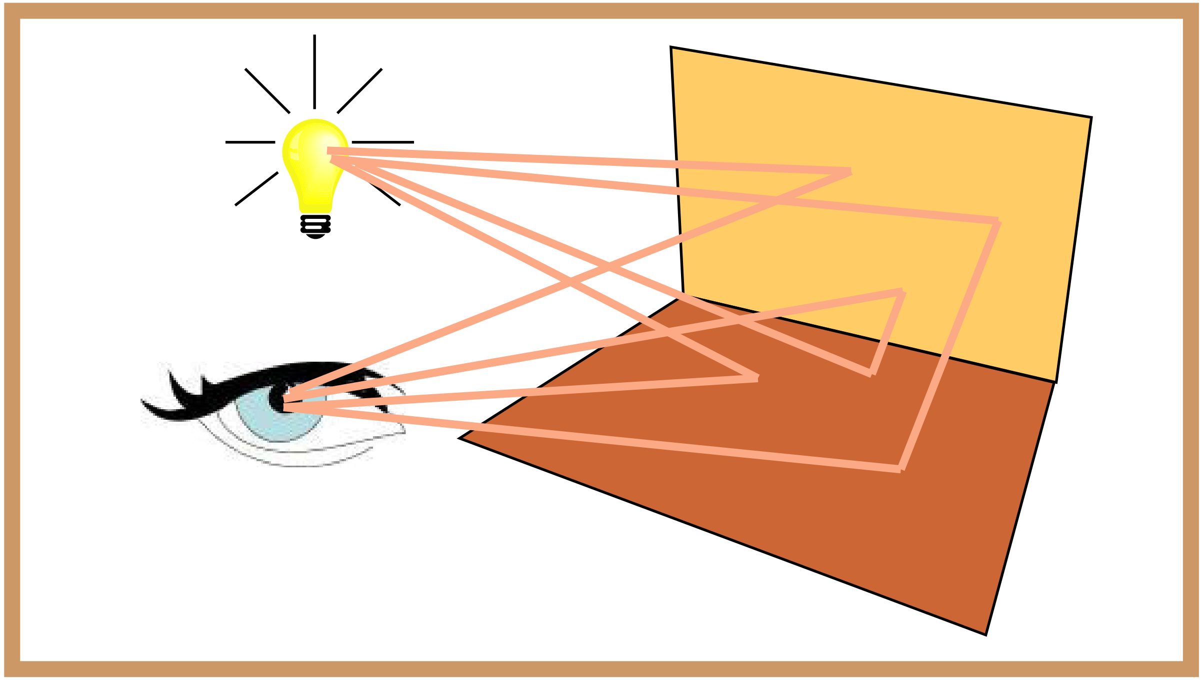
Local Illumination

Outline

- Introduction
- Radiometry
- Reflectance
- Reflectance Models

The Big Picture



Light sources emit photons.
We assume that photons travel along straight lines.
When these photons hit the surfaces they are absorbed (changed to heat).
We assume ray optics as opposed to e.g. wave optics.
Our eye collects these photons.

Radiometry

- Energy of a photon

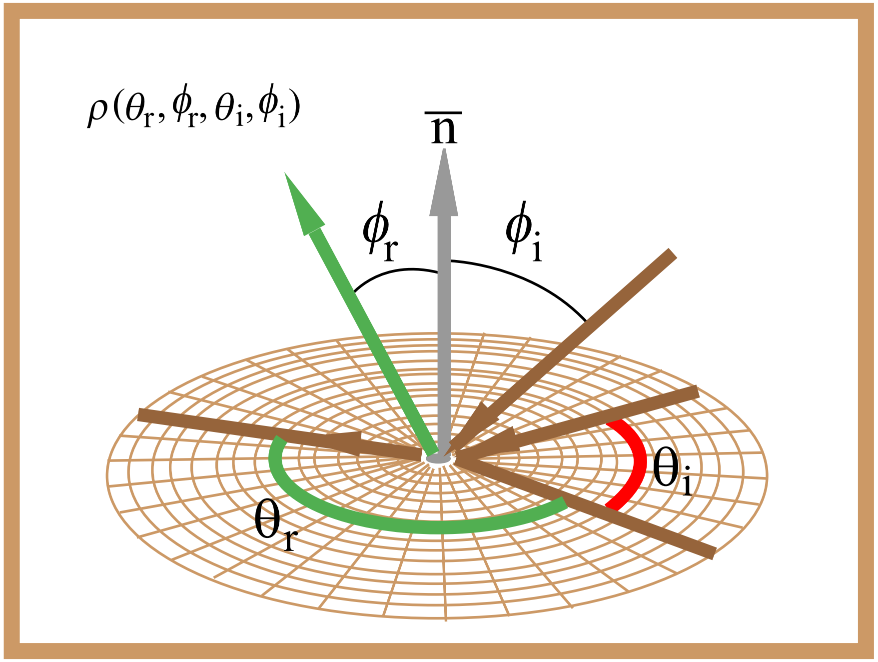
$$e_\lambda = \frac{hc}{\lambda} \qquad h \approx 6.63 \cdot 10^{-34} J \cdot s \quad c \approx 3 \cdot 10^{8} m/s$$

- Radiant Energy of *n photons*

$$Q = \sum_{i=1}^{n} \frac{hc}{\lambda_i}$$

- Radiation flux (electromagnetic flux, radiant flux) Units: Watts

$$\Phi = \frac{dQ}{dt}$$



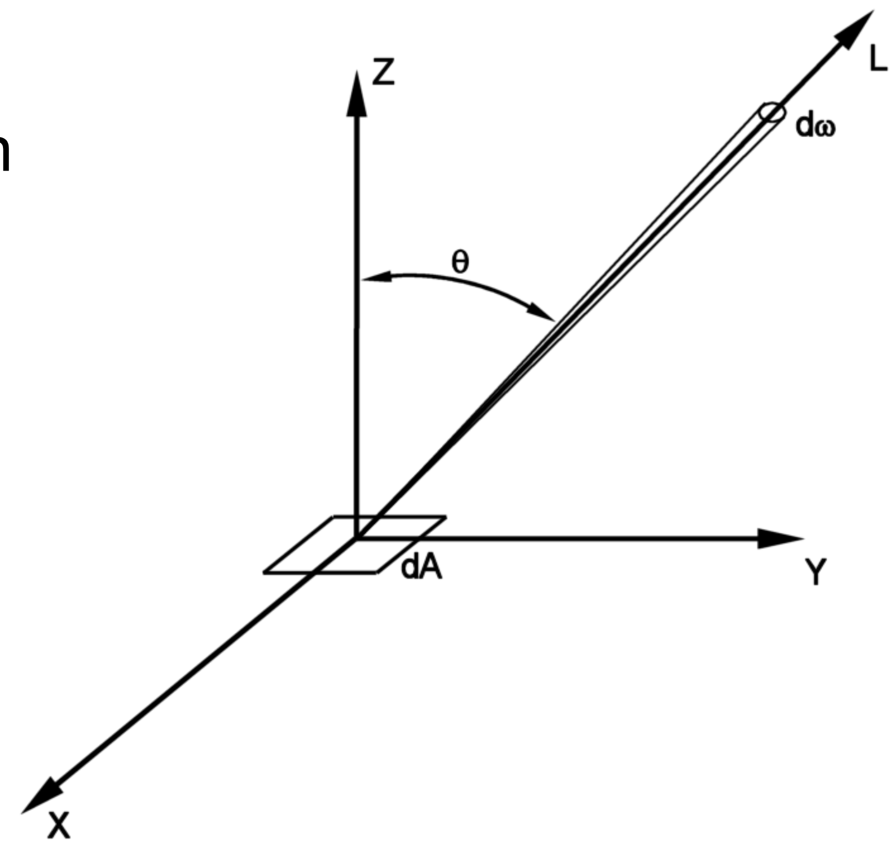
Radiometry is the measurement of the electromagnetic radiation in the ultraviolet, the visible, and the infrared frequency spectrum.

Radiation flux (or radiant flux, electromagnetic flux), usually denoted with ϕ , is equivalent to power. It measures the time-rate flow of light energy where Q denotes the energy of a collection of photons across all wavelengths and t denotes

time. The unit of flux is the Watt [W].

Radiometry

- **Radiance** – radiant flux per unit solid angle per unit projected area
  - Number of photons arriving per time at a small area from a particular direction



$$L(\omega) = \frac{d^2\Phi}{\cos\theta \, dA \, d\omega}$$

$$Units: \frac{Watt}{meter^2 \; steradian}$$

This is the most fundamental concept in radiometry. It is a physical quantity equivalent to the psychological concept of brightness observed by humans. It is usually denoted with the letter L and defined for all directions ω. It measures electromagnetic flux Φ travelling in the small range of directions through the solid angle element *d* ⌉ and crossing an element of projected area dA

Radiometry

- **Irradiance** – differential flux falling onto differential area

$$E = \frac{d\Phi}{dA} \qquad Units: \frac{Watt}{meter^2}$$

- Irradiance can be seen as a density of the incident flux falling onto a surface.
- It can be also obtained by integrating the radiance over the solid angle.

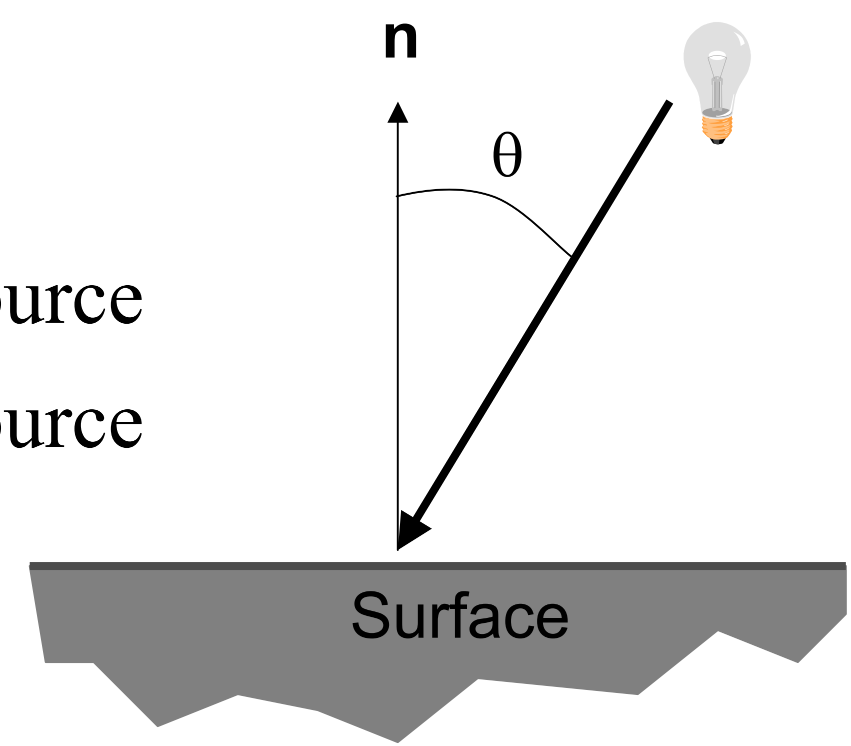
Light Emission

- Light sources: sun, fire, light bulbs etc.
- Consider a point light source that emits light uniformly in all directions

$$E = \frac{\Phi_s \cos\theta}{4\pi d^2} \qquad L = \frac{\Phi_s}{4\pi d^2}$$

Φ_s – power of the light source

d – distance to the light source



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Reflection & Reflectance

- Reflection - the process by which electromagnetic flux incident on a surface leaves the surface without a change in frequency.
- Reflectance – a fraction of the incident flux that is reflected
- We do not consider:
  - absorption, transmission, fluorescence
  - diffraction

Reflectance

- Bidirectional scattering-surface distribution Function (BSSRDF)

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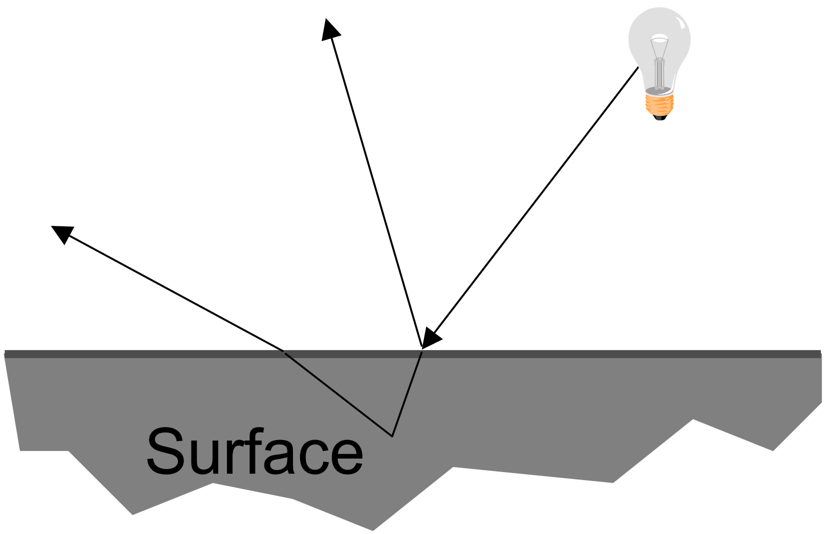
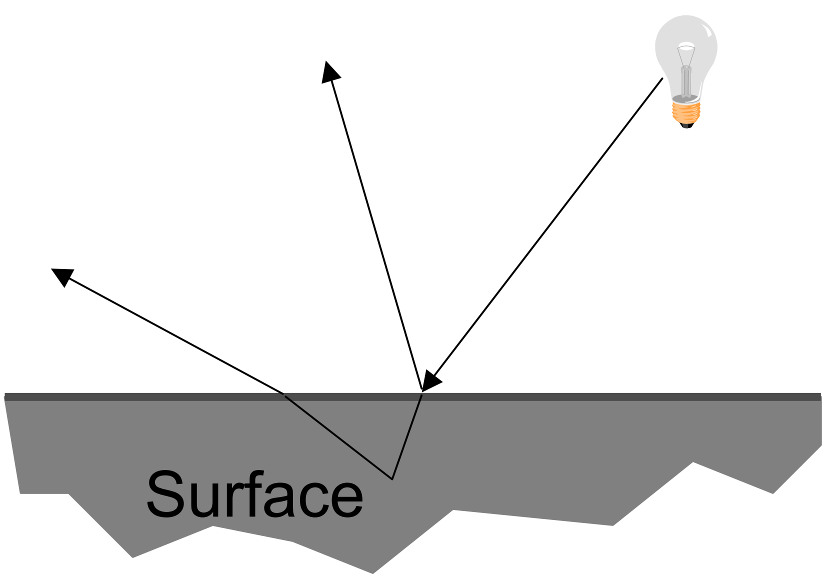


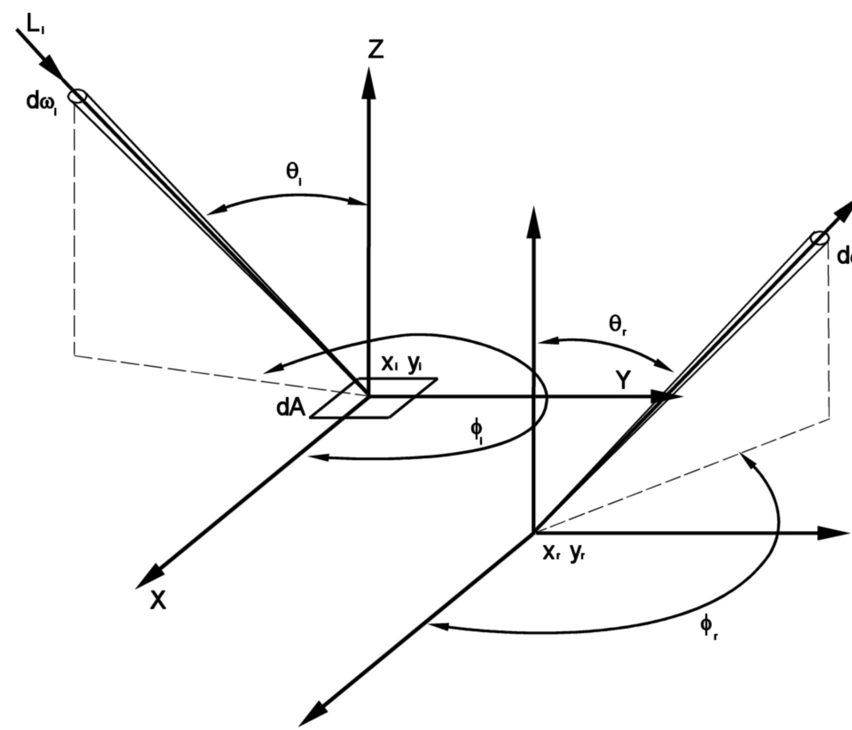
Reflectance

- Bidirectional scattering-surface distribution Function (BSSRDF)

$$S(\theta_i,\phi_i,\theta_r,\phi_r,x_i,y_i,x_r,y_r) = \frac{dL_r(\theta_r,\phi_r,x_r,y_r)}{d\Phi_i(\theta_i,\phi_i,x_i,y_i)}$$

$$Units: \frac{1}{meter^2\ steradian}$$



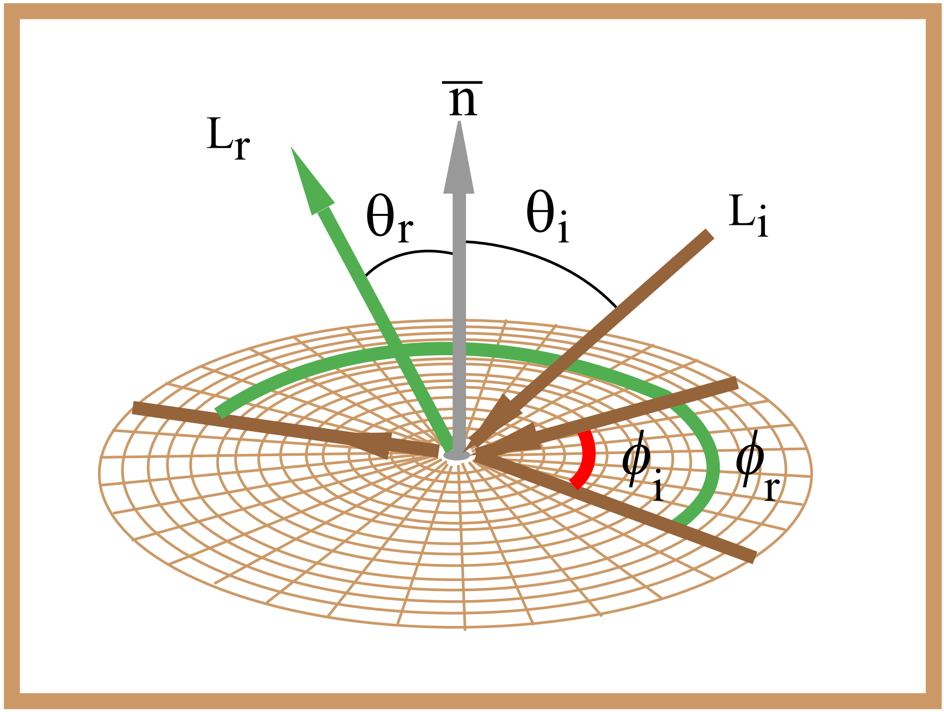


Reflectance

- Bidirectional Reflectance Distribution Function (BRDF)

$$Units : \frac{1}{steradian}$$

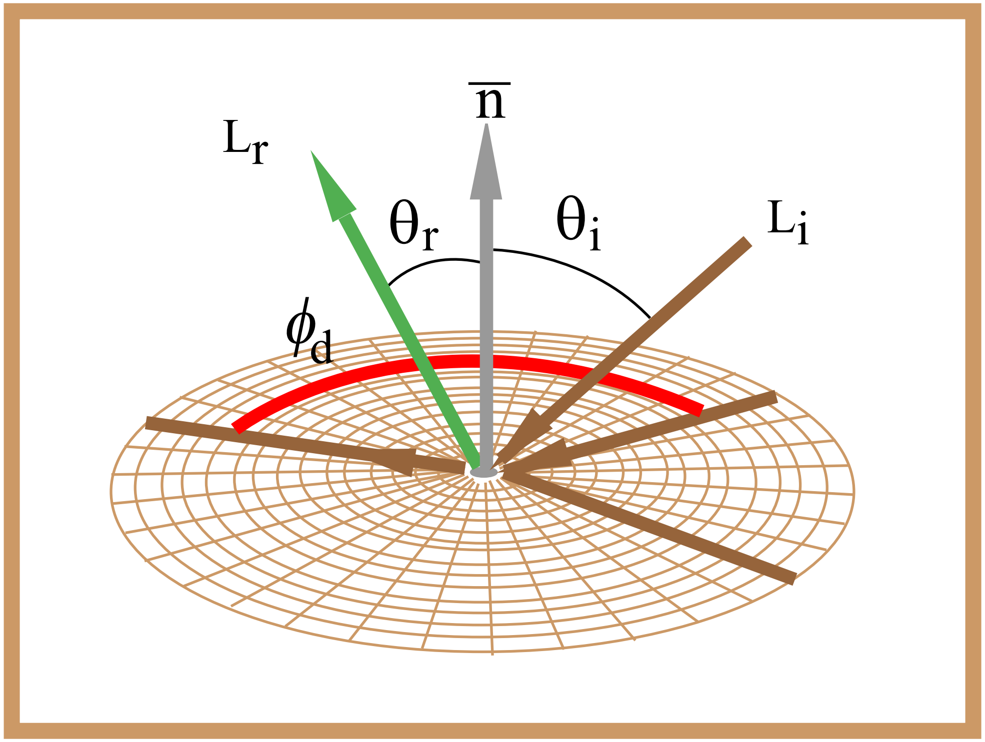
$$f_r(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{dL_r(\theta_r, \phi_r)}{dE_i(\theta_i, \phi_i)}$$



Isotropic BRDFs

- Rotation along surface normal does not change reflectance

$$f_r(\theta_i, \theta_r, \phi_r - \phi_i) = f_r(\theta_i, \theta_r, \phi_d) = \frac{dL_r(\theta_r, \phi_d)}{dE_i(\theta_i, \phi_d)}$$



Anisotropic BRDFs

- Surfaces with strongly oriented microgeometry elements
- Examples:
  - brushed metals,
  - hair, fur, cloth, velvet

Images removed due to copyright considerations.

Properties of BRDFs

- Non-negativity

$$f_r(\theta_i, \phi_i, \theta_r, \phi_r) \geq 0$$

- Energy Conservation

$$\int_\Omega f_r(\theta_i, \phi_i, \theta_r, \phi_r) d\mu(\theta_r, \phi_r) \leq 1 \quad \text{for all} \, (\theta_i, \phi_i)$$

- Reciprocity

$$f_r(\theta_i, \phi_i, \theta_r, \phi_r) = f_r(\theta_r, \phi_r, \theta_i, \phi_i)$$

How to compute reflected radiance?

- Continuous version

$$L_r(\omega_r) = \int_{\Omega} f_r(\omega_i, \omega_r) dE_i(\omega_i) =$$

$$= \int_{\Omega} f_r(\omega_i, \omega_r)\, dL_i(\omega_i) \cos(\omega_i \cdot n)\, d\omega_i \qquad \omega = (\theta, \phi)$$

- Discrete version – *n* point light sources

$$L_r(\omega_r) = \sum_{j=1}^{n} f_r(\omega_{ij}, \omega_r) E_j =$$

$$= \sum_{j=1}^{n} f_r(\omega_{ij}, \omega_r) \cos\theta_j \frac{\Phi_{sj}}{4\pi\, d_j^{\,2}}$$

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How do we obtain BRDFs?

- Measure BRDF values directly

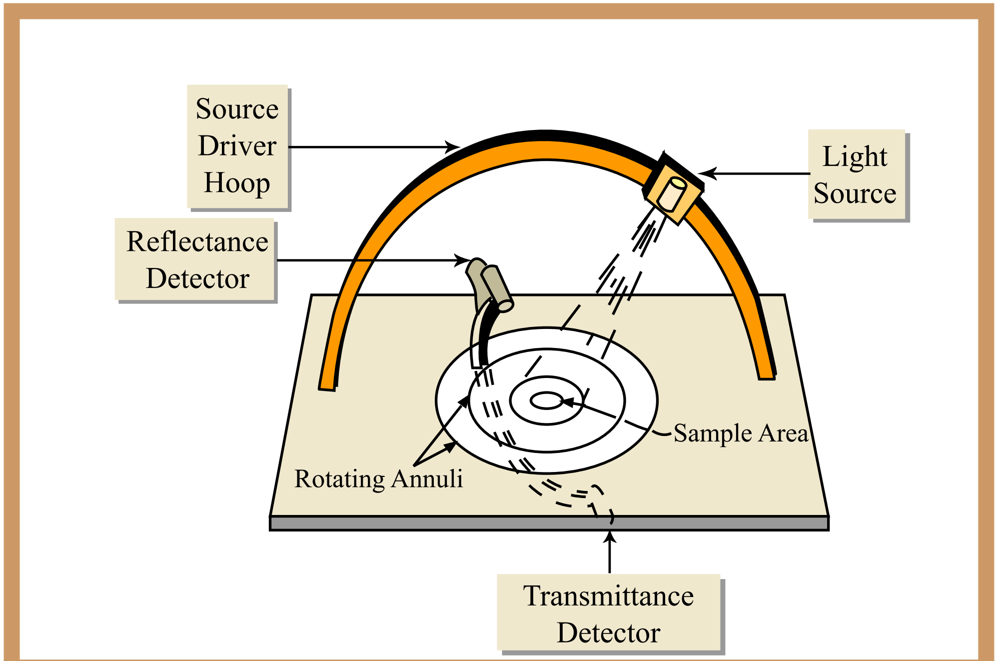
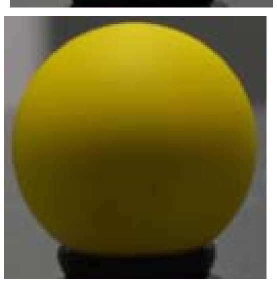


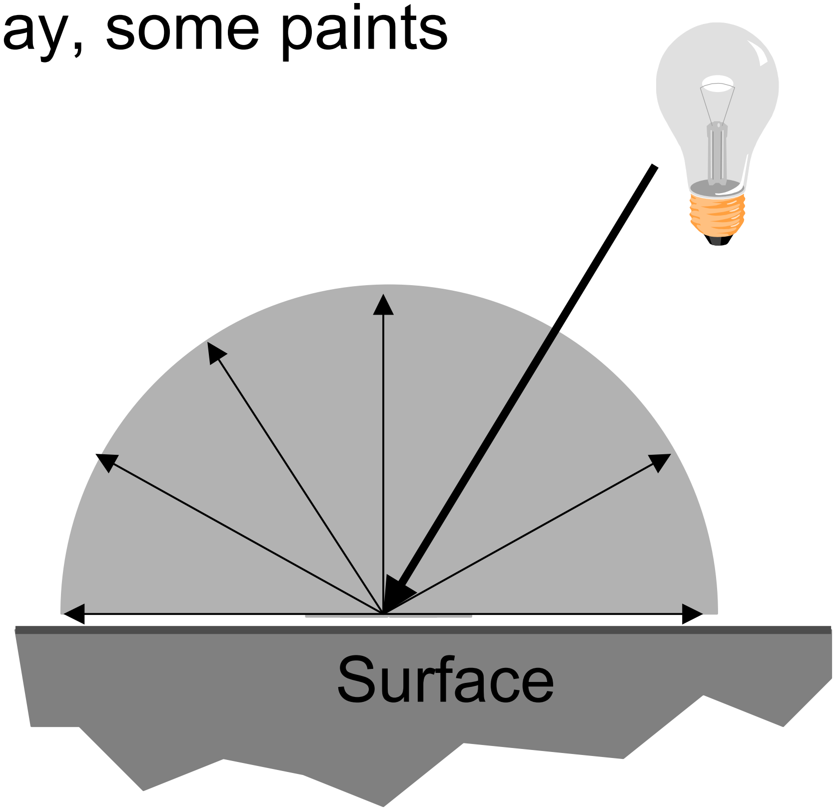
Image adapted from:
Ward, Gregory J. “Measuring and Modeling Anisotropic Reflection.” *ACM SIGGRAPH Computer Graphics. 26*, no.2 (July 1992): 265-272.

- Analytic Reflectance Models
  - Physically-based models
    - based on laws on physics
  - Empirical models
    - “ad hoc” formulas that work

Ideal Diffuse Reflectance

- Assume surface reflects equally in all directions.
- An ideal diffuse surface is, at the microscopic level, a very rough surface.
  - Example: chalk, clay, some paints





Ideal Diffuse Reflectance

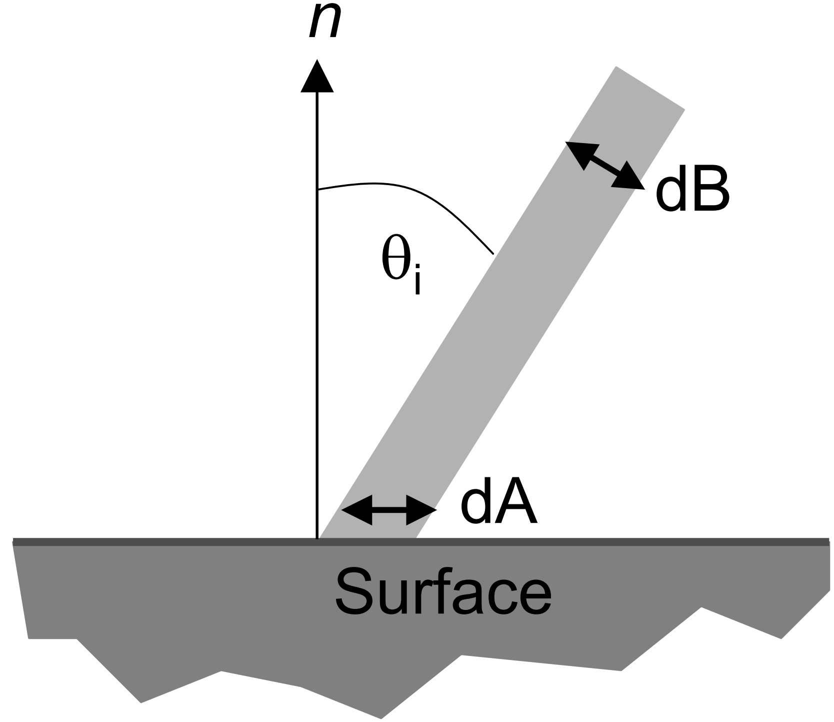
- BRDF value is constant

$$L_r(\omega_r) = \int_\Omega f_r(\omega_i, \omega_r) dE_i(\omega_i) =$$

$$= f_r \int_\Omega dE_i(\omega_i) =$$

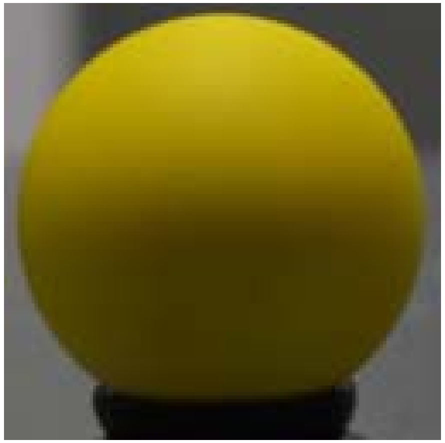
$$= f_r E_i$$

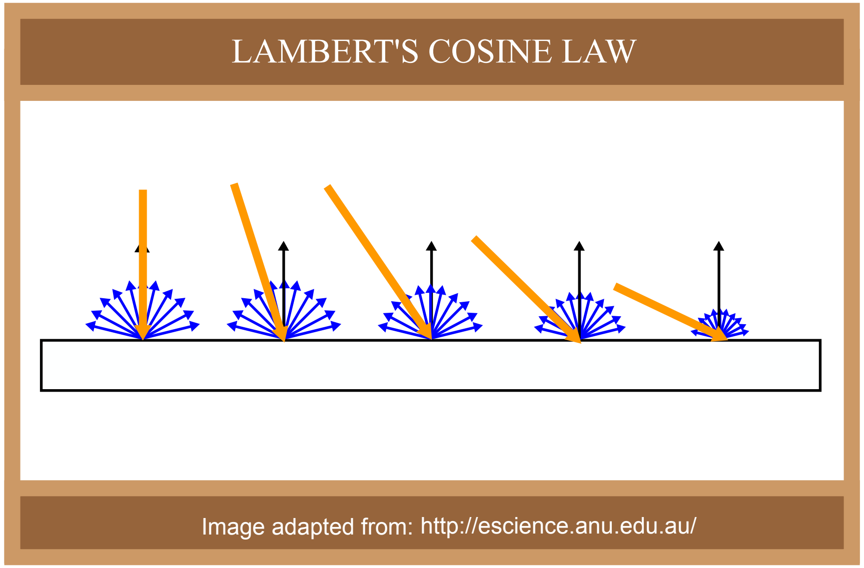
$$dB = dA \cos\theta_i$$



Ideal Diffuse Reflectance

- Ideal diffuse reflectors reflect light according to Lambert's cosine law.



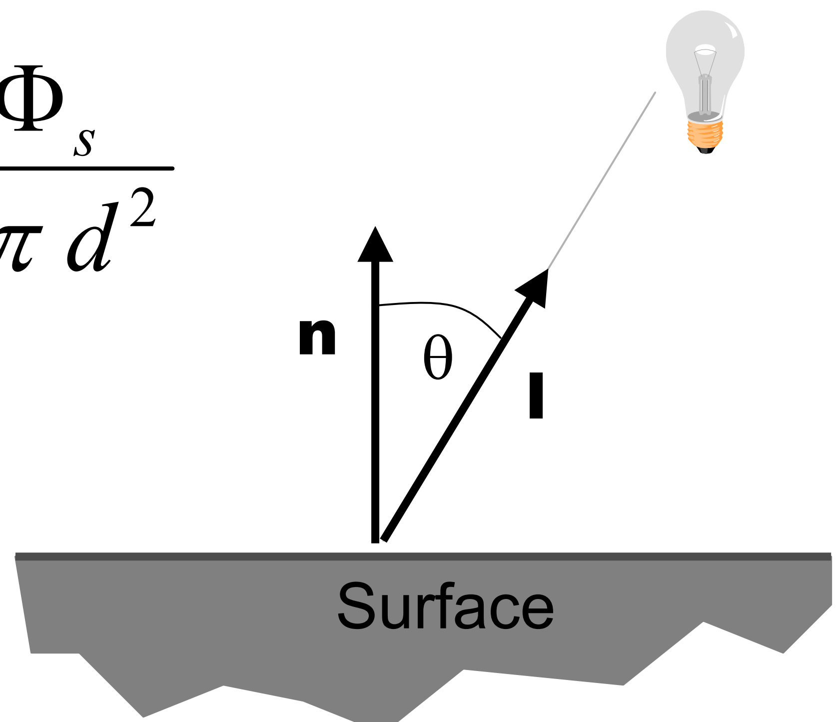


Ideal diffuse reflectors reflect light according to Lambert's cosine law.
Lambert's law determines how much of the incoming light energy is reflected.
The reflected intensity is independent of the viewing direction.

Ideal Diffuse Reflectance

- Single Point Light Source
  - k_d: The diffuse reflection coefficient.
  - **n**: Surface normal.
  - **l**: Light direction.

$$L(\omega_r) = k_d (\mathbf{n} \cdot \mathbf{l}) \frac{\Phi_s}{4\pi d^2}$$

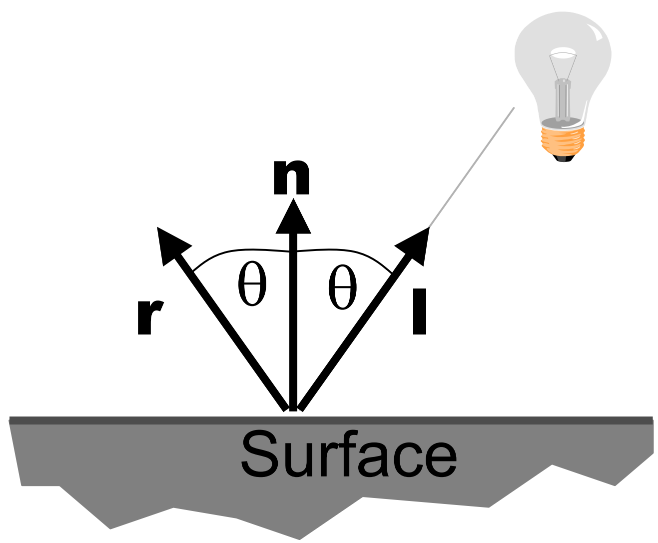


Ideal Diffuse Reflectance – More Details

- If **n** and **l** are facing away from each other, **n** • **l** becomes negative.
- Using max((**n** • **l**),0) makes sure that the result is zero.
  - From now on, we mean max() when we write •.
- Do not forget to normalize your vectors for the dot product!

Ideal Specular Reflectance

- Reflection is only at mirror angle.
  - View dependent
  - Microscopic surface elements are usually oriented in the same direction as the surface itself.
  - Examples: mirrors, highly polished metals.



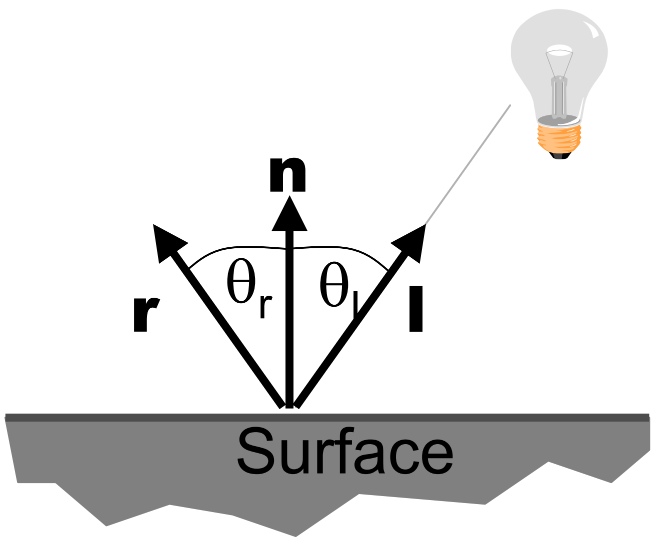
A second surface type is called a *specular reflector*. When we look at a shiny surface, such as polished metal or a glossy car finish, we see a highlight, or bright spot. Where this bright spot appears on the surface is a function of where the surface is seen from. This type of reflectance is view dependent.

At the microscopic level a specular reflecting surface is very smooth, and usually these microscopic surface elements are oriented in the same direction as the surface itself. Specular reflection is merely the *mirror reflection* of the light source in a surface. Thus it should come as no surprise that it is viewer dependent, since if you stood in front of a mirror and placed your finger over the reflection of a light, you would expect that you could reposition your head to look around your finger and see the light again. An ideal mirror is a purely specular reflector.

In order to model specular reflection we need to understand the physics of reflection.

Ideal Specular Reflectance

- Special case of Snell's Law
  - The incoming ray, the surface normal, and the reflected ray all lie in a common plane.



$$n_l \sin\theta_l = n_r \sin\theta_r$$

$$n_l = n_r$$

$$\theta_l = \theta_r$$

The angle that the reflected ray forms with the surface normal is determined by the angle that the incoming ray forms with the surface normal, and the relative speeds of light of the mediums in which the incident and reflected rays propagate according to the following expression. (Note: n_l and n_r are the indices of refraction)

Reflection is a very special case of Snell's Law where the incident light's medium and the reflected rays medium is the same. Thus we can simplify the expression to:

Non-ideal Reflectors

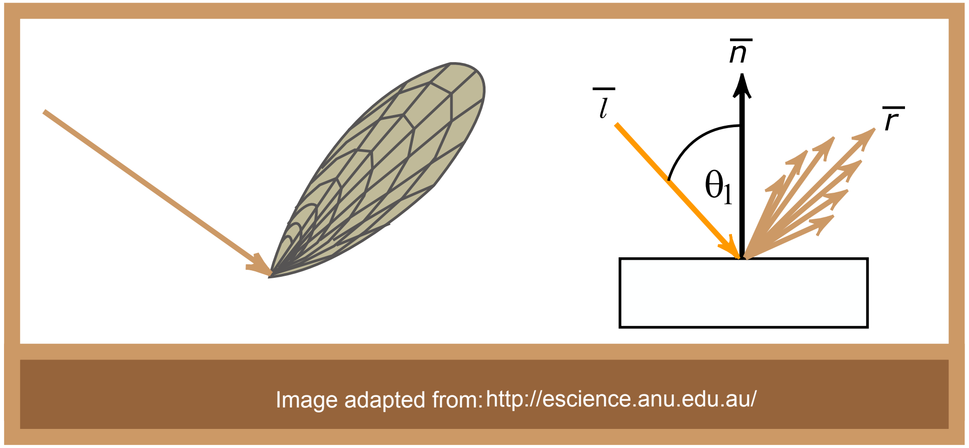
- Snell's law applies only to ideal mirror reflectors.
- Real materials tend to deviate significantly from ideal mirror reflectors.
- They are not ideal diffuse surfaces either …

At this point we will introduce an empirical model that is consistent with our experience, at least to a crude approximation.

In general, we expect most of the reflected light to travel in the direction of the ideal ray. However, because of microscopic surface variations we might expect some of the light to be reflected just slightly offset from the ideal reflected ray. As we move farther and farther, in the angular sense, from the reflected ray we expect to see less light reflected.

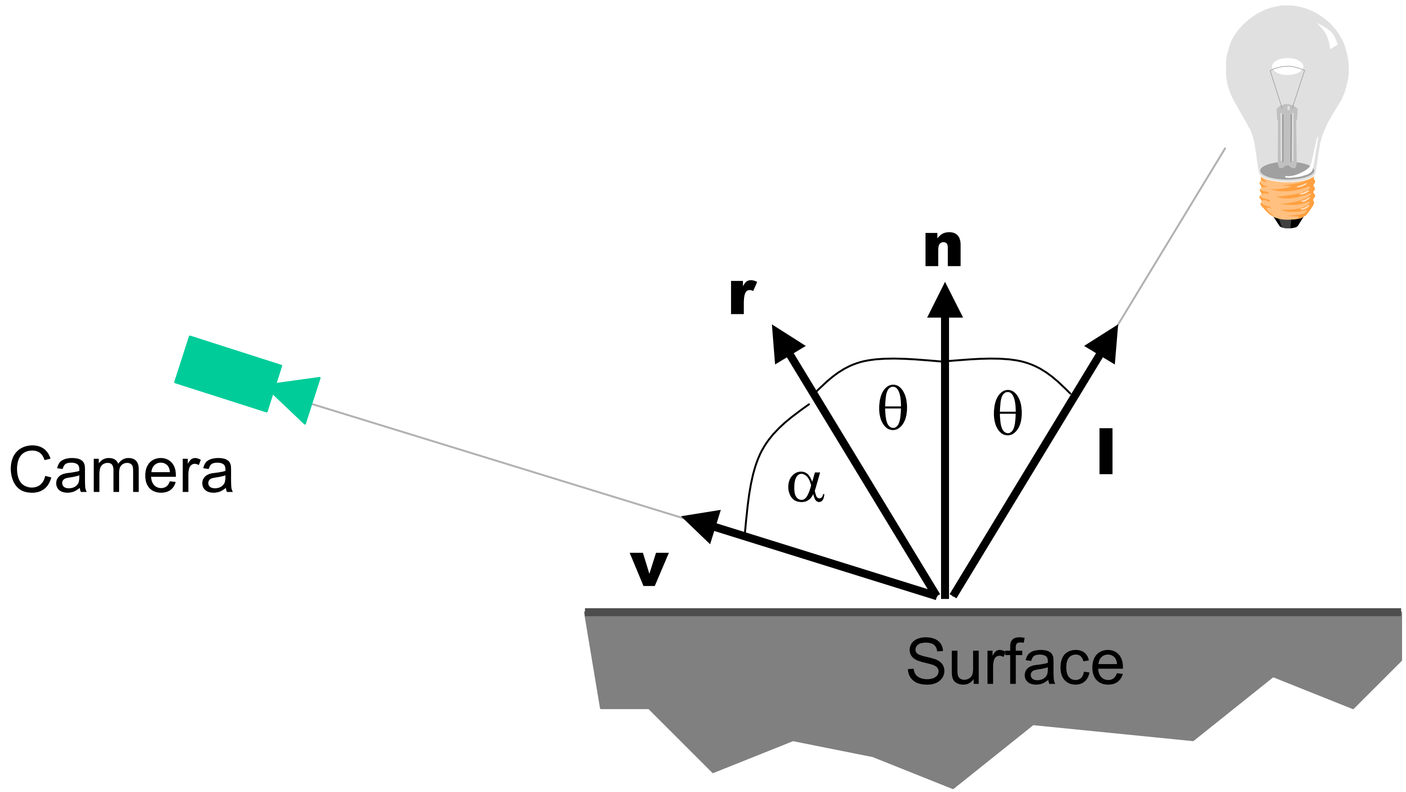
Non-ideal Reflectors

- Simple Empirical Model:
  - We expect most of the reflected light to travel in the direction of the ideal ray.
  - However, because of microscopic surface variations we might expect some of the light to be reflected just slightly offset from the ideal reflected ray.
  - As we move farther and farther, in the angular sense, from the reflected ray we expect to see less light reflected.



The Phong Model

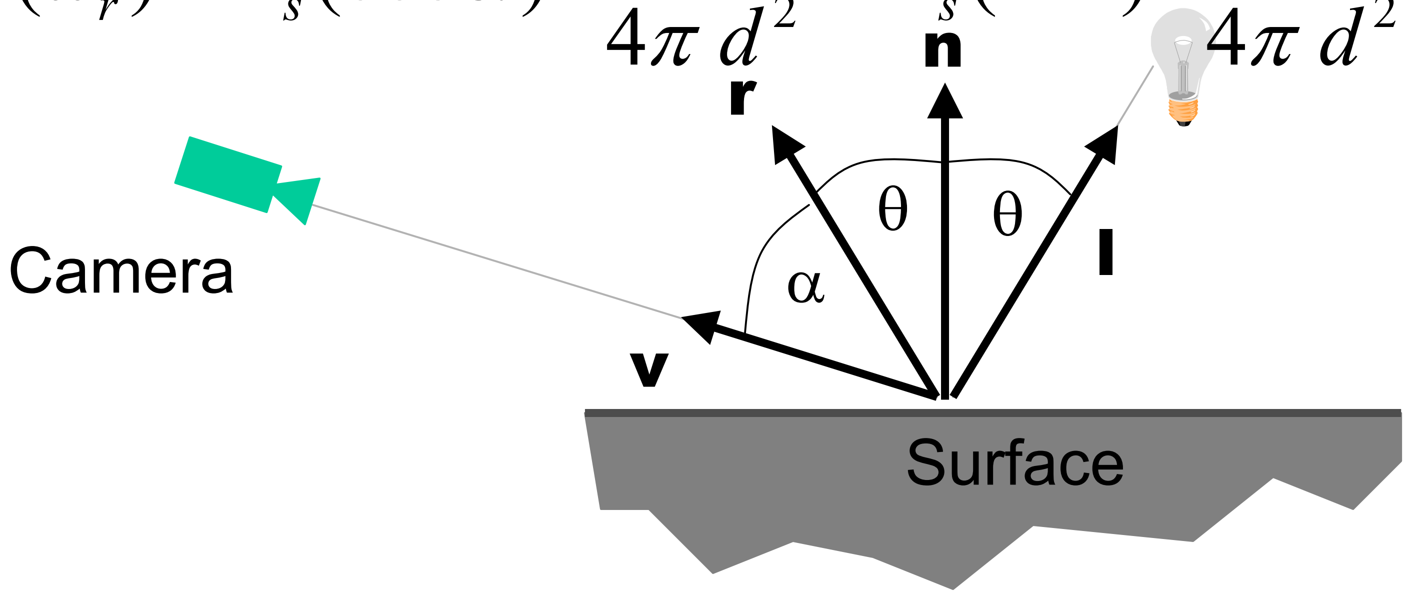
- How much light is reflected?
  - Depends on the angle between the ideal reflection direction and the viewer direction α.



The Phong Model

- Parameters
  - k_s: specular reflection coefficient
  - q : specular reflection exponent

$$L(\omega_r) = k_s(\cos\alpha)^q \frac{\Phi_s}{4\pi d^2} = k_s(\mathbf{v}\cdot\mathbf{r})^q \frac{\Phi_s}{4\pi d^2}$$

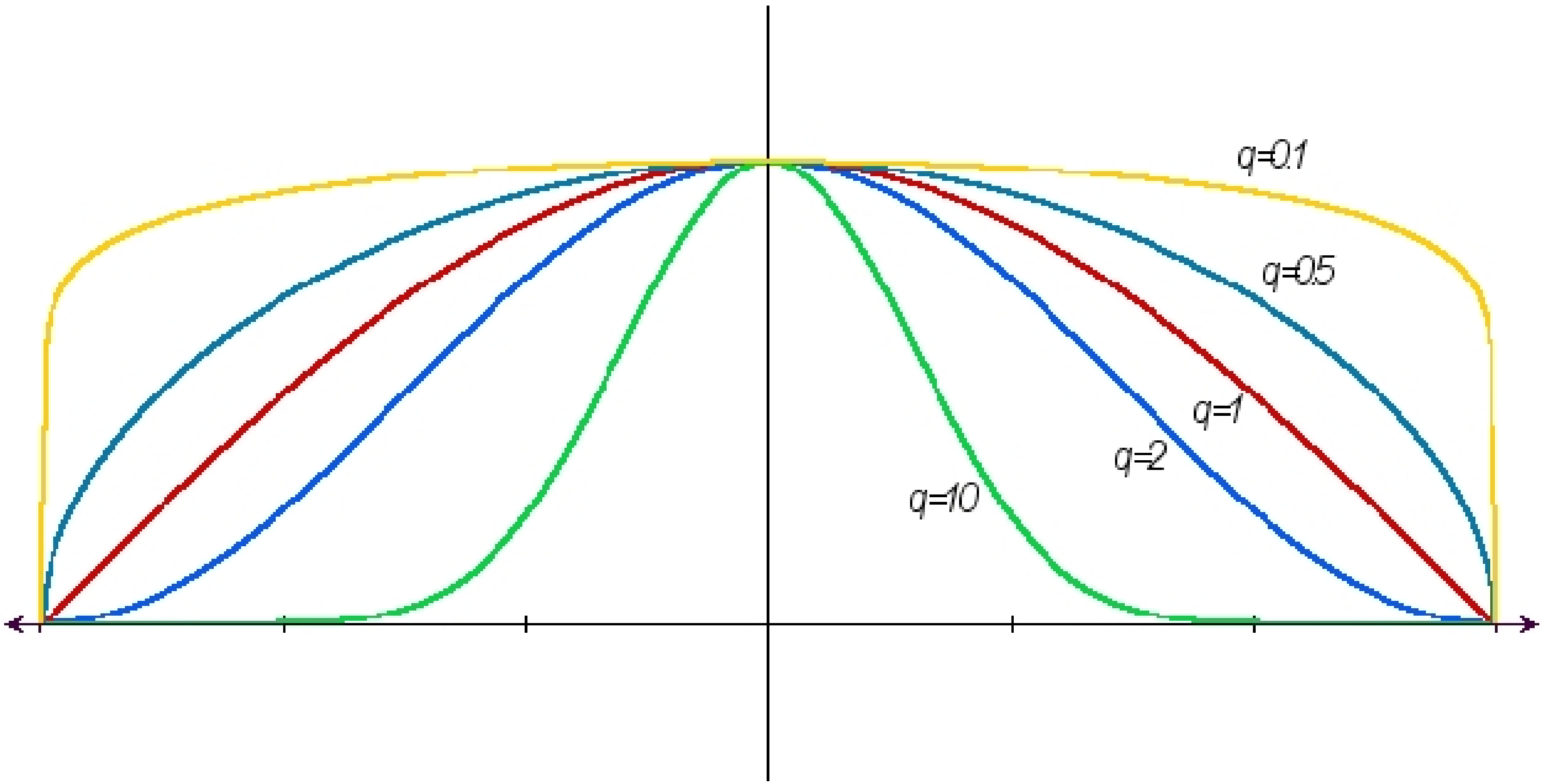


One function that approximates this fall off is called the Phong Illumination model.
This model has no physical basis, yet it is one of the most commonly used illumination models in computer graphics.

The cosine term is maximum when the surface is viewed from the mirror direction and falls off to 0 when viewed at 90 degrees away from it. The scalar n_{shiny} controls the rate of this fall off.

The Phong Model

- Effect of the q coefficient



The diagram below shows the how the reflectance drops off in a *Phong illumination* model.
For a large value of the n_{shiny} coefficient, the reflectance decreases rapidly with increasing viewing angle.

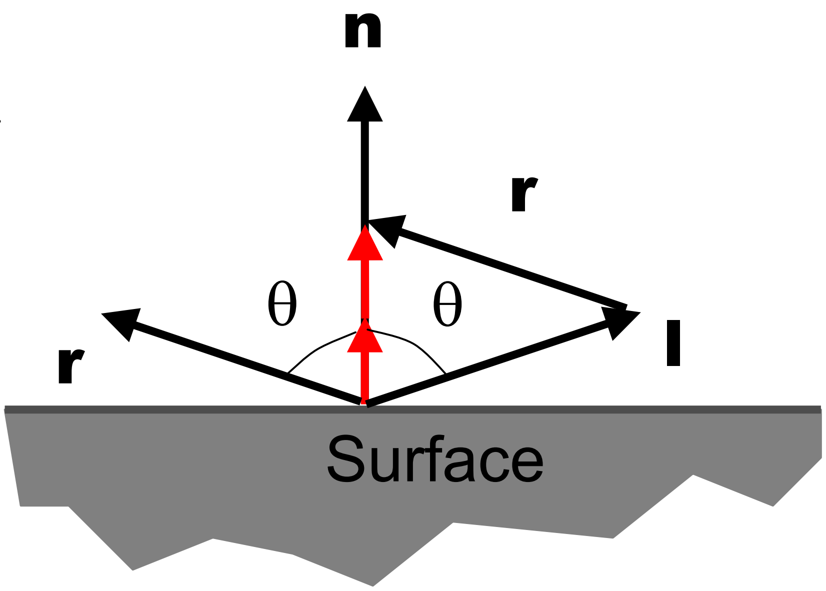
The Phong Model

$$\mathbf{r} + \mathbf{l} = 2 \cos\theta\, \mathbf{n}$$

$$\mathbf{r} = 2(\mathbf{n} \cdot \mathbf{l})\mathbf{n} - \mathbf{l}$$

$$L(\omega_r) = k_s (\mathbf{v} \cdot \mathbf{r})^q \frac{\Phi_s}{4\pi\, d^2} =$$

$$= k_s (\mathbf{v} \cdot (2(\mathbf{n} \cdot \mathbf{l})\mathbf{n} - \mathbf{l}))^q \frac{\Phi_s}{4\pi\, d^2}$$

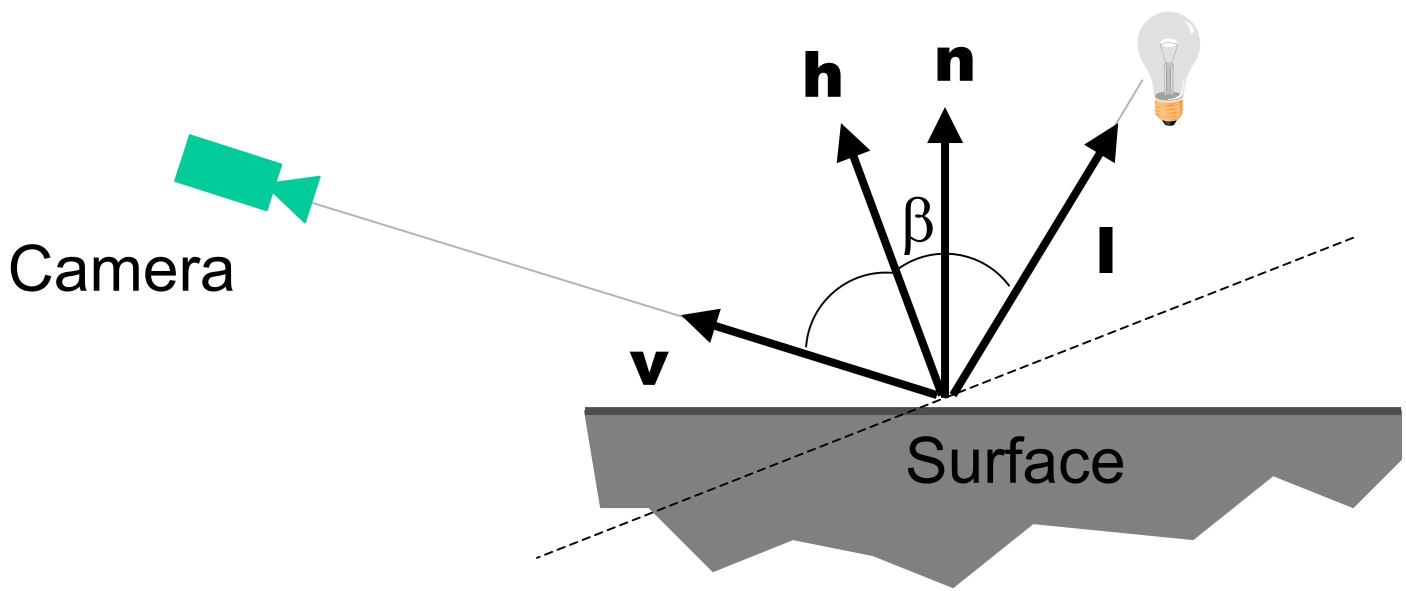


Blinn-Torrance Variation

- Uses the halfway vector **h** between **l** and **v**.

$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{\|\mathbf{l} + \mathbf{v}\|}$$

$$L(\omega_r) = k_s (\cos \beta)^q \frac{\Phi_s}{4\pi d^2} = k_s (\mathbf{n} \cdot \mathbf{h})^q \frac{\Phi_s}{4\pi d^2}$$



Jim Blinn introduced another approach for computing Phong-like illumination based on the work of Ken Torrance.

His illumination function uses the following equation:

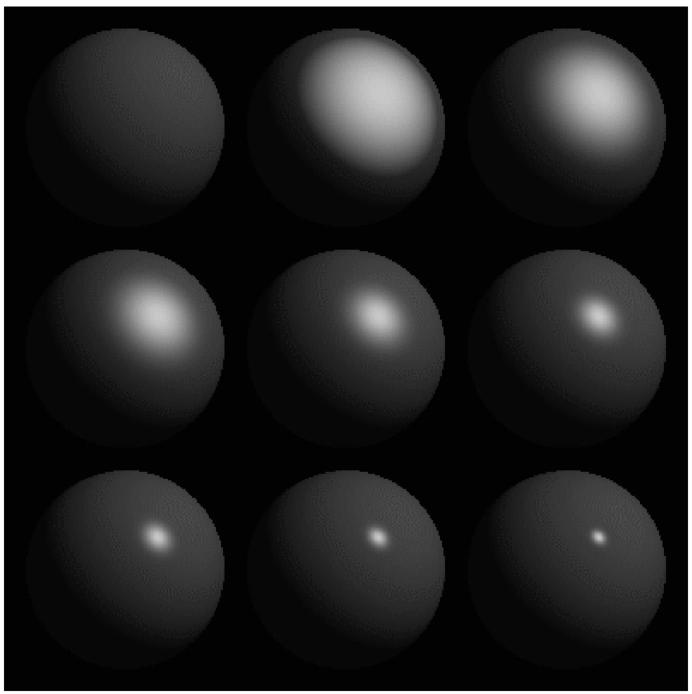
H is the normal to the (imaginary) surface that maximally reflects light in the V direction

In this equation the angle of specular dispersion is computed by how far the surface's normal is from a vector bisecting the incoming light direction and the viewing direction.

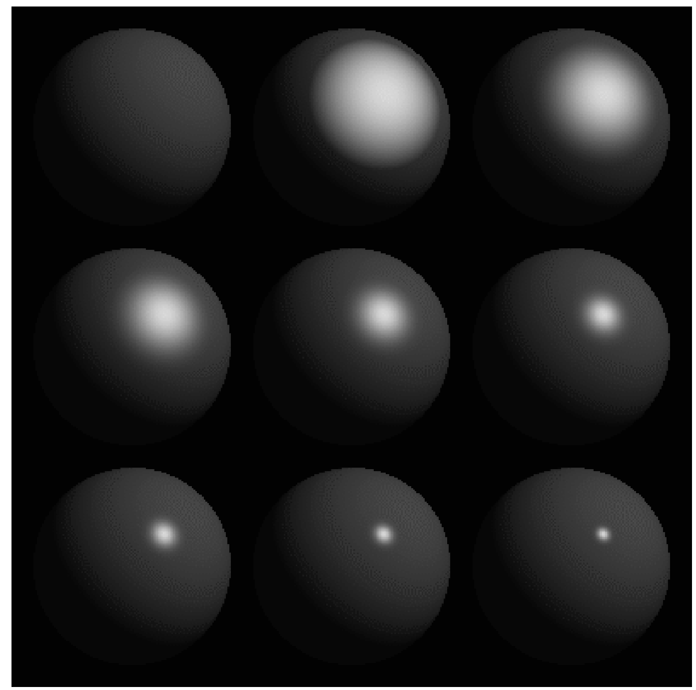
On your own you should consider how this approach and the previous one differ.

Phong Examples

- The following spheres illustrate specular reflections as the direction of the light source and the coefficient of shininess is varied.



Phong



Blinn-Torrance

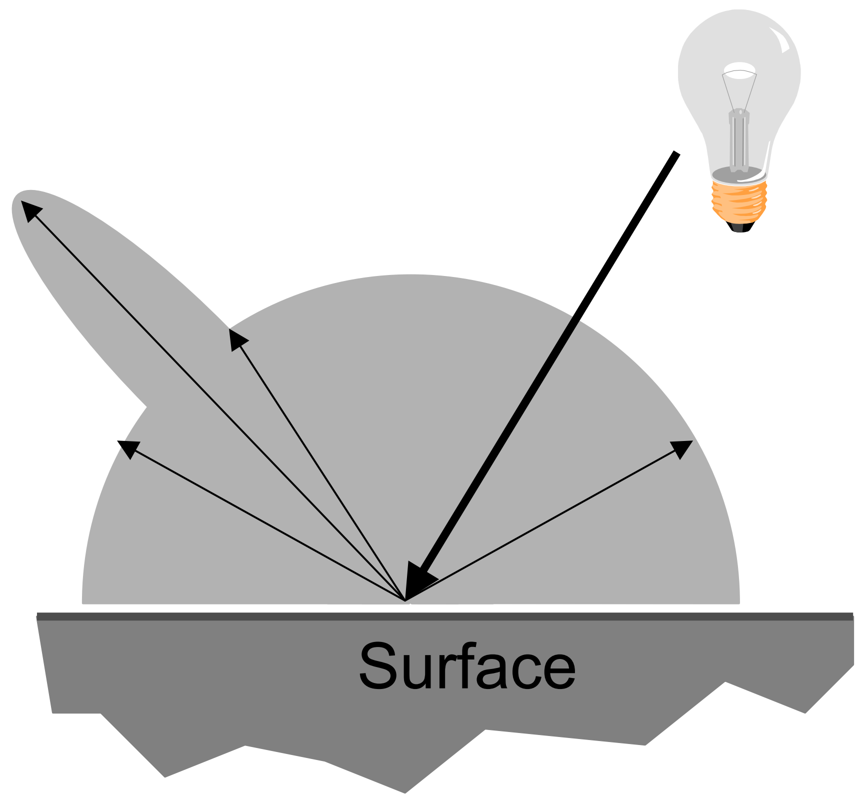
The Phong Model

- Sum of three components:

  diffuse reflection +

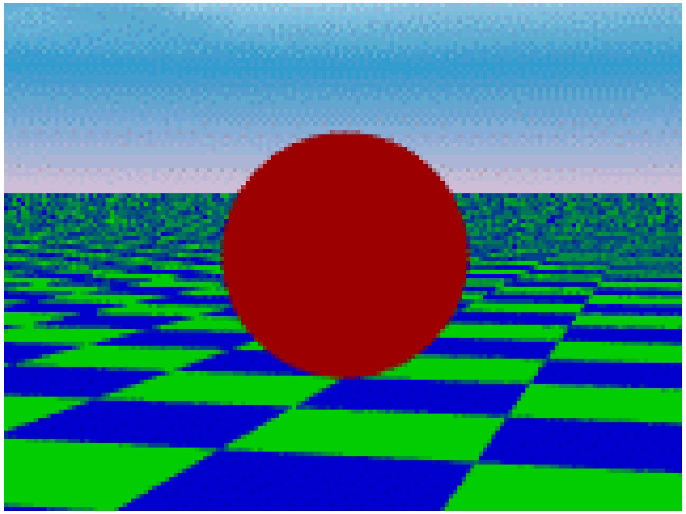
  specular reflection +

  "ambient".



Ambient Illumination

- Represents the reflection of all indirect illumination.
- This is a total hack!
- Avoids the complexity of global illumination.

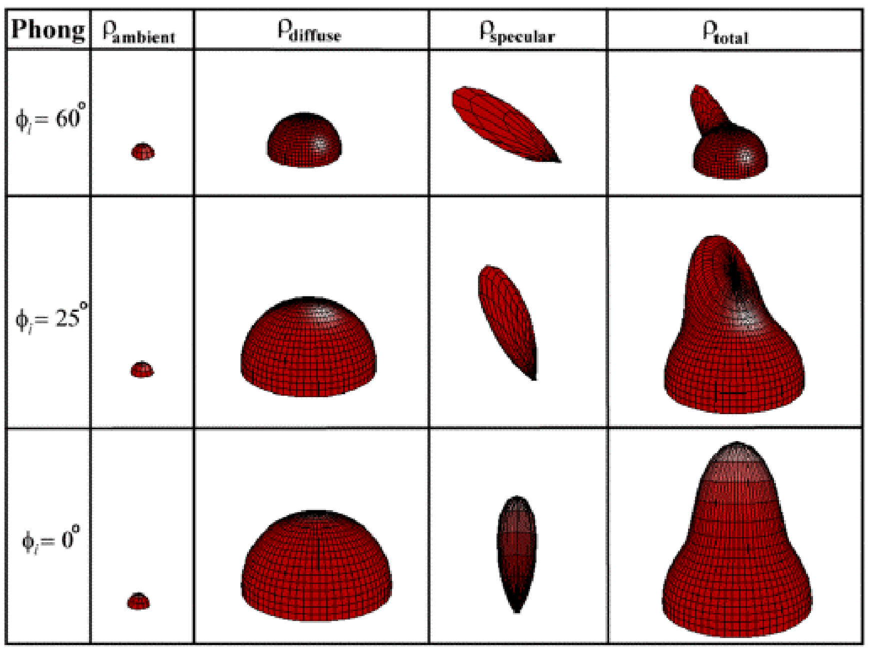


$$L(\omega_r) = k_a$$

Putting it all together

- Phong Illumination Model

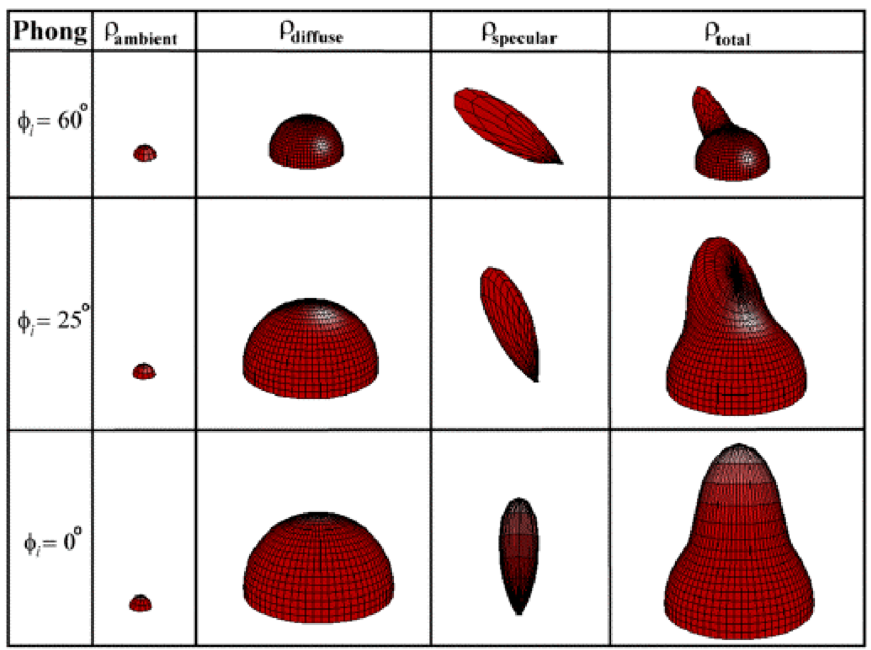
$$L(\omega_r) = k_a + \left(k_d (\mathbf{n} \cdot \mathbf{l}) + k_s (\mathbf{v} \cdot \mathbf{r})^q \right) \frac{\Phi_s}{4\pi d^2}$$



For Assignment 3

- Variation on Phong Illumination Model

$$L(\omega_r) = k_d L_a + \left(k_d (\mathbf{n} \cdot \mathbf{l}) + k_s (\mathbf{v} \cdot \mathbf{r})^q \right) L_i$$



Adding color

- Diffuse coefficients:
  - $k_{d\text{-}red}$, $k_{d\text{-}green}$, $k_{d\text{-}blue}$
- Specular coefficients:
  - $k_{s\text{-}red}$, $k_{s\text{-}green}$, $k_{s\text{-}blue}$
- Specular exponent:

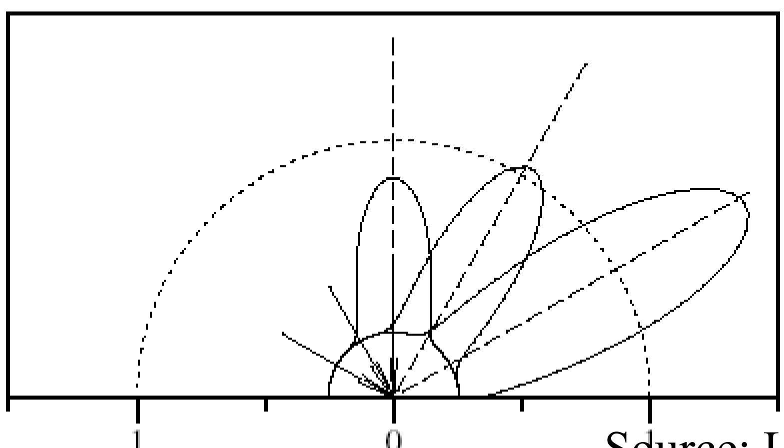
  q

Phong Demo

Fresnel Reflection

- Increasing specularity near grazing angles.

(Images removed due to copyright considerations.)



Source: Lafortune et al. 97

Can Phong model handle this case?

Off-specular & Retro-reflection

- Off-specular reflection
  - Peak is not centered at the reflection direction
- Retro-reflection:
  - Reflection in the direction of incident illumination
  - Examples: Moon, road markings

The Phong Model

- Is it non-negative?
- Is it energy-conserving?
- Is it reciprocal?
- Is it isotropic?

Shaders (`Material` class)

- Functions executed when light interacts with a surface
- Constructor:
  - set shader parameters
- Inputs:
  - Incident radiance
  - Incident & reflected light directions
  - surface tangent (anisotropic shaders only)
- Output:
  - Reflected radiance

Questions?