_2 \parallel R_3}{R_1 + (R_2 \parallel R_3)} \qquad \text{...(10)}$$

The output voltage in such a case becomes even more attenuated. The attenuation above the $+V_1$ voltage level is more effective than below the level $+V_1$. This causes slight steep in the output voltage unlike the previous case (that is, in the absence of D_1 and R_3). Whenever it gets a value less than $+V_1$, diode D_1 becomes reverse biased, as a result of which, the parallel connection of R_2 and R_3 no more exists. Thus, the output voltage once again becomes the same as expressed by Equation (9). This case remains the same for a negative half cycle at the input voltage until the output voltage attains a value less than $-V_1$. This causes diode D_2 to operate in forward biased condition, thereby making a parallel connection between resistors R_2 and R_4. The output voltage in this case can be given as:

$$v_o = v_i \frac{R_2 \parallel R_4}{R_1 + (R_2 \parallel R_4)} \qquad \text{...(11)}$$

It must be noted here that the value of resistors R_3 and R_4 must be the same in order to make the attenuation identical to the positive and negative half-cycles. This process is repeated again and again to get an approximation of a continuous sine wave. If more diodes and resistors (with different bias levels) are connected to the circuit, an improved approximation of the sine wave can be obtained at the output. Figure 12.11 depicts such a configuration.

In the figure, it is evident that three positive and three negative bias voltage levels are obtained when the loading is done using six diodes and six resistors. This causes the output voltage slope to change thrice in every quarter cycle, thereby producing a better sine wave.



(a) Circuit (b) Input and Output Waveforms

Fig. 12.11 Improved Sine-Wave Conversion

Example 2 An integrator circuit with R_1 = 500 Ω, R_2 = 4.7 kΩ, C_1 = 0.3 μF, and I_2 = 1.17 mA has to be connected to a Schmitt trigger circuit with its components given as $R_{1\,\text{Sch}}$ = 2.7 kΩ, $R_{2\,\text{Sch}}$ = 15 kΩ, and V_{CC} = ±12 V. Obtain:

(a) UTP and LTP of the square waveform

(b) peak-to-peak amplitude of the triangular waveform

(c) frequency of the triangular waveform.

Solution: Given that: V_{CC} = ±12 V

For integrator circuit: R_1 = 500 Ω, R_2 = 4.7 kΩ, C_1 = 0.3 μF

For Schmitt trigger: $R_{1\,\text{Sch}}$ = 2.7 kΩ, $R_{2\,\text{Sch}}$ = 15 kΩ

At the trigger points of the square waveform, the voltage is given as:

$$V_+ = 0 \text{ V} \quad \text{and} \quad V_{R2\,\text{Sch}} \approx V_{CC} - 1 \text{ V} = 12 \text{ V} - 1 \text{ V} = 11 \text{ V}$$

The current through resistor $R_{2\,\text{Sch}}$ can be obtained as:

$$I_{2\,\text{Sch}} = \frac{V_{R2\,\text{Sch}}}{R_{2\,\text{Sch}}}$$

Substituting the required values, we get:

$$I_{2\,\text{Sch}} = \frac{11}{15 \times 10^3} = 733\ \mu\text{A}$$

(a) UTP and LTP of the square wave can now be calculated as:

$$|UTP| = |LTP| = \pm(I_{2\,Sch}\, R_{1\,Sch})$$

Substituting the values, we get:

$$|UTP| = |LTP| = 733 \times 10^{-6} \times 2.7 \times 10^{3} = 1.98 \text{ V} \approx 2 \text{ V}$$

(b) The peak-to-peak amplitude of the triangular waveform can be found as:

$$V_{p-p} = 2 \text{ UTP} = 2 \times 2 = 4 \text{ V}$$

(c) The frequency of the triangular waveform can be calculated as:

$$f = \frac{1}{2t} \qquad \text{[Refer to Eqn.(3)]} \quad ...(2a)$$

For this, the value of t is obtained as:

$$t = \frac{C_1 \Delta V}{I_2} \qquad \text{[Refer to Eqn.(8)]}$$

where $\Delta V = UTP - LTP = 2 - (-2) = 4$ V

Substituting the required values, it becomes:

$$t = \frac{0.3 \times 10^{-6} \times 4}{1.17 \times 10^{-3}} = 1.025 \text{ ms}$$

Now using the value of t in Equation (2a), we get:

$$f = \frac{1}{2t} = \frac{1}{2 \times 1.02 \times 10^{-3}} = 488 \text{ Hz}$$

12.4 ARBITRARY WAVEFORM GENERATOR

An arbitrary waveform is virtually any desired waveform used for test purpose of various electronic instruments. Therefore, an arbitrary waveform generator is one which allows the user to produce any desired waveform for some specific applications, say for testing the communication equipments.

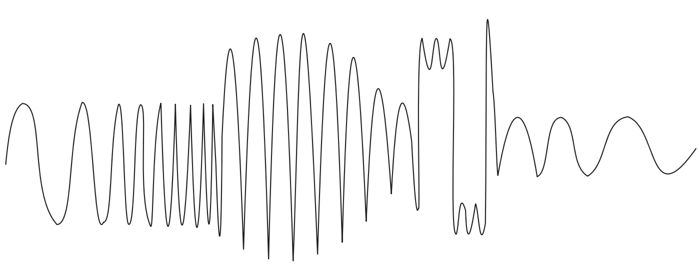


Fig. 12.12 Arbitrary Waveform

For testing purpose we consider a modulated signal as shown in Figure 12.12 that varies over the entire bandwidth and amplitude range of the equipment. To investigate the response of the equipment, noise could be superimposed on the signal and gaps could be introduced between waveform bursts. Once generated, these waveforms can be stored and used again and again.

A programmable function generator (a type of arbitrary waveform generator) is capable of producing all pulse wave, sine wave, triangular wave, and ramp wave. This generator offers phase, frequency, and amplitude modulation of different waveforms. DC offset voltages can also be superimposed on the signals by this generator. Any waveform can be selected via a menu button and its amplitude, phase, and frequency can be adjusted precisely through a control knob. The liquid crystal display represents the status of the generated waveform to be displayed. In addition, there are various functions for waveform generation, such as linear and logarithmic amplitude and frequency sweeps. A number of generated waveforms, generally up to six can be stored in the memory of the instrument for future reference. The amplitude of the output signal of the generator may be in the range of 1 mV to 20 V (peak-to-peak) as well as a frequency in the range of 0.1 mHz to 20 MHz.

12.5 AUDIO FREQUENCY SIGNAL GENERATOR

The audio frequency (AF) signal generators are meant to produce an output signal in the audio frequency range, that is, between 20 Hz to 20 kHz. The main components of an AF signal generator are *sinusoidal oscillator*, *sine-to-square wave converter*, and *attenuator circuit*. Either the sine or the square wave output can be obtained at the output of the generator. The block diagram of an AF signal generator is depicted in Figure 12.13.

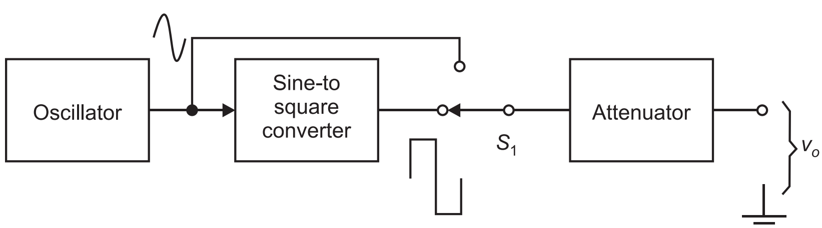


Fig. 12.13 Block Diagram of an AF Signal Generator

12.5.1 Oscillator Circuit

The sinusoidal oscillator is basically an RC network with amplification and feedback features and which produces a low frequency signal at its output as well as offers a controlled phase shift of the signal. A Wien bridge oscillator can be used for this purpose since it generates a clean sine-wave signal with low distortion, good amplitude, and frequency stability. Figure 12.14 illustrates the configuration of a Wien bridge oscillator circuit.

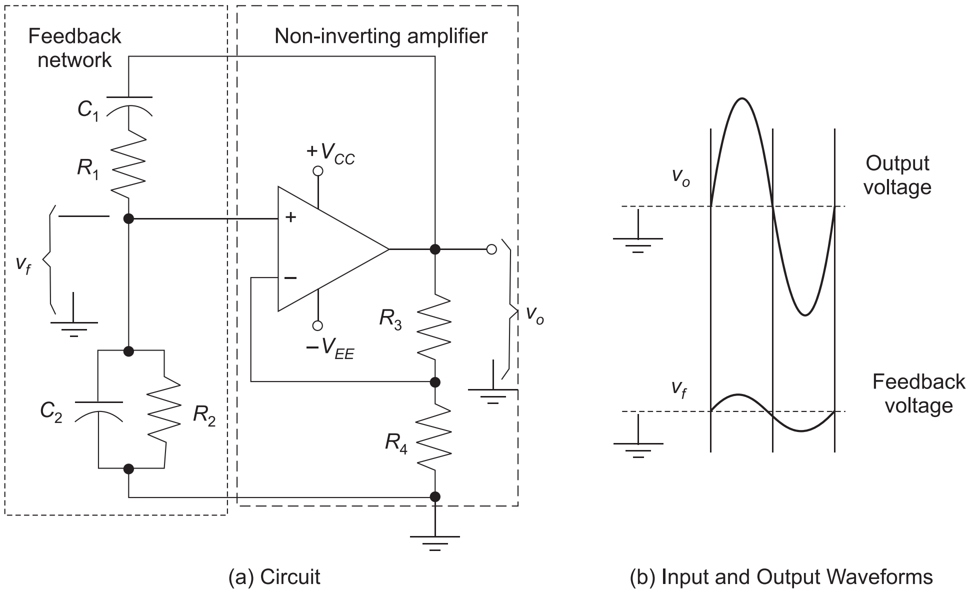


Fig. 12.14 Wien Bridge Oscillator

As studied earlier in Section 7.5.1, depending upon the bridge components, the Wien bridge is balanced at a particular frequency. The balance condition for this bridge circuit as obtained in Chapter 7, can be written as:

$$\frac{R_3}{R_4} = \frac{R_1}{R_2} + \frac{C_2}{C_1} \qquad \text{...(12)}$$

Also, the operating frequency at which the balance can be achieved is given as:

$$f = \frac{1}{2\pi\sqrt{C_2R_1C_1R_2}} \qquad \text{...(13)}$$

Now suppose resistors R_1 and R_2 possess an equal value and so capacitors C_1 and C_2, so Equation (12) yields:

$$R_3 = 2R_4 \qquad \text{...(14)}$$

Then Equation (13) then reduces to:

$$f = \frac{1}{2\pi CR} \qquad \text{...(15)}$$

The input and output voltages of the op-amp in Figure 12.14(a), remain in phase only at the balance frequency of the bridge. The feedback voltage V_f becomes out of phase with the output voltage at all other frequencies as the bridge is off balance. Here, note that the input of the op-amp is the voltage generated across R_2 and C_2 network.

The non-inverting amplifier provides a voltage gain given as:

$$A_v = \frac{R_3 + R_4}{R_4} \qquad ...(16)$$

Here, using the value of R_3 from Equation (14), the above equation gives:

$$A_v = 3$$

However, in order to sustain the oscillations of the circuit, the voltage gain must be slightly higher than this value. It must be noted here that an excess value of A_v may result in signal distortion since at such a high gain, the amplitude of the output signal tends to become equal to the supply voltages, that is, $+V_{CC}$ and $-V_{EE}$. To deal with such a situation, resistor R_3 is divided into two resistors (resistors R_{3a} and R_{3b}) as shown in Figure 12.15 as well as two diodes D_1 and D_2 are also employed in the circuit.

For small output amplitude, the voltage drop across resistor R_{3b} is not sufficient to make the diodes D_1 and D_2 operate in forward biased condition. The voltage gain in that case can be expressed as:

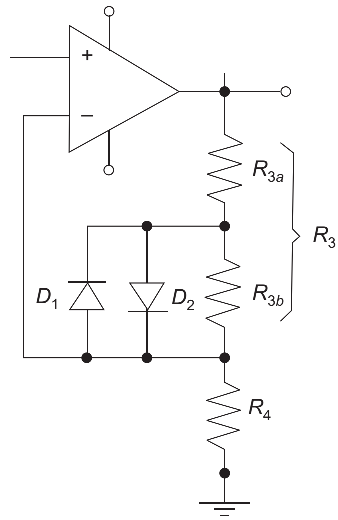


Fig. 12.15 Improved Non-Inverting Amplifier Section

$$A_v = \frac{R_{3a} + R_{3b} + R_4}{R_4} \qquad ...(17)$$

On the contrary, when the amplitude of the voltage becomes sufficiently high to forward bias D_1 and D_2, resistor R_{3b} is short-circuited, thereby providing a gain expressed as:

$$A_v = \frac{R_{3a} + R_4}{R_4} \qquad ...(18)$$

Note that if Equation (18) yields a voltage gain less than 3, then oscillations with small amplitude will only be sustained and those with high amplitude will vanish.

12.5.2 Sine-to-Square Wave Converter

A Schmitt trigger circuit (as discussed in Section 12.3.2) produces a square wave output. This circuit can be used as a sine-to-square wave converter circuit in an AF signal generator. However, it should be noted here that the UTP and LTP of the Schmitt trigger are kept close to the ground level. This ensures that whenever the sine wave input reaches a zero crossing of the voltage level, the output of the Schmitt trigger circuit may switch between the extreme levels of the supply voltage.

12.5.3 Attenuator Circuit

The output of the oscillator circuit can be adjusted using a voltage divider and a potentiometer attenuator circuit. The circuit is shown in Figure 12.16.

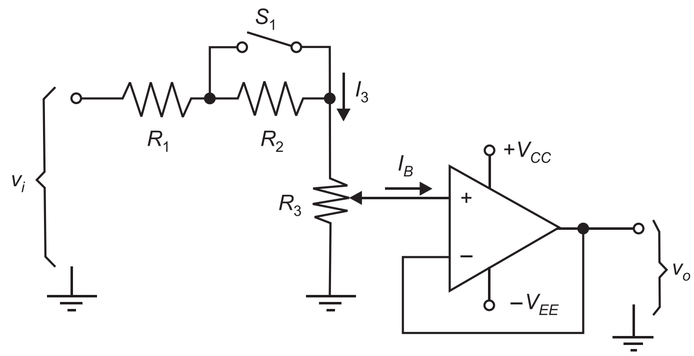


Fig. 12.16 Attenuator Circuit

It can be seen from the figure that resistors R_1, R_2 and R_3 make a voltage divider circuit which is meant to attenuate the output of the incoming signal. Among these resistors, R_3 is a potentiometer which controls the amplitude of the output voltage. Furthermore, a switch S_1 is incorporated in order to allow the output amplitude to switch between two amplitude ranges by shorting resistor R_2. An op-amp is connected to this voltage divider circuit and acts as a voltage follower circuit, thereby providing low impedance at the output of the signal generator.

The audio frequency signal generators provide an adjustable output voltage in the range of 0 to 10 V as well as a maximum output frequency of 100 kHz. The function generators are an example of AF signal generator.

Example 3 Consider Figure 12.15 with the following values and determine the maximum amplitude of the output voltage.

$$R_{3a} = 450\ \Omega, \quad R_{3b} = 350\ \Omega, \quad \text{and } R_4 = 400\ \Omega$$

Solution: It is evident from the figure that the voltage across resistor R_{3b} is equal to the forward bias voltage of the diode. Thus, we may write as:

$$V_{R3b} = V_{FB} = 0.7\text{ V} \quad \text{(assuming the silicon diodes)}$$

The current flowing through R_{3b} can now be easily calculated as:

$$I_{R3b} = \frac{V_{R3b}}{R_{3b}} \qquad \text{...(3a)}$$

Substituting the required values in Equation (3a), we get:

$$I_{R3b} = \frac{0.7}{350} = 2\text{ mA}$$

Now, the output voltage can be obtained as:

$$V = I_{R3b}(R_{3a} + R_{3b} + R_4)$$

Substituting the given values, we obtain:

$$V_o = 2 \times 10^{-3}(450 + 350 + 400) = 2.4\text{ V}$$

12.6 RADIO FREQUENCY SIGNAL GENERATOR

Radio frequency (RF) signal generators are used to generate a sine-wave output with a frequency range of 100 kHz to 40 GHz. The main components of this generator are an *RF oscillator*, an *amplifier circuit*, an *attenuator circuit*, and an *output level meter*. The basic arrangement of this generator is depicted in Figure 12.17.

The oscillator circuit provides a frequency control. Also, it incorporates a frequency range switch which facilitates the adjustment of the output signal to a desired frequency. On the other hand, the amplifier circuit provides adjustment for the output amplitude. This ensures that a voltage can be set at a calibration point before applying it to the output level meter. It must be noted here that whenever the frequency is changed, the output signal must be reset to the specific calibration point.

In addition, the RF oscillators also employ some means to modulate the amplitude as well as the frequency of the input signal through the switches shown in Figure 12.17. The switches S_1 and S_2 are used to select internal or external frequency or amplitude modulation or no modulation. Moreover, to prevent the RF interference between the components and RF emission from any point except the output terminals, each section of the system is shielded in a metal box and complete shielding of the whole system is also done.

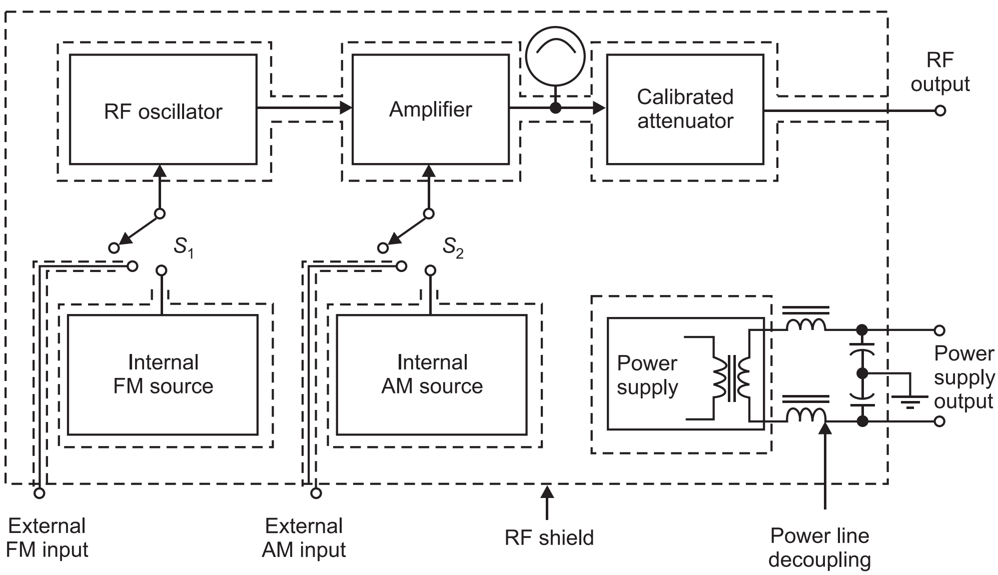


Fig. 12.17 Radio Frequency Signal Generator

12.6.1 RF Oscillator Circuit

The RF signal generator employs a Hartley oscillator or a Colpitts oscillator. The oscillator circuit consists of an amplifier circuit as well as a feedback network which provides the

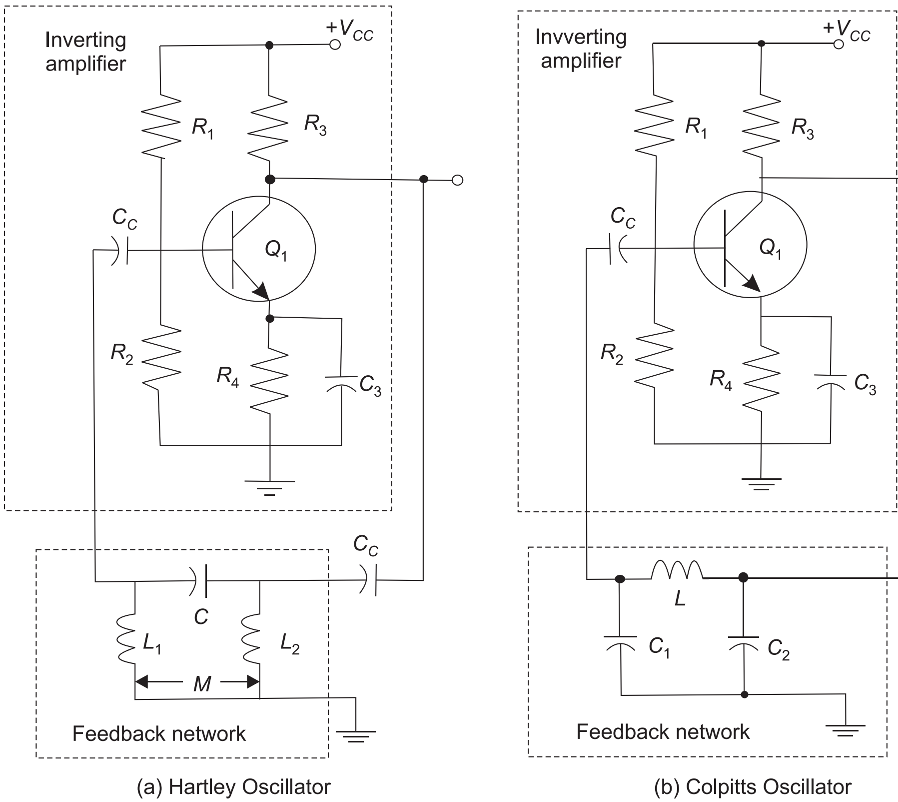


Fig. 12.18 Radio Frequency Oscillator

phase-shifting feature. Figure 12.18 illustrates the configuration of both Hartley and Colpitts oscillators.

The figure reveals that the construction of both the oscillators is approximately the same, except the feedback network. There are two inductors L_1 and L_2, and one capacitor C in the feedback network of Hartley oscillator while two capacitors C_1 and C_2, and one inductor L in Colpitts oscillator. The coupling capacitor C_C is included in both the circuits. The oscillation or resonant frequency f for both of these oscillators can be given as:

$$f = \frac{1}{2\pi\sqrt{C_T L_T}} \qquad \text{...(19)}$$

where C_T and L_T represent the total or equivalent capacitance and inductance of these oscillators, respectively. The resonant frequency can be adjusted or altered by changing the values of the components in the feedback network.

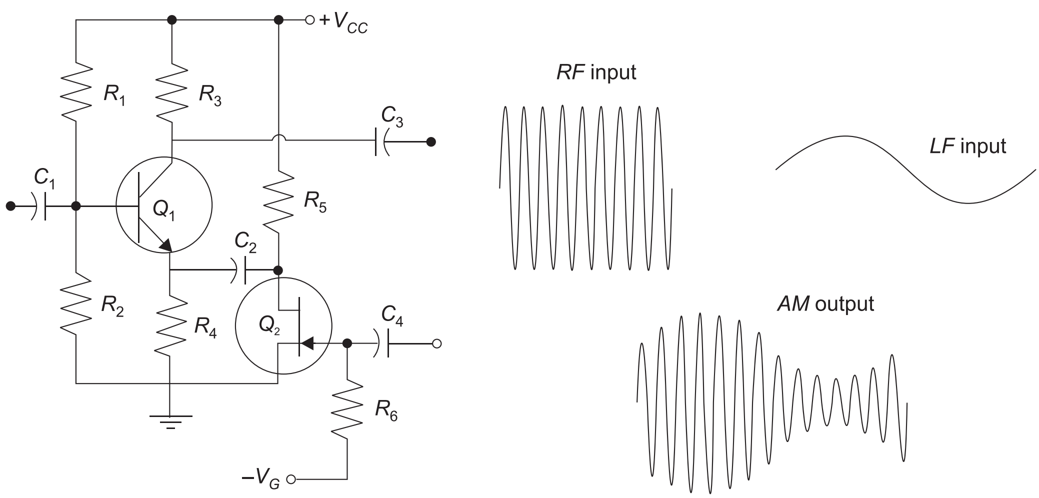
The inverting amplifier amplifies the signal along with providing a phase shift of 180° to the signal before it is applied to the feedback network where it is again phase shifted by 180° and fed to the input terminals of the amplifier. It is to be noted that the inverting amplifier offers a gain equal to the attenuation provided by the feedback network. Thus, we may conclude that the loop gain of these oscillators is unity while a loop phase shift of 360° is provided to the input signal. These specifications are necessary to keep the oscillations of the circuit sustained.

12.6.2 Internal AM and FM Source Circuits

Almost every RF signal generator has some mechanism to modulate the input RF signal in terms of its amplitude as well as frequency. In the block diagram shown in Figure 12.17, there are two circuits, namely, *internal AM source* and *internal FM source,* which provide the AM and FM modulating signals, respectively. The internal FM source can be connected to the RF oscillator circuit while the internal AM source can be connected to the amplifier circuit.

Amplitude modulation

The modulation of the output signal amplitude can be performed at the amplification stage of the generator. The circuit shown in Figure 12.19 performs the amplitude modulation of the RF signal generated by the RF oscillator.



(a) Circuit Arrangement

(b) Input and Output Waveforms

Fig. 12.19 Amplitude Modulation

In the above circuit, suppose that field-effect transistor (FET) Q_2 is initially not connected to the circuit. The gain of the amplifier in such a case comes to be:

$$A_v = \frac{R_3}{R_4}$$

Now let Q_2 be connected to resistor R_4 via a capacitor C_2 in order to keep the DC bias condition of bipolar junction transistor (BJT) Q_1 unaffected. The amplifier gain can now be given as:

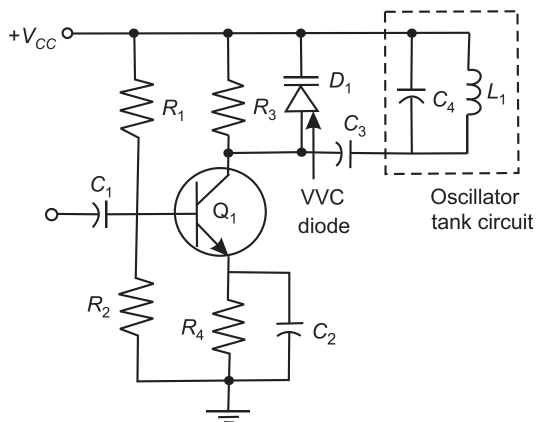
$$A_v = \frac{R_3}{R_4 \parallel R_D} \qquad \text{...(20)}$$

where R_D represents the drain resistance of Q_2. Equation (20) reveals that resistance R_D and R_4 are parallel to each other.

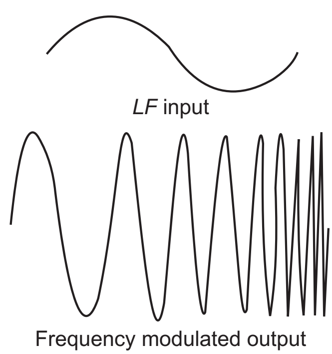
When a low frequency signal is applied to the gate terminal of the FET via capacitor C_4, it changes its drain resistance R_D, thereby changing the amplifier gain. It causes the amplitude of the output RF signal to increase and decrease in accordance with the low frequency signal at its input. Therefore, it may be concluded that the amplitude of the RF signal gets modulated by the LF input signal. This can be seen from the corresponding waveforms in the figure.

Frequency modulation

The frequency of the RF signal is modulated at the oscillator section of the generator. An approach for modulation is by utilizing a special semiconductor diode, known as **variable voltage capacitor diode** (VVC diode). This diode is specially constructed and works on reverse bias. Any change in its reverse bias, changes the value of its capacitance. Figure 12.20 shows an arrangement employing such a diode and the corresponding components to provide frequency modulation of the RF signal.



(a) Circuit Arrangement



(b) Input and Output Waveforms

Fig. 12.20 Frequency Modulation

A BJT Q_1 is incorporated in the above circuit to alter the voltage across the VVC diode with the aid of other circuit components. An LF input is applied to the VVC diode (marked as D_1) via these components and the transistor. Capacitor C_4 along with inductor L_1 constitutes an oscillator tank circuit which generates oscillations at the output. A capacitor C_3 couples diode D_1 to this tank circuit. The capacitance of this tank circuit C_{TC} comes out to be as:

$$C_{TC} = C_D \parallel C_4$$

where C_{TC} represents the tank circuit capacitance and C_D represents the diode capacitance. Thus, the resonant frequency f of this tank circuit can be calculated as:

$$f = \frac{1}{2\pi\sqrt{(C_D \parallel C_4)\, L_1}} \qquad ...(21)$$

As the capacitance of the diode changes, it changes the resonance frequency of the tank circuit. Therefore, it may be concluded that the output frequency of the tank circuit gets modulated by the low frequency signal applied at the input of the circuit.

12.6.3 Output Level Meter and Calibrated Attenuator

As stated earlier, output of the amplifier circuit is adjustable so that the voltage at the calibrated attenuator input should be at some specific calibration point. This calibration point is indicated by the output level meter. The amplifier output must always be checked for its level to be equal to this calibration point whenever the frequency is altered. However, for the actual output voltage of the calibrated attenuator to be at the indicated level, the RF generator should be loaded correctly. The output of the attenuator circuit is only correct when the specified load is connected. Any difference in the connected load and the specified load must be adjusted through parallel- or series-connected resistors. It must be noted here that if the series connected resistors are included in the circuit, they cause the RF generator output to be further attenuated. In this case, the actual signal level applied to the load must be calculated.

The RF generators used in laboratories provide an output frequency in the range of 0.15 MHz to 50 MHz in eight ranges with a frequency error less than 1%. The impedance at the output of the generator is usually 75 Ω. An output voltage of 50 mV can be obtained if a 75 Ω load is connected to the circuit. This voltage can be attenuated by up to 80 dB. The output signal can be amplitude-modulated by an internal source of 1 kHz. This modulation can be done at a depth of 30%. However, if the source is external, the frequency range changes to 20 Hz to 20 kHz. For frequency modulation, the internal source may be 1 kHz or power frequency. Also, if the source is external, the frequency range changes to 0 to 5 kHz.

Example 4 Determine the required inductance of a Colpitts oscillator if it possesses an oscillating frequency of 1MHz and the values of its capacitances are given as $C_1 = C_2 =$ 1.05 pF.

Solution: Given that: f = 1MHz, $C_1 = C_2$ = 1.05 pF

The oscillating frequency of a Colpitts oscillator is expressed as:

$$f = \frac{1}{2\pi\sqrt{C_T L_T}} \qquad \text{[Refer to Eqn. (19)]} \quad ...(4a)$$

Here, the total capacitance C_T of the circuit can be obtained as:

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{1}{\frac{1}{1.05 \times 10^{-12}} + \frac{1}{1.05 \times 10^{-12}}} = \frac{1}{1.904 \times 10^{-12}} = 0.525 \text{ pF}$$

Substituting the required values in Equation (4a), we get:

$$1 \times 10^6 = \frac{1}{2 \times 3.14\sqrt{0.525 \times 10^{-12}\ L_T}}$$

On solving, it comes out to be:

$$L_T = 48.2 \text{ mH}$$

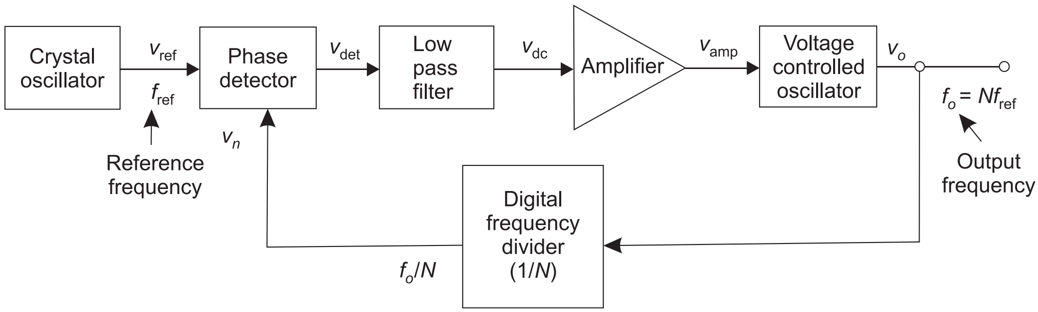
12.7 FREQUENCY-SYNTHESIZED SIGNAL GENERATOR

The frequency of an RF oscillator sometimes needs to be stabilized. A piezoelectric crystal can be used for this purpose. Except at its own resonance frequency, this crystal provides high impedance to all other frequencies. Thus, if one of the coupling capacitors in the RF oscillator circuit (see Figure 12.18) is replaced by a piezoelectric crystal, the circuit would certainly oscillate at the resonance frequency of the crystal which is very stable. However, the problem arises in adjusting the frequency of the crystal oscillator. For this, the oscillator is applied as an input to a *frequency synthesizer* which uses a *phase-locked loop* circuit to multiply the reference frequency of the oscillator f_{ref} by a selected factor N. Let us consider the phase-locked loop circuit shown in Figure 12.21 to understand its functioning.

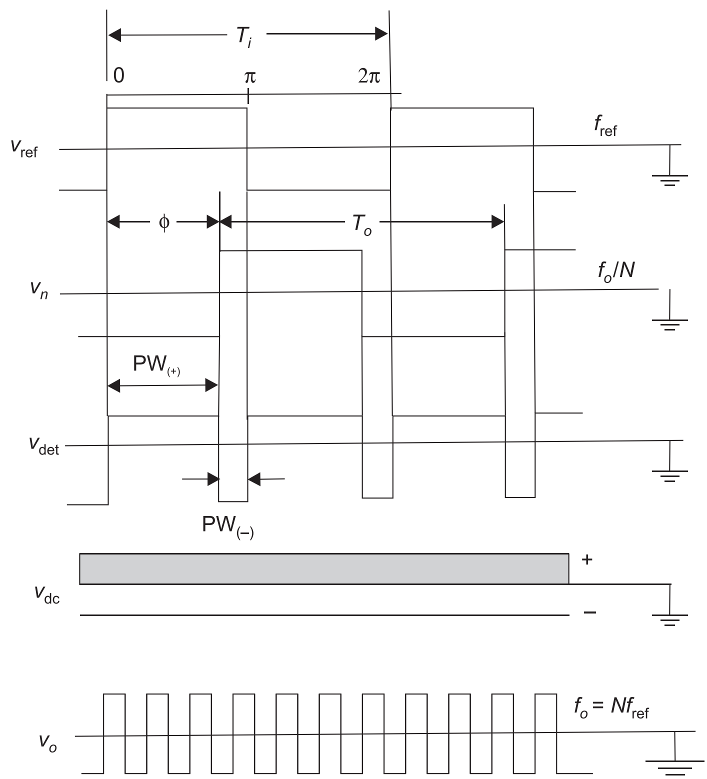
The output signal v_{ref} of the oscillator circuit with reference frequency f_{ref} is converted into a square wave prior to applying it to the phase detector circuit. On the other input terminal of the phase detector, another square wave signal v_n is applied from the frequency divider circuit as shown in the figure. These two square waves have phase difference ϕ between them with equal amplitudes and frequencies. The output of the phase detector is a pulse waveform whose pulse widths are controlled by phase difference ϕ between v_{ref} and v_n. The low-pass filter receives this pulse waveform and produces a DC voltage v_{dc}. It can be seen from Figure 12.21(b) that a symmetrical (about ground level) square wave with an average voltage of zero volts is produced at the output of the phase detector for a phase difference of $\pi/2$ radians. For a phase difference less than $\pi/2$ radians, the DC output becomes a negative quantity, while for ϕ greater than $\pi/2$ radians, it is a positive quantity [see Figure 12.21(b)]. The sensitivity of the phase detector is denoted as K_{det} and is defined as the ratio of the change in the DC output voltage to that in the input phase angle, measured in volts/radian. Mathematically, it can be expressed as:

$$K_{\text{det}} = \frac{v_{dc}}{\Delta\phi} \qquad \text{...(22)}$$

The amplifier amplifies the signal coming from the filter output and provides the amplified signal v_{amp} at its output which is used as a control voltage for the voltage-controlled oscillator (VCO). Thus, the frequency of the VCO is determined by v_{amp} and its output is used to trigger the digital frequency divider circuit. It must be noted here that the VCO oscillates at



(a) Block Diagram



(b) Corresponding Waveforms

Fig. 12.21 Phase-Locked Loop

a free-running frequency f_{FR} when a zero input voltage v_{amp} is applied to it. This frequency changes in accordance with the applied input voltage, that is, it increases for a positive input and decreases for a negative input voltage. The sensitivity of the VCO is denoted as K_{VCO} and is defined as the ratio of the change in output frequency to that in the control voltage, measured in hertz/volt. Mathematically, it can be expressed as:

$$K_{\text{VCO}} = \frac{\Delta f_o}{v_{\text{amp}}} \qquad ...(23)$$

The frequency divider circuit operates in the way described in Section 5.7. However, the ratio or factor, by which the frequency of the VCO is divided, is determined by a bank of switches. For a factor $N = 500$, output frequency of frequency divider of 1 kHz, and a reference frequency of 1 kHz, the output frequency of the VCO comes out to be:

$$f = N \times f_{\text{ref}} = 500 \times 1 \text{ kHz} = 500 \text{ kHz}$$

This implies that the output signal of the amplifier is capable enough to produce an output frequency of 500 kHz. Now let the frequency f_o at the output of the circuit increase and become greater than 500 kHz. This in turn increases the output frequency of the frequency divider which becomes greater than the reference frequency f_{ref}. As a consequence of this, the phase difference ϕ rapidly decreases which further causes the width of the pulses at the output of the phase detector also to decrease. The amplitude of the filter and amplifier output signal subsequently gets decreased which causes the output frequency of the VCO to fall back to its previous value, that is, 500 kHz. But, when the output frequency falls below 500 kHz, the frequency divider output frequency decreases and becomes lower than the reference frequency f_{ref}. This increases the phase difference ϕ due to which the output pulse width of the phase detector is also increased. Further, the amplitude of the filter and amplifier circuit gets increased, driving back the frequency of the VCO to 500 kHz again.

Therefore, it may be concluded that the output frequency of the VCO can be made stable at a value equal to N times the reference frequency f_{ref}. Thus, by selecting any frequency divider ratio N, the output frequency of the frequency synthesizer can be stabilized to any multiple of the reference frequency f_{ref}.

Example 5 The free-running frequency f_{FR} and reference frequency f_{ref} of a PLL frequency-synthesized signal generator are given as 450 kHz and 250 kHz, respectively. Determine its output frequency f_o, and phase difference $\Delta\phi$, if factor $N = 5$, phase detector sensitivity $K_{\text{det}} = 0.5$ V/rad, VCO sensitivity $K_{\text{VCO}} = 250$ kHz/V, DC voltage (at filter output) $v_{\text{dc}} = 0.4$ V and the amplifier gain $A_v = 8$.

Solution: Given that: $f_{FR} = 450$ kHz, $f_{\text{ref}} = 250$ kHz, $N = 5$, $K_{\text{det}} = 0.5$, $K_{\text{VCO}} = 250$ kHz/V, and $A_v = 8$

The output frequency f_o can be obtained as:

$$f_o = N \times f_{\text{ref}}$$

Substituting the required values, we get:

$$f_o = 5 \times 250 \times 10^3 = 1.25 \text{ MHz}$$

Now the amplifier output voltage v_{amp} can be obtained as:

$$v_{\text{amp}} = \frac{\Delta f_o}{K_{\text{VCO}}} \qquad \text{[Refer to Eqn. (23)]} \quad ...(5a)$$

where $\Delta f_o = f_o - f_{FR} = 1.25 \times 10^6 - 450 \times 10^3 = 800$ kHz

Substituting the values in Equation (5a), we get:

$$v_{\text{amp}} = \frac{\Delta f_o}{K_{\text{VCO}}} = \frac{800 \times 10^3}{250 \times 10^3} = 3.2 \text{ V}$$

The phase difference ϕ can be determined as:

$$\Delta\phi = \frac{v_{\text{dc}}}{K_{\text{det}}} = \frac{0.4}{0.5} = 0.8 \text{ rad} \qquad \text{[Refer to Eqn.(22)]}$$

12.8 SWEEP FREQUENCY GENERATOR

The sweep frequency generator, also known as **sweeper**, produces an output frequency which is cyclically swept over a range of frequencies. This generator simplifies and speeds up the frequency response testing process of filters and amplifiers.

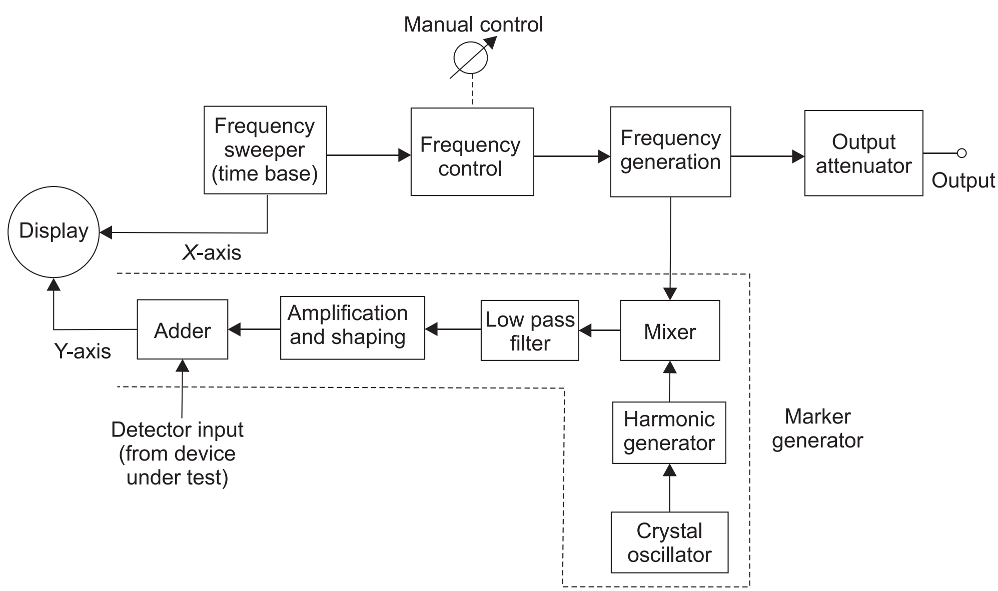


Fig. 12.22 Block Diagram of Sweep Frequency Generator

Figure 12.22 shows a sweep generator circuit that has display unit built into it to show variation of amplitude with frequency. The time base generally gives a triangular or sawtooth waveform which can be adjusted manually from the instrument panel to give the sweep times in the range from 10 ms to greater than 100 s. This generator sweeps a wide band of frequencies ranging from low-frequency to audio frequency, that is, 0.001 Hz to 100 kHz, radio frequency range, that is, 100 kHz to 1500 MHz and microwave range, that is, 1 GHz to 200 GHz. The manual switching between different frequency oscillators may create problem as overlapping between two bands may occur for the required frequency range. The alternate switching method is stacked switching in which electronic switches automatically select the frequency band so the complete frequency range is swept.

To generate the marker frequencies that act as reference onto the screen, the output from a stable crystal oscillator is passed to a harmonic generator that generates a series of narrow pulses spaced at harmonic intervals. The sample of radio frequency output from the sweeper is then mixed with these harmonics to produce a series of low difference frequency output bursts, known as **birdies** through a mixer. These birdies are produced when sweeper frequency approaches and passes through each harmonic frequency which is then shaped, amplified and combined with the signal received through the detector from device under test before it is applied to the display. Therefore, we get a composite picture of frequency response characteristic of frequency calibration markers and the device under test.

12.9 VIDEO SIGNAL GENERATOR

A video signal generator produces video signals either directly or with RF modulation. These signals are in the form of geometrical patterns such as cross-hatch, dots, vertical and horizontal bars on TV channels for testing, alignment, and servicing of TV receivers. This generator is also known as **pattern generator**. The patterns generated are used for adjusting linearity and video amplification. Moreover, the video signal generators generate an FM signal to perform alignment of the sound sections of the receiver.

Figure 12.23 shows video signal generator along with sound signal generator consisting of two stable chains of multivibrators, pulse shaping circuits, and dividers. One chain is below the line frequency and produces horizontal bars while the other chain is held above 15625 Hz frequency and produces vertical bars. After modifying the signals into short duration pulses, they are fed to the video section of the receiver along with the synchronous pulse train to produce fine lines on the screen. The square wave video signals are produced by multivibrators at m times the horizontal frequency that provides m vertical white and black bars. The multivibrators are then triggered by the horizontal blanking pulse after every m cycles to synchronize the bar signal on every line. The frequency of multivibrator is varied by a control knob on the front panel of the video generator to change the number of bars.

Another set of multivibrator is triggered either by master oscillator or from 50 Hz mains to produce square wave video signals at n times the vertical frequency. These video signals are then fed to the video amplifier to produce horizontal black and white bars. The switching rate of the corresponding multivibrator is controlled by a potentiometer. This potentiometer can vary the number of horizontal bars generated. A video adder combines the bar pattern signal with synchronous and blanking pulses to produce composite video signals which is then fed to the modulator.

Various patterns can be generated by the use of switches employed in the signal path of the two multivibrators. There are two switches, namely mH and nV to produce different patterns on the screen. If only mH switch is *on*, vertical bars are produced whereas horizontal bars are produced only with the nV switch *on*. When both switches are *on*, a cross-hatch pattern is produced whereas a blank white raster is produced with both the switches *off*. All these patterns are produced for some specific purposes. To check the vertical linearity of the signals, the horizontal bar pattern must be spaced equally throughout the screen. Similarly, to check the horizontal linearity, a vertical bar pattern is used. The linearity can be adjusted more precisely by the cross-hatch pattern. Also, the number of squares on the vertical and horizontal sides of the screen in a cross-hatch pattern is counted for picture centring and checking the aspect ratio. The modulated picture signals are fed into the RF section of the receiver and are available on limited channels.

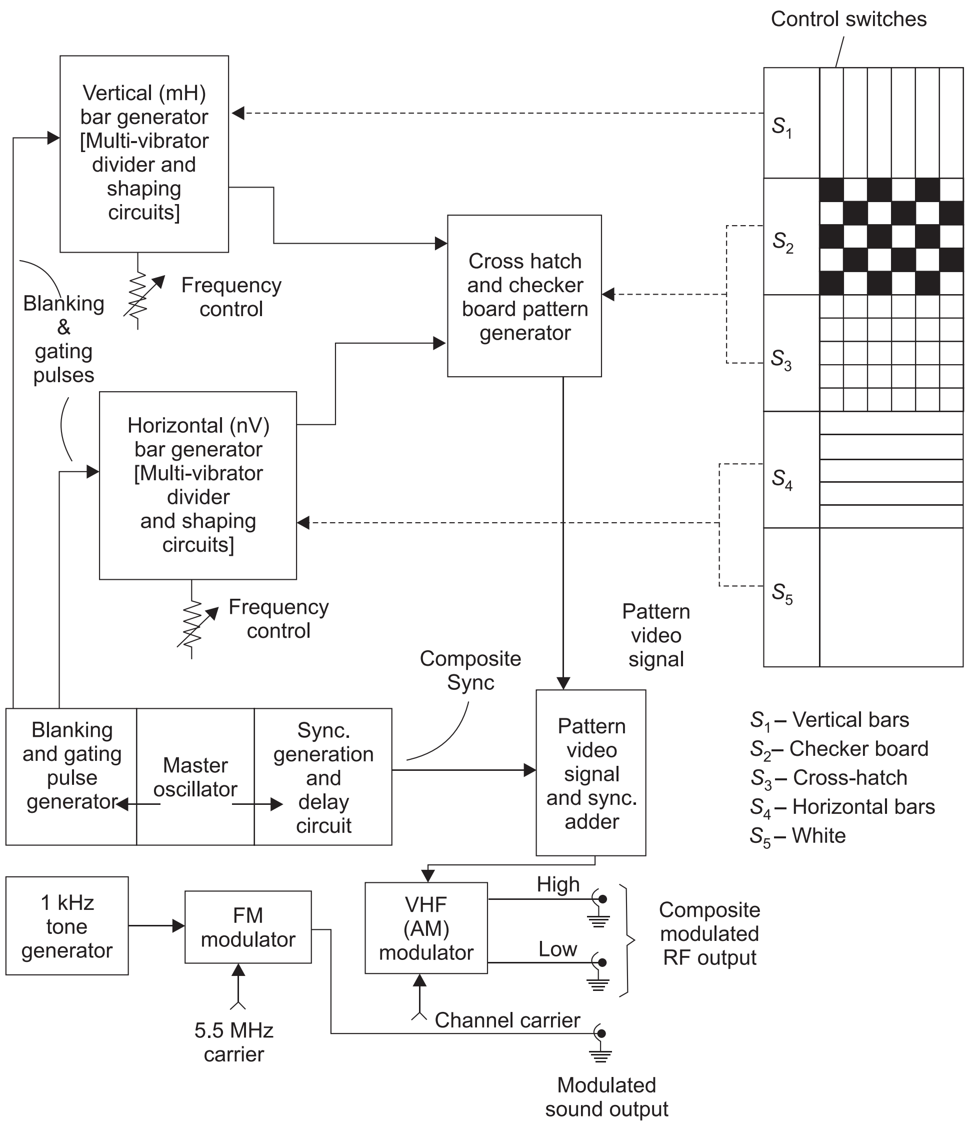


Fig. 12.23 Block Diagram of Video Signal Generator

The sound IF and discriminator circuits are aligned by an FM sound signal which is modulated by a 1 kHz tone, having a carrier frequency of 5.5 MHz ±100 kHz. A standard accessory, that is, 75/300 Ω very high frequency (VHF) balun is provided with the pattern generator.

The video signal generators are used in varied applications as follows.

- In sweep generation circuits to detect any spurious oscillations.
- To interact between two oscillators.

Let us summarize

1. Signals are generated by signal generators in various forms, such as sine wave, square wave, pulse wave, and sweep wave. These signals may lie in low-frequency range to radio-frequency range.
2. Based on the type of signals generated, the signal generators can be categorized as sine-wave generators, pulse generators, sweep frequency generators, oscillators, and function generators.
3. Pulse- and square-wave generators are generally used with an oscilloscope for measurement purposes. The major difference between these two generators lies in their duty cycle which can be defined as the ratio of the width of the pulse and the time period or pulse repetition time it takes.
4. A pulse-wave generator basically consists of a square-wave generator, a monostable multivibrator, and an output attenuator stage.
5. Function generator is an instrument capable of producing various waveforms such as triangular wave, pulse wave, sinusoidal wave, square wave, and also ramp wave. Any of these waveforms can be generated with the desired frequency and amplitude ranges.
6. The function generator consists of an integrator, Schmitt trigger, sine-wave converter, and attenuator.
7. An arbitrary waveform is virtually any desired waveform used for test purpose of various electronic instruments. Therefore, an arbitrary waveform generator is one which allows the user to produce any desired waveform for some specific applications, say for test purposes.
8. The audio frequency (also known as AF) signal generators are meant to produce an output signal in the audio frequency range, that is, between 20 Hz to 20 kHz. The main components of an AF signal generator are a sinusoidal oscillator, a sine-to-square wave converter, and an attenuator circuit.
9. The radio frequency (also known as RF) signal generators are used to generate a sine-wave output with a frequency range of 100 kHz to 40 GHz. The main components of this generator are an RF oscillator, an amplifier circuit, an attenuator circuit, and an output level meter.
10. The frequency of an RF oscillator sometimes needs to be stabilized. For this, the oscillator is applied as an input to a frequency synthesizer which uses a phase-locked loop circuit to multiply the reference frequency of the oscillator f_r by a selected factor N.
11. The sweep frequency generator, also known as sweeper, produces an output frequency which is cyclically swept over a range of frequencies.
12. A video signal generator produces video signals directly and also with RF modulation in the form of geometrical patterns such as cross-hatch, dots, vertical and horizontal bars on TV channels for testing, alignment, and servicing of TV receivers. This generator is also known as pattern generator.

EXERCISES

Fill in the Blanks

1. The duty cycle of a pulse-wave generator lies in the range of _________.
2. _________ are used to detect any spurious oscillations in sweep waveform generation circuits.
3. The output frequency of VCO in frequency-synthesized signal generators is expressed as N times the _________ frequency.
4. The RF oscillator used in an RF signal generator is either a _________ oscillator or a _________ oscillator.
5. An astable multivibrator provides _________ stable state.

Multiple Choice Questions

1. The sensitivity of VCO in frequency-synthesized signal generator is expressed as

   (a) $K_{\text{VCO}} = \dfrac{\Delta f_o}{v_{\text{amp}}}$ (b) $K_{\text{VCO}} = \dfrac{v_{\text{amp}}}{\Delta f_o}$

   (c) $K_{\text{VCO}} = \dfrac{\Delta f_o}{v_{\text{dc}}}$ (d) $K_{\text{VCO}} = \dfrac{v_{\text{dc}}}{\Delta f_o}$

2. To produce a triangular waveform, which of the following is correct?

   (a) a square wave is to be differentiated (b) a sine wave is to be differentiated

   (c) a square wave is to be integrated (d) a sine wave is to be integrated

3. Which of the following is an example of audio frequency signal generator?

   (a) video signal generator (b) sweep signal generator

   (c) pulse signal generator (d) function generator

4. Which of these signal generators is used to interact between two oscillator circuits?

   (a) sweep signal generator (b) video signal generator

   (c) RF signal generator (d) None of these

5. The expression for the frequency of an RF oscillator circuit is given as

   (a) $f = \dfrac{1}{\sqrt{C_T L_T}}$ (b) $f = \dfrac{1}{2\pi\sqrt{C_T L_T}}$

   (c) $f = \dfrac{1}{2\pi\sqrt{\dfrac{C_T}{L_T}}}$ (d) none of these

State True or False

1. The time period between two consecutive trigger pulses is known as recovery time.
2. Function generator is used to produce any desired waveform for the testing of various electronic instruments.
3. The ratio of the change in the DC output voltage to the change in the phase angle of the input voltage is referred to as phase detector sensitivity.

4. The width of a pulse waveform can be expressed as $PW = RC \ln\left(\dfrac{V_o - LTP}{V_o - UTP}\right)$.
5. The RF oscillators provide a loop phase shift of 360° to the input signal.

Descriptive/Numerical Questions

1. Describe the working of a sweep frequency generator.
2. Discuss in detail the RF signal generators.
3. Draw the block diagram of a pulse generator instrument and explain its operation.
4. Determine the frequency of Colpitts oscillator with L =100 mH, C_1 = 0.005 μF, and C_2 = 0.01 μF.
5. Draw the block diagram of a function generator and explain it in detail.
6. Determine the output pulse width of a monostable multivibrator with following specifications:

   $V_{CC} = \pm 12$ V, $V_{D1} = \pm 0.7$ V, $R_1 = 10$ kΩ, $C_1 = 0.33$ μF, $R_2 = 100$ kΩ, and $R_3 = 10$ kΩ
7. Explain the working of a frequency-synthesized signal generator.
8. A PLL system has reference frequency as well as free-running frequency of 120 kHz, detector sensitivity is 0.9 V/rad, VCO sensitivity is 120 kHz/V and factor N as 8. Determine its output frequency, DC voltage, and phase difference.
9. Draw a Wien bridge oscillator circuit diagram. Explain the construction and working of its major components.
10. Draw suitable circuit diagram of a Hartley and Colpitts oscillator and explain its oscillating frequency.

13

Analysis of Signals

Chapter Objectives

After reading this chapter, you will be able to:

- Appreciate the need for the analysis of waveforms
- Describe wave analyzers along with their types including frequency-selective and heterodyne wave analyzers
- Describe the spectrum analyzer and spectral displays on CRO along with amplitude, frequency, and phase modulation of signals
- Appreciate the concept of power analyzer and its types including logic analyzer, communications signal analyzer, and network monitoring system
- Describe the harmonic distortion analyzer and its types including tuned circuit, heterodyne harmonic, and fundamental-suppression harmonic distortion analyzers

13.1 INTRODUCTION

In the previous chapter, we studied the different types of signal generators. These generators were used for the generation of various waveforms. The waveforms generated at the output need to be tested for quality, stability, and distortion. In addition, waveform analysis also includes determination of amplitude, frequency, and phase angle of its harmonic components. Signal analyzers such as *wave analyzer*, *spectrum analyzer*, *power analyzer,* and *distortion analyzer* are required for this purpose. All these analyzers measure the basic frequency properties of a signal using different techniques. We will discuss all these analyzers in this chapter.

13.2 WAVE ANALYZER

Wave analyzers are the type of instruments that measure the relative amplitude of single-frequency components within the band of 10 Hz to 40 MHz. The amplitude of the frequency component is displayed on a CRO or a suitable voltmeter. Therefore, this instrument is basically considered as a voltmeter that selects or tunes only one signal frequency component and rejects all others.

Any periodic waveform consists of a series of sinusoidal harmonics along with a DC component. This wave is analyzed to obtain the values of its frequency, amplitude, and phase angle of its harmonic components. It uses the band-pass filter network to filter out various harmonic components and passes only a narrow band of frequency offering high attenuation to all other frequencies. The wave analyzers are categorized in two types depending on their frequency range as *frequency-selective wave analyzer* and *heterodyne wave analyzer*.

Applications

Wave analyzers are used in various fields to measure sound, vibration, and electrical parameters.

- The vibrations generated by rotating electrical machines and apparatuses, and noise are first identified by the wave analyzers and then reduced or eliminated.
- Along with the wave analyzer, a fine spectrum analyzer is used which shows various discrete frequencies and resonances which can be related to motions within the machine.

13.2.1 Frequency-selective Wave Analyzer

In frequency-selective wave analyzer, the waveform to be analyzed is applied first to the input attenuator, set by the meter range switch on the front panel. The waveform is then fed to the driver amplifier whose output is the attenuated waveform, applied to the high-Q active filter. A cascaded arrangement of RC resonant sections and filter amplifiers makes up the filter section. The capacitors used in selecting frequency range are close-tolerance polystyrene capacitors. These capacitors are switched in the RC sections and the pass band of the total filter section is covered in decade steps over the entire audio range. The filter can be tuned to any desired frequency within the selected pass band through a precision potentiometer. This wave analyzer is suitable for audio frequency range, that is, from 20 Hz to 20 kHz as shown in Figure 13.1.

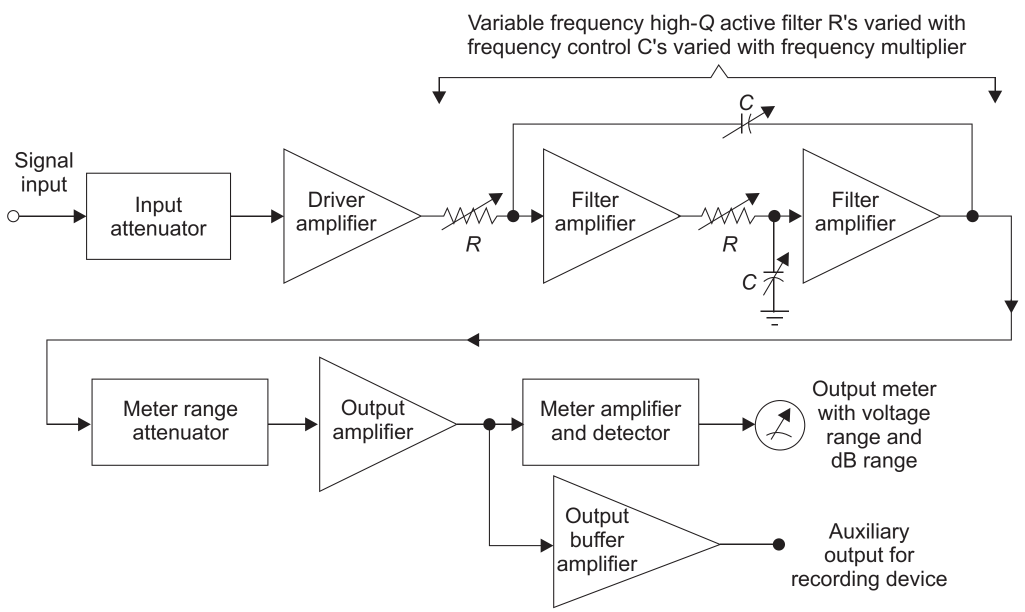


Fig. 13.1 Block Diagram of Frequency-selective Wave Analyzer

The selected signal from the final amplifier stage is applied to the meter circuit and to an unturned buffer amplifier. The buffer amplifier in turn drives an electronic counter or a recorder. An average-type detector drives the meter having several voltage ranges as well as the decibel scale.

This wave analyzer has a very narrow bandwidth of about 1% of the selected frequency. Figure 13.2 shows attenuation curve for this analyzer with initial attenuation rate of approximately 600 db per octave. At one-half and twice the selected frequency, the attenuation is about 75 dB. The attenuation still increases as we go far from the centre frequency.

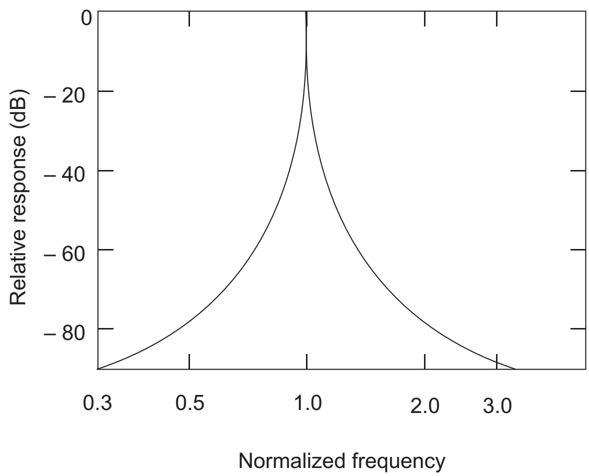


Fig. 13.2 Attenuation Curve of Frequency-selective Wave Analyzer

13.2.2 Heterodyne Wave Analyzer

The heterodyne wave analyzer is well suited for radio frequency range, that is, in the megahertz range, unlike the frequency-selective wave analyzer (discussed earlier), that works in the audio frequency range. In this analyzer, heterodyning principle is used in which the input signal to be analyzed is mixed with the internal tunable local oscillator signal at the mixer stage that produces a high intermediate frequency (abbreviated as IF).

The input signal after being fed to an input attenuator and an unturned amplifier through the probe connector, then mixes with the local oscillator where the oscillator is adjusted to give a fixed frequency output. This output lies in the pass band of the IF amplifier and is amplified by a 30 MHz IF amplifier. The signal from the IF amplifier is then fed to the second mixer provided with a 30 MHz crystal oscillator signal. The output of this mixer is the information centred on zero frequency which is applied to an active filter that has a controllable bandwidth. It passes the selected frequency component to the meter amplifier and the detector circuit. The meter detector output is applied to a recording device or read on a decibel calibrated scale.

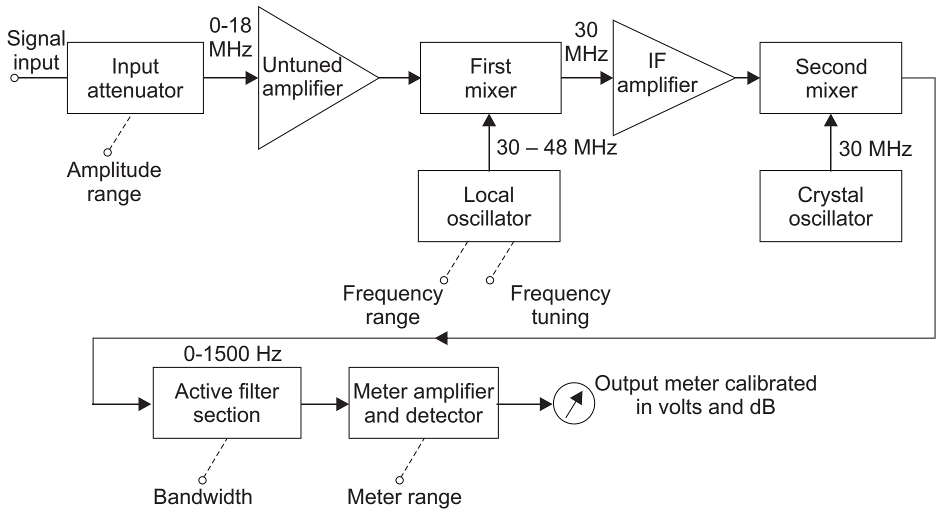


Fig. 13.3 Block Diagram of Heterodyne Wave Analyzer

13.3 SPECTRUM ANALYZER

The spectrum analyzer is an instrument which is used to analyze the distribution of energy over a frequency spectrum of a given electrical signal. It provides information about the bandwidth, spurious signal generation, and effects of various modulation techniques. This information is useful in design and testing of radio frequency (that is, RF) and pulse circuitry. The spectrum analyzers are of two types—*high-frequency* and *low-frequency spectrum analyzers*. The spectrum analysis of high-frequency spectrum analyzers can be of further two sub-types namely, *audio frequency analysis* (also known as AF analysis) and *radio frequency analysis* (also known as RF analysis). The RF spectrum covers a majority of fields like communication, radar, industrial instrumentation, and navigation and thus its frequency range is from 10 MHz to 40 GHz. The spectrum analyzer gives a graphical representation of amplitude of signal as a function of its frequency in the RF spectrum range.

13.3.1 High-frequency Spectrum Analyzer

The basic swept-tuned radio-frequency spectrum analyzer is shown in Figure 13.4 which consists of a sawtooth generator, voltage-tuned local oscillator, mixer, intermediate frequency (IF) amplifier, and detector and video amplifier. The sawtooth generator produces a ramp voltage and feeds it to the frequency control element of the voltage tuned local oscillator. The horizontal plates of CRO is also applied with same sawtooth voltage. The mixer input is provided with the RF signal to be tested. The local oscillator sweeps at a recurring linear rate in its frequency band to beat with the input signal so that the intermediate frequency is generated. The RF input signal generates the IF component only when it is present in the

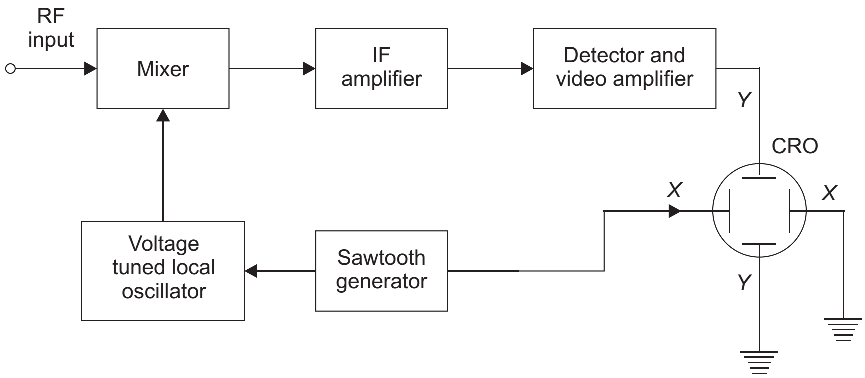


Fig. 13.4 Block Diagram of Swept Radio Frequency Spectrum Analyzer

band. The IF signal is then amplified, detected, and applied to the vertical deflection plates of the CRO thereby producing an amplitude versus frequency display on the screen.

The general purpose spectrum analyzer is shown in Figure 13.5. It works on the principle of superheterodyne receiver with the frequency range of 10 kHz to 300 MHz. It generates an IF signal higher than the highest input frequency, that is, 400 MHz in our case. The low-pass filter removes the input image, that is, a band of frequencies from 800 to 1100 MHz and also attenuates the signal at the first IF of 400 MHz. This spectrum analyzer has a selectivity of 1 kHz when set at the narrowest band. However, this selectivity cannot be achieved at 400 MHz. Therefore, the first IF of 400 MHz is heterodyned to a relatively lower frequency while the second IF in our case is at 21.4 MHz. Here, the crystal filters are used to achieve the desired selectivity. The image components in the second oscillator must be removed in the same manner. The second local oscillator is at 421.4 MHz, that is, 21.4 MHz above the first IF which sets the image frequency at 442.8 MHz to be removed by the first IF filter. The first local oscillator frequency is swept by the varactor diodes and the frequency spectrum which has been swept is called **dispersion** of the spectrum analyzer. It is the band

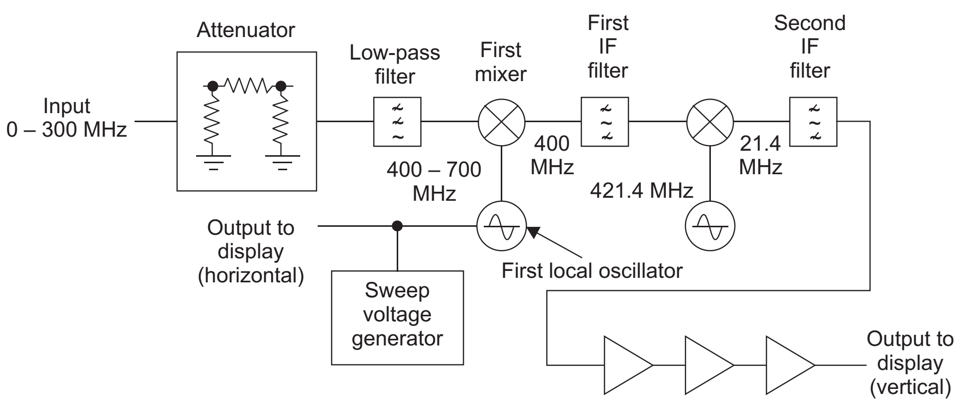


Fig. 13.5 Block Diagram of General Purpose Spectrum Analyzer

of frequency that can be displayed on the screen. The spectrum analyzer basically sweeps the narrow frequency ranges where there is a possibility that the frequency instabilities of the first local oscillator may destroy the display of the analyzer.

The frequency instabilities that occur in the instrument are of two types, namely, *long-term instability* and *phase noise*.

- **Long-term instability:** This instability arises due to drift of the first local oscillator frequency and appears as the movement of the spectrum across spectrum analyzer screen. This instability can be compensated by bringing back the spectrum analyzer to centre the display. If the frequency drift is too fast, the display cannot be centred by the operator.
- **Phase noise:** The noise voltages in the tuned circuit or those picked up by the varactor circuit cause rapid variation in frequency resulting in phase noise. The tuning range of first local oscillator is of several hundred megahertz or microvolts of noise on the varactor tuning voltage which may cause significant frequency modulation. The frequency modulation cannot be corrected due to phase noise, therefore, some electronic means must be applied to the first local oscillator.

13.3.2 Low-frequency Spectrum Analyzer

The low-frequency spectrum analyzer (also termed as LF spectrum analyzer) is another type of spectrum analyzer with the frequency range from 7–30 Hz. Figure 13.6 shows an LF spectrum analyzer which consists of an amplifier to which input signal is applied. This amplifier has an overall gain of 200 depending on the magnitude of the input signal along with the number of stages. Then the signal is fed to the band pass filter (also known as **BPF**) whose centre frequency can be varied at a rate by the ramp and applied to the *X*-plates of a CRO. The BPF gives a full-wave rectified output which is filtered to give an absolute value of the amplitude of the frequency component present at that moment.

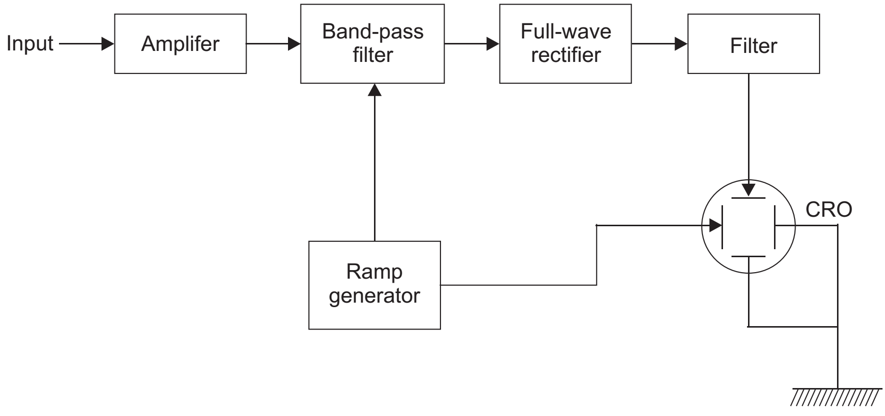


Fig. 13.6 Block Diagram of LF Spectrum Analyzer

It is to be noted here that low-frequency signals are difficult to mix so RF spectrum analyzers are not used for them. Also, unlike RF spectrum analyzers, LF spectrum analyzers do not use local oscillators.

The LF spectrum analyzers are very uncommon and are used for measuring repetition rate, pulse width, and FM deviation.

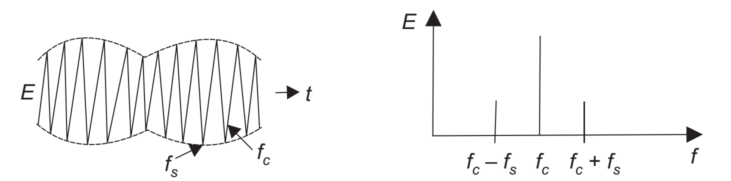
13.3.3 Spectral Displays

Spectral displays of the electrical signals are made on CRO in terms of *time domain* and *frequency domain*. In time domain, the *X*-axis of the CRO is calibrated to read pulse width, rise time, and repetition rate directly. However, in frequency domain, applicable for spectrum analyzers, the signals are broken into individual frequency components and displayed along *X*-axis of CRO which is calibrated in terms of frequency. The different types of signals are applied to the spectrum analyzer whose spectra are displayed on the CRO as discussed further.

The input signal in a spectrum analyzer is a continuous wave which is swept by its local oscillator. The signal displayed on the screen is a plot of an output of IF amplifier pass band. The screen display is a single spike as the sweep width or spectrum width of total RF is wide in comparison to IF bandwidth in the analyzer. The signal could be amplitude, frequency, or phase modulated by an input signal.

Amplitude modulation

When an input signal of frequency f_s modulates a continuous wave signal of frequency f_c in its amplitude, an **amplitude modulated signal** is produced with two side band frequencies, $f_c + f_s$ and $f_c - f_s$ (see Figure 13.7). The magnitude of the signal displayed depends on the percentage of modulation.



(a) Plot of Amplitude Versus Time (b) Plot of Amplitude Versus Frequency

Fig. 13.7 Amplitude Modulation

Frequency modulation

When a continuous signal of frequency f_c is modulated in frequency at a rate of f_r, a **frequency modulated** signal is produced with infinite number of sidebands located at intervals of $f_c + nf_r$, where $n = 1, 2, 3\ldots$ It is to be noted here that only the significant power signals are considered.

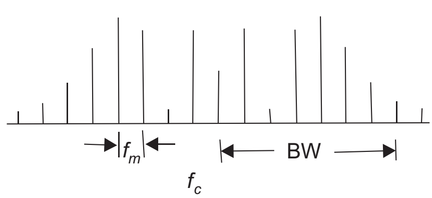
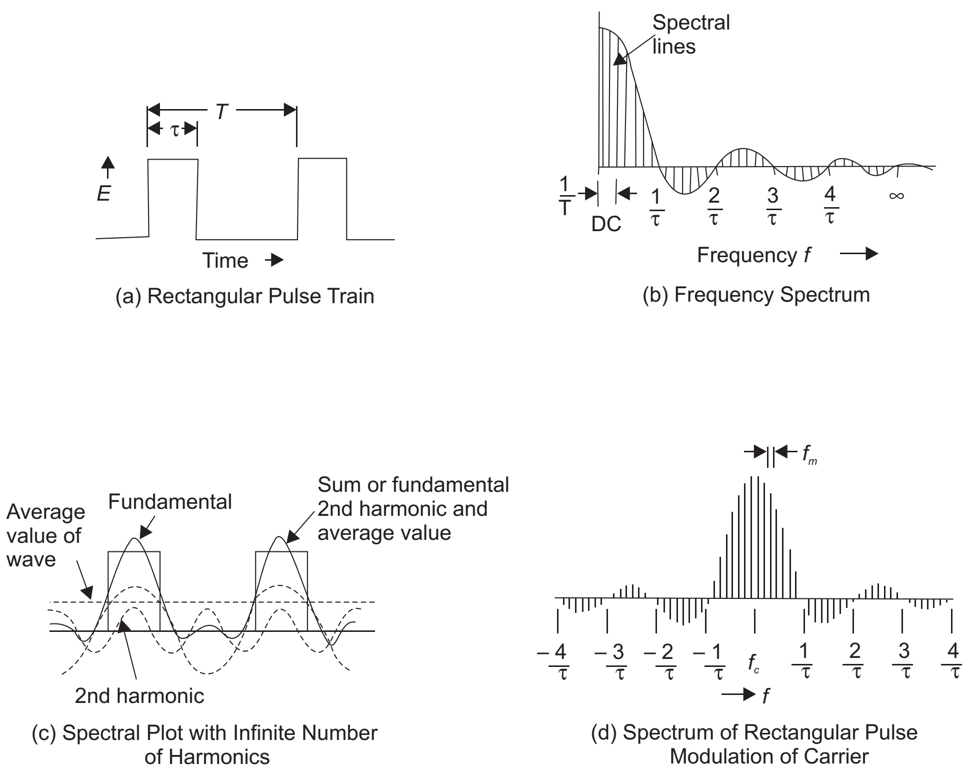


Fig. 13.8 Frequency Modulation

Pulse modulation

A time domain pulse with zero rise time and no overshoot is shown in Figure 13.9(a) which is an idealized rectangular waveform. This waveform is broken into individual frequency components to give a frequency spectrum to be analyzed as shown in Figure 13.9(b). As a constant voltage, fundamental frequency, and a third harmonic are added algebraically to the above frequency spectrum, it becomes a wave and eventually results in a square wave with more odd harmonics added in phase with the fundamental frequency. When an infinite number of odd harmonics are added in phase with the fundamental frequency, the pulse becomes perfectly rectangular. Figure 13.9(c) shows a spectral plot in frequency domain in which the amplitude and phases of an infinite number of harmonics are plotted thereby resulting in a smooth envelope. The spectral plot from the rectangular pulse modulation of the carrier is shown in Figure 13.9(d) where individual lines represent the modulation product of the carrier and the modulating pulse frequency with its harmonics. The center of the spectral plot contains the carrier frequency which represents the main lobe and a group of spectral lines above and below the base line are shown as side lobes with the spectral lines spaced in frequency equal to the pulse repetition rate of the original pulse waveform. It is to be noted here that sidebands or spectral lines are twice the harmonic frequencies present in the modulating pulse.



(a) Rectangular Pulse Train

(b) Frequency Spectrum

(c) Spectral Plot with Infinite Number of Harmonics

(d) Spectrum of Rectangular Pulse Modulation of Carrier

Fig. 13.9 Pulse Modulation

13.4 POWER ANALYZER

The power analyzers measure and analyze power, harmonics, and pulse width modulation (also known as PWM) motor drive triggering. These instruments are used to check the transformer efficiency and power supply performance, test and verify the correct operation of motors, and also as oscilloscopes to control high voltage power electronic circuits by troubleshooting and verifying complicated electronic control circuits. The oscilloscopes used for this purpose have bandwidth of 100 MHz and a sampling rate of 500 samples/s per channel. Such instruments measure harmonics up to thirty-first harmonic with the fundamental frequency lying between 30 and 450 Hz.

The different types of power analyzers are *logic analyzer*, *communications signal analyzer*, and *network monitoring system*.

13.4.1 Logic Analyzer

A combination of an oscilloscope and a single logic analyzer probe can capture, display, and analyze time-correlated analog and digital measurements. Such instruments are used for real-time digital system analysis. Multiple-base digital systems can be debugged and verified by performing timing analysis and a single synchronous base state. Some of the advanced analyzers available are 1.25 Gb/s data rate or 800 MHz data acquisition.

13.4.2 Communications Signal Analyzer

The design, evaluation, and testing of Telecom and Datacom components, transceiver sub-assemblies, and transmission systems can be done with the help of communication signal analyzer. Also, these analyzers perform testing and manufacturing for IEEE conformance. Such instruments provide results that include data along with amplitude and time histograms, mask testing, statistical measurements, and communication-tailored measurement with jitter, duty cycle, noise, undershoot and overshoot *Q*-factor, mean optical power, and amplitude measurements for non-return to zero and return to zero signals.

Some of the features of communication signal analyzer are as follows.

- It has wide bandwidth ranging from DC to 70 GHz.
- It has fast acquisition rate.
- It has automated communication measurements
- It supports optical and electrical standards

13.4.3 Network Monitoring System

The network monitoring system provides a complete network management and supervision capabilities that gives a broad view of the entire network via centralized remote monitoring system. Such systems with 150 protocols for second generation (that is, 2G, 2.5G) and third generation (3G) fixed and mobile networks are used for monitoring of Global System for Mobile Communication (GSM) and General Packet Radio Service (GPRS) non-intrusive networks.

Such systems are capable of traffic monitoring and signal measurements, real-time status monitoring, signal accounting, call and transaction data record, billing verification, protocol analysis, fraud detection, roaming supervision, GPRS monitoring, and performance analysis of intelligent network services.

13.5 HARMONIC DISTORTION ANALYZER

When a sinusoidal signal is applied as an input to an electronic device, such as an amplifier, ideally the output should also be a sinusoidal wave. However, due to the inherent non-linear characteristics of the device, it is not an exact replica of the input signal. Thus, we may conclude that non-linearities of different circuit elements produce harmonics in the output wave. Hence, this distortion is known as **harmonic distortion** (abbreviated as HD).

The total harmonic distortion (abbreviated as THD) can be expressed in terms of the harmonic content of the wave as harmonics get produced in the input sine wave and these harmonics are the multiples of the fundamental frequency of the input signal. Mathematically, total harmonic distortion, denoted as D, is given as:

$$D = \sqrt{D_2^2 + D_3^2 + D_4^2 + ...}$$

where D_n (n = 2, 3, 4…) is the distortion of the nth harmonic .

Here, each harmonic distortion can be obtained by the ratio of the amplitude of the harmonic to that of the fundamental frequency, represented as:

$$D_2 = \frac{H_2}{H_1}, \quad D_3 = \frac{H_3}{H_1}, \quad D_4 = \frac{H_4}{H_1}$$

where H_n is the amplitude of the nth harmonic.

There are different methods by which harmonic distortion can be measured, namely, *tuned-circuit harmonic analyzer*, *heterodyne harmonic analyzer* or *wavemeter*, and *fundamental-suppression harmonic distortion analyzer*.

13.5.1 Tuned-circuit Harmonic Analyzer

In tuned-circuit harmonic analyzer, a circuit formed by the series combination of an inductor L and a capacitor C, that is, *series-resonant circuit* is tuned to a specific harmonic frequency. The input of the amplifier is then transformer-coupled to this harmonic component. The amplifier output is rectified and applied to a meter circuit. Once the reading is obtained on the meter, the resonant circuit returns to the next harmonic frequency for the next reading. Figure 13.10 shows the block diagram of a tuned circuit harmonic analyzer.

It is to be noted here that the compensation for the variation in the AC resistance of the series-resonant circuit as well as for the variation in the amplifier gain over the frequency range of the instrument is provided by the parallel-resonant circuit made of inductor L_1, resistance R_1, and capacitor C_1.

Disadvantages

This analyzer has many disadvantages as follows.

- For the analyzer to work at low frequencies, the values required for L and C would be very large. Therefore, the physical size of these components would be impractical.
- It is difficult to separate and distinguish the harmonics of the signal frequency as they are very close in frequency.

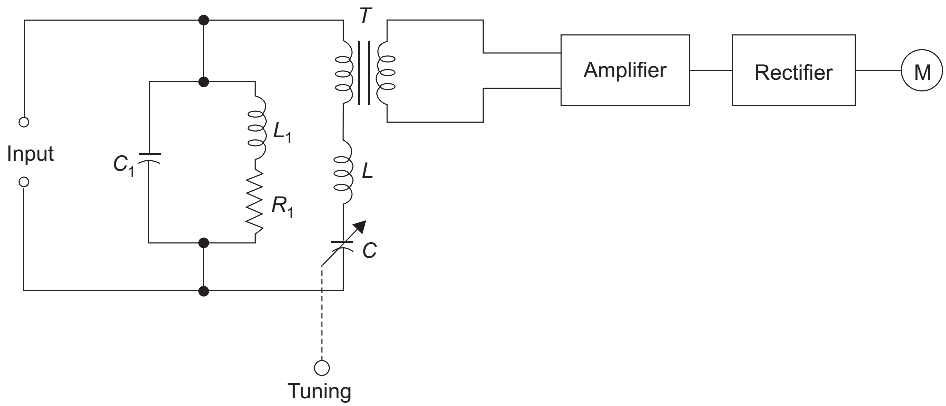


Fig. 13.10 Tuned-circuit Harmonic Analyzer

13.5.2 Heterodyne Harmonic Analyzer or Wavemeter

The heterodyne harmonic analyzer, also known as **wavemeter** is used to overcome the limitations of the tuned-circuit harmonic analyzer by using a highly selective fixed-frequency filter. Figure 13.11 shows functional sections of this analyzer in which the variable frequency oscillator output is heterodyned with each harmonic of the input signal efficiently. The frequency of the filter is made equal to the sum or the difference of the frequency of the signal output from the mixer. Highly selective quartz-crystal type filters are used as each harmonic frequency is converted to constant frequency. At the output, the metering circuit receives a constant frequency signal corresponding to the particular harmonic being measured. The meter is calibrated in terms of voltage. The instruments that provide direct reading are known as **frequency-selective voltmeters**. They are of heterodyne type instruments. In these instruments, the input signal frequency is read by a calibrated dial. Only the difference frequency is passed by a low-pass filter in the input circuit while rejects the sum of the mixed

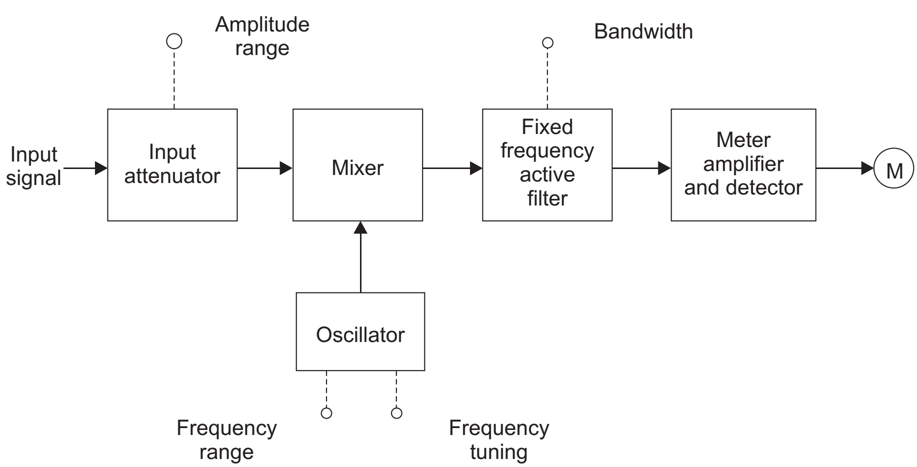


Fig. 13.11 Heterodyne Harmonic Analyzer

frequencies. This voltage is compared with the input signal and read by a voltmeter calibrated in dBm and volts. The range of such instruments lies between –90 dBm and +32 dBm.

Advantages

The advantages of this analyzer are as follows.

- The mixer is generally a balanced modulator, so eliminating the original frequency of the harmonic is easier through it.
- The harmonic distortion generated by the balanced modulator is low as compared to different types of mixers.
- Quartz crystal filters or inverse feedback filters are used as they give excellent selectivity.

13.5.3 Fundamental-suppression Harmonic Distortion Analyzer

In fundamental-suppression harmonic distortion analyzer, total harmonic distortion is measured instead of the distortion caused by each component. The network shown in Figure 13.12 passes all the harmonics components of the frequency and rejects or suppresses the fundamental frequency on application of an input waveform. This network consists of four sections, namely, *input circuit with impedance convertor*, *rejection amplifier*, *metering circuit*, and *power supply*. The input circuit with impedance convertor is placed at the input terminals of the instrument that provides a low-noise, high input impedance circuit whose gain is independent of the signal source impedance. The input signal fundamental frequency is rejected by the rejection amplifier while the remaining frequency components are passed on to the metering circuit that measures the harmonic distortion. A visual indication of total harmonic distortion is provided by the metering circuit in terms of percentage of total input voltage.

The distortion analyzer operates in two modes of operation based on the position of the function switch, whether it is connected in *voltmeter position* or *distortion position*. On connecting in voltmeter position, the instrument works as a conventional AC voltmeter in which the input signal passes through the 1/1 and 100/1 attenuator and applied to the impedance convertor that selects the appropriate range of the meter. In this mode, the rejection amplifier is bypassed by the output signal of the impedance convertor and the signal then goes directly to the metering circuit. It is to be noted here that for general purpose voltage and gain measurements, the voltmeter section can be used separately.

On connecting the switch in distortion position, the input signal is applied to a 1 MΩ attenuator providing 50-dB attenuation in 10-dB steps and the sensitivity switch marked on the front panel controls it. The signal is then fed to the impedance convertor after selecting the desired attenuation. The signals with high source impedance are measured accurately without distorting the input signal, using sensitivity selector in the high impedance position. The signal is then fed to the rejection amplifier consisting of a preamplifier, Wien bridge, and bridge amplifier. The signal is received by the preamplifier that adds amplification to the extremely low distortion levels. The input signal fundamental frequency is rejected by the Wien bridge acting as rejection filter. This bridge acts as an interstage coupling element connected between preamplifier and the bridge amplifier. The frequency range selector tunes the bridge to the input signal fundamental frequency and is balanced by the coarse and fine balance controls for zero output. The voltage and phase of the fundamental frequency

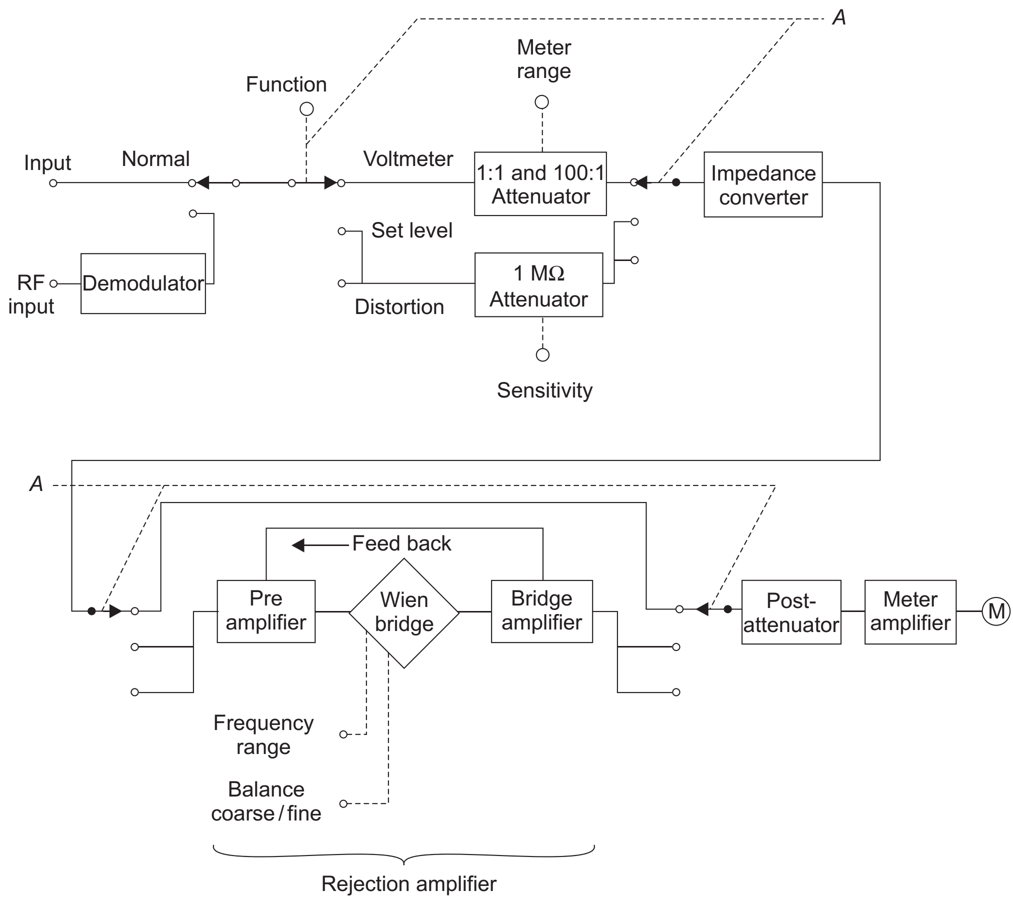


Fig. 13.12 Fundamental-suppression Harmonic Distortion Analyzer

appearing at the junction of the series and shunt reactances are same as the voltage and phase at the midpoint of the resistive branch on bridge being tuned and balanced. No output signal appears when the two voltages are equal and in phase.

The Wien bridge passes all frequencies other than the fundamental frequency by offering varying degrees of phase shift and attenuation and the output voltage is then amplified by the bridge amplifier whose output then goes to the meter circuit through a post-attenuator and displayed on the front panel meter. The post-attenuator limits the signal level to the meter amplifier to 1 mV for a full scale deflection on all ranges. The meter amplifier is basically a multistage circuit with flat response characteristics, designed for low drift and low noise. The meter scale is calibrated to read the rms value of the sine wave.

Advantages

The advantages of fundamental-suppression distortion analyzer are as follows.

- Since all the frequency components are passed and only the fundamental frequency component is suppressed, the selectivity requirement is not very severe.
- A very small amount of harmonic distortion is generated in the instrument.

Let us Summarize

1. The waveform analysis includes waveforms generated at the output of the signal generators to be tested for quality, stability, and distortion. In addition, it also includes determination of amplitude, frequency, and phase angle of its harmonic components. The signal analyzers such as wave analyzer, spectrum analyzer, power analyzer, and distortion analyzer are required for this purpose.
2. The wave analyzers measure the relative amplitude of single-frequency component within the band of 10 Hz to 40 MHz.
3. The wave analyzers are categorized in two types depending on their frequency range as frequency-selective wave analyzer and heterodyne wave analyzer.
4. The spectrum analyzer is an instrument which is used to analyze the distribution of energy over a frequency spectrum of a given electrical signal. It gives information about the bandwidth, spurious signal generation, and effects of various modulation techniques. This information is used in design and testing of radio frequency (that is, RF) and pulse circuitry.
5. The spectrum analyzers are of two types—high-frequency and low-frequency spectrum analyzers. The spectrum analysis of high-frequency spectrum analyzers can be of further two sub-types, namely, audio frequency analysis (also known as AF analysis) and radio frequency analysis (also known as RF analysis).
6. The spectral displays of the electrical signals are made on CRO in terms of time and frequency domain in which the *X*-axis of the CRO is calibrated to read pulse width, rise time, repetition rate, and frequency directly.
7. The input signal in a spectrum analyzer is a continuous wave. The carrier signal could be amplitude, frequency, or phase modulated by an input signal.
8. The power analyzers measure and analyze power, harmonics, and pulse width modulation (also known as PWM) motor drive triggering.
9. The different types of power analyzers are logic analyzer, communications signal analyzer, and network monitoring system.
10. Harmonic distortion is due to non-linearities of different circuit elements that produce harmonics in the output wave.
11. The total harmonic distortion (abbreviated as THD) can be expressed in terms of harmonic content of the wave as harmonics get produced in the input sine wave. These harmonics are the multiples of the fundamental frequency of the input signal.
12. There are different methods by which harmonic distortion can be measured, namely, tuned-circuit harmonic analyzer, heterodyne harmonic analyzer or wavemeter, and fundamental-suppression harmonic distortion analyzer.

EXERCISES

Fill in the Blanks

1. The wave analyzer is suitable for _________ frequency range.
2. In heterodyne harmonic analyzer, the input signal as well as RF signal is combined with the _________
3. The harmonics of the signal frequency are difficult to distinguish in tuned-circuit harmonic analyzer since the harmonics are very close in _________.
4. _________ analyzers are used as real-time digital system analysis.
5. The frequency range of a low-frequency spectrum analyzer is _________.

Multiple Choice Questions

1. The long term instability can be defined as
   (a) drift of the first local oscillator frequency
   (b) drift of the second local oscillator frequency
   (c) both of the above
   (d) none of the above
2. At one half and twice the selected frequency, the attenuation in frequency-selective wave analyzer is about
   (a) 65 dB (b) 75 dB
   (c) 50 dB (d) 80 dB
3. The frequency range of the RF spectrum lies between
   (a) 10 Hz to 40 MHz (b) 10 MHz to 40 GHz
   (c) 20 Hz to 20 kHz (d) none of the above
4. The expression for total harmonic distortion is given as
   (a) $D = \sqrt{D_2^2 + D_3^2 + D_4^2 + \ldots}$ (b) $D = \sqrt{D_3^2 + D_5^2 + D_7^2 + \ldots}$
   (c) $D = \sqrt{D_2^2 + D_4^2 + D_6^2 + \ldots}$ (d) none of the above
5. Which of these signal analyzers can be used for AF applications?
   (a) Harmonic distortion analyzer (b) Frequency-selective wave analyzer
   (c) Power analyzer (d) Heterodyne wave analyzer

State True or False

1. The wave analyzers work in audio frequency range.
2. A broad view of the entire network can be produced by network monitoring system using centralized remote monitoring system.
3. The drift in the frequency of the first local oscillator is known as phase noise.
4. A power analyzer cannot be used to check the transformer efficiency.
5. The design and testing of datacom and telecom components can be performed using a logic analyzer.

Descriptive/Numerical Questions

1. Draw the block schematic of tuned-circuit harmonic distortion analyzer and explain its working. What are the advantages and disadvantages of this instrument?
2. Draw and discuss the spectral displays of various modulations using spectrum analyzer.
3. Draw the block schematic of a low-frequency spectrum analyzer and explain its principle and working.
4. What is meant by distortion? Explain the available types of distortion.
5. Distinguish between spectrum analyzer and harmonic distortion analyzer.
6. Explain the operation of a frequency-selective wave analyzer with a neat diagram.
7. Discuss the working of a wave analyzer using heterodyning principle. Also list the applications of the wave analyzer.
8. Explain the significance of power analyzer and its applications.
9. Differentiate between heterodyne and fundamental-suppression harmonic distortion analyzers.
10. Why is wavemeter called frequency-selective voltmeter?

14

DC and AC Potentiometers

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Explain the basic principle of potentiometers
- Discuss the various advantages of potentiometers
- Describe the two types of potentiometers—DC and AC
- Differentiate between the different types of DC potentiometers that include basic slide wire, Crompton's, multi-range, precision type, and deflection potentiometers
- Differentiate between the two types of AC potentiometers that include polar and co-ordinate AC potentiometers
- Explain the various applications of both DC and AC potentiometers
- Describe the self-balancing potentiometer

14.1 INTRODUCTION

In chapters 3 and 4, we discussed measuring voltage directly, using different types of instruments. Now we shall discuss measuring voltage by indirect means using a potentiometre by comparing it with some known voltage. Here, the known voltage can be provided by any known voltage reference source or a standard cell may be used for this purpose.

A potentiometer consists of a galvanometer which compares the known voltage with the unknown voltage. The galvanometer is connected between the unknown voltage and the sliding contact or **wiper**. The sliding contact of the potentiometer is adjusted and the corresponding galvanometer reading is noted down until the galvanometer shows a zero deflection. At this point, the current flowing through the circuit is zero and the position of sliding contact gives the magnitude of voltage. This voltage measurement is independent of the source resistance.

In this chapter we will discuss about two types of potentiometers, namely, *DC potentiometers* and *AC potentiometers* and their types along with the applications of both potentiometers.

14.2 DC POTENTIOMETERS

DC potentiometers are widely used in educational as well as industrial applications. Some of the advantages of DC potentiometers are described as follows.

- Since the results obtained from the potentiometer do not depend upon any pointer deflection, they give more accurate results and depend only on the accuracy of the known voltage reference source.
- Potentiometers are based on balance condition under which no current flows through then. Thus, a potentiometer circuit has zero power consumption and also the result is independent of the source resistance.
- In addition to measuring voltage, they can be used for measuring current by measuring the voltage drop produced by the unknown current through a known standard resistance.

14.2.1 Types of DC Potentiometers

There are many types of DC potentiometers which include the *basic slide wire DC potentiometer*, *Crompton's potentiometer*, *multiple range potentiometer*, *precision type potentiometer* (*vernier potentiometer*), and *deflection potentiometer*.

Basic Slide-wire potentiometer

The basic slide wire potentiometer is shown in Figure 14.1. This circuit gives the basic principle of operation for all potentiometers. It consists of a slide wire or **resistance wire** *PQ* placed alongside a scale. Its resistance per unit length is uniform.

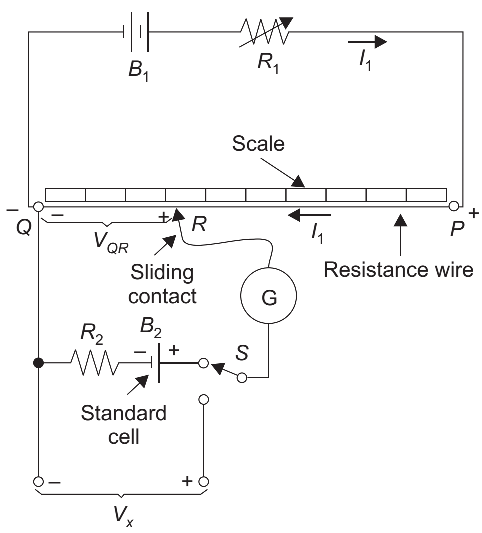


Fig. 14.1 Slide Wire Potentiometer

A battery B_1 connected in series with a variable resistance R_1 supplies a current I_1 to the wire. Resistance R_1 is used to control this current. A very sensitive galvanometer G is also connected in the circuit. It is connected to the slide wire at one end through a sliding contact R and to the switch S at the other end. The switch S moves between two positions, one facilitating contact with unknown voltage V_x and the other with a standard cell B_2. The standard cell B_2 is protected by a resistor R_2 in case of excessive current flow.

Standardization

The standardization of a potentiometer must be done before measuring the voltage. The process of setting the voltage drop per cm along the slide wire to the desired level is known as **standardizing** a potentiometer. To standardize the potentiometer, the switch S is set to the standard cell position. Now, the resistance R_1 is adjusted along with the position of the sliding contact on the slide wire until the galvanometer shows zero deflection. At this position on the sliding wire, the voltage of sliding wire is equal to the voltage of standard cell, that is, $V_{QR} = V_{B2}$. At this point, the voltage/cm is given by:

$$\text{Voltage/cm} = \frac{V_{QR}}{QR}\ \text{V/cm} \qquad \text{...(1)}$$

Here, we have assumed that the length of the slide wire is exactly 100 cm, however, it may vary. The potentiometer is now said to be standardized or calibrated.

Now, after standardizing the potentiometer, the unknown voltage V_x is measured by connecting the switch S to an unknown voltage position. The slide wire contact is again moved to obtain zero deflection in the galvanometer. This new value of the length QR_{new} is noted down. The unknown voltage V_x is then given by:

$$V_x = QR_{\text{new}} \times \text{voltage/cm} \qquad \text{...(2)}$$

Note: *The basic slide wire potentiometer cannot provide a great degree of precision.*

Crompton's potentiometer

As a basic slide wire potentiometer cannot be used for high precision measurements, it is used for education purpose only. Another type of potentiometer, known as **Crompton's potentiometer** is used in modern laboratories. The circuit diagram of this type of potentiometer is shown in Figure 14.2.

This potentiometer consists of additional resistors which increase the accuracy of the instrument and are connected in series with the slide wire. These precision resistances are designated from R_6 to R_{12} and are connected to a rotary switch. Here, the slide wire is constructed in a circular shape, represented by R_{13} in the figure. Thus, a circular scale and a pointer may indicate the position of the sliding contact. When the zero deflection is obtained on the galvanometer, if the rotary switch is at position U then the unknown voltage V_x is given by the expression:

$$V_x = V_{QR} + V_{R12} + V_{R11}$$

The position of switched contact (here, U) can also be shown on a circular scale using a pointer. When the potentiometer is standardized, the voltages may be indicated directly on the scales instead of giving positions of moving contacts. The current I in the slide wire flows through two variable resistors R_1 and R_2 where $R_1 << R_2$. The current I is supplied from battery B_1 and it is approximately adjusted by R_1 followed by a fine adjustment by resistor R_2.

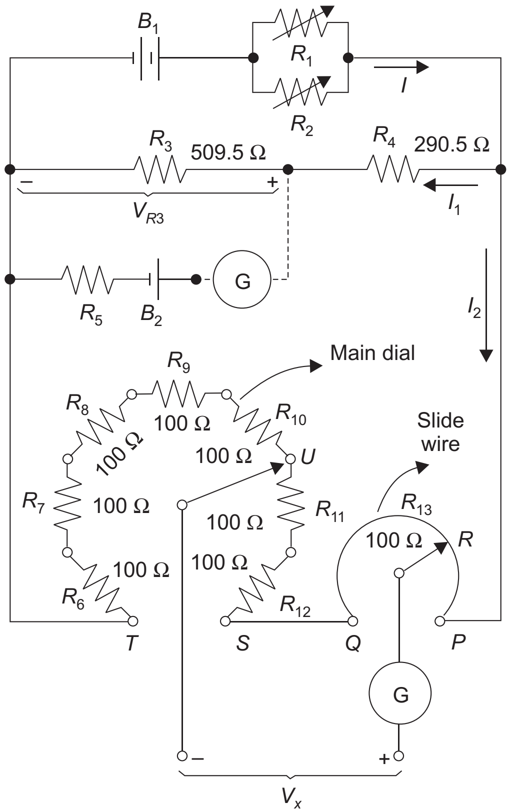


Fig. 14.2 Crompton's Potentiometer

The standardization of this potentiometer is done using two precision resistors R_3 and R_4. When the voltage V_{R3} is equal to the standard cell voltage, the instrument is said to be **standardized**. A pushbutton is used to connect the galvanometer to the standard cell B_2. When the button is pressed, the galvanometer gets connected to B_2 and the junction of R_3 and R_4 as shown by dotted lines in the figure. Null deflection is then obtained on the galvanometer by adjusting resistors R_1 and R_2. Now, the galvanometer is connected to the unknown voltage V_x by releasing the pushbutton. The approximate null deflection in galvanometer is obtained by moving the switched contact U and precise null is obtained by moving the sliding contact R. Since the potentiometer drifts away from its calibration owing to a slight fall in battery voltage, its standardization is checked again by pressing the pushbutton and adjusting R_1 and R_2. This procedure is repeated many times until a null is obtained in both conditions of the pushbutton, pressed and released. Now, the unknown voltage is read from the scales of the potentiometer.

Multiple range potentiometer

The circuit of Crompton's potentiometer shown in Figure 14.2 can be modified to extend its range from a single range of 1.6 V. The modified circuit is shown in Figure 14.3. This circuit is of a duo range potentiometer in which resistors R_A and R_B and a selecting switch S are provided to obtain the two different ranges.

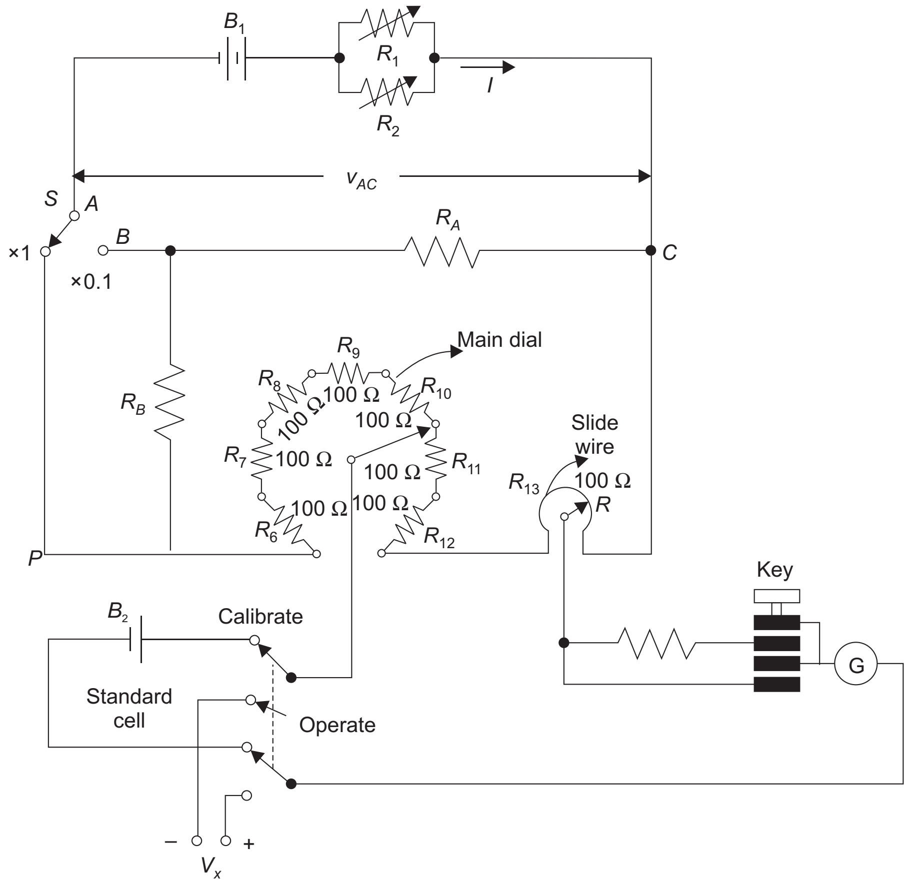


Fig. 14.3 Duo Range Potentiometer

Let us redraw the circuit shown above by omitting standardization circuit and galvanometer for understanding purpose. The simplified circuit is shown in Figure 14.4.

As shown in Figure 14.3 the main dial consists of seven steps of 100 Ω each having a total resistance of 700 Ω and the slide wire has a resistance of 100 Ω. Thus, the total measuring circuit resistance, consisting of main dial and slide wire in series connection comes out to be 800 Ω (same as shown in Figure 14.4). Now, to get a voltage drop of 1.6 V across the measuring circuit, the current I_m through it must be 2 mA ($I = V/R = 1.6/800 = 2$ mA). This value of current corresponds to the position A of switch S, however, when switch is in position B, that is, × 0.1 range, the current I_m becomes 1/10 of its previous value. Now, the current I_m is 0.2 mA so that a voltage of 0.16 V can be developed across the measuring circuit resistance R_m.

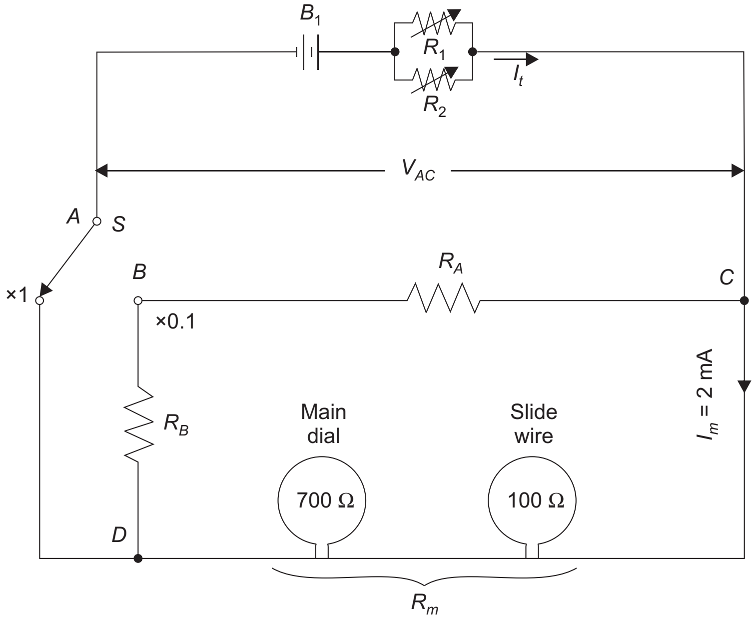


Fig. 14.4 Simplified Circuit of Duo Range Potentiometer

The requirement of the duo range potentiometer circuit is that the changing of measuring range does not require a readjustment of battery voltage or variable resistances R_1 and R_2. Thus, after performing standardization for $\times 1$ range, the standardization of $\times 0.1$ range is not required. To achieve this condition the voltage V_{AC} must be the same for both ranges which require same current I_m in both cases.

The simple circuit diagrams for ranges $\times 1$ and $\times 0.1$ are shown in Figure 14.5(a) and 14.5(b), respectively. When the circuit is on range $\times 1$, the series combination of R_A and R_B is connected in parallel with measuring circuit resistance R_m. When the circuit is on range $\times 0.1$, the series combination of R_B and R_m is connected in parallel with the range resistor R_A.

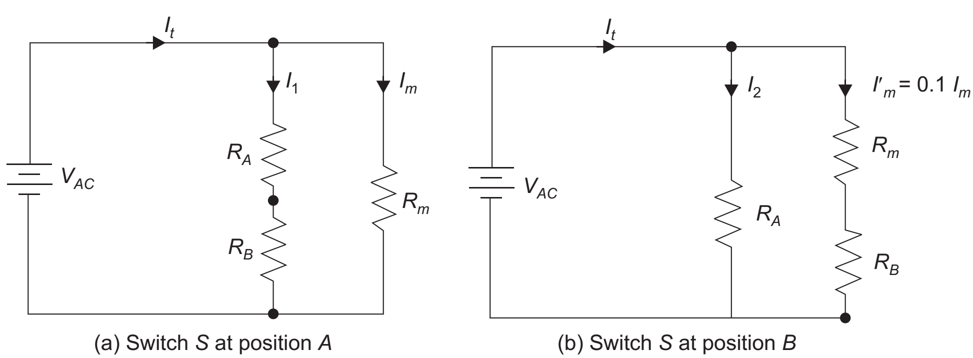


Fig. 14.5 Simplified Circuit

The total current I_t supplied by the battery will be same if the following condition is met:

$$\frac{R_m(R_A + R_B)}{R_A + R_B + R_m} = \frac{R_A(R_m + R_B)}{R_A + R_B + R_m}$$

On solving, the above equation yields:

$$R_A = R_m \qquad ...(3)$$

Thus, the measuring circuit resistance R_m must be equal to the range resistor R_A.

Now, we know that when the switch is at position *B*, the current through the measuring circuit is 1/10 times of the current when the switch is at position *A*. Thus, we have:

$$I'_m = 0.1\ I_m \qquad ...(4)$$

where

$$I_m = \frac{V_{AC}}{R_m} \qquad ...(5)$$

and

$$I'_m = \frac{V_{AC}}{R_m + R_B} \qquad ...(6)$$

From Equations (4) and (6), we get:

$$0.1 I_m = \frac{V_{AC}}{R_m + R_B} \qquad ...(7)$$

Now, from Equations (5) and (7), we get:

$$0.1\,(R_m + R_B) = R_m$$

$$\Rightarrow \qquad R_B = 9R_m = 9R_A \qquad ...(8)$$

Thus, for the circuit shown in Figure 14.4, we have the following values:

$$R_A = R_m = 800\ \Omega$$

$$R_B = 9R_m = 7200\ \Omega$$

The advantages of duo potentiometer are higher precision and higher accuracy. Here, the potentiometer has a voltage ratio of 10:1. However, by selecting proper values of R_A and R_B, any ratio of potentiometer can be constructed.

Precision type (vernier) potentiometer

Earlier discussed potentiometers can provide a precision of 100 μV for voltages up to 1.6 V. However, some applications may need higher precision and accuracy. The precision and accuracy problems in a potentiometer occur due to poor maintenance in contacts and non-uniformity of slide wire. Vernier potentiometer is a precision type potentiometer as it removes the limitation due to non-uniformity of slide wire. It works on two voltage ranges: normal range (from 1.6 V down to 10 μV) and lower range (from 0.16 V down to 1 μV). The circuit diagram of a vernier potentiometer is shown in Figure 14.6.

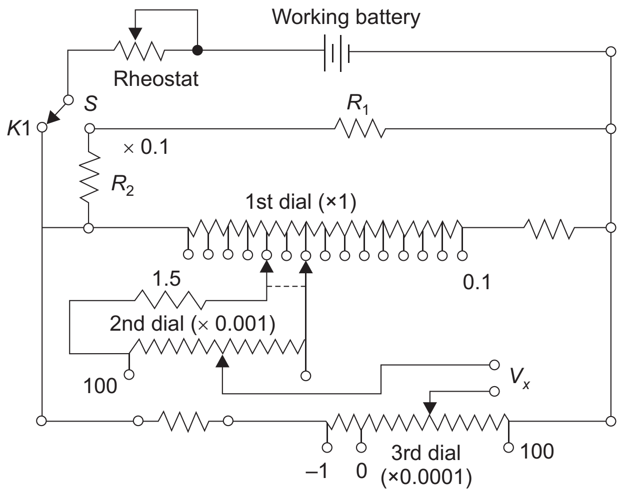


Fig. 14.6 Vernier Potentiometer

It consists of three measuring dials out of which the first dial measures voltage up to 1.5 V in steps of 0.1 V, second or middle dial measures voltage up to 0.1 V in steps of 0.001 V, and the third dial measures voltage ranging from – 0.0001 V to 0.001 V in steps of 0.00001 V (that is, 10 μV). The middle and third dial both have 102 studs. As can be seen, this potentiometer does not have any slide wire.

The moving arm of the middle dial carries two arms which are spaced two studs apart and its resistances are used to shunt two coils of the first dial. The first dial is the main dial and the resistance between its two studs is smaller than the resistance of the second dial. This is to ensure that the voltage drop across the second dial is larger than 0.1 V, otherwise switch contact resistances and leads may produce errors. A shunt circuit is used to obtain the third dial. The third dial allows a true zero and also lets a small negative quantity to be measured.

The measurements in a vernier potentiometer may get affected by stray contact and thermal voltages in the potentiometer, the measuring circuits, and the galvanometer. These voltages are difficult to locate and control and may be of the range of one to several microvolts. To minimize these voltages, the metals used for the construction of terminals, resistors, and connecting leads must be properly selected. Also, the thermal shields may be used for the same.

Deflection potentiometer

Deflection potentiometer is used for the measurement of continuously changing voltage. Since conventional potentiometers consist of dials, they are not able to follow changes in the voltage even if changes are slow. Thus, deflection potentiometers are used for such voltages. The circuit diagram of a deflection potentiometer is shown in Figure 14.7.

This potentiometer consists of one or two main dials containing decade resistance boxes. It also consists of a centre-zero type galvanometer to indicate deflection and two

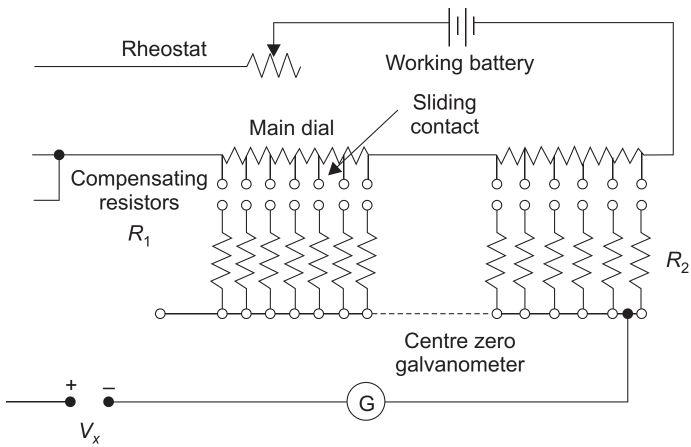


Fig. 14.7 Deflection Potentiometer

compensating resistors R_1 and R_2. These resistances are so selected that the resistance value as seen from the terminals where unknown voltage is applied is constant and does not depend on the position of the sliding contacts. This implies that the current passing through the galvanometer is also independent of the dial settings and is proportional to the out-of-balance current. Thus, the value of out-of-balance voltage can be read directly from the galvanometer scale by calibrating it. The final result of the unknown voltage is obtained by algebraic sum of galvanometer reading and main dial setting where the main dial setting is always kept nearly equal to the unknown voltage.

The deflection potentiometers are used to monitor temperature in the circuit which employs a thermocouple. The unknown voltage terminals are connected to the output of the thermocouple.

Example 1 A simple slide wire is used for the measurement of current in a circuit. The voltage drop across a standard resistance of 1 Ω is balanced at 75 cm. Find the magnitude of the current if the standard cell having an emf of 145 volts is balanced at 50 cm.

Solution: Given that: $V_{B2} = V_{QR} = 145$ V, $QR = 50$ cm, $QR_{\text{new}} = 75$ cm, and $S = 1\ \Omega$

The voltage/cm can be given as:

$$\text{Voltage/cm} = \frac{V_{QR}}{QR} \qquad \text{[Refer to Eqn. (1)]}$$

Substituting the given values, we obtain:

$$\text{Voltage/cm} = \frac{145}{50} = 2.9$$

Now, the unknown voltage is given by:

$$V_x = QR_{\text{new}} \times \text{Voltage/cm} \qquad \text{[Refer to Eqn. (2)]}$$

Substituting the given values, we obtain:

$$V_x = 75 \times 2.9 = 217.5 \text{ V}$$

Now, the current can be given using Ohm's law as:

$$I = \frac{V_x}{S} = \frac{217.5}{1} = 217.5 \text{ A}$$

Example 2 Consider the Crompton's potentiometer as shown in Figure 14.2 with the following parameters: main dial resistors of 100 Ω each, slider wire of resistance 100 Ω, R_3 = 509.5 Ω, R_4 = 290.5 Ω, and voltage of standard cell is 2 V. Calculate the maximum voltage that can be measured using this potentiometer.

Solution: Given that: R_3 = 509.5 Ω, R_4 = 290.5 Ω, and V_{B2} = 2 V

Now, we know that the potentiometer is standardized when the voltage across R_3 is equal to V_{B2}. Thus, we have:

$$V_{R3} = V_{B2} = 2 \text{ V}$$

The current I_1 can be calculated as:

$$I_1 = \frac{V_{B2}}{R_3} \qquad \text{[Refer to Fig. 14.2]}$$

Substituting the values, we get:

$$I_1 = \frac{2}{509.5} = 3.9 \text{ mA}$$

Thus, the maximum voltage $V_{\max}$ that can be measured is calculated using the expression:

$$V_{\max} = I_1(R_3 + R_4)$$

$$\Rightarrow \qquad V_{\max} = 3.9(290.5 + 509.5) = 3120 \text{ mV} = 3.12 \text{ V}$$

14.2.2 Applications of DC Potentiometers

The DC potentiometers have many applications which include *calibration of ammeter*, *calibration of voltmeter*, *calibration of wattmeter*, *measurement of resistance,* and *measurement of power*.

Calibration of ammeter

The circuit required for calibrating an ammeter is shown in Figure 14.8. The ammeter to be calibrated is connected in series with a standard resistance *S*. The resistance *S* is chosen suitably according to the current requirement.

The potentiometer is connected across the standard resistance to measure the voltage across it. Let the voltage be denoted by V_S. The current I_S through the resistance *S* is then given by the expression:

$$I_S = \frac{V_S}{S}$$

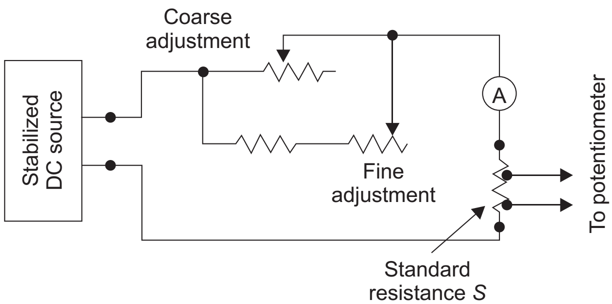


Fig. 14.8 Circuit for Calibration of Ammeter using a Potentiometer

Since resistor S and ammeter are connected in series, the same current flows through them which can be determined by the above expression. This method of calibrating ammeter using a potentiometer is a very accurate method.

Calibration of voltmeter

For voltages below 1.6 V, direct measurement can be done using a potentiometer having high precision. However, a precise voltage divider is used for higher voltages. The circuit diagram for voltmeter calibration using voltage divider is shown in Figure 14.9.

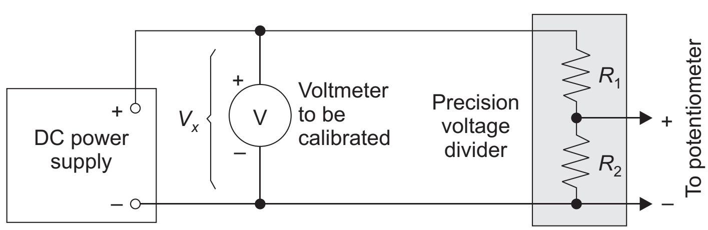


Fig. 14.9 Circuit Diagram for Voltmeter Calibration Using Voltage Divider

The potentiometer is connected to the resistance R_2 to measure the voltage across it. The voltage across the voltmeter V_x can then be calculated by the expression:

$$V_x = \frac{V_{R2}(R_1 + R_2)}{R_2}$$

A volt-box is available which contains precision resistors for the purpose of calibrating voltmeter using potentiometer. A typical volt-box is shown in Figure 14.10 in which the rotary switch and resistors are shown in a straight line formation.

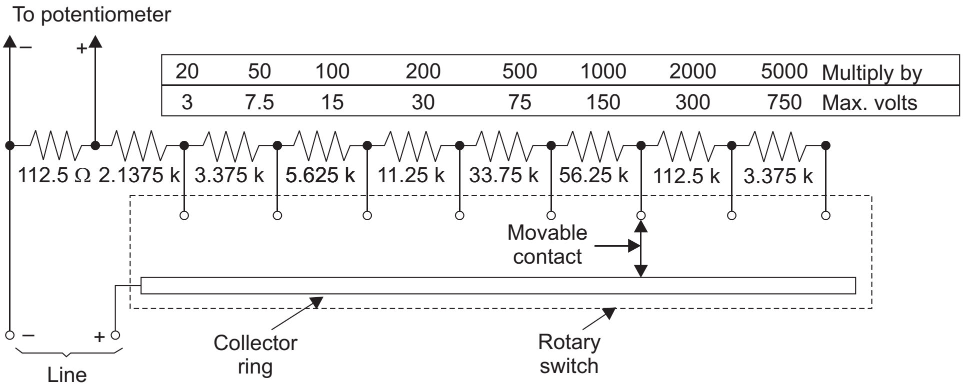


Fig. 14.10 Volt-box

As can be seen from the figure, the potentiometer is connected across 112.5 Ω resistor at the left side to measure voltage drop. The voltmeter terminal voltage, that is, the line voltage is applied across at least two resistors including the 112.5 Ω resistor and one serially connected resistor. The number of resistors connected can be altered by switching the position of the moving contact and thus, the voltage division ratio is altered. Suppose the position in volt-box is set to multiply-by-20 position, the voltage division ratio can be calculated by the expression:

$$\frac{\text{Line voltage}}{\text{Potentiometer voltage}}$$

$$\Rightarrow \qquad \text{Voltage division ratio} = \frac{112.5 + 2.1375}{112.5} = 20$$

Thus, the voltmeter terminal voltage is given by the multiplication of measured potentiometer voltage by the factor 20. For the general case, at any position, the voltmeter terminal voltage or line voltage V_x can be written as:

$$V_x = \text{Measured voltage} \times \text{Multiplier}$$

It is to be noted that with every multiplier position, a maximum voltage is indicated which must be applied to the volt-box in every case. These maximum voltages are applied to minimize heating and power dissipation in precision resistors. For maximum applied voltage, the volt-box output reading is 150 mV.

Calibration of wattmeter

The circuit required for calibrating a wattmeter using a potentiometer is shown in Figure 14.11. A low voltage supply is used to feed the current coil of the wattmeter and the value of current is adjusted using a rheostat (or variable resistance) connected in series with it. The voltage coil of the wattmeter is supplied from the supply. The potentiometer is connected to the circuit through a double pole double throw switch (also known as DPDT).

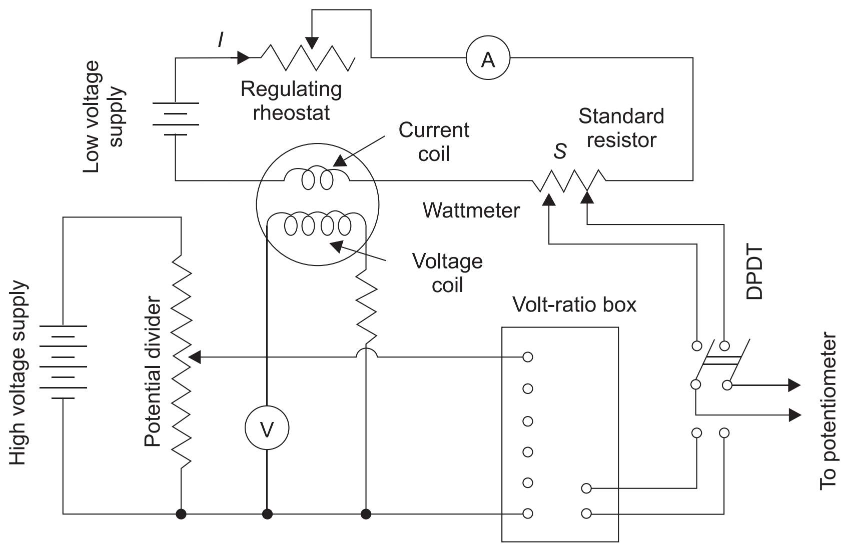


Fig. 14.11 Circuit for Calibration of Wattmeter using a Potentiometer

A volt-ratio box is also used in the circuit that steps down the voltage which is then read by the potentiometer. As we know, the circuit shown above is known as phantom loading circuit (as studied in Section 6.11.1). The potentiometer measures the current I and voltage V many times turn by turn. The wattmeter reading is then compared with the true power, that is, VI.

Measurement of resistance

The measurement of resistance using potentiometers is suitable for low resistances only. The circuit used for this purpose is illustrated in Figure 14.12.

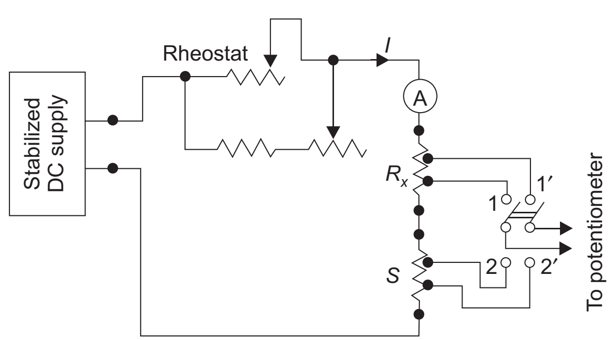


Fig. 14.12 Circuit for Measurement of Resistance using a Potentiometer

The unknown resistance is shown by R_x and the standard resistor is shown by S, both of them are of the same order and connected in series. A potentiometer is connected through a double pole double throw switch to the circuit. A rheostat or a variable resistor is also connected in the circuit to control the current I flowing through it.

When the switch is in position 1, 1′ the unknown resistance R_x gets connected to the potentiometer. Let the potentiometer reading be V_{Rx} at this point. Thus, we have:

$$V_{Rx} = IR_x \qquad ...(9)$$

Now, the switch position is changed to 2, 2′ which results in connecting the potentiometer and standard resistor S. Let the potentiometer reading at this point be V_S. Thus, we have:

$$V_S = IS \qquad ...(10)$$

From Equations (9) and (10), we get:

$$R_x = \frac{V_{Rx}}{V_S} S \qquad ...(11)$$

Thus, the value of unknown resistance R_x can be determined very easily. A stable DC supply is an essential component of this circuit as it helps in maintaining the current to be same in both switch positions. Otherwise, the results obtained will not be accurate.

Here, it is to be noted that the actual value of the current flowing through the circuit need not to be determined. However, an ammeter is connected in the circuit to check if the current flowing is within limits or not. The current I is adjusted to a value so that the voltage drop across each resistance R_x and S is equal to 1 V.

Measurement of power

The circuit required for measurement of power using a potentiometer is shown in Figure 14.13. It consists of a standard resistance S, serially connected with the load and a volt-box. The voltage drop across both standard resistance S and output terminals of volt-box is measured.

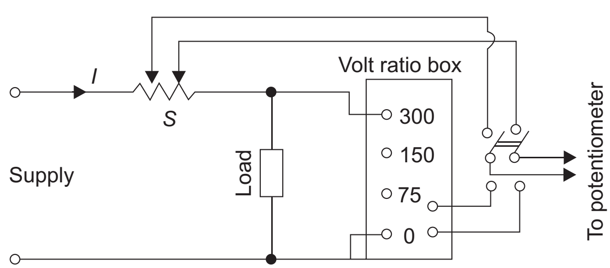


Fig. 14.13 Circuit for Measurement of Power using a Potentiometer

Initially, the potentiometer is connected across the standard resistance S and its reading at that point be denoted by V_S. When the potentiometer is connected across the output terminals of volt-box, let the reading be denoted by V_R. The current flowing through the load I can be given as:

$$I = \frac{V_S}{S}$$

The voltage across the load can be given by the potentiometer reading when it is connected across the output terminals of volt-box. Thus, we have:

$$V = KV_R$$

where V is the voltage across load and K is the multiplying factor of volt-box.

Hence, the power consumed by the load P can be expressed as:

$$P = VI = \frac{KV_R V_S}{S}$$

Example 3: In the measurement of a low resistance by means of a potentiometer, the following readings were obtained: voltage drop across low resistance under test is 0.83942 V, voltage drop across a standard resistance connected in series with the unknown is 1.01575 V. If the value of the standard resistance is 0.10014 Ω, find the value of the unknown resistance.

Solution: Given that: V_{Rx} = 0.83942 V, V_S = 1.01575, and S = 0.10014 Ω

The value of unknown resistance R_x can be determined as:

$$R_x = \frac{V_{Rx}}{V_S} S \qquad \text{[Refer to Eqn.(11)]}$$

Substituting the given values, we get:

$$R_x = \frac{0.83942}{1.01575} \times 0.10014 = 0.08275 \ \Omega$$

14.3 AC POTENTIOMETERS

The principle of operation of an AC potentiometer is similar to that of a DC potentiometer. However, in AC potentiometer, in addition to the magnitude, we need to consider the phase of the unknown voltage and slide wire voltage as well. Thus, in AC potentiometers, the balance is obtained when the magnitudes as well as phase of both the voltages are equal. One more difference between AC and DC potentiometers is that in AC potentiometers, the voltage or current which is to be measured is supplied with the same source as that for the AC supply for the slide wire. This ensures no waveform or frequency errors.

14.3.1 Types of AC Potentiometers

Based on the manner in which the value of the unknown voltage is displayed by the scale or dial of AC potentiometers, they are classified into two types, namely, *polar type* and *co-ordinate type* potentiometers.

Polar AC potentiometers

In polar AC potentiometers, the unknown voltage is expressed in polar form, that is, the magnitude and phase of the voltage is displayed. Consider Figure 14.14 in which the unknown voltage is represented by a vector OB. The length of the vector OA is varied with the help of sliding contact on the slide wire while its phase angle is varied with the help of a phase-shifter.

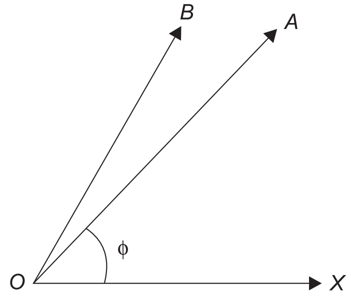


Fig. 14.14 Voltage by Polar AC Potentiometer

An example of polar type AC potentiometer is *drysdale potentiometer*, discussed in this section.

Co-ordinate AC potentiometers

In co-ordinate AC potentiometers, the unknown voltage is expressed in rectangular co-ordinates form. Consider Figure 14.15 in which the unknown voltage is represented by the vector OB. The in-phase component, that is, X component and quadrature component, that is, Y component are obtained from the vector OB and noted down from the scale reading. These two components are 90° out of phase with each other. The magnitude R and phase ϕ of the unknown voltage can be determined as:

$$R = \sqrt{X^2 + Y^2}$$

and
$$\phi = \tan^{-1} \frac{X}{Y}$$

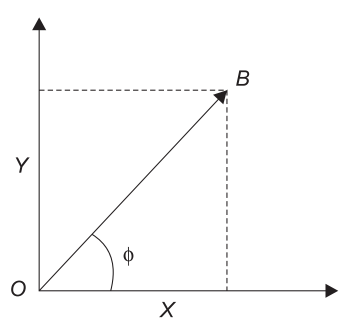


Fig. 14.15 Voltage by Co-ordinate AC Potentiometer

An example of co-ordinate type AC potentiometer is *Gall Tinsley potentiometer*, discussed in this section.

14.3.2 Drysdale Polar Potentiometer

The circuit diagram of Drysdale polar potentiometer using a phase shifting transformer is shown in Figure 14.16. This phase transformer changes the phase of the AC supply which is applied to the DC potentiometer.

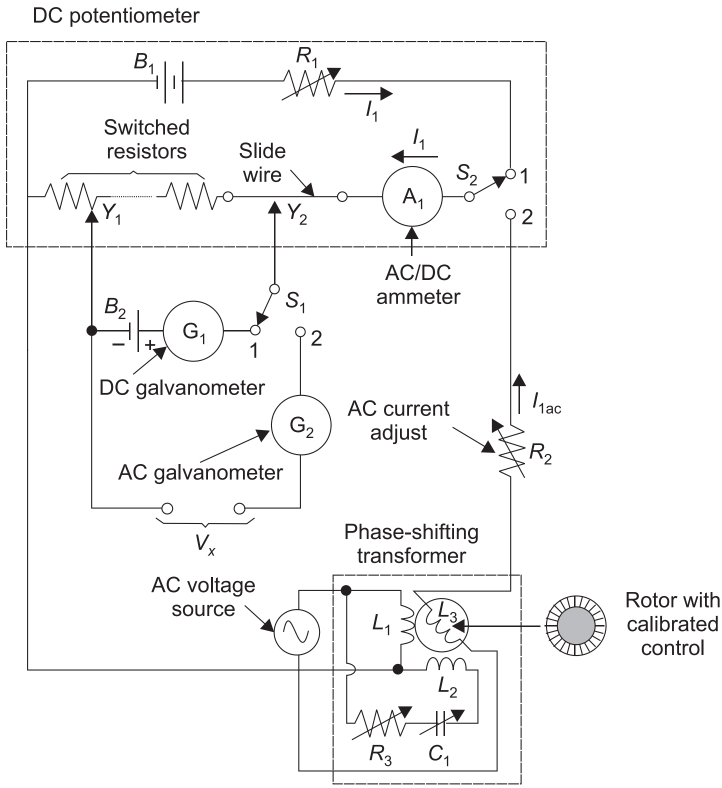


Fig. 14.16 Drysdale Polar Potentiometer

As shown in the figure, the primary winding of the transformer consists of two stationary windings L_1 and L_2. The magnetic axes of the stationary windings are perpendicular to each other. Winding L_2 is connected to the supply through a variable resistor R_3 and capacitor C_1 while the winding L_1 is connected directly to the AC supply. C_1 and R_3 determine the phase relationship between the currents in windings L_1 and L_2 and it is adjusted to 90° phase difference while preparing for the measurement. The secondary winding of the transformer consists of one rotator winding L_3. Winding L_3 can be rotated by hand and a constant amplitude voltage is induced in it from L_1 and L_2. In addition to C_1 and R_3, the phase between the supply and secondary voltage also depends upon the relative positions of rotors L_3 and stators L_1 and L_2. Hence, we can say that the output voltage of the phase shifting transformer is a sinusoidal voltage having constant amplitude and adjustable phase. To indicate this phase relationship between the supply and output voltage, a scale and pointer is provided with the rotor winding.

Initially the DC potentiometer is standardized and the value of standardized current I_1 is noted down from ammeter A_1. Now, both the switches S_1 and S_2 are moved to position 2 for AC measurements and phase shifter is set to 0° so that a maximum input is provided to L_3 from L_1. Now, the resistance R_2 is adjusted so that the same current (I_{1AC}) flows as determined above during DC standardization. After this the phase shifter is set to 90° and resistance R_3 is adjusted to give the potentiometer current. Now, the phase shifter is set to 45° and capacitor C_1 is adjusted to give potentiometer current. These steps with different positions of phase shifter are repeated continuously until the rotor position has no effect on the potentiometer current. Now, the sliding contacts Y_1 and Y_2 of the potentiometer are adjusted so that a minimum reading is obtained in galvanometer G_2. To reduce the galvanometer G_2 reading to zero, the position of the rotor is adjusted. This process is repeated again and again to obtain null deflection in galvanometer G_2. Now, the rms value of the voltage can be noted down from the potentiometer scale and its phase relationship with AC supply is noted down from the position of the rotor on phase shifting transformer.

14.3.3 Gall-Tinsley Potentiometer

The circuit diagram for Gall-Tinsley potentiometer is shown in Figure 14.17. It consists of two potentiometers—one **in-phase** potentiometer denoted by P_1 and other **quadrature** potentiometer denoted by P_2. The AC supply to both the potentiometers is provided by two step-down transformers denoted as T_1 and T_2. As can be seen from the figure, the primary winding of T_2 consists of variable resistor R_1 and capacitor C_1 to provide a phase shift of 90° to quadrature potentiometer.

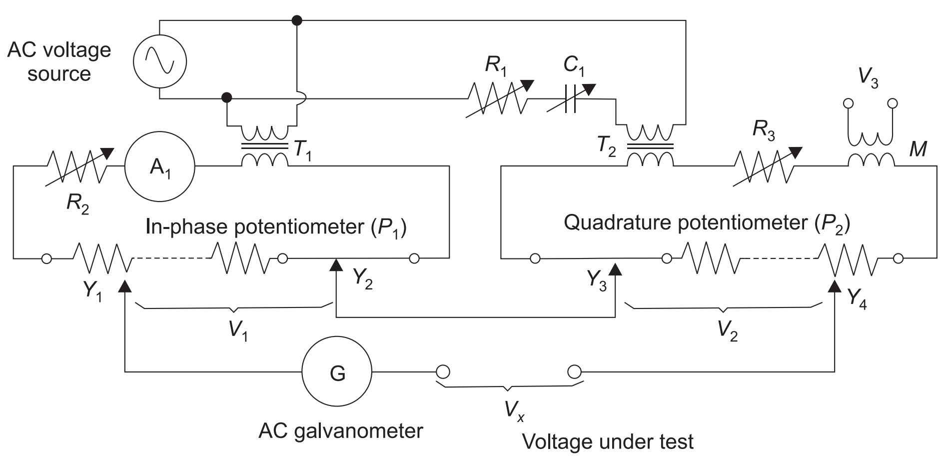


Fig. 14.17 Gall–Tinsley Potentiometer

The in-phase potentiometer is first standardized by using DC supply, standard cell, DC galvanometer, and DC/AC ammeter. The DC standardization current is then noted down.

Now, the AC supply is connected to the in-phase potentiometer and R_2 is adjusted until the same current flows through it as noted during DC standardization. This process standardizes the in-phase potentiometer on AC. After this, the standardization of quadrature potentiometer is performed using in-phase potentiometer.

The mutual inductance M in the circuit gives a particular output voltage which can be easily measured when the standardized current is flowing through potentiometer P_2. The voltage across M (that is, V_3) is in phase with the supply P_1 as the mutual inductance M introduces a phase shift of 90°. Now, the components R_1, R_3, and C_1 are adjusted until a null deflection is obtained in the galvanometer. At this point the potentiometer P_2 is said to be standardized as the current flowing through it is equal to and in quadrature to the current flowing through potentiometer P_1.

After standardizing both potentiometers P_1 and P_2, the unknown voltage V_x is measured by adjusting sliding contacts Y_1, Y_2, Y_3, and Y_4 until null is obtained on the AC galvanometer. Both the in-phase and quadrature components of V_x can be read separately from the potentiometer scales.

Here it is to be noted that if the phase difference between the in-phase component and V_x is more than 180°, the terminal connections of the unknown voltage and connections of both potentiometers must be reversed. It can be done by using reversing switches.

14.3.4 Applications of AC Potentiometers

AC potentiometers have many applications which include *calibration of ammeter*, *calibration of voltmeter*, *testing of wattmeter and energy meter*, and *measurement of self reactance of a coil*.

Calibration of ammeter

The procedure of calibrating an ammeter using AC or DC potentiometers is the same and it may be performed by using a non-inductive standard resistance. The successive values of alternating current passing through the resistance are then noted down.

Calibration of voltmeter

The procedure of calibrating a voltmeter using AC or DC potentiometers is also the same. For voltages up to 1.5 V, the measurement can be done directly. However, for medium value voltages, a volt box must be used along with the AC potentiometer. For higher value voltages, two capacitors are connected in series with the AC potentiometer.

Testing of wattmeter and energy meter

Here also, the testing of wattmeter and energy meters is done using the same set-up as used for DC potentiometers. However, in AC potentiometer circuits, a phase shifting transformer is added in the potential divider circuit. It enables the testing at different values of power factors by varying the phase of the voltage with respect to the current.

Measurement of self reactance of a coil

The circuit diagram for the measurement of self reactance of a coil using an AC potentiometer is shown in Figure 14.18. As shown in the figure, the coil whose reactance is to be determined is connected serially with a standard resistor S. The potentiometer is connected to the circuit through a double pole double throw switch.

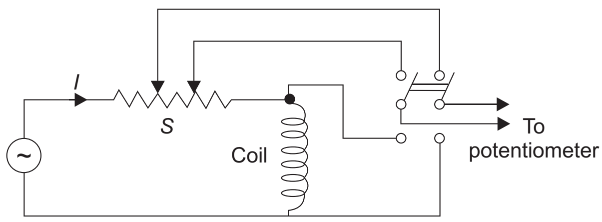


Fig. 14.18 Circuit for Measuring Self Reactance of a Coil using an AC Potentiometer

Initially the voltage across the standard resistance S is measured and then the voltage across the coil is measured. Assuming the potentiometer is a polar type, the reading of the potentiometer, that is, the voltage across standard resistor V_S is given by:

$$V_S = V_{So} \angle\theta_S$$

The voltage across the coil V_C is given by the relation:

$$V_C = V_{Co} \angle\theta_C$$

The current through the coil I is given as:

$$I = \frac{V_{So}}{S} \angle\theta_S \qquad \text{...(12)}$$

Impedance of the coil can be written as:

$$Z_C = \frac{V_C}{I}$$

Substituting the value of I from Eqn. (12), we get:

$$Z_C = \frac{SV_{Co}\angle\theta_C}{V_{So}\angle\theta_S} = \frac{SV_{Co}}{V_{So}} \angle(\theta_C - \theta_S) \qquad \text{...(13)}$$

Thus, the resistance of the coil R_C can be written in terms of its impedance as:

$$R_C = Z_C \cos(\theta_C - \theta_S)$$

Substituting the value from Eqn. (13), we get:

$$R_C = \frac{SV_{Co}}{V_{So}} \cos(\theta_C - \theta_S) \qquad \text{...(14)}$$

Similarly, using Eqn. (13), we can write the reactance of the coil X_C as:

$$X_C = Z_C \sin(\theta_C - \theta_S) = \frac{SV_{Co}}{V_{So}} \sin(\theta_C - \theta_S) \qquad \text{...(15)}$$

14.4 SELF-BALANCING POTENTIOMETER

Self-balancing potentiometers are widely used in instrumentation industries as they have many advantages which are listed as follows.

- They are automatic in their action and do not require regular attention of the operator.
- They also have a recording mechanism which helps in drawing the curve of the quantity being measured.
- They can also act as monitoring devices as they can be easily mounted on a panel or a switchboard.
- They can also be used in special function generators.
- They can also be used in dividing and multiplying networks in analog computers.

In this potentiometer also, the voltage or emf to be measured is compared with the known voltage or emf. However, unlike normal potentiometers if the known and unknown voltages are not equal, the unbalance voltage is not deflected by the galvanometer. Instead, it is applied to an amplifier which amplifies it and the amplified output is used to drive a motor. This motor moves a sliding contact which is used to balance the potentiometer. Here, it is to be noted that the unbalance voltage is DC in nature and therefore must be applied to a DC amplifier. Since the output of a DC amplifier is unstable, the working of DC amplifier is unsatisfactory and thus, the working of this set up is also not satisfactory.

The above mentioned difficulty can be overcome by using a convertor, consisting of a vibrating reed and is placed between the amplifier and the potentiometer. The vibrating reed in a convertor is excited with AC supply and it operates a double throw switch. The current through the primary winding of a transformer is reversed by double throw switch for each cycle of the vibrating reed. This continuous reversal of primary winding current of the transformer induces an alternating voltage in its secondary winding. The induced secondary winding voltage is proportional to the unbalance DC voltage or emf input to the convertor which is then converted to AC and applied to an AC amplifier.

Now, the output of the amplifier is applied to the control winding of a two-phase induction motor whose second winding is supplied by an AC line voltage. The phase difference between the AC line voltage and the voltage supplied by the converter is 90° due to the presence of a capacitor in converter driving circuit. Thus, the voltage (that is, output of amplifier), applied to the control winding of the two-phase induction motor will either lead or lag the voltage (that is, line voltage), applied to its second winding by 90°. This will rotate the motor and the direction of rotation depends on the amplifier output voltage phase which further depends on the polarity of the input (that is, the unbalance DC voltage) applied to the converter. Hence, if the voltage to be measured is larger than the potentiometer-balancing voltage, the motor will rotate in one direction. However, if the voltage to be measured is smaller than the potentiometer-balancing voltage, the motor will rotate in the opposite direction. This is due to shifting in output voltage of the amplifier by 180° owing to reversal in polarity of the unbalance voltage.

The shaft of the motor and slide wire are connected mechanically so as to reduce the unbalance in the potentiometer by the rotation of the motor. When both the potentiometer voltage and the voltage to be measured are equal, the output voltage of the amplifier becomes zero and the motor stops rotating. Thus, the unbalance voltage causes the motor to rotate and thereby, sliding contact comes to its balance position.

The self-balancing potentiometer as shown in Figure 14.19, is used to measure the temperature of thermocouples.

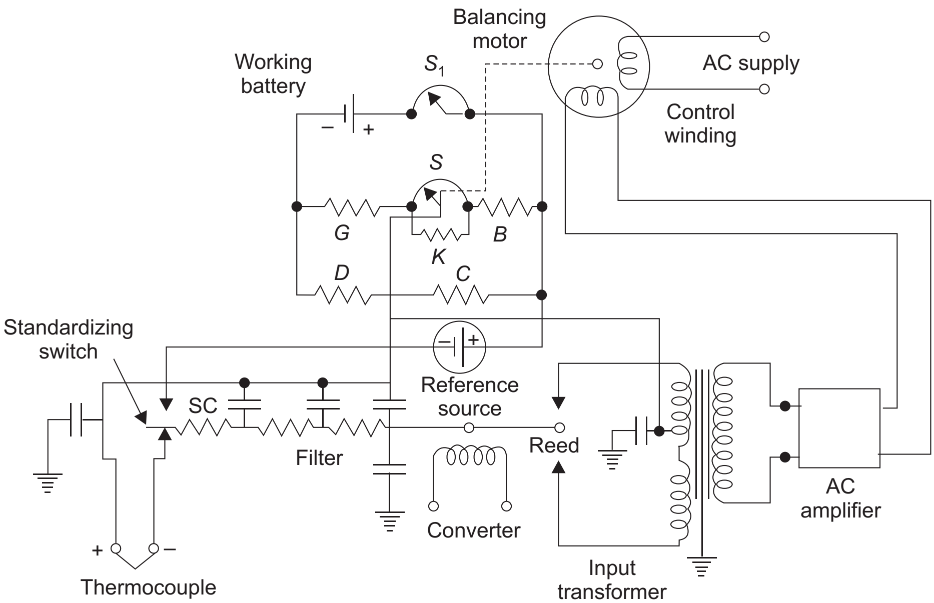


Fig. 14.19 Self-Balancing Potentiometer

As we know, due to the difference in temperatures of hot and cold junctions of a thermocouple, an emf is produced. An electrical compensating circuit is used to compensate the variation in temperature of reference junction. This electrical circuit is made up of two resistors *D* and *G*, where the resistor *D* (made up of nickel copper alloy) is used to compensate the temperature variation in reference junction. Resistor *G* is used to balance the voltage drop across resistor *D* at the required base temperature. Resistor *G* may also include zero suppression. The measuring circuit consists of a slide wire *S* and resistors *K* and *B*. Resistor *B* is used to calibrate the circuit with the reference voltage by producing correct voltage drop. A zener source is used to provide reference voltage and a rheostat S_1 is used to adjust the working current (see Figure 14.19).

Before applying the signal to the potentiometer, it is passed through a low-pass filter which does not affect the DC component but smoothens out any AC component, if present. The motor which is connected to the slide wire to obtain balance is also connected mechanically to a pen mechanism. The pen moves on a chart when actuated by the mechanism. This chart is controlled using a separate clock motor and hence, the temperature is noted down on a strip chart.

Let us Summarize

1. A potentiometer measures an unknown voltage by comparing it with some known voltage. The known voltage can be provided by any known voltage reference source, or a standard cell may be used for this purpose.
2. There are two types of potentiometers, namely, DC potentiometers and AC potentiometers.
3. DC potentiometers are widely used in educational as well as industrial applications with many advantages.
4. The types of DC potentiometers include the basic slide wire DC potentiometer, Crompton's potentiometer, multiple range potentiometer, precision type (vernier) potentiometer, and deflection potentiometer.
5. DC potentiometers have many applications which include calibration of ammeter, calibration of voltmeter, calibration of wattmeter, measurement of resistance, and measurement of power.
6. In AC potentiometers, the balance is obtained when the magnitudes as well as phase of both the voltages (unknown voltage and slide wire voltage) are equal.
7. Based on the manner in which the value of the unknown voltage is displayed by the scale or dial of AC potentiometers, they are classified into two types, namely, polar type and co-ordinate type potentiometers.
8. A polar type AC potentiometer is drysdale potentiometer using a phase shifting transformer.
9. A co-ordinate type AC potentiometer is Gall–Tinsley potentiometer consisting of two potentiometers—in-phase potentiometer and quadrature potentiometer.
10. AC potentiometers have many applications which include calibration of ammeter, calibration of voltmeter, testing of wattmeter and energy meter, and measurement of self reactance of coil.
11. Self-balancing potentiometers are widely used in instrumentation industries as they have many advantages.
12. In a self-balancing potentiometer also, the voltage or emf to be measured is compared with the known voltage or emf. However, unlike in normal potentiometer if the known and unknown voltages are not equal, the unbalance voltage is not deflected by galvanometer. Instead it is applied to an amplifier which amplifies it and the amplified output is used to drive a motor. This motor moves a sliding contact which balances the potentiometer.

EXERCISES

Fill in the Blanks

1. A potentiometer is basically a _________ type of instrument.
2. Ideally the power consumed in the circuit of unknown source while measuring its voltage using a potentiometer is _________.
3. Two types of AC potentiometer are _________ and _________.
4. To obtain balance in AC potentiometers, both _________ and _________ are considered.
5. By mounting a self-balancing potentiometer on a switchboard, it can be used as a _________ device.

Multiple Choice Questions

1. Standardization of potentiometers is done to make them

   (a) accurate (b) precise

(c) both (a) and (b) (d) direct reading and accurate

2. Gall–Tinsley potentiometer is a type of
   (a) DC potentiometer (b) co-ordinate AC potentiometer
   (c) polar AC potentiometer (d) none of these
3. Deflection potentiometer is used for voltage which is
   (a) varying slowly (b) constant
   (c) varying rapidly (d) all of these
4. The resistance R_x, measured using a potentiometer, is given by the expression
   (a) $R_x = \dfrac{V_{Rx}}{V_S S}$ (b) $R_x = \dfrac{V_{Rx}}{V_S} S$
   (c) $R_x = \dfrac{V_S}{V_{Rx}} S$ (d) none of these
5. When the maximum voltage is applied at terminals in a volt-box, the potentiometer reading is
   (a) 120 mV (b) 120.45 mV
   (c) 150 mV (d) 150.45 mV

State True or False

1. AC supply in AC potentiometers is obtained from the voltage to be measured.
2. Volt-box is used during calibration of ammeter using a potentiometer.
3. Self-balancing transformers can draw the curve of the quantity being measured.
4. Drysdale potentiometer is a co-ordinate AC potentiometer.
5. Self-balancing potentiometer does not require an operator for its action.

Descriptive/Numerical Questions

1. Draw the diagram of a laboratory type (Crompton's) DC potentiometer and explain how voltage is measured with this potentiometer.
2. How can a potentiometer be used for:
   (a) calibration of a voltmeter
   (b) calibration of a wattmeter?
3. What is meant by standardization of a potentiometer?
4. Explain the construction and operation of a slide wire DC potentiometer.
5. List the applications of DC potentiometers.
6. What are the types of AC potentiometers?
7. A simple slide wire is used for measuring the current in a circuit. The voltage drop across a standard resistance of 5 Ω is balanced at 55 cm. Find the magnitude of the current if the standard cell having a voltage of 105 volts is balanced at 30 cm.
8. Explain the procedure of measurement of high voltage by a DC potentiometer.
9. Explain in detail the construction and working principle of an AC potentiometer.
10. Describe the construction and working of a polar type potentiometer. Explain the methods for standardizing it.
11. Describe how co-ordinate type potentiometer can be used for the calibration of a voltmeter and AC energy meters.

12. Discuss the working of a Drysdale polar type of potentiometer with the help of a neat diagram.
13. List the applications of AC potentiometers.
14. Explain in detail the working of a self-balancing potentiometer.

15

Transformers

CHAPTER OBJECTIVES

After reading this chapter, you will be able to:

- Appreciate the basic concept of transformers and their parameters
- Describe instrument transformers along with their advantages and disadvantages
- Discuss different types of instrument transformers, namely, current and potential transformers
- Describe the construction, equivalent circuit, phasor diagrams, characteristics, errors and their reduction methods, and testing methods for both current and potential transformers
- Differentiate between current and potential transformers

15.1 INTRODUCTION

In earlier chapters, we have studied the measurement of different parameters of DC as well as AC instruments. These parameters include current, voltage, energy, power, frequency, and power factor. In this chapter, we will study an instrument named **transformer** that works on AC systems and measures the same parameters. Apart from measuring the parameters mentioned above, a transformer can be used in protection circuits of power systems to control the operation of undervoltage, overcurrent, earth fault, and various other types of relays. The transformers are known as **instrument transformers** when they are used for the measuring purpose while the actual measurement is done using a measuring instrument.

There are two types of instrument transformers, namely, **current transformers** (also known as CT) and **voltage** or **potential transformers** (also known as PT). In this chapter, we will study the advantages and disadvantages, construction, equivalent circuit, phasor diagram, characteristics, errors, causes of errors, methods to reduce errors, and testing methods for these transformers.

15.2 INSTRUMENT TRANSFORMERS

The direct measurement of high currents and voltages in power systems is not possible as the design constraints make the meters for such large currents and voltages very costly and large in size. Since the meters with such large range cannot be designed so transformers are used to step down these currents and voltages. Therefore, meters or instruments of moderate sizes can measure large current, voltage, power, and energy with the help of instrument transformers.

The current transformer measures the current that passes through its primary winding and an ammeter is connected to the secondary winding as shown in Figure 15.1(a). It steps down the current to the ammeter level. The voltage transformer measures the voltage that is connected to its primary winding and voltmeter is connected to the secondary winding as shown in Figure 15.1(b). It also steps down the voltage to the voltmeter level.

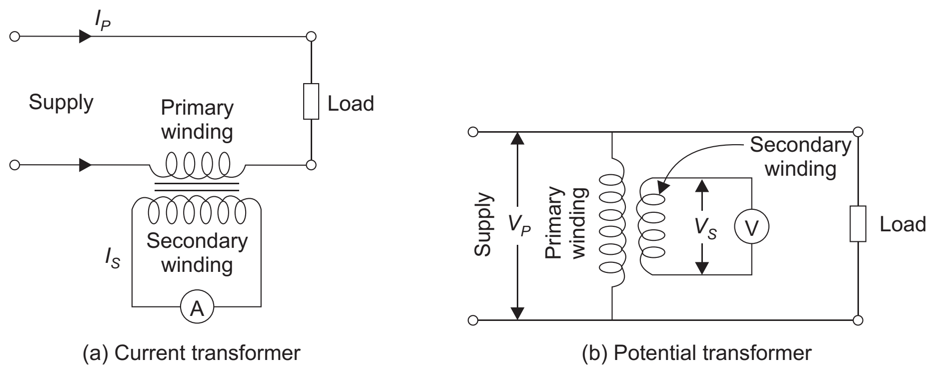


Fig. 15.1 Basic Configuration of Transformers

Similar to DC measurements, the range of instruments can be extended for current and voltage measurement with the use of shunts and multipliers, respectively. However, shunts and multipliers have many disadvantages. Also, they are suitable only for small values of current and voltage.

15.2.1 Advantages and Disadvantages

Instrument transformers are used for both routine and precise measurements with many advantages discussed as follows.

- The cost of instruments and instrument transformers can be reduced with the standardization of secondary winding ratings of current and potential transformers. Also, the damaged instruments can be easily replaced.
- The instruments with moderate ratings can be used to measure large currents and voltages with the help of instrument transformers. Instrument transformers have been standardized at very moderate ratings, which is 5 A secondary winding current for current transformer and 100 to 120 V secondary winding voltage for voltage transformer. Thus, a 1000 A current can be measured using a 5 A ammeter with the help of 1000/5 A ratio current transformer.
- The instrument transformers produce readings independent of the constants *R*, *L*, and *C* so they produce same readings as produced by the instrument.
- The meter circuit in secondary winding is isolated from primary winding. This becomes very advantageous while working with high voltage power systems as it provides good insulation and safety to the operators.

The disadvantages in instrument transformers are due to multipliers (for voltage measurements) and shunts (for current measurements) which are used in the circuit for range extension.

- The multipliers work easily below 1000 V but at high voltages, the power consumed by them is very large.
- In high voltage multipliers, the leakage currents should be kept negligible. However, it is very difficult to insulate multipliers for prevention of leakage currents and reduce distributed capacitance to avoid shunt capacitance currents. Therefore, the use of multipliers becomes costly and complicated by using special structures to prevent leakage effects.
- The error occurs as power circuit is not electrically isolated from the measuring circuit.
- At high voltage measurements, insulation of the instrument and shunt is quite difficult.
- The power consumed by shunts at large currents is considerably large so the method of using shunts is limited to a few hundred amperes only.
- The accuracy cannot be achieved easily by using shunt on ac, as current division between the shunt and meter depends on the ratio of reactance to resistance of the two paths.

15.2.2 Rating and Ratio

The term **rating** of an instrument transformer is represented by two groups of numbers. One represents the nominal current or voltage applied to its primary winding and the other represents the current or voltage induced in its secondary winding. For example, a transformer designated with a rating of 360:120 volts means that 360 volts is applied to the primary winding and 120 volts is induced in its secondary winding.

The **ratio** of an instrument transformer is defined in terms of rating, expressed as the relationship between its primary and secondary ratings. The ratio is of different types, namely, *turns ratio*, *nominal ratio*, *transformation ratio*, and *ratio correction factor*.

Turns ratio

Turns ratio, denoted as N, is expressed as the ratio of number of turns of the two windings—primary and secondary. Mathematically, it is expressed as:

For current transformer:

$$N = \frac{\text{Number of turns of secondary winding}}{\text{Number of turns of primary winding}} \qquad ...(1)$$

For potential transformer:

$$N = \frac{\text{Number of turns of primary winding}}{\text{Number of turns of secondary winding}} \qquad ...(2)$$

Nominal ratio

The nominal ratio, denoted as N_k, is defined as the ratio of the rated value of primary winding current (or voltage) to the rated value of secondary winding current (or voltage). Mathematically, it can be expressed for both current and potential transformer as:

For current transformer:

$$N_k = \frac{\text{Rated primary winding current}}{\text{Rated secondary winding current}} \qquad \text{...(3)}$$

For potential transformer:

$$N_k = \frac{\text{Rated primary winding voltage}}{\text{Rated secondary winding voltage}} \qquad \text{...(4)}$$

Transformation ratio

Transformer ratio, denoted as *TR*, is expressed as the ratio of the magnitude of phasor of primary winding to the phasor of secondary winding. Mathematically, it can be expressed as:

$$TR = \frac{|\text{Primary phasor}|}{|\text{Secondary phasor}|}$$

Thus, we have, for current transformer:

$$TR = \frac{\text{Primary winding current}}{\text{Secondary winding current}} \qquad \text{...(5)}$$

For potential transformer:

$$TR = \frac{\text{Primary winding voltage}}{\text{Secondary winding voltage}} \qquad \text{...(6)}$$

Ratio correction factor

The ratio correction factor, denoted as *RC*, is obtained from transformer ratio and nominal ratio. When transformer ratio is divided by nominal ratio, we get ratio correction factor. Mathematically, it is expressed as:

$$\text{Ratio correction factor } (RC) = \frac{\text{Transformer ratio}}{\text{Nominal ratio}} = \frac{TR}{N_k} \qquad \text{...(7)}$$

Note: *The ratio marked on the transformer is the nominal ratio.*

15.2.3 Rated Burden

We know that the load is always connected on the output side across the secondary winding terminals. The output is expressed in volt-ampere at rated secondary winding voltage. The term **rated burden** is used to express the volt-ampere loading which is considered without errors that exceeds the limit for a particular class of accuracy. Mathematically, it is expressed in terms of current and induced voltage as:

$$\text{Secondary winding burden due to load} = \begin{pmatrix}\text{Secondary winding}\\ \text{current}\end{pmatrix}^2 \times \begin{pmatrix}\text{Impedance of load on}\\ \text{secondary winding}\end{pmatrix}$$

$$= \frac{(\text{Secondary winding induced voltage})^2}{(\text{Impedance of load on secondary winding})} \quad ...(8)$$

$$\text{Total secondary winding burden} = \begin{pmatrix}\text{Secondary winding}\\ \text{current}\end{pmatrix}^2 \times \begin{pmatrix}\text{Impedance of secondary}\\ \text{winding circuit including}\\ \text{secondary winding}\end{pmatrix}$$

$$= \frac{(\text{Secondary winding induced voltage})^2}{\begin{pmatrix}\text{Impedance of secondary winding circuit including}\\ \text{impedance of secondary winding}\end{pmatrix}} \quad ...(9)$$

15.2.4 Testing Method

Testing of an instrument transformer is a prime requisite to find phase angle error and also to find their ratio. There are two groups of methods used for testing that include *absolute methods* and *comparison methods*.

Absolute methods

Absolute methods determine the transformer errors in terms of resistance, capacitance, and inductance of the circuit under test.

Comparison methods

The comparison method performs comparison of transformer errors which is under test with the known errors of the standard current transformer.

The two types of comparison methods employed according to the measurement technique are *null* and *deflection methods*.

Null methods

A network is used in these methods with appropriate phasor quantities balanced against each other. The impedance elements of the network help in determining the phase angle errors and ratio. This method is a direct method which gives the results in terms of calibrated scales marked on the adjustable elements of the network.

Deflection methods

In deflection methods, the phase angle errors and ratios are determined from the magnitudes of deflection of instruments like electrodynamometer wattmeters. Therefore, these methods measure quantities related to their deflection or phasors. It is to be noted here that these methods could be made direct in some cases.

15.3 CURRENT TRANSFORMER

Current transformer is a type of instrument transformer that is used to measure current. The current to be measured passes through the primary winding having very few turns. The secondary winding has a large number of turns and its terminals are connected directly to the ammeter or current coil of wattmeter that can measure current as well as power. To protect the equipment from insulation breakdown, one terminal of the secondary winding is earthed (see Figure 15.2).

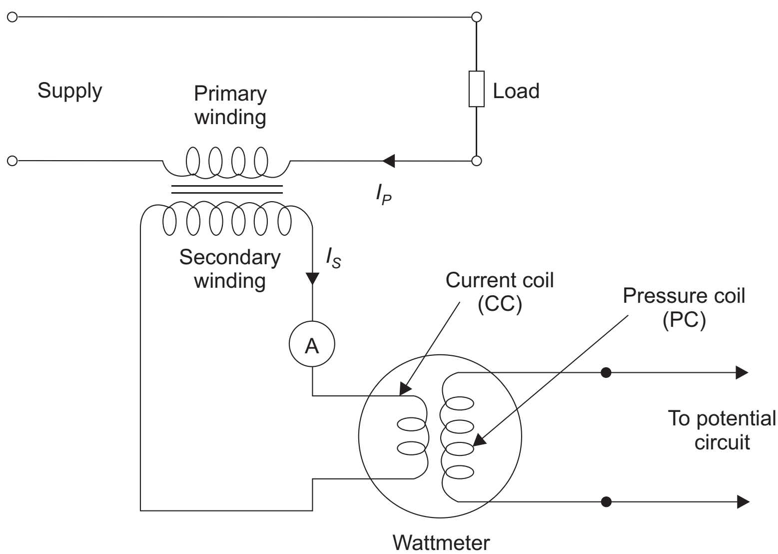


Fig. 15.2 Wattmeter Connected to the Secondary Winding of a Current Transformer

The current transformer is also known as **series transformer** as the current-carrying conductor is in series with the primary winding.

15.3.1 Construction

Current transformers are of three basic types, namely, *bar type*, *wound type*, and *window* or *ring type*.

Bar type current transformer

In bar type current transformer, the primary winding is a bar of suitable size and material that forms an integral part of the transformer. The primary winding consists of a single turn with an insulation of bakelized paper tube or a resin which is moulded directly on the bar (see Figure 15.3).

Wound type current transformer

In wound type current transformer, the primary winding of more than one full turn is wound on the core and the secondary winding is wound on a bakelite former or bobbin as shown in Figure 15.4. The suitable insulation is provided between primary and secondary winding.

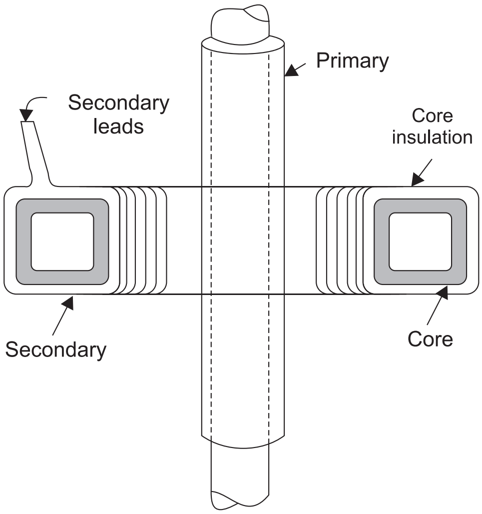


Fig. 15.3 Bar Type Current Transformer

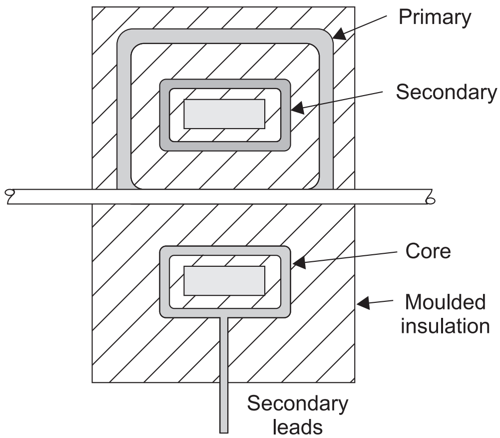


Fig. 15.4 Wound Type Current Transformer

Window type current transformer

In window or **ring type** current transformer, the core is insulated by end collars and circumferential wraps of elephantide or presspahn. These pressboards protect the secondary winding from the mechanical damage due to sharp corners. The secondary winding is put on the core using a toroidal winding machine or by hand winding in case of a small number of turns. Then the ring type transformer is completed by exterior taping. The core of this transformer is made of nickel-iron alloy or an electrical sheet. The window type current transformer may have three kinds of shapes such as stadium, rectangular, and circular.

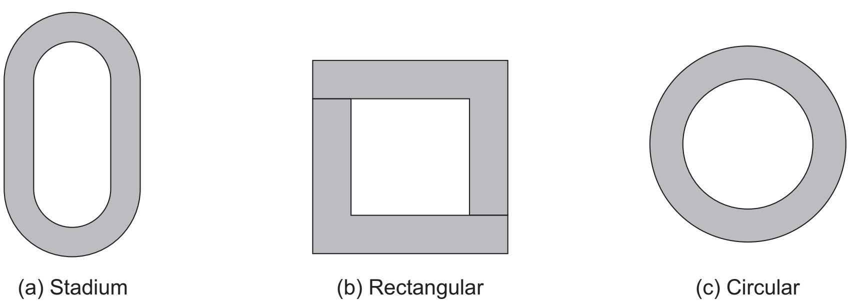


Fig. 15.5 Window Type Current Transformer

15.3.2 Equivalent Circuit and Phasor Diagram

The equivalent circuit of a current transformer is shown in Figure 15.6 with its phasor diagram shown in Figure 15.7. The equivalent circuit shows the primary and secondary windings with flux linked between them. We can find the transformer ratio and phase angle from the phasor diagram.

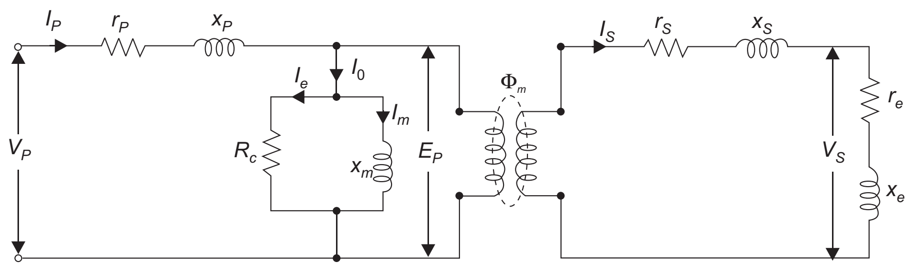


Fig. 15.6 Equivalent Circuit of Current Transformer

In Figure 15.6, we have the following terms, primary winding current I_P, secondary winding current I_S, resistances of primary and secondary windings as r_P and r_S, reactances of primary and secondary windings as x_P and x_S, resistance and reactance of external burden are r_e and x_e, primary and secondary induced voltages as E_P and E_S, voltage at the secondary winding terminal as V_S, number of primary and secondary winding turns as N_P and N_S, turn ratio (ratio of secondary winding turn to primary winding turn) as N, exciting current I_0, magnetizing component and loss component of exciting current as I_m and I_e, phase angle of transformer as θ, working flux of transformer as Φ, angle between exciting current and working flux is α, angle between secondary winding induced voltage and secondary winding current as δ.

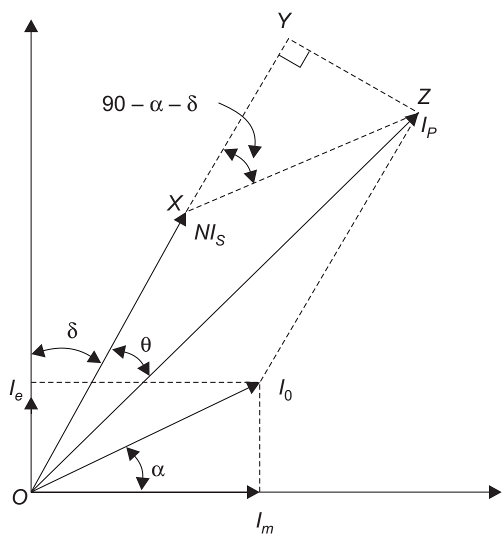


Fig. 15.7 Phasor Diagram of Current Transformer

Transformation ratio

The transformation ratio *TR* is computed for which primary winding and secondary winding currents are to be determined from the phasor diagram shown in Figure 15.7. From the figure, we have $OZ = I_P$, $OX = NI_S$, $XZ = I_0$, $\angle YXZ = 90° - \delta - \alpha$, $XY = I_0 \cos(90° - \delta - \alpha) = I_0 \sin(\delta + \alpha)$, $YZ = I_0 \sin(90° - \delta - \alpha) = I_0 \cos(\delta + \alpha)$.

Using Pythagoras theorem, we have:

$$(OZ)^2 = (OX + XY)^2 + (YZ)^2$$

$$(I_P)^2 = [NI_S + I_0 \sin(\delta + \alpha)]^2 + [I_0 \cos(\delta + \alpha)]^2$$

$$= N^2 I_S^2 + I_0^2 \sin^2(\delta + \alpha) + 2NI_S I_0 \sin(\delta + \alpha) + I_0^2 \cos^2(\delta + \alpha)$$

$$= N^2 I_S^2 + 2NI_S I_0 \sin(\delta + \alpha) + I_0^2$$

$$I_P = [N^2 I_S^2 + 2NI_S I_0 \sin(\delta + \alpha) + I_0^2]^{1/2} \qquad ...(10)$$

From Eqn. (5), we have the transformation ratio *TR* as:

$$TR = \frac{I_P}{I_S} = \frac{[N^2 I_S^2 + 2NI_S I_0 \sin(\delta + \alpha) + I_0^2]^{1/2}}{I_S}$$

When $I_0 << NI_S$ for a current transformer, the above equation becomes:

$$TR \approx \frac{[N^2 I_S^2 + 2NI_S I_0 \sin(\delta + \alpha) + I_0^2 \sin^2(\delta + \alpha)]^{1/2}}{I_S} \approx \frac{NI_S + I_0 \sin(\delta + \alpha)}{I_S} \approx N + \frac{I_0}{I_S} \sin(\delta + \alpha)$$

$$TR \approx N + \frac{I_0}{I_S}(\sin\delta \cos\alpha + \cos\delta \sin\alpha) \approx N + \frac{I_m \sin\delta + I_e \cos\delta}{I_S} \qquad ...(11)$$

where $I_m = I_0 \cos\alpha$ and $I_e = I_0 \sin\alpha$

Phase angle

The phase angle θ of the transformer is defined as the phase difference between the primary current and secondary current on being reversed. When the primary current lags the reversed secondary current, this angle is taken to be positive. On the contrary, when the primary current leads the reversed secondary current, this angle is taken to be negative. From the phasor diagram in Figure 15.7, we can compute the phase angle θ as:

$$\tan\theta = \frac{YZ}{OY} = \frac{YZ}{OX + XY} = \frac{I_0 \cos(\delta + \alpha)}{NI_S + I_0 \sin(\delta + \alpha)} \text{ radian}$$

When θ is very small, we have $\tan\theta \approx \theta$. The term $I_0 \sin(\delta + \alpha)$ can be neglected from the above equation as I_0 is very small in reference to NI_S. Therefore, θ becomes:

$$\theta \approx \frac{I_0 \cos(\delta + \alpha)}{NI_S} \approx \frac{I_0 \cos\delta\cos\alpha - I_0 \sin\delta\sin\alpha}{NI_S} \approx \frac{\mathrm{I}_m \cos\delta - I_e \sin\delta}{NI_S} \text{ radian}$$

The phase angle θ can be written in terms of degrees as:

$$\theta \approx \frac{180}{\pi}\left(\frac{I_m \cos\delta - I_e \sin\delta}{NI_S}\right) \text{ degrees} \qquad ...(12)$$

Example 1 A current transformer having a 1 turn primary having frequency 50 Hz supplies a current of 15 A through its secondary. At the rated load with non-inductive burden of 2 Ω, the exciting mmf is given as 82 A. The number of turns in the secondary is 98. Neglecting the effects of iron losses, I^2R losses, and magnetic leakage, calculate from the fundamentals the

(a) ratio

(b) phase angle

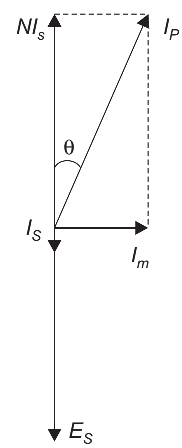
Solution: Given that: $N_P = 1$, $I_S = 15$ A, $N_S = 98$, magnetizing mmf = 82 A, and secondary winding resistance = 2 Ω

Now, secondary winding voltage E_S is given by:

$$E_S = I_S r_S \qquad \text{[As secondary burden is purely resistive]}$$

Substituting the values, we get:

$$E_S = 15 \times 2 = 30 \text{ V}$$



Now, δ = 0 due to resistive secondary burden making secondary power factor to be unity. Also, as loss components are to be neglected, we have $I_e = 0$. The magnetizing component is given by the expression:

$$I_m = \frac{\text{Magnetizing mmf}}{N_P} = \frac{82}{1} = 82 \text{ A}$$

Now, the secondary current is given to be $I_S = 15$ A. Thus, the reflected secondary current is given by:

$$NI_S = 98 \times 15 = 1470 \text{ A}$$

The primary current I_P is expressed from the figure as:

$$I_P = \sqrt{(NI_S)^2 + (I_m)^2} = \sqrt{(1470)^2 + (82)^2} \qquad \text{(Since } N = N_S \text{ as } N_P = 1\text{)}$$

$$\Rightarrow \qquad I_P = 1472.28 \text{ A}$$

(a) The transformation ratio is given by the expression:

$$TR = \frac{\text{Primary winding current}}{\text{Secondary winding current}} = \frac{I_P}{I_S} = \frac{1472.28}{15} = 98.15 \qquad \text{[Refer to Eqn.(5)]}$$

(b) The phase angle is given by the figure:

$$\theta = \tan^{-1}\left(\frac{I_m}{N_S I_S}\right) = \tan^{-1}\left(\frac{82}{1470}\right) = 3.19°$$

Example 2 Determine the actual transformation ratio of a bar primary type current transformer having the following parameters: Secondary turns = 305, secondary circuit resistance = 1.7 Ω, secondary circuit reactance = 0.98 Ω, magnetizing mmf = 98 A, secondary winding current = 4.75 A, and iron loss = 1.34 W.

Solution: Given that: $N_P = 1$, $N_S = 305$, $r_S = 1.7\ \Omega$, $x_S = 0.98\ \Omega$, magnetizing mmf = 98 A, $I_S = 4.75$ A and iron loss = 1.34 W

Turns ratio N is given as:

$$N = \frac{N_S}{N_P} = 305$$

Burden impedance of secondary circuit is calculated as:

$$\sqrt{r_S^2 + x_S^2} = \sqrt{1.7^2 + 0.98^2} = 1.96\ \Omega$$

For secondary circuit, we have:

$$\cos\delta = \frac{r_s}{\text{burden impedance}} = \frac{1.7}{1.96} = 0.867$$

$$\sin\delta = \frac{x_s}{\text{burden impedance}} = \frac{0.98}{1.96} = 0.5$$

Induced voltage in secondary winding E_S comes out to be:

$$E_S = I_S \times \text{Burden impedance} = 4.75 \times 1.96 = 9.31 \text{ V}$$

And, induced voltage in primary winding E_P is given as:

$$E_P = \frac{E_S}{N} = \frac{9.31}{305} = 0.030 \text{ V}$$

Loss component of current I_e is given as:

$$I_e = \frac{\text{Iron loss}}{E_P} = \frac{1.34}{0.03} = 44.67 \text{ A}$$

And, magnetizing component of current I_m is given as:

$$I_m = \frac{\text{Magnetizing mmf}}{N_P} = \frac{98}{1} = 98 \text{ A}$$

Now, the actual transformation ratio is given by the expression:

$$TR = N + \frac{I_m \sin\delta + I_e \cos\delta}{I_S} \qquad \text{[Refer to Eqn. (11)]}$$

Substituting the values in the above expression, we get:

$$TR = 305 + \frac{98 \times 0.5 + 44.67 \times 0.867}{4.75} = 323.46$$

15.3.3 Characteristics

There are various characteristics that affect the working of current transformers. They are *change in burden of secondary winding circuit*, *change in frequency*, *change in power factor of secondary winding burden,* and *change in current of primary winding*.

Change in burden of secondary winding circuit

When the secondary winding circuit burden is increased, the voltage induced in the secondary winding must be increased which can be achieved by increasing the flux and flux density. For this, loss component I_e and magnetizing component I_m are increased which results in increased errors. Thus, to summarize, it can be said that with an increase in secondary winding burden, the transformation ratio increases and phase angle between primary current and reversed secondary current is shifted to more positive values.

Change in frequency

When the frequency increases, the flux density decreases proportionally. A current transformer operates at a frequency for which it is designed. Therefore, the effect due to change in frequency is not important to consider.

Change in power factor of secondary winding burden

As already mentioned, δ is the angle between secondary winding induced voltage E_S and secondary winding current I_S. There are two types of secondary winding burden to be considered, that are, capacitive and inductive burdens. In case of **capacitive burdens**, the secondary induced voltage E_S lags the secondary current I_S, therefore, the angle δ is negative. Here, actual transformation ratio TR becomes less than the turns ratio N for δ approaching $-90°$. The phase angle θ is always positive for all negative values of δ.

In case of **inductive burdens**, the secondary induced voltage E_S leads the secondary current I_S, therefore, the angle δ is positive. Here, actual transformation ratio TR becomes greater than turns ratio N. For small values of δ, phase angle θ is positive and becomes negative as δ approaches 90° with more inductive burden.

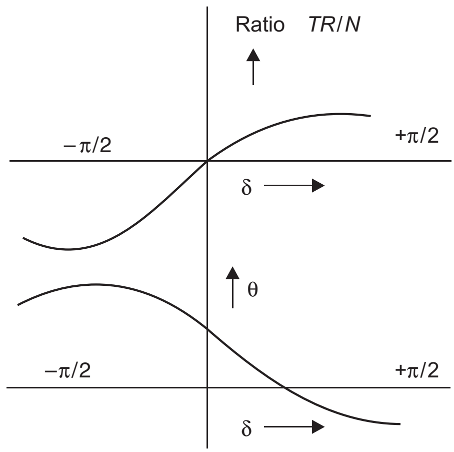


Fig. 15.8 Graph Showing Variation of Transformation Ratio and θ with δ

Change in current of primary winding

For the change in current of primary winding, the current in secondary winding changes proportionally. The magnetizing component I_m and loss component I_e of exciting current are greater at low values of primary I_P or secondary current I_S. Hence, it results in large errors. When the primary current increases, secondary current also increases thereby reducing the ratio error and phase angle.

15.3.4 Errors and Their Causes

The transformation ratio (that is, actual ratio) TR depends on the magnetizing and loss components of the exciting current, secondary winding load current and its power factor and is not equal to the turn ratio N. Therefore, the errors are introduced in the current measurements as secondary winding current is not a constant fraction of primary winding current. There are two kinds of errors that creep in power measurements—the phase difference between secondary winding current and primary winding current is θ instead of 180°, and actual transformation ratio is different from the turns ratio.

Phase angle error

The phase angle error is expressed in terms of magnetizing component I_m and loss component I_e of the exciting current and is given by the relation as:

$$\text{Phase Angle } \theta = \frac{180}{\pi}\left[\frac{I_m \cos\delta - I_e \sin\delta}{NI_S}\right] \quad \text{...(13)}$$

Ratio error

The ratio error is defined in terms of nominal ratio and transformation ratio as:

$$\text{Percentage ratio error} = \frac{\text{Nominal ratio} - \text{Transformation ratio}}{\text{Transformation ratio}} \times 100 = \frac{N_k - TR}{TR} \times 100 \quad \text{...(14)}$$

A current transformer is said to be ideal if it has a phase angle of zero and its actual transformation ratio is equal to turns ratio. However, in practice, errors are caused due to many reasons since ideal transformers are not available. Some of the causes of errors are discussed as follows.

- The magnetic leakage is always present in the transformer due to which flux linkages of primary and secondary windings are not equal.
- The transformer draws a magnetizing current I_m as some exciting magneto-motive force (mmf) is required by the primary winding to produce the flux.
- The core of the transformer becomes saturated as the flux density in the core does not depend linearly on magnetizing force.
- As the exciting current I_0 is fed to the transformer, it must have some component to supply core losses (that are, hysteresis and eddy current losses) and I^2R losses of transformer windings.

15.3.5 Methods to Reduce Errors

To reduce errors in current transformers some design features are adopted which are discussed as follows.

Turns compensation

The actual transformation ratio can be made equal to the nominal ratio by decreasing the number of turns of the secondary winding. Usually these turns are one or two less than the number at which actual transformation ratio is equal to nominal ratio. However, this correction is applicable for a particular value of current and burden impedance. The current transformer is then said to be compensated.

It is to be noted here that change in the number of turns of the secondary winding by one or two has negligible effect on phase angle error.

Leakage reactance

To minimize ratio error, the leakage reactance must be reduced by reducing the spacing between primary and secondary windings. Also, ring-shaped cores surrounded by uniform distribution of toroidal windings can be used to reduce leakage reactance.

Core designing

The reluctance and losses in the core must be low so as to minimize the errors in a current transformer. The flux path reluctance can be reduced by using high permeability materials, core having large cross-sectional area, short magnetic paths, and keeping a low flux density. Also, the number of joints should be kept as small as possible since the air gaps produced by the joint offer high reluctance to the flux.

To reduce the losses in the core materials with low hysteresis and low eddy current losses should be selected. Keeping flux density to be low also helps in reducing core losses.

Current ratings of primary winding

In a current transformer, the ratio of exciting current to primary current must be small, irrespective of the circuit applied at the input. This requires the ratio of excitation mmf to the primary winding mmf to be small. However, generally the value of primary winding mmf is low resulting in a problem and hence, it must be increased to achieve the desired results. The suitable values for primary winding mmf at which no problem occurs is 500 A at rated current. Therefore, single turn primary winding is used if the primary winding mmf of the current transformer has a value of 500 A or more. For transformers having rating less than 500 A, primary winding with multi turns are used keeping the core size in mind.

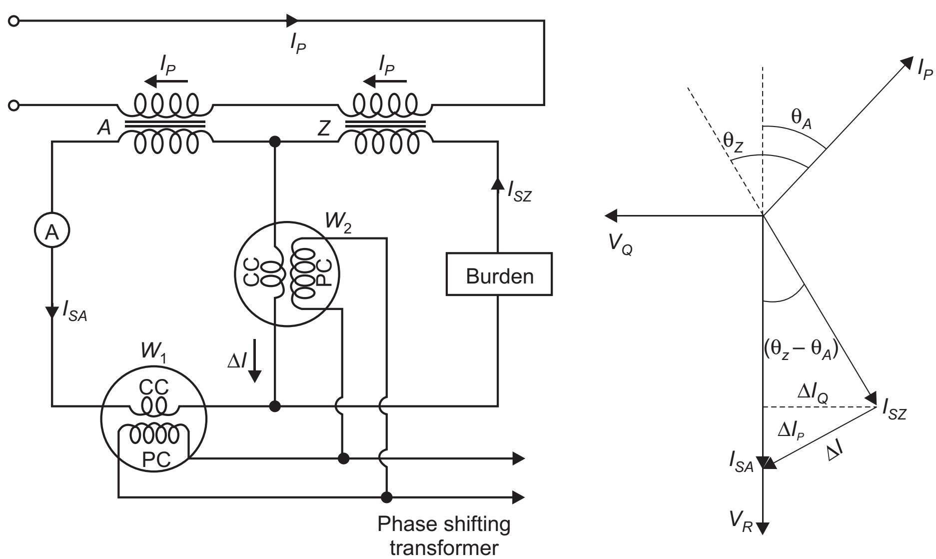
However, with the advancement in magnetic materials used and methods to improve permeability, a single turn primary winding can be used for a current transformer having rating 100 A.

15.3.6 Testing Methods

The testing of current transformers employs three methods, namely, *Silsbee's method*, *mutual inductance method*, and *Arnold's method*.

Silsbee's method

Silsbee's method is a type of comparison method based on deflection and null. Figure 15.9(a) shows a circuit consisting of test transformer *Z* whose phase angle and ratio can be determined in terms of standard transformer *A*. Both the test transformer *Z* and standard transformer *A* have same nominal ratio.



(a) Circuit diagram (b) Phasor diagram

Fig. 15.9 Silsbee's Method

The current in the standard transformer *A* can be set to the desired value by connecting an ammeter in its secondary winding. The secondary winding of the test transformer *Z* is connected to the adjustable burden. The primary winding of both the transformers are connected in series. The current coil of wattmeter W_1 carries the secondary current of standard transformer A whereas the current coil of wattmeter W_2 carries the difference of secondary currents of standard and test transformers. The pressure coils of both wattmeters are connected to a constant voltage V of a phase shifting transformer. From the phasor diagram of Figure 15.9(b), we can find the reading of wattmeters. We consider two cases to adjust the phase of voltage V and then note the readings of wattmeters.

- First case is when the phase of voltage V is adjusted to give wattmeter W_1 reading zero. Then the voltage V is in quadrature with current I_{SA}. This voltage is represented as V_Q in the figure. The wattmeter readings would be taken as:

  For wattmeter W_1, $W_{1Q} = V_Q I_{SA} \cos 90° = 0$ ...(15)

  For wattmeter W_2, $W_{2Q} = V_Q I_Q = V_Q I_{SZ} \sin(\theta_Z - \theta_A)$ ...(16)

  Here, θ_A is the phase angle of standard transformer and θ_Z is the phase angle of transformer under test.
- Second case is when the phase of voltage V is shifted through 90° such that it is represented by V_R and is in phase with I_{SA}. The wattmeter readings would be taken as:

  For wattmeter W_1, $W_{1R} = V_R I_{SA} \cos\theta = V_R I_{SA}$ ...(17)

  For wattmeter W_2, $W_{2R} = V_R \Delta I_R = V_R[I_{SA} - I_{SZ}\cos(\theta_Z - \theta_A)]$ ...(18)

Determination of phase angle and ratio

When the voltage is same for both sets of readings then $V = V_Q = V_R$. From Eqn. (15) and (16), we have:

$$W_{2Q} = V I_{SZ} \sin(\theta_Z - \theta_A) \quad ...(19)$$

and

$$W_{1R} = V I_{SA} \quad ...(20)$$

Now, putting the value from Eqn. (16), (18) in (19), we get:

$$W_{2Q} = V[I_{SA} - I_{SZ}\cos(\theta_Z - \theta_A)] = VI_{SA} - VI_{SZ}\cos(\theta_Z - \theta_A) = W_{1R} - VI_{SZ} \quad ...(21)$$

As $\cos(\theta_Z - \theta_A) = 1$ and $(\theta_Z - \theta_A)$ is very small.

$$\Rightarrow \quad VI_{SZ} = W_{1R} - W_{2R} \qquad [\text{Since } W_{2Q} = W_{2R}] \quad ...(22)$$

To determine the **phase angle** θ of the transformer, we consider the above equations. We have:

$$\sin(\theta_Z - \theta_A) = W_{2Q}/VI_{SZ} \qquad [\text{Refer to Eqn. (19)}] \quad ...(23)$$

$$\cos(\theta_Z - \theta_A) = (VI_{SA} - W_{2R})/VI_{SZ} = (W_{1R} - W_{2R})/VI_{SZ} \qquad [\text{Refer to Eqn. (21)}] \quad ...(24)$$

On dividing Eqn. (23) by (24), we get:

$$\tan(\theta_Z - \theta_A) = W_{2Q}/(W_{1R} - W_{2R})$$

or $$(\theta_Z - \theta_A) = W_{2Q}/(W_{1R} - W_{2R}) \text{ radians}$$

Therefore, phase angle θ of the transformer is given as:

$$\theta_Z = [W_{2Q}/(W_{1R} - W_{2R})] + \theta_A \approx [W_{2Q}/W_{1R}] + \theta_A \text{ radians} \quad [\text{Since } W_{2R} \text{ is small}] \quad ...(25)$$

To determine the **ratio** TR of the transformer, we consider the transformation ratio TR_Z of the transformer under test as:

$$TR_Z = I_P/I_{SZ} \quad ...(26)$$

and transformation ratio TR_A of the transformer under test as:

$$TR_A = I_P/I_{SA} \quad ...(27)$$

Therefore, the transformation ratio TR_Z can be determined by dividing Eqn. (26) by (27) as:

$$\frac{TR_Z}{TR_A} = \frac{I_{SA}}{I_{SZ}} = \frac{VI_{SA}}{VI_{SZ}} = \frac{W_{1R}}{W_{1R} - W_{2R}} \approx 1 + \frac{W_{2R}}{W_{1R}} \quad [\text{Refer to Eqn. (20) and (22)}]$$

$$\Rightarrow \quad TR_Z = TR_A\left(1 + \frac{W_{2R}}{W_{1R}}\right) \quad ...(28)$$

Arnold's method

Arnold's method is a type of comparison method based on null technique. The circuit diagram for this method is shown in Figure 15.10.

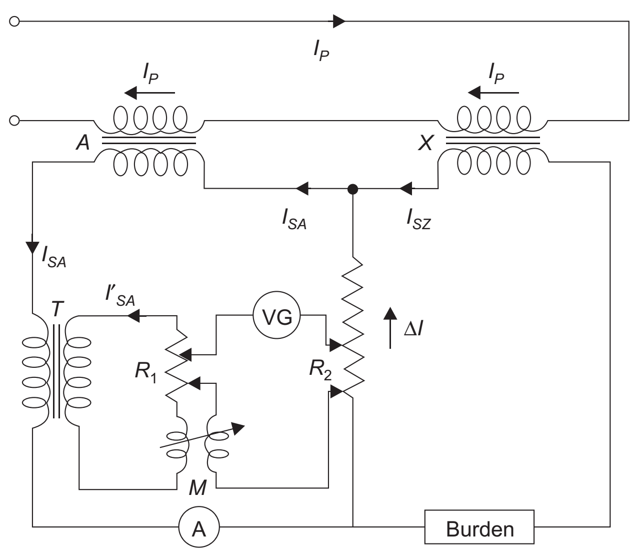


Fig. 15.10 Configuration of Arnold's Method

As can be seen from the figure that a 5/5 current transformer T is included in the circuit to isolate the secondary windings of current transformer from the measuring circuit.

To achieve the balance, we have:

$$\Delta I R_2 + I'_{SA} - j\omega M = 0 \quad ...(29)$$

Now, we have:

$$\Delta I = I_{SA} - I_{SZ} \quad ...(30)$$

where $\quad I_{SA} = -\dfrac{I_P}{R_A}(\cos\theta_A + j\sin\theta_A)$

or, since θ_A is very small, it can be written as:

$$I_{SA} = -\frac{I_P}{R_A}(1 + j\theta_A) \quad ...(31)$$

and

$$I_{SZ} = -\frac{I_P}{R_Z}(1 + j\theta_Z) \quad ...(32)$$

Thus, from Eqn. (31) and (32), we get:

$$I_{SZ} = \frac{R_A}{R_Z}[1 + j(\theta_Z - \theta_A)]\, I_{SA} \quad ...(33)$$

Substituting Eqn. (33) in Eqn. (30), we get:

$$\Delta I = I_{SA}\left[1 - \frac{R_A}{R_Z}\{1 + j(\theta_Z - \theta_A)\}\right] \quad ...(34)$$

Now, since transformer T is 1:1 current transformer, we have:

$$I'_{SA} = I_{SA} \quad ...(35)$$

Substituting Eqn. (34) and (35) in Eqn. (29), we get:

$$I_{SA}\left[1 - \frac{R_A}{R_Z}\{1 + j(\theta_Z - \theta_A)\}\right]R_2 + I_{SA}R_1 - I_{SA}\, j\,\omega M = 0$$

Equating real parts, we get:

$$R_2\left(1 - \frac{R_A}{R_Z}\right) = R_1$$

$$\Rightarrow \quad 1 - \frac{R_A}{R_Z} = \frac{R_1}{R_2}$$

Equating imaginary parts, we get:

$$\frac{R_A R_2}{R_Z}(\theta_Z - \theta_A) = \omega M$$

$$\Rightarrow \qquad \theta_Z - \theta_A = \frac{\omega M}{R_2}\frac{R_Z}{R_A} \text{ radians}$$

As we know, $R_Z \approx R_A$, we get:

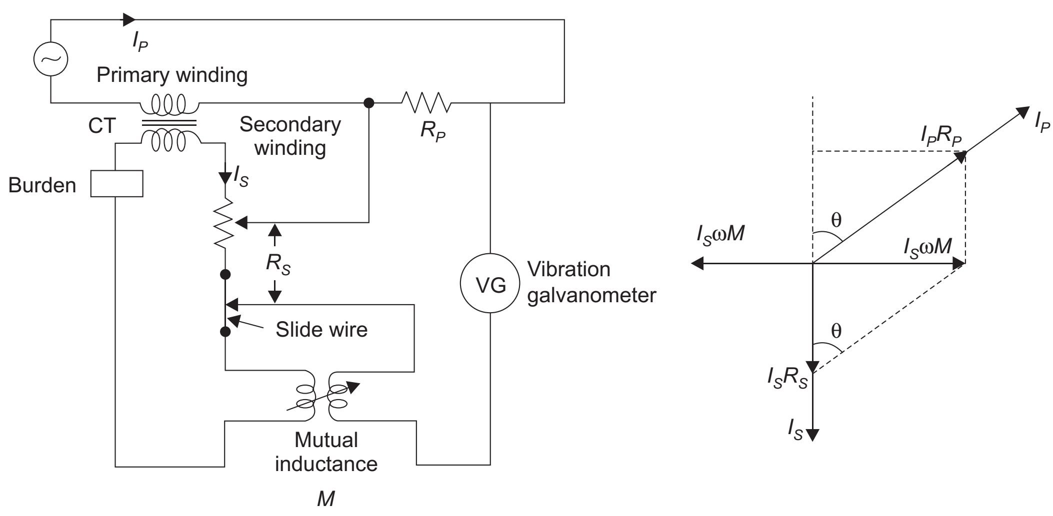
$$\theta_Z - \theta_A = \frac{\omega M}{R_2} \text{ radians}$$

Thus, the phase angle of the transformer can be written as:

$$\theta_Z = \frac{\omega M}{R_2} + \theta_A \text{ radians} \qquad ...(36)$$

Mutual inductance method

Mutual inductance method is an absolute method based on null technique. The setup for this test is shown in Figure 15.11(a) along with its phasor diagram in Figure 15.11(b).



(a) Circuit diagram (b) Phasor diagram

Fig. 15.11 Configuration of Mutual Inductance Method

Here, R_P and R_S are non-inductive shunts having very low value of resistance. A slide-wire is present on the resistor R_S for its resistance adjustment thereby making it variable. However, the resistance R_P is fixed. The voltage drop across both the resistances R_P and R_S is matched which is indicated using a vibration galvanometer (VG).

For zero deflection in vibration galvanometer, we have:

$$I_P R_P = I_S R_S$$

Or, it can be written as:

$$\frac{I_P}{I_S} = \frac{R_S}{R_P}$$

This relation is possible when the phase difference between I_P and I_S is zero. Thus, R_P and R_S are so selected that their ratio R_S/R_P is equal to the nominal ratio of the current transformer.

From the figure it can be seen that a mutual inductance M is also connected in the circuit. It is provided to compensate for the phase difference between I_P and I_S as it is difficult to obtain a balance with resistance only without having any phase compensating device in the circuit.

Determination of phase angle and ratio

Under balance conditions, we have:

$$\tan\theta = \frac{I_S \omega M}{I_S R_S} = \frac{\omega M}{R_S} \quad \text{...(37)}$$

and

$$\cos\theta = \frac{I_S R_S}{I_P R_P} \quad \text{...(38)}$$

From Eqn. (37) and (38), we get the phase angle θ as:

$$\theta \approx \frac{\omega M}{R_S} \text{ radian} \quad \text{...(39)}$$

Actual ratio of the transformer TR can be calculated as:

$$TR = \frac{I_P}{I_S}$$

$$\Rightarrow \quad TR = \frac{R_S}{R_P \cos\theta}$$

Since θ is very small, we have:

$$TR \approx \frac{R_S}{R_P} \quad \text{...(40)}$$

15.4 POTENTIAL TRANSFORMER

Potential transformers consist of two windings, namely, primary and secondary winding. The primary winding is made up of a fine wire and has very large number of turns. It is connected across the line which carries the voltage to be measured. Secondary winding is made up of a heavy wire and has very few turns. The voltage circuit is connected across the secondary winding. The voltage delivered by the secondary winding of potential transformer is in the range of 100 to 120 V with a value of 110 V. Loading of potential transformers is very small, about a few volt-amperes.

Potential transformers are used to operate potential coils of wattmeters, voltmeters, and relays from high voltage lines.

15.4.1 Construction

While designing potential transformers, desired performance criteria are to be kept in mind. They must have small phase angle and constant ratio. The potential transformers are generally large in size with small output. Due to this, there are no thermal problems in a potential transformer and its loading is limited by accuracy considerations. On a thermal basis, they can carry loads which are many times their rated loads. For a low voltage potential transformer, the load can be 2 to 3 times while for a high voltage transformer it can be 30 times of its rated value. Different parts of a potential transformer are described as follows.

Core

The core of a potential transformer can be of core or shell type. For low voltage transformers, shell type core is preferred. Also, to minimize the effects of air gaps at the joints, assembling and interleaving of the core laminations must be done carefully.

Insulation

Coil is insulated using cotton tape and varnished cambric and separated from each other using hard libre. When the potential transformers are used at voltages above 7 kV, they are oil immersed. However, no such compound is filled when they are working at low voltages. For voltages up to 45 kV, dry type and porcelain-insulated transformers are used.

Windings

The primary and secondary windings of a potential transformer are made coaxial so as to minimize the leakage reactance. For low voltage transformers, the primary winding is a single coil. However, for high voltage transformers, primary winding is divided into a number of short coils. This is done to decrease the insulation needed between the layers of coil. The secondary winding, that is, the low voltage winding, is placed next to the core to reduce insulation problems.

Bushings

In potential transformers when the neither side of the line is connected to ground potential, two bushings are used. However, in some potential transformers, the line is connected to neutral of the grounded neutral systems and thus, only one high voltage bushing is used.

For oil-filled transformers, to reduce the overall size of the transformer, oil-filled bushings are used.

Here, it is to be noted that in current transformers, only one bushing is used. A typical two-winding single-phase potential transformer is shown in Figure 15.12.

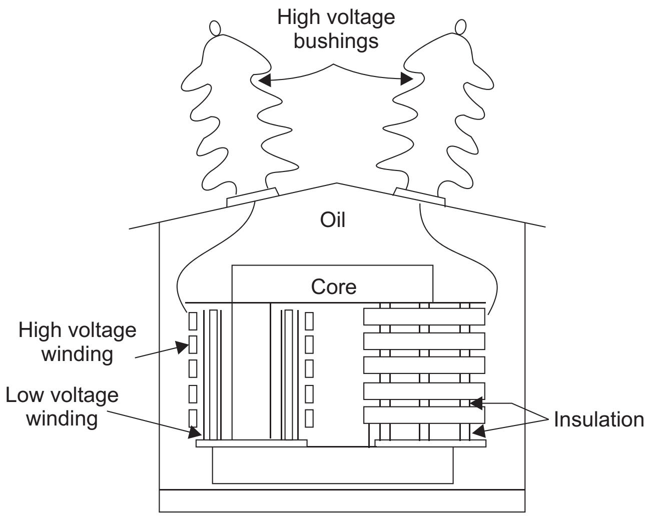


Fig. 15.12 Two-winding Single-phase Potential Transformer

15.4.2 Equivalent Circuit and Phasor Diagram

The equivalent circuit of a potential transformer is shown in Figure 15.13 along with its phasor diagram in Figure 15.14. Due to small power loading of a potential transformer, the exciting and secondary winding current that flows in it are of the same order.

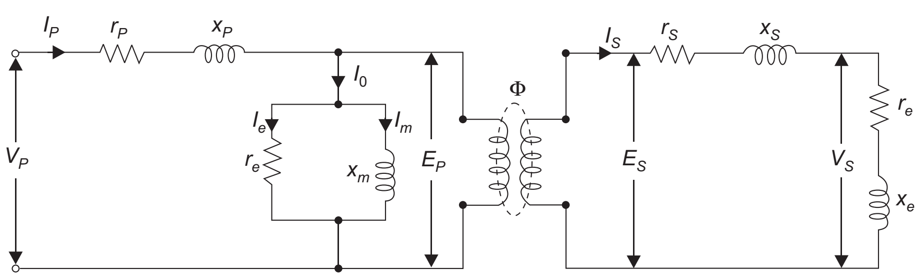


Fig. 15.13 Equivalent Circuit of Potential Transformer

The terms marked in Figure 15.13 are: I_P is the primary winding current, I_S is the secondary winding current, r_P and r_S are resistances of primary and secondary windings, respectively, x_P and x_S are reactances of primary and secondary windings, respectively, r_e and x_e are resistance and reactance of secondary load circuit, respectively, E_P and E_S are primary and secondary induced voltages, respectively, voltage at the secondary winding terminal is V_S, number of primary and secondary winding turns are N_P and N_S, turn ratio (ratio of secondary winding turn to primary winding turn) is N, magnetizing component and loss component of exciting current are I_m and I_e, working flux of transformer is Φ.

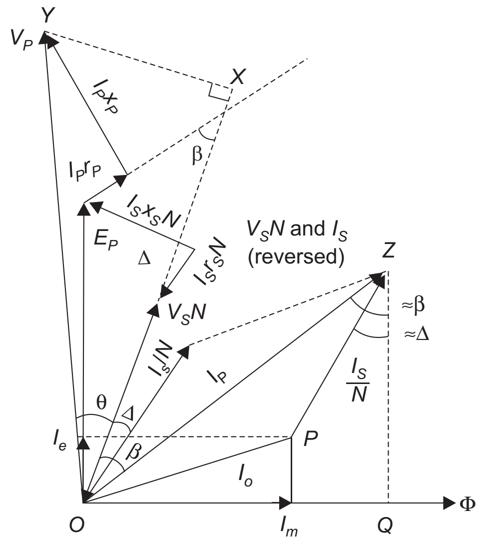


Fig. 15.14 Phasor Diagram of Potential Transformer

Here, Δ is the phase angle of the secondary load circuit, θ is phase angle of transformer (angle between V_P and reversed V_S), and β is the phase angle between I_P and reversed V_S.

Transformation ratio

Consider the phasor diagram of potential transformer shown in Figure 15.14, we have:

$$OX = V_P \cos\theta$$

From the figure, it can be written as:

$$OX = NV_S + NI_S r_S \cos\Delta + NI_S x_S \sin\Delta + I_P r_P \cos\beta + I_P x_P \sin\beta$$

From the above two equations, we get:

$$V_P \cos\theta = NV_S + NI_S r_S \cos\Delta + NI_S x_S \sin\Delta + I_P r_P \cos\beta + I_P x_P \sin\beta$$

Or, it can be written as:

$$V_P \cos\theta = NV_S + NI_S(r_S \cos\Delta + x_S \sin\Delta) + I_P r_P \cos\beta + I_P x_P \sin\beta \qquad ...(41)$$

Since θ is a very small phase angle, it can be assumed that V_P and reversed V_S are perpendicular to Φ and thus, on approximation we have:

$$\angle OZQ = \beta \qquad ...(42)$$

$$\angle PZQ = \Delta$$

Also,

$$I_P \cos\beta = I_e + \frac{I_S}{N}\cos\Delta \qquad ...(43)$$

$$I_P \sin\beta = I_m + \frac{I_S}{N}\sin\Delta$$

As the value of θ is generally less than 1°, we have cos θ = 1 and thus, we get:

$$V_P \cos\theta = V_P \qquad ...(44)$$

Substituting Eqn. (43) and (44) in Eqn. (41), we get:

$$V_P = NV_S + NI_S\,(r_S \cos\Delta + x_S \sin\Delta) + \left(I_e + \frac{I_S}{N}\cos\Delta\right) r_P + \left(I_m + \frac{I_S}{N}\sin\Delta\right) x_P$$

$$= NV_S + I_S\left(Nr_S + \frac{r_P}{N}\right)\cos\Delta + I_S\left(Nx_S + \frac{x_P}{N}\right)\sin\Delta + I_e r_P + I_m x_P \qquad ...(45)$$

Or, it can be written as:

$$V_P = NV_S + \frac{I_S}{N}(N^2 r_S + r_P)\cos\Delta + \frac{I_S}{N}(N^2 x_S + x_P)\sin\Delta + I_e r_P + I_m x_P$$

$$= NV_S + \frac{I_S}{N}R_P\cos\Delta + \frac{I_S}{N}X_P\sin\Delta + I_e r_P + I_m x_P$$

$$\Rightarrow \qquad V_P = NV_S + \frac{I_S}{N}(R_P\cos\Delta + X_P\sin\Delta) + I_e r_P + I_m x_P \qquad ...(46)$$

where R_P and X_P are the equivalent resistance and reactance of the transformer referred to the primary side, respectively.

Now, the actual transformation or **voltage ratio** is given as:

$$TR = \frac{V_P}{V_S}$$

Substituting the value from Eqn. (46), we get:

$$TR = \frac{NV_S + \frac{I_S}{N}(R_P\cos\Delta + X_P\sin\Delta) + I_e r_P + I_m x_P}{V_S}$$

Or, $$TR = N + \frac{\frac{I_S}{N}(R_P \cos\Delta + X_P \sin\Delta) + I_e r_P + I_m x_P}{V_S} \quad \text{...(47)}$$

From Eqn. (47), it can be seen that the transformation ratio is expressed in terms of equivalent resistance and reactance referred to the primary side. However, it can be expressed in terms of equivalent resistance and reactance referred to the secondary side. For this, rewriting Eqn. (45) as:

$$V_P = NV_S + NI_S\left(r_S + \frac{r_P}{N^2}\right)\cos\Delta + NI_S\left(x_S + \frac{x_P}{N^2}\right)\sin\Delta + I_e r_P + I_m x_P$$

$$\Rightarrow \quad V_P = NV_S + NI_S(R_S \cos\Delta + x_S \sin\Delta) + I_e r_P + I_m x_p \quad \text{...(48)}$$

where R_S and X_S are the equivalent resistance and reactance of the transformer referred to the secondary side, respectively.

Now, the actual transformation or voltage ratio is given as:

$$TR = \frac{V_P}{V_S}$$

Substituting value from Eqn. (48), we get:

$$TR = \frac{NV_S + NI_S\,(R_S \cos\Delta + X_S \sin\Delta) + I_e r_P + I_m x_P}{V_S}$$

Or, $$TR = N + \frac{NI_S\,(R_S \cos\Delta + X_S \sin\Delta) + I_e r_P + I_m x_P}{V_S} \quad \text{...(49)}$$

Now, the difference between actual transformation ratio *TR* and turns ratio *N* can be written from Eqn. (47) as:

$$TR - N = \frac{\frac{I_S}{N}(R_P \cos\Delta + X_P \sin\Delta) + I_e r_P + I_m x_P}{V_S} \quad \text{...(50)}$$

From Eqn. (49), we have:

$$TR - N = \frac{NI_S\,(R_S \cos\Delta + X_S \sin\Delta) + I_e r_P + I_m x_P}{V_S} \quad \text{...(51)}$$

Phase angle

The phase angle is obtained as:

$$\tan\theta = \frac{XY}{OX}$$

$$= \frac{I_P x_P \cos\beta - I_p r_P \sin\beta + NI_S x_S \cos\Delta - NI_S r_S \sin\Delta}{NV_S + NI_S r_S \cos\Delta + NI_S x_P \sin\Delta + I_p r_P \cos\beta + I_P x_P \sin\beta}$$

Since I_P and I_S are very small as compared to NV_S, they can be neglected and we get:

$$\tan\theta = \frac{I_P x_P \cos\beta - I_p r_P \sin\beta + NI_S x_S \cos\Delta - NI_S r_S \sin\Delta}{NV_S} \quad ...(52)$$

Substituting Eqn. (43) in Eqn. (52), we get:

$$\tan\theta = \frac{x_P\left(I_e + \frac{I_S}{N}\cos\Delta\right) - r_P\left(I_m + \frac{I_S}{N}\sin\Delta\right) + NI_S x_S \cos\Delta - NI_S r_S \sin\Delta}{NV_S}$$

$$= \frac{I_S\left(\frac{x_P}{N} + Nx_S\right)\cos\Delta - I_S\left(\frac{r_P}{N} + Nr_S\right)\sin\Delta + I_e x_P - I_m r_P}{NV_S}$$

$$= \frac{\frac{I_S}{N}(x_P + N^2 x_S)\cos\Delta - \frac{I_S}{N}(r_P + N^2 r_S)\sin\Delta + I_e x_P - I_m r_P}{NV_S}$$

Or, it can be written as:

$$\tan\theta = \frac{\frac{I_S}{N} X_P \cos\Delta - \frac{I_S}{N} R_P \sin\Delta + I_e x_P - I_m r_P}{NV_S}$$

$$\tan\theta = \frac{\frac{I_S}{N}(X_P \cos\Delta - R_P \sin\Delta) + I_e x_P - I_m r_P}{NV_S}$$

Now, we know that θ is very small, therefore tan θ = θ. Hence, the phase angle may be written as:

$$\theta = \frac{\frac{I_S}{N}(X_P \cos\Delta - R_P \sin\Delta) + I_e x_P - I_m r_P}{NV_S} \quad ...(53)$$

Or, it can be written as:

$$\theta = \frac{I_S}{V_S}(X_S \cos\Delta - R_S \sin\Delta) + \frac{I_e x_P - I_m r_P}{NV_S} \text{ radians} \quad ...(54)$$

Example 3 A potential transformer with ratio of 1000/100 volts has the following constants: primary resistance = 94.5 Ω, secondary resistance = 0.86 Ω, primary reactance = 60.2 Ω, equivalent reactance = 60.2 Ω, and magnetizing current = 0.60 A at 0.45 power factor. Calculate the phase angle at no load between primary and secondary voltage.

Solution: Given that: Turns ratio, $N = 1000/100 = 10$, primary resistance $r_P = 94.5\ \Omega$, secondary resistance $r_S = 0.86\ \Omega$, primary reactance $x_P = 60.2\ \Omega$, equivalent reactance $X_P = 60.2\ \Omega$, and magnetizing current $I_m = 0.60$ A, and power factor = 0.45

Now at no load, the power factor is given by:

$$\cos\alpha = 0.45$$

$$\Rightarrow \qquad \sin\alpha = \sqrt{1^2 - 0.45^2} = 0.893$$

The magnetizing component I_m is given by the relation:

$$I_m = I_o \sin\alpha = 0.60 \text{ A} \qquad \text{[Refer to Fig. 15.14]}$$

$$\Rightarrow \qquad I_o = \frac{I_m}{\sin\alpha} = \frac{0.60}{0.893} = 0.672 \text{ A}$$

The loss component of current I_e is given by:

$$I_e = I_o \cos\alpha \qquad \text{[Refer to Fig. 15.14]}$$

On substituting values, we get:

$$I_e = 0.672 \times 0.45 = 0.302 \text{ A}$$

The phase angle θ is given by the expression:

$$\theta = \frac{\frac{I_S}{N}(X_P \cos\Delta - R_P \sin\Delta) + I_e x_P - I_m r_P}{NV_S} \qquad \text{[Refer to Eqn. (53)]}$$

When no load is applied, $I_S = 0$. Thus, we get:

$$\theta = \frac{I_e x_P - I_m r_P}{NV_S}$$

Substituting the values, we get:

$$\theta = \frac{0.302 \times 60.2 - 0.60 \times 94.5}{10 \times 1000}$$

$$\theta = -0.0038 \text{ radians}$$

15.4.3 Characteristics

There are various characteristics that affect the working of a potential transformer. These include *change in power factor of secondary burden*, *change in frequency*, *change in current of secondary winding* and *change in voltage of primary winding*.

Change in power factor of secondary burden

When the power factor of secondary circuit burden is decreased, angle Δ increases which results in a shifting of current I_P towards current I_O. Also, the phase difference between V_P and E_P and V_S and E_S decreases and they align more in phase with each other, respectively. Due to this, a relative reduction in E_P with respect to V_P occurs as V_P is constant. Similarly, a relative reduction in V_S with respect to E_S occurs. Thus, with the reduction in power factor of secondary winding burden, the transformation ratio increases, while the phase angle decreases.

Change in frequency

The flux and frequency are inversely proportional to each other for a constant voltage. Therefore, with an increase in frequency there will be a decrease in flux. This results in a reduction in I_m and I_e, which further leads to a reduction in the voltage ratio. However, as frequency increases, the leakage reactance increases which results in an increase in voltage ratio. These two effects counteract on each other and hence, the change in voltage ratio due to change in frequency depends upon the relative values of leakage reactance and currents (I_m and I_e).

Here, it is to be noted that the phase angle increases with an increase in frequency.

Change in current of secondary winding

When the secondary burden is increased, it results in an increase in secondary current and also primary current. With an increase in primary and secondary currents, primary and secondary voltage drops also increase. Thus, the value of V_S decreases for a particular value of V_P and thereby resulting in an increase in actual transformation ratio. Thus, with an increase in burden of secondary winding, the actual ratio increases and the ratio error also increases thereby becoming more negative. Also, the phase angle between V_P and reversed V_S increases, making it more negative.

Change in voltage of primary winding

The primary winding voltage which is connected to supply voltage usually lies in a small range and there is no wide variation in it. Thus, it is not important to consider the effect of primary voltage on ratio and phase angle errors.

15.4.4 Errors and their Causes

The errors introduced by a potential transformer in any measurement are *phase angle error* and *voltage ratio error*, described as follows.

Phase angle error

For ideal transformers, the phase difference between primary winding voltage V_P and reversed secondary winding voltage V_S is 0°. However, practical transformers have some phase difference other than 0°. When primary winding voltage lags the reversed secondary winding voltage, the phase angle is positive and expressed by Eqn. (53) and (54) while it is taken as negative when the primary winding voltage leads the reversed secondary winding voltage.

Voltage ratio error

The percentage error in secondary voltage of the potential transformer is expressed as:

$$\text{Percentage ratio error} = \frac{\text{Nominal ratio} - \text{Transformation ratio}}{\text{Transformation ratio}} = \frac{N_k - TR}{TR} \times 100$$

The ratio of a transformer depends upon its operating conditions. Here, it is to be noted that both phase angle and ratio errors are important while measuring power. However, in voltage measurements, only ratio error is involved.

15.4.5 Methods to Reduce Errors

In addition to using new materials for the construction of transformers, some modifications in the design may be done to reduce errors. These modifications are discussed as follows.

Compensation of turns

From Eqn. (47), it can be seen that when no load is applied, the difference between actual and turns ratio is given by $(I_e r_P + I_m x_P)/V_S$. Now, when a resistive or inductive load is applied there will be some voltage drop across resistance and leakage reactance of the windings which results in a further increase in ratio. Also, if turns ratio and nominal ratio are equal, there will be errors as the actual ratio will not be equal to nominal ratio.

This error can be reduced by either increasing the number of turns of secondary winding or decreasing the number of turns of primary winding. This results in a less turns ratio as compared to nominal ratio. Thus, actual transformation ratio becomes equal to nominal ratio. This also results in the reduction of ratio error for the whole range of burden.

Reduction of leakage reactance and resistance

To reduce the leakage reactance, the primary and secondary windings must be kept as close to each other as possible since the magnitudes of the primary and secondary leakage flux determine the leakage reactance. However, the spacing must be in agreement with the insulation requirements. The resistance of the winding can also be reduced by the use of thick conductors and small length mean turn.

To reduce leakage reactance and resistance, the flux density in the core must be kept as high as possible without resulting in saturation. High flux density in the core implies fewer turns in the windings, thus reducing the leakage reactance. Also, high flux density implies lesser core area and lesser mean turn length which results in smaller resistance of the winding.

Reduction of loss and magnetizing components

From Eqn. (51), it can be seen that the difference between transformation ratio and turns ratio is dependent on secondary winding current and loss and magnetizing components of no load current. In potential transformers, the components of no load current, that is I_e and I_m have values almost in the range of load current. Thus, these currents can be reduced to improve the performance of the transformer. This is achieved by using core material of good quality, keeping magnetic paths short, maintaining low flux density in the core, taking precautions while assembling and interleaving of core.

15.4.6 Testing Methods

The testing of potential transformers employs four methods, namely, *Clothier and Medina comparison method*, *comparison method using wattmeters*, *absolute null method*, and *method using capacitance dividers*.

Clothier and Medina comparison method

The circuit diagram for Clothier and Medina comparison method is shown in Figure 15.15. It is a portable method and employs an iron cored dynamometer for detection purpose. It consists of two transformers whose primary windings are connected in parallel.

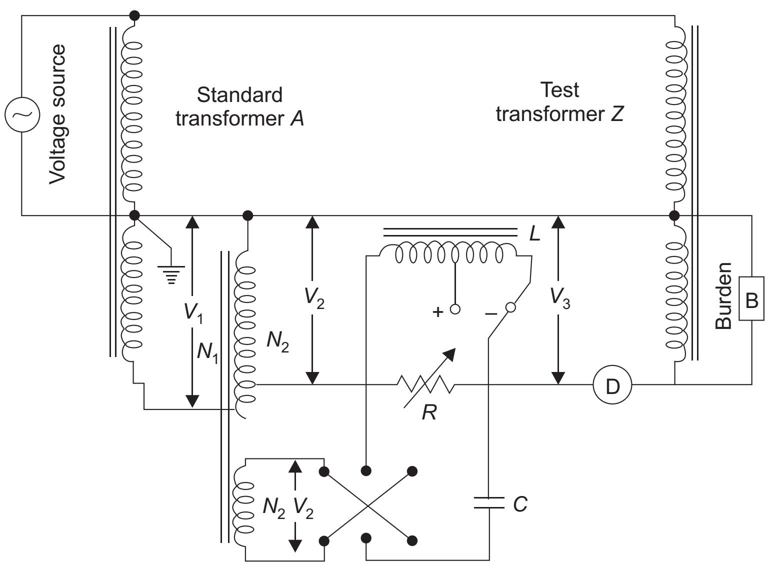


Fig. 15.15 Circuit Diagram for Clothier and Medina Comparison Method

From the figure we can say that the additional circuit is also provided which adjusts the magnitude and phase of secondary voltage of standard transformer *A* so that it becomes equal to that of test transformer *Z*. An autotransformer is connected to the secondary winding of the current transformer to adjust the number of turns of primary winding *N*. A variable resistor *R* is adjusted to balance the phase angle. This resistance *R* is connected in the circuit through which current flows and is in quadrature with the secondary voltage. A reversing switch is provided to determine the sign of secondary voltage.

At balance condition, we have:

$$\frac{R_Z}{R_A} = \frac{N_1}{N_2} \qquad ...(55)$$

and

$$\theta_Z = \theta_A \pm \omega RC \qquad ...(56)$$

To keep the ratio and phase relations independent of each other, inductor L is connected in parallel with the resistor R. The value of L is given differently for both positive and negative phase angles as:

$$L = \frac{2}{3\omega^2 C} \quad \text{for positive phase angles}$$

and
$$L = \frac{2}{\omega^2 C} \quad \text{for negative phase angles}$$

Comparison method using wattmeters

The circuit diagram for comparison method using wattmeters is shown in Figure 15.16(a). This method is similar to Silsbee's method of testing current transformers. The standard transformer A and test transformer Z both have the same nominal ratio and their primary windings are connected in parallel. The secondary winding of test transformer is connected to a burden, while the secondary winding of standard transformer is connected to the potential coil of wattmeter W_1. The difference between the secondary voltages of test and standard transformer is applied across the potential coil of wattmeter W_2. The current coils of both wattmeters are connected serially and carry a constant current I, supplied from a phase shifting transformer.

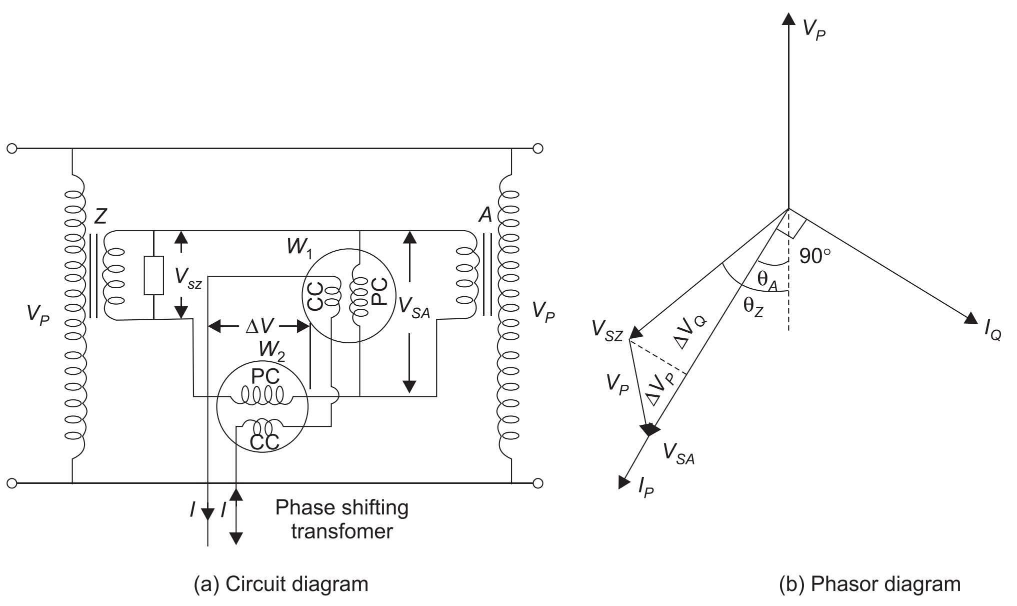


Fig. 15.16 Comparison Method using Wattmeters

- The first case is when the phase of current I is adjusted so that the reading of wattmeter W_1 is zero. At this position, current I is in quadrature with voltage V_{SA}, and is shown as I_Q in the phasor diagram [see Figure 15.16(b)]

Reading of the wattmeter W_1 can be given as:

$$W_{1Q} = V_{SA} I_Q \cos 90° = 0$$

And, reading of the wattmeter W_2 can be given as:

$$W_{2Q} = (\text{Component of voltage } \Delta V \text{ in phase with } I_Q) \times I_Q$$

$$W_{2Q} = \Delta V_Q I_Q = V_{SZ} \sin(\theta_Z - \theta_A) \times I_Q$$

$$\Rightarrow \qquad W_{2Q} = V_{SZ} I_Q \sin(\theta_Z - \theta_A) \qquad ...(57)$$

- The second case is when the phase of current I is shifted by an angle of 90°. Now, the current I is shown as I_P which is in phase with V_{SA}.
  Reading of the wattmeter W_1 can be given as:

$$W_{1P} = V_{SA} I_P \cos 0° = V_{SA} I_P \qquad ...(58)$$

And, reading of the wattmeter W_2 can be given as:

$$W_{2P} = (\text{Component of voltage } \Delta V \text{ in phase with } I_P) \times I_P$$

$$W_{2P} = \Delta V_P I_P = [V_{SA} - V_{SZ} \cos(\theta_Z - \theta_A)] \times I_P \qquad ...(59)$$

Determination of phase angle and ratio

Let the current be the same for both cases, we have:

$$I = I_P = I_Q$$

Thus, Eqn. (57), (58), and (59) can be written as:

$$W_{2Q} = V_{SZ} I \sin(\theta_Z - \theta_A), \qquad ...(60)$$

$$W_{1P} = V_{SA} I, \qquad ...(61)$$

and

$$W_{2P} = [V_{SA} - V_{SZ} \cos(\theta_Z - \theta_A)]\, I \qquad ...(62)$$

Now, from Eqn. (61) and (62), we get:

$$W_{2P} = [V_{SA} I - V_{SZ} I \cos(\theta_Z - \theta_A)] = W_{1P} - V_{SZ} I \cos(\theta_Z - \theta_A) \qquad ...(63)$$

As $(\theta_Z - \theta_A)$ is a very small quantity, we have $\cos(\theta_Z - \theta_A) = 1$. Thus, the above equation can be written as:

$$W_{2P} = W_{1P} - V_{SZ} I$$

Or, it can be written as:

$$V_{SZ} I = W_{1P} - W_{2P} \qquad ...(64)$$

The actual transformation ratio of the test transformer TR_Z is given as:

$$TR_Z = \frac{V_P}{V_{SZ}} \quad \text{...(65)}$$

The actual transformation ratio of the standard transformer TR_A is given as:

$$TR_A = \frac{V_P}{V_{SA}} \quad \text{...(66)}$$

Dividing Eqn. (65) by Eqn. (66), we get:

$$\frac{TR_Z}{TR_A} = \frac{V_{SA}}{V_{SZ}} = \frac{V_{SA}\, I}{V_{SZ}\, I} \quad \text{...(67)}$$

Substituting the values from Eqn. (61) and (64), Eqn. (67) becomes:

$$\frac{TR_Z}{TR_A} = \frac{W_{1P}}{W_{1P} - W_{2P}} \approx 1 + \frac{W_{2P}}{W_{1P}}$$

Thus, the ratio of the transformer under test can be written as:

$$TR_Z = TR_A \left(1 + \frac{W_{2P}}{W_{1P}} \right)$$

Now, the phase angle can be determined using Eqn. (60) as:

$$\sin(\theta_Z - \theta_A) = \frac{W_{2Q}}{V_{SZ}\, I} \quad \text{...(68)}$$

Also, from Eqn. (63), we have:

$$\cos(\theta_Z - \theta_A) = \frac{W_{1P} - W_{2P}}{V_{SZ}\, I} \quad \text{...(69)}$$

Dividing Eqn. (68) by Eqn. (69), we get:

$$\tan(\theta_Z - \theta_A) = \frac{W_{2Q}}{W_{1P} - W_{2P}}$$

As $(\theta_Z - \theta_A)$ is a very small quantity, we have $\tan(\theta_Z - \theta_A) \approx (\theta_Z - \theta_A)$. Thus, the above equation can be written as:

$$(\theta_Z - \theta_A) \approx \frac{W_{2Q}}{W_{1P} - W_{2P}}$$

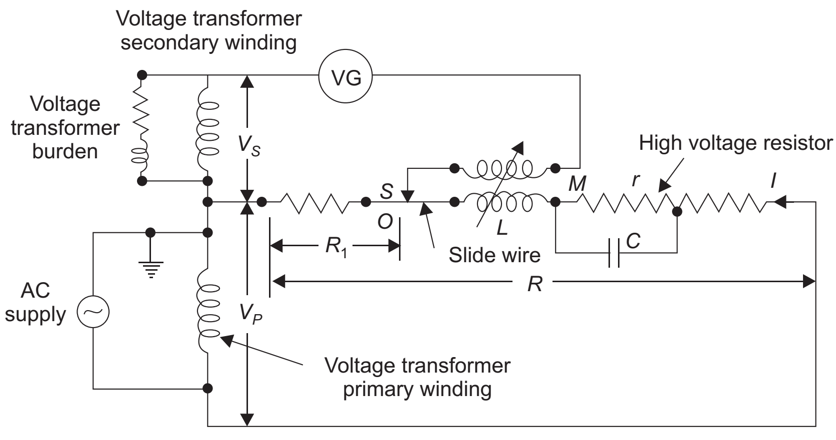
Thus, the phase angle of the transformer under test can be written as:

$$\theta_Z \approx \frac{W_{2Q}}{W_{1P} - W_{2P}} + \theta_A \approx \frac{W_{2Q}}{W_{1P}} + \theta_A \text{ radians}$$

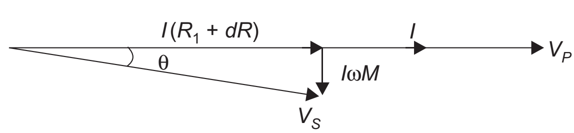
Here, it is to be noted that the voltage range of wattmeter W_2 should be small as a very small voltage ΔV is applied to its pressure coil.

Absolute null method

In absolute null method, testing of the transformer is done using a resistance potential divider. The secondary winding is connected to the burden (which is used for transformer testing) while the primary winding is connected to a normal voltage at normal frequency. Primary winding is connected to the secondary winding at one end. A non-inductive potential divider is connected to the primary winding whose lower part is a fixed resistor R_1 while the upper part is a shielded high voltage resistor r. The resistor R_1 is connected in series with a slide wire S while the resistor r is connected in series with the primary winding of a variable mutual inductor M [see Figure 15.17(a)]. A capacitor C is used to shunt a small portion of resistor r and also to compensate for the phase angle error introduced by the primary winding inductance of mutual inductance M.



(a) Circuit diagram



(b) Phasor diagram

Fig. 15.17 Absolute Null Method

The values of resistance are so selected that the nominal ratio of the test transformer is equal to R/R_1. The impedance of the divider is given by the expression:

$$Z = R - r + j\omega L + \frac{r}{1 + j\omega Cr}$$

$$= R - r + j\omega L + \frac{r}{1 + \omega^2 C^2 r^2}(1 - j\omega Cr)$$

Or, it can be written as:

$$Z = R - r + \frac{r}{1 + \omega^2 C^2 r^2} + j\omega\left(L - \frac{Cr^2}{1 + \omega^2 C^2 r^2}\right)$$

The divider will be non-reactive at the operating frequency if $L = \dfrac{Cr^2}{1 + \omega^2 C^2 r^2}$. The total impedance in that case will be given as:

$$Z = R - r + \frac{r}{1 + \omega^2 C^2 r^2}$$

Generally, $\omega^2 C^2 r^2 << 1$ and thus, it can be neglected. Therefore, the impedance becomes equal to R.

Now, the balance is obtained by adjusting the contacts on slide wire and mutual inductor until the vibration galvanometer (VG) gives no deflection. The balanced phasor diagram is shown in Figure 15.17(b). Here, the phase angle θ is a very small quantity. We have:

$$V = I(R_1 + dR) = \frac{V_P}{R}(R_1 + dR)$$

where I is the current flowing through the divider.

The transformation ratio TR is given by the relation:

$$TR = \frac{V_P}{V_S} = \frac{R}{R_1 + dR}$$

Now, the ratio error is expressed as:

$$\text{Ratio error} = \frac{\text{Nominal ratio} - \text{Actual ratio}}{\text{Actual ratio}}$$

Substituting the values, we get:

$$= \frac{\dfrac{R}{R_1} - \dfrac{R}{R_1 + dR}}{\dfrac{R}{R_1 + dR}} = \frac{R_1 + dR}{R} - 1 = \frac{dR}{R_1}$$

Thus, from the above expression we can say that the slide wire can be calibrated directly in terms of ratio error. Now, the phase angle θ is given as:

$$\tan\theta \approx \theta = \frac{\omega M}{R_1 + dR} \approx \frac{\omega M}{R_1} \qquad \text{[Refer to Fig. 15.17(b)]}$$

Thus, the mutual inductor can be calibrated directly in terms of phase angle. It must be noted that to avoid errors, high voltage resistor must be shielded properly. A simple earthed shield may result in errors as in that case capacitance currents may flow between the shield and the resistor. Therefore, special shielding techniques must be employed. Also, by reducing the resistance of the divider to its minimum possible value, errors can be minimized. The generally adopted value is 20 Ω/V.

Method using capacitance dividers

For voltages above 40 kV, testing using resistance dividers is practically not possible. Hence, capacitance dividers are used at high voltages. The circuit diagram for testing potential transformers using capacitance voltage dividers is shown in Figure 15.18.

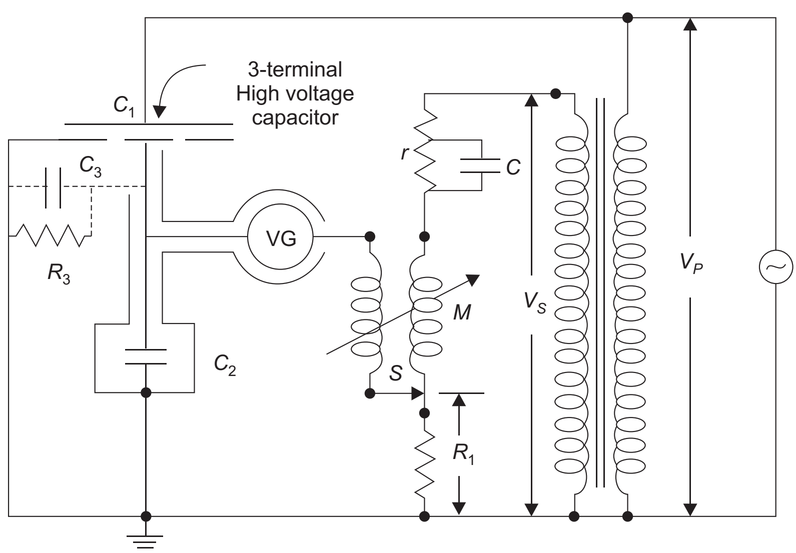


Fig. 15.18 Circuit Diagram for Testing PT using Capacitance Dividers

As can be seen from the circuit, it consists of two capacitors C_1 and C_2. The capacitor C_1 is a three-terminal capacitor of value 10 pF. C_1 is a high voltage, compressed gas capacitor while C_2 is a low power factor, high-quality mica capacitor. The secondary winding of the potential transformer is connected to a resistance divider having the ratio of 10/1. The divider is used for ratio and phase angle adjustments and consists of a mutual inductor M and a variable potential slide wire S. Like in resistance divider circuit discussed in absolute null method, the mutual inductor M and the slide wire S can be calibrated directly to read phase angle and ratio error, respectively.

However, one problem occurs with capacitance dividers and that is the presence of capacitance C_3 (shown by dotted line in Figure 15.18) of low voltage electrode and guard ring of C_1. A loss resistance R_3 is also associated with this capacitor. Therefore, it becomes necessary to compensate for this by making C_3 large so that R_3 and $C_2 + C_3$ have negligible power factor. Since a loss-free capacitance exists between the high voltage and low voltage of electrodes of C_1, the phase error is solely determined by C_2 shunted by C_3 and R_3. The phase angle error must be negligible and thus, a limit is put on the minimum value of C_2 and on the minimum ratio of divider. To overcome this difficulty, a circuit is used as shown in Figure 15.19.

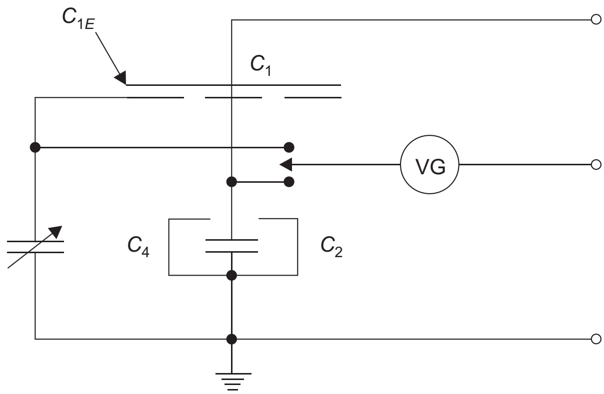


Fig. 15.19 Error Correction in Capacitance Divider Method

In the circuit shown in Fig. 15.18, the guard ring is earthed via a capacitor C_4 whose value is so adjusted that $C_1/C_2 = C_{1E}/C_4$. This makes the guard ring and low voltage electrode at the same potential. Also, using 10/1 ratio resistance divider helps in reducing the error due to C_3 and R_3. It makes the ratio of capacitance divider to be 10 times the nominal ratio of potential transformer due to which a large value of C_2 can be used. In addition to it, by using a 10/1 ratio resistance divider, we can measure ratio errors of any sign.

15.5 DIFFERENCE BETWEEN CURRENT AND POTENTIAL TRANSFORMERS

The operations of current and potential transformers vary in many respects from each other. The differences are explained as follows.

- The current in the primary winding of a current transformer is independent of the conditions of the secondary winding circuit, whereas the current in the primary winding of a potential transformer depends on the burden of the secondary circuit.
- In current transformer, a small voltage is connected across its terminals and this transformer is connected in series with one line to carry the full line current. On the other hand, in potential transformer, a full line voltage is connected upon its terminals.
- In normal operation conditions, the current in primary winding and excitation of current transformer varies over large limits. In potential transformer, the line voltage is constant and hence, excitation current and flux density varies only over a particular range.

- The current transformer is considered as a series transformer with virtual short circuit conditions whereas the potential transformer is considered as a parallel transformer with its open-circuited secondary winding, operating with no damage to the transformer or to the operator.

Let us Summarize

1. The transformer works on AC systems and measures current, voltage, energy, power, frequency, and power factor.
2. A transformer can also be used in protection circuits of power systems to control the operation of undervoltage, overcurrent, earth fault and various other types of relays. Transformers are known as instrument transformers when they are used for measuring purpose while the actual measurement is done using a measuring instrument.
3. The two types of instrument transformers are current transformer (also known as CT) and voltage or potential transformers (also known as PT).
4. The meters or instruments of moderate sizes can measure large current, voltage, power, and energy with the help of instrument transformers through the step down technique.
5. The instrument transformers are used for both routine and precise measurements with many advantages.
6. The term rating of an instrument transformer is represented by two groups of numbers. One represents the nominal current or voltage applied to its primary winding and the other represents the current or voltage induced in its secondary winding.
7. The ratio of an instrument transformer is defined in terms of rating, expressed as the relationship between its primary and secondary rating. The ratio is of different types, namely, turns ratio, nominal ratio, transformation ratio, and ratio correction factor.
8. Testing of an instrument transformer is a prime requisite to find phase angle error and also to find their ratio. There are two groups of methods used for testing—absolute methods and comparison methods. The two types of comparison methods employed according to the measurement technique are null and deflection methods.
9. The current transformer is a type of instrument transformer that is used to measure current.
10. The current to be measured passes through the primary winding having very few turns. The secondary winding has large number of turns and its terminals are connected directly to the ammeter or current coil of wattmeter that can measure current as well as power.
11. The current transformers are of three basic types, namely, bar type, wound type, and window or ring type.
12. There are various characteristics that affect the working of current transformers—change in burden of secondary winding circuit, change in frequency, change in power factor of secondary winding burden, and change in current of primary winding.
13. The testing of current transformers employs three methods, namely, Silsbee's method, mutual inductance method, and Arnold's method.
14. Potential transformers consist of two windings, namely, primary and secondary windings. The primary winding is made up of a fine wire and has very large number of turns. It is connected across the line which carries the voltage to be measured. Secondary winding is made up of a heavy wire and has very few turns.
15. While designing potential transformers, the desired performance criteria to be kept in mind are to have a small phase angle and a constant ratio.
16. There are various characteristics that affect the working of potential transformers—change in power factor of secondary burden, change in frequency, change in current of secondary winding and change in voltage of primary winding.

17. The testing of potential transformers employs four methods, namely, Clothier and Medina comparison method, comparison method using wattmeters, absolute null method, and method using capacitance dividers.
18. The operations of current and potential transformers vary in many respects from each other.

EXERCISES

Fill in the Blanks

1. Use of oil-filled bushings in oil-filled potential transformers _________ their size.
2. With a reduction in frequency of a potential transformer, its phase angle _________.
3. Two types of core shape that can be used in a potential transformer are _________ and _________.
4. CT and PT are abbreviations of _________ and _________, respectively.
5. Percentage ratio error of a potential transformer is given by the expression _________.

Multiple Choice Questions

1. Transformer ratio is given by
   (a) $\dfrac{\text{primary winding current or voltage}}{\text{secondary winding current or voltage}}$ (b) $\dfrac{\text{number of turns of secondary winding}}{\text{number of turns of primary winding}}$
   (c) $\dfrac{\text{secondary winding current or voltage}}{\text{primary winding current or voltage}}$ (d) $\dfrac{\text{number of turns of primary winding}}{\text{number of turns of secondary winding}}$
2. The secondary winding of a current transformer is
   (a) never left short-circuited (b) never left open-circuited
   (c) always kept open-circuited (d) none of these
3. The ratio error in the current transformer is largely dependent upon
   (a) iron loss component of magnetizing current
   (b) magnetizing component of the magnetizing current
   (c) both (a) and (b)
   (d) either (a) or (b)
4. In case of potential transformers, the phase angle error is
   (a) always negative (b) always positive
   (c) zero (d) none of these
5. The ratio marked on a transformer is
   (a) turns ratio (b) transformer ratio
   (c) nominal ratio (d) none of these

State True or False

1. Transformers are used for DC systems.
2. While using a transformer, the meter circuit connected in secondary winding is isolated from its primary.
3. For ideal current transformer, the transformation ratio must be equal to turns ratio.

4. Silsbee's method of testing current transformers is an absolute method.
5. The number of turns in secondary winding of a potential transformer is larger than the number of turns in its primary winding.

Descriptive/Numerical Questions

1. What is the purpose of instrument transformers? Define their nominal rate.
2. Explain the term burden of an instrument transformer. What are the advantages of instrument transformers?
3. Explain the method of testing a current transformer using mutual inductance.
4. Draw the equivalent circuit and phasor diagram of a current transformer. Derive the expression for ratio and phase angle errors.
5. Explain the difference between current transformer and potential transformer.
6. Draw the equivalent circuit and phasor diagram of a potential transformer. Derive the expression for ratio and phase angle errors.
7. Explain the method of testing of potential transformer using wattmeters.
8. Describe the design and constructional features used in current transformer for reductions of ratio and phase angle errors.
9. Describe the design and constructional features used in potential transformer for reduction of ratio and phase angle errors.
10. What are the characteristics of current and potential transformers?
11. A current transformer with the ratio of 1000 / 5 A and 50 Hz frequency has the following constants: secondary burden impedance = 1.5 Ω, exciting mmf = 150 A, iron loss at full load = 5.2 W, and primary winding turn = 1. Calculate the ratio error at full load. Also, find the actual transformation ratio.
12. A potential transformer with the ratio of 2000/200 volts has the following constants: primary resistance = 100.5 Ω, secondary resistance = 0.65 Ω, primary reactance = 57.2 Ω, equivalent reactance = 59.4 Ω, and loss current = 0.005 A at 0.50 power factor. Calculate the phase angle at no load between primary and secondary voltages.

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