MIT 6.837
Monte-Carlo
Ray Tracing

Schedule

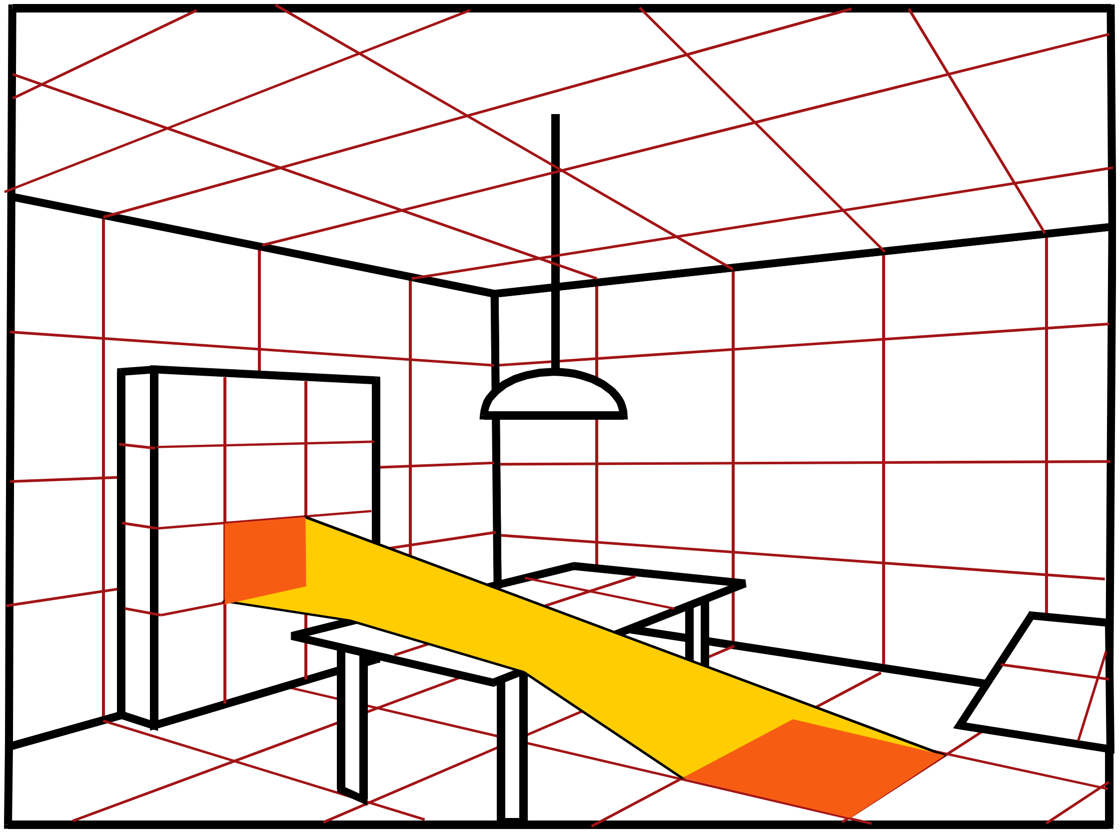
- Review Session:
  Tuesday November 18th, 7:30 pm
  bring lots of questions!

- Quiz 2: Thursday November 20th, in class
  (one weeks from today)

Review of last week?

Radiosity

- Diffuse surfaces
- Subdivide scene
- Radiosity assumed constant over a patch
- Form-factor between patches
- Geometry and visibility
- Big Matrix system



Radiosity

- Smoothing and other gimmicks

Limitations of radiosity

- Diffuse only for basic method
  - Costly extension to specular
- Requires meshing
- Cost of visibility computation
  - If you send rays, why not use ray tracing?
- Memory consumption vs. time scalability

Why still learn radiosity?

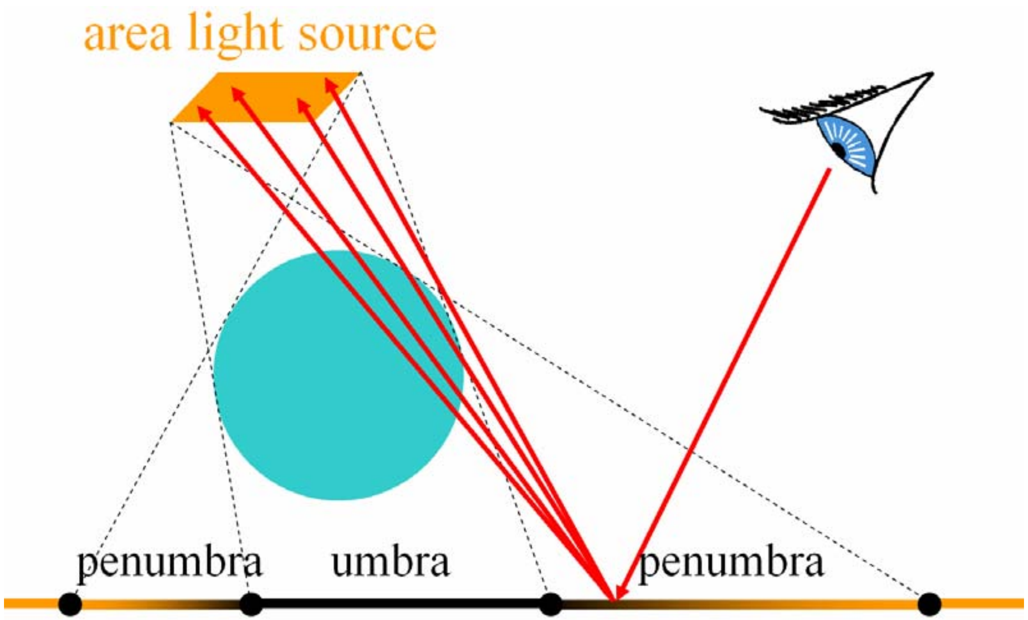
- Still used in architecture (Lightscape)
- Introduction to finite element method
  - Project the problem onto a finite basis of functions
    - In the case of radiosity: piecewise constant
  - Express interaction between elements
  - Get a big matrix system
  - Same as deformable object simulation
- Pre-computed radiance transfer: same strategy
  - Use finite basis function
  - Precompute lighting as a function of primary sources
  - Use in a simplified version in Max Payne 2

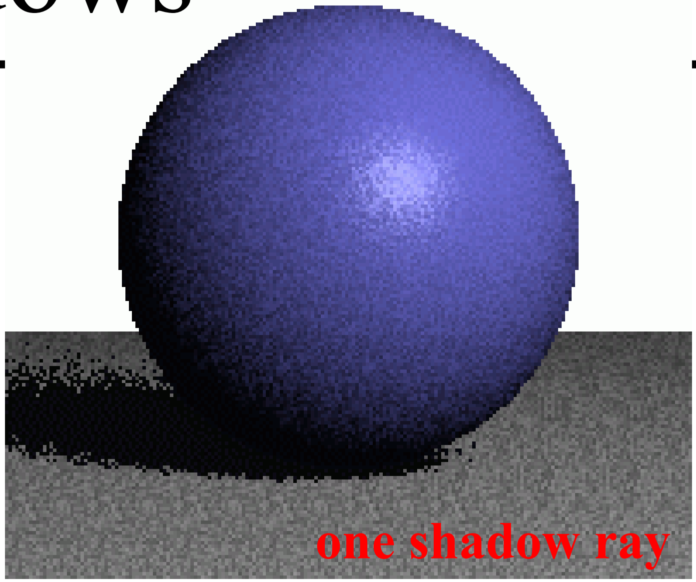
Today: Monte Carlo Ray Tracing

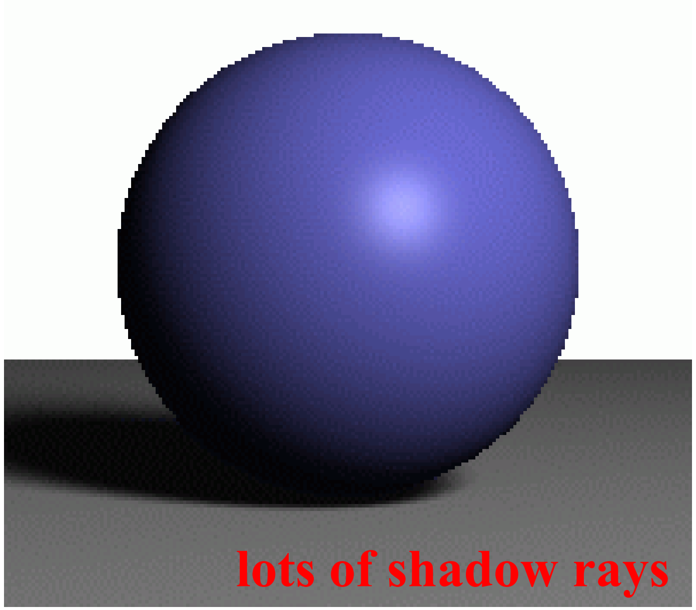
- Principle of Monte-Carlo Ray Tracing
- Monte Carlo integration
- Review of Rendering equation
- Advanced Monte Carlo Ray Tracing

Probabilistic Soft Shadows

- Multiple shadow rays to sample area light source
- Monte-Carlo ray tracing generalizes this

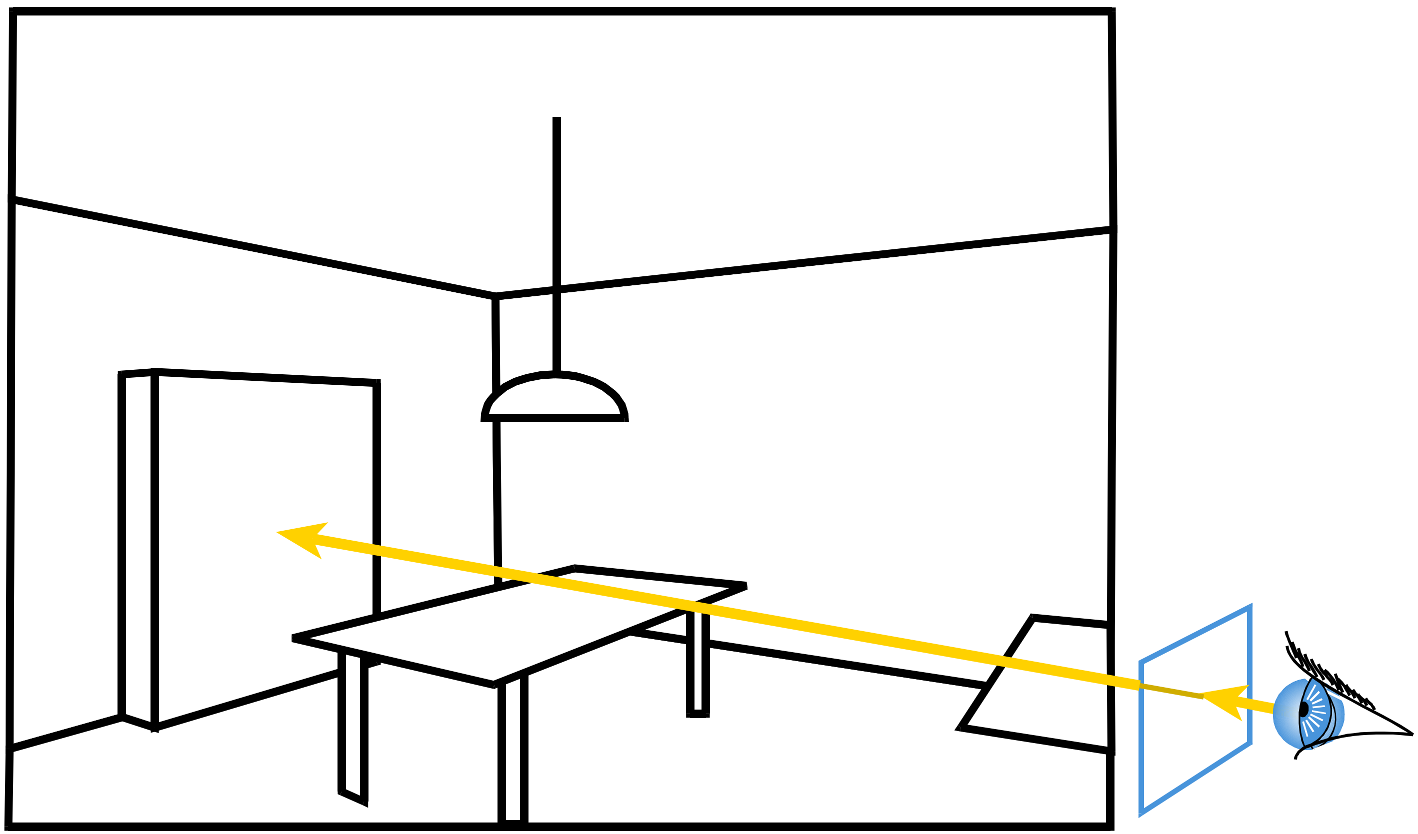






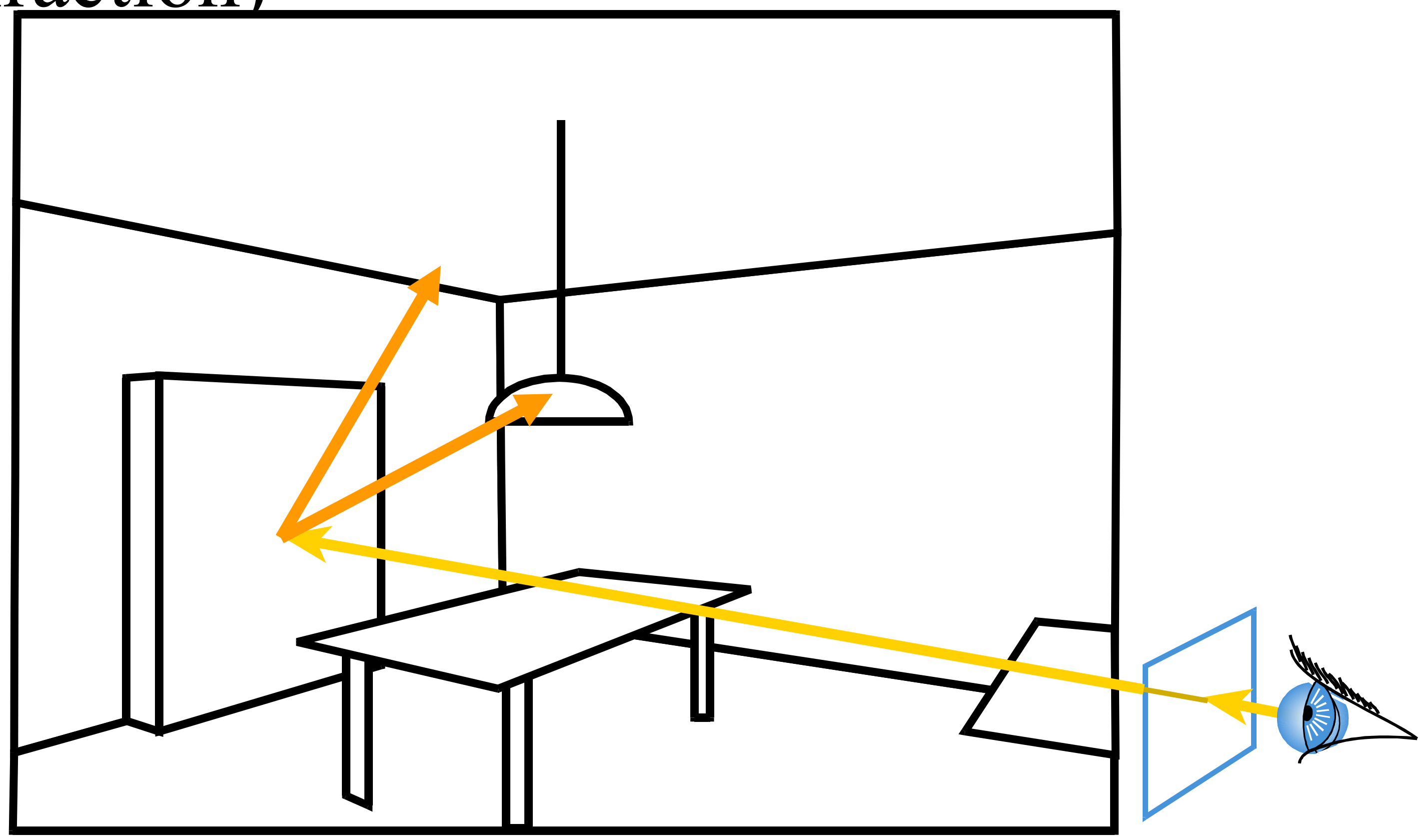
Ray Casting

- Cast a ray from the eye through each pixel



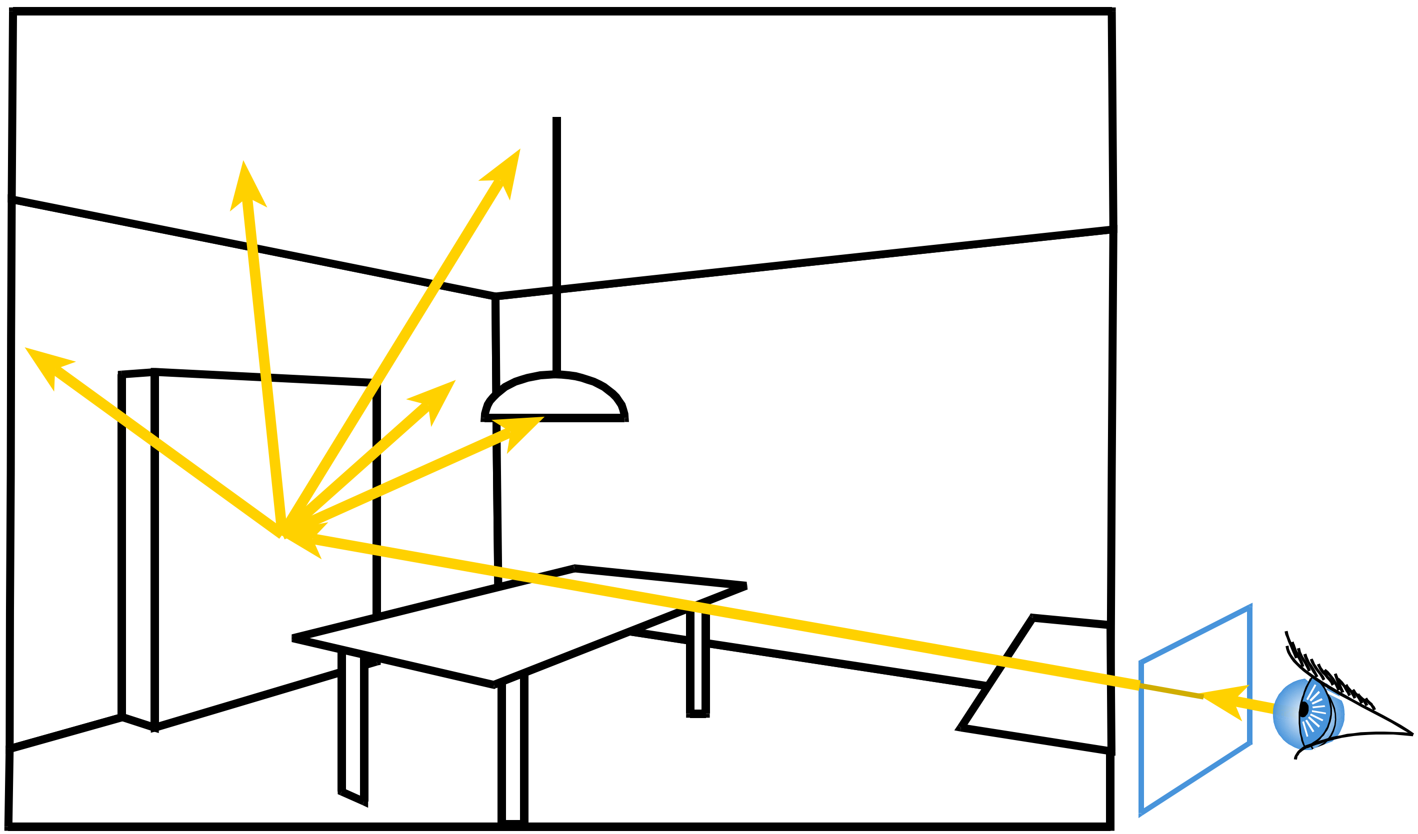
Ray Tracing

- Cast a ray from the eye through each pixel
- Trace secondary rays (light, reflection, refraction)



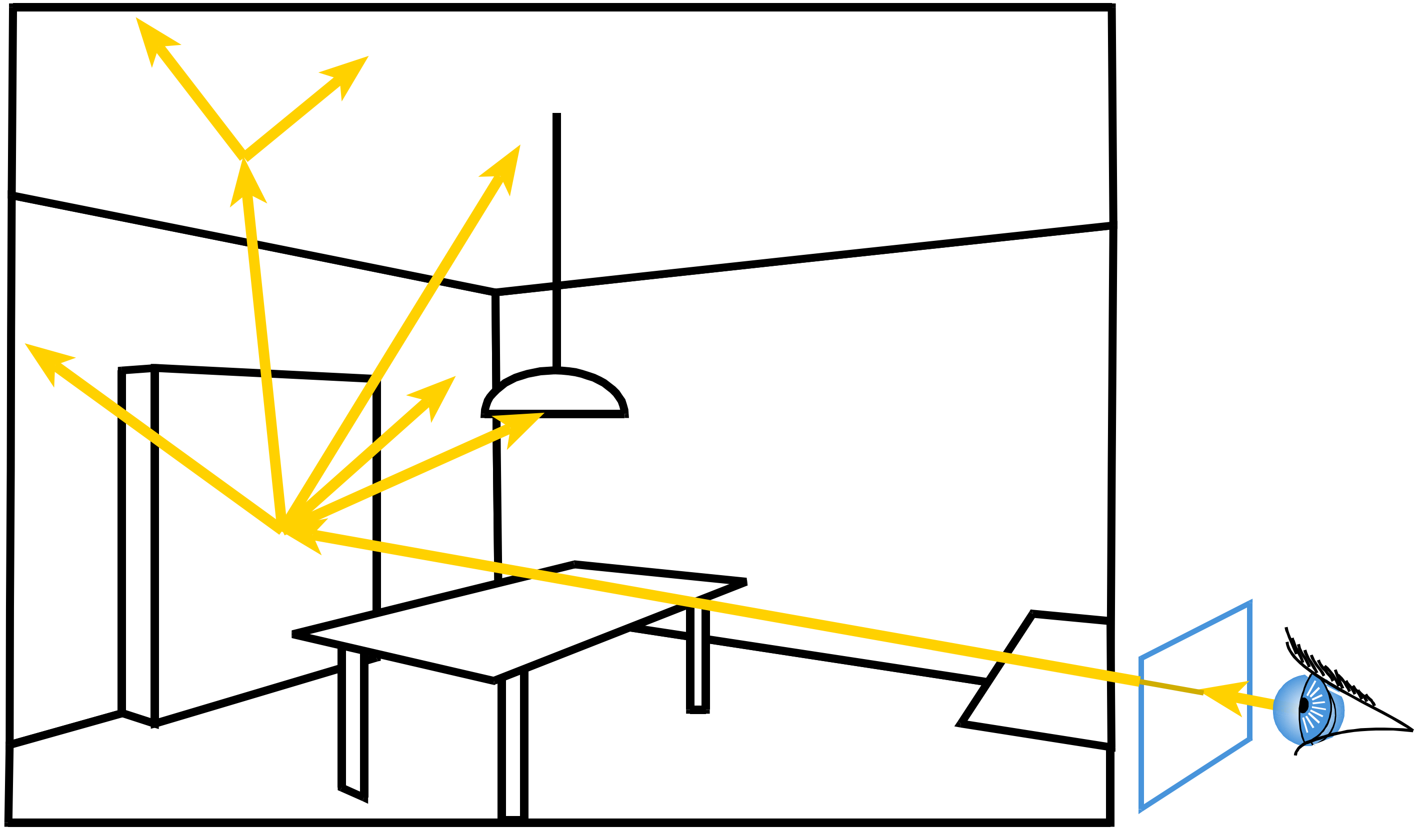
Monte-Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
  - Accumulate radiance contribution



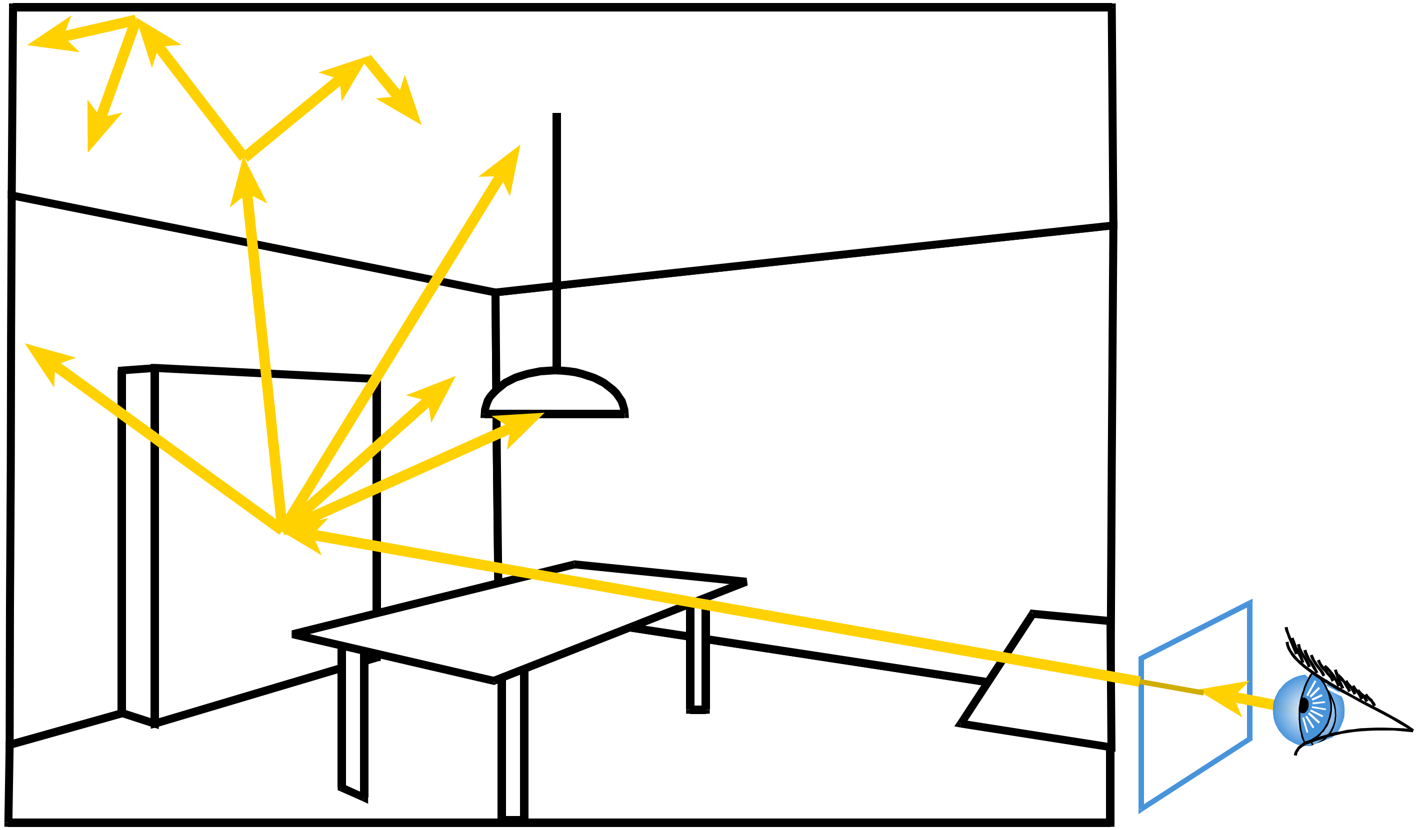
Monte-Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



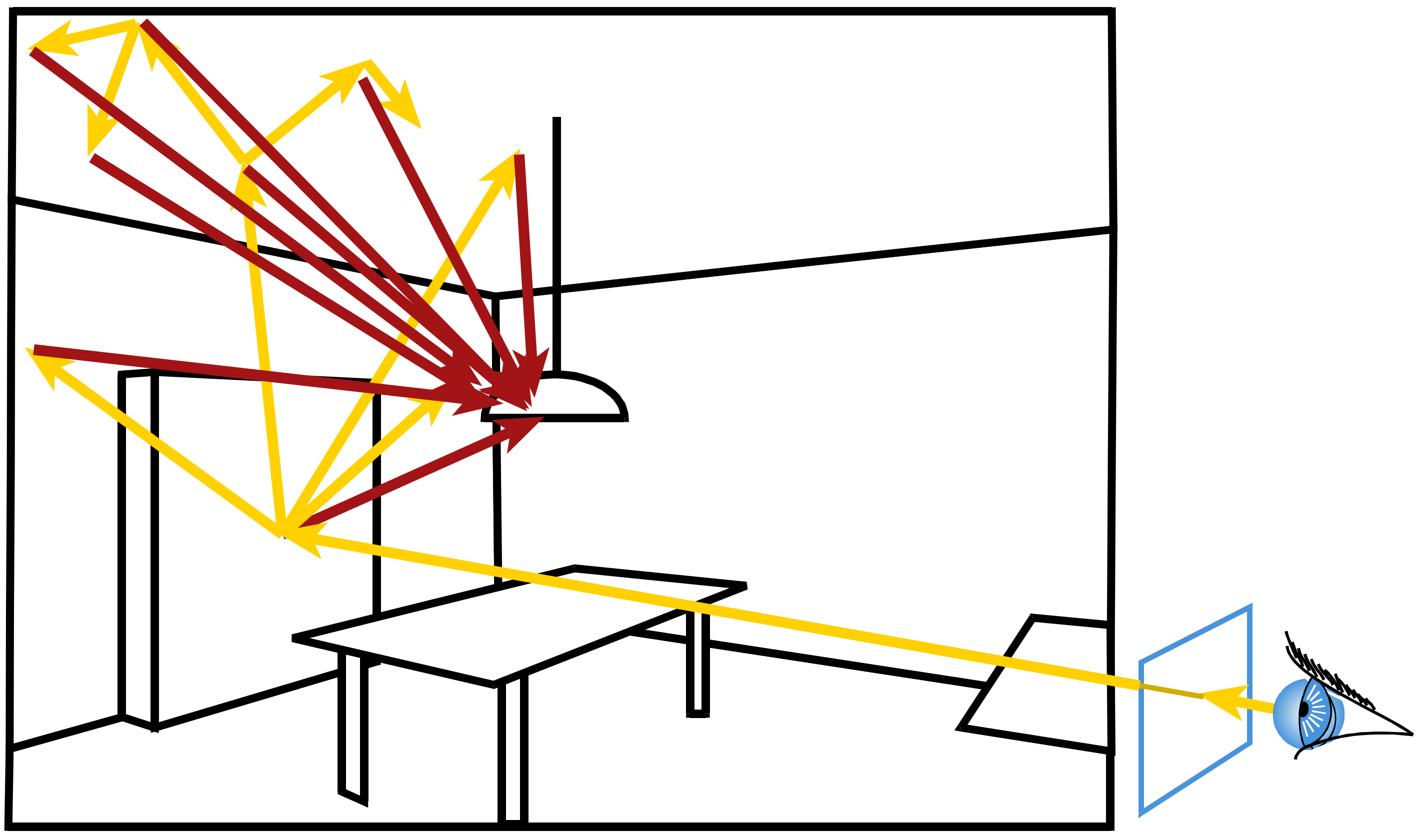
Monte-Carlo

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



Monte-Carlo

- Systematically sample primary light



Results

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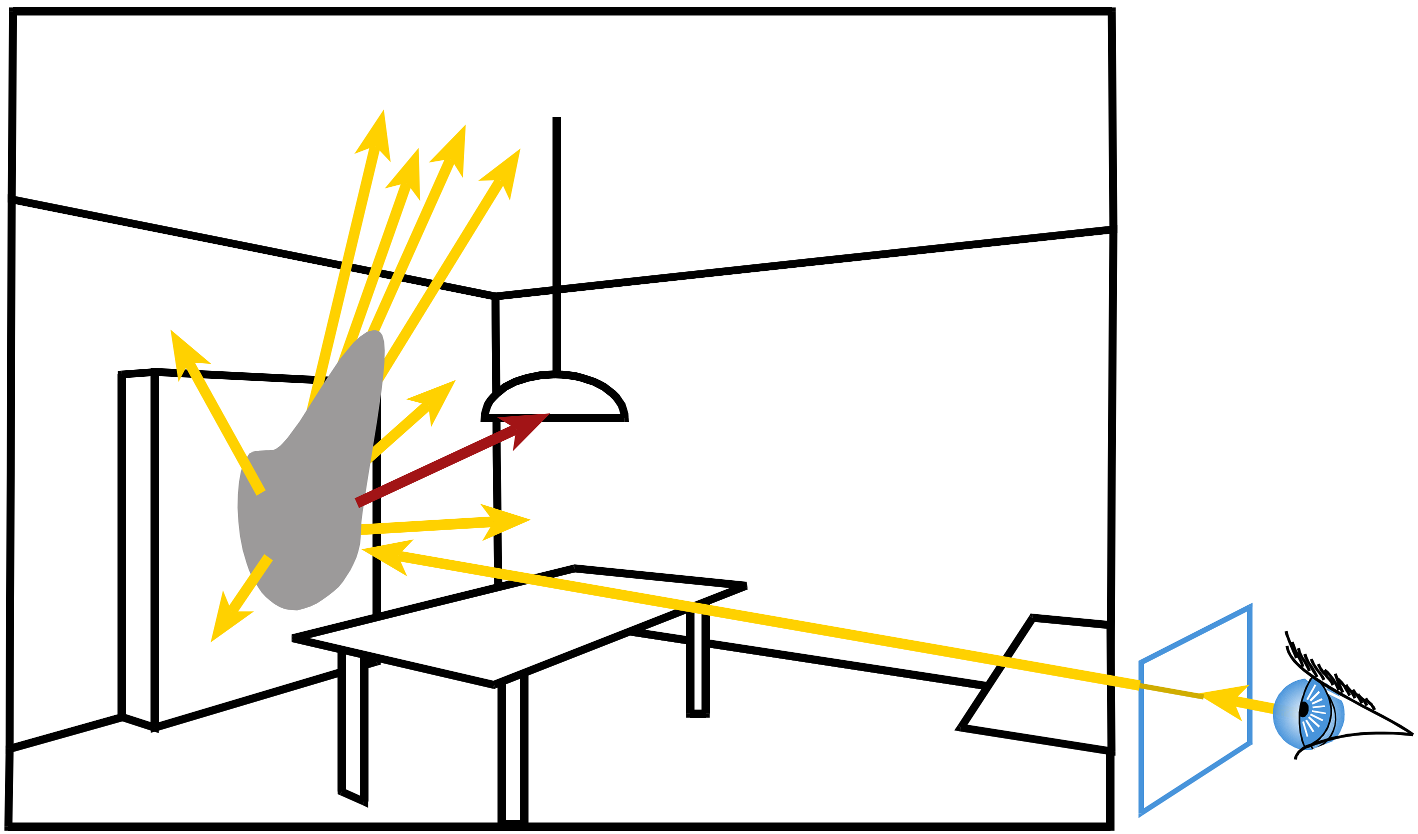
Monte-Carlo

- Take reflectance into account
  - Multiply incoming radiance by BRDF value



- 1 sample per pixel

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- 256 samples per pixel

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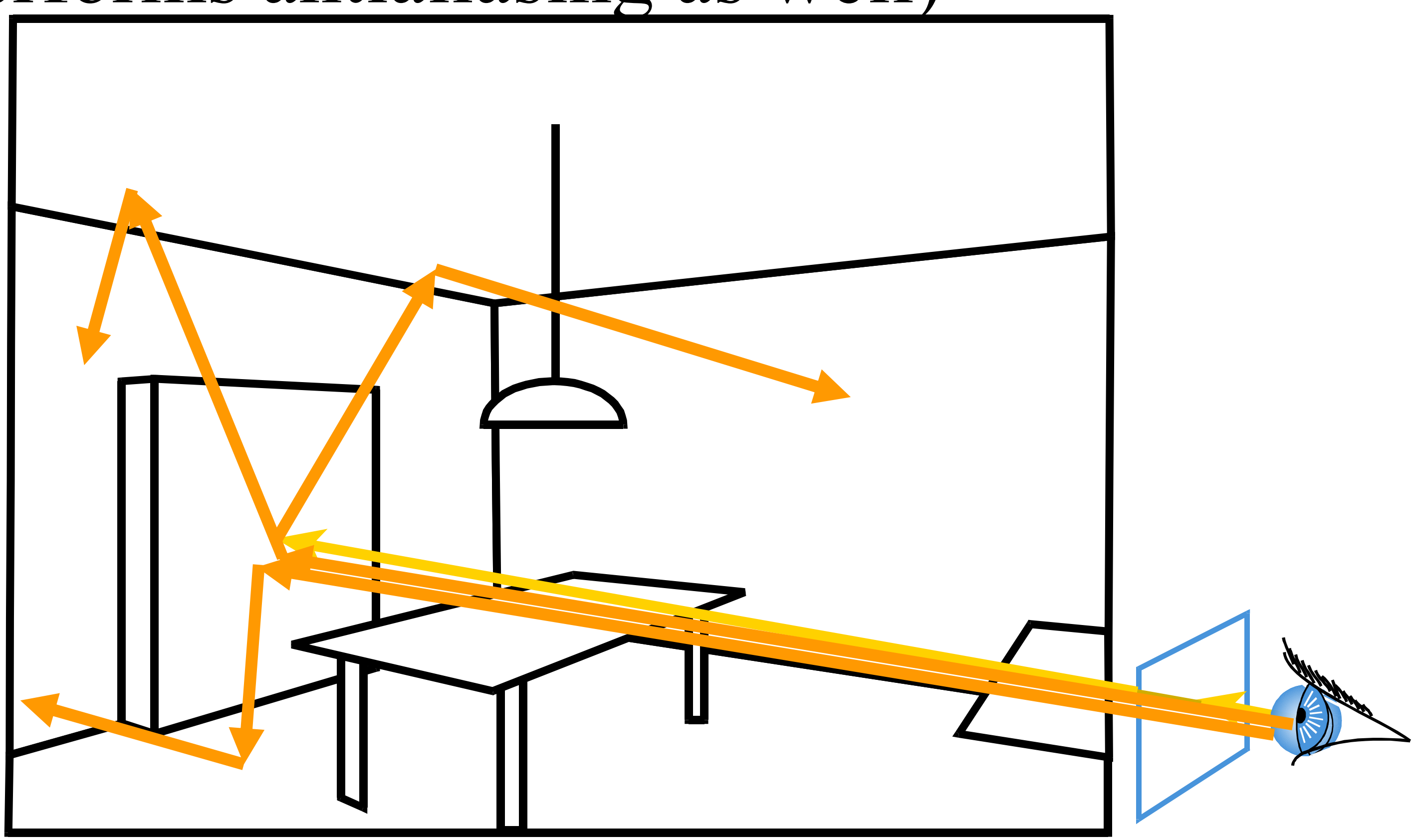
Monte Carlo Path Tracing

- Trace only one secondary ray per recursion
- But send many primary rays per pixel
- (performs antialiasing as well)



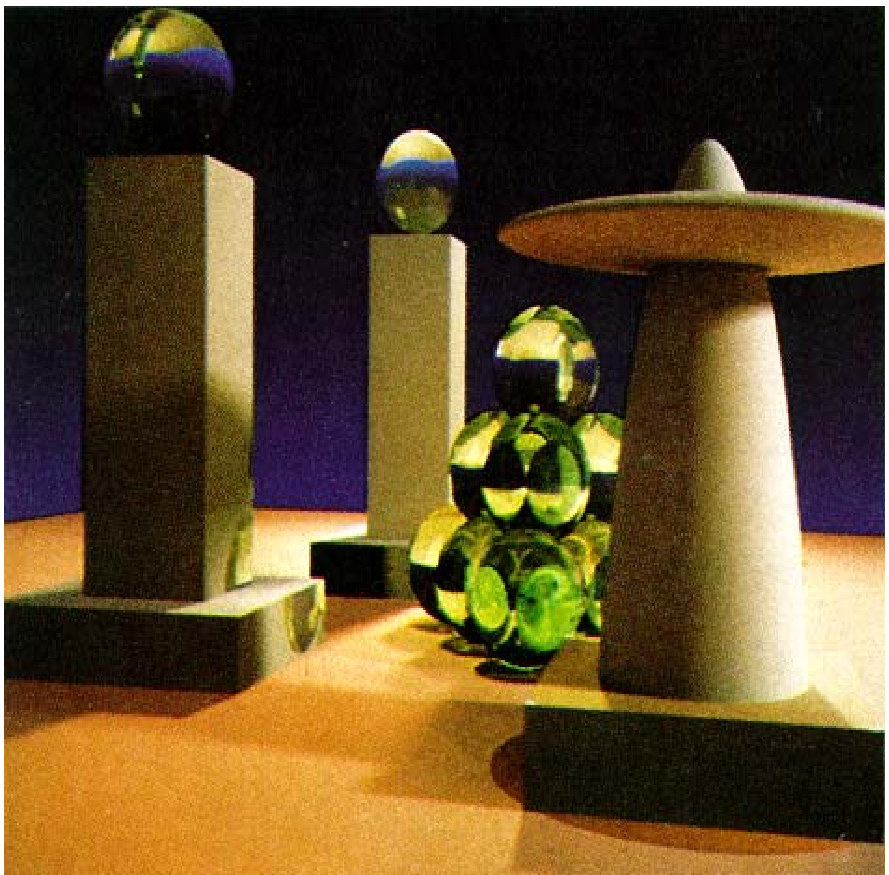
Monte Carlo Path tracing

traceRay
- Intersect all objects
- Shade visible object

Shade visible object
- Trace random ray to sample incoming light
- Shade using brdf

Vintage path tracing image

- by Jim Kajiya (1986)
- Also introduced the rendering equation



Convergence speed

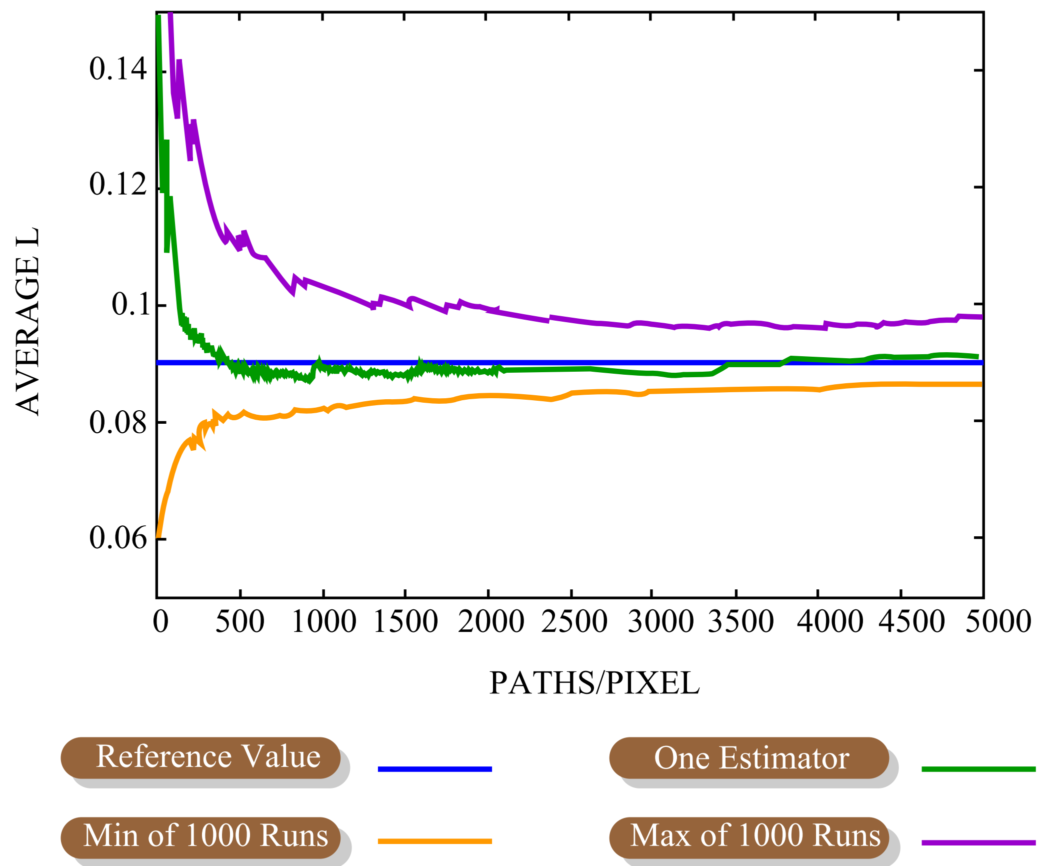


Image adapted from:
Henrik Wann Jensen. *Realistic Image Synthesis Using Photon Mapping.* 1st Ed.
A K Peters, Ltd. July 2001. ISBN: 1568811470.

Results

- 10 paths/pixel

(Image removed due to copyright considerations.)

Results

- 10 paths/pixel

(Image removed due to copyright considerations.)

Results

- 100 paths/pixel

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Why use random numbers?

- Fixed random sequence
- We see the structure in the error

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Recap

- Send random rays
- Sample the rendering equation
- No meshing required, no special storage
- No limitation
  - On reflectance
  - On geometry

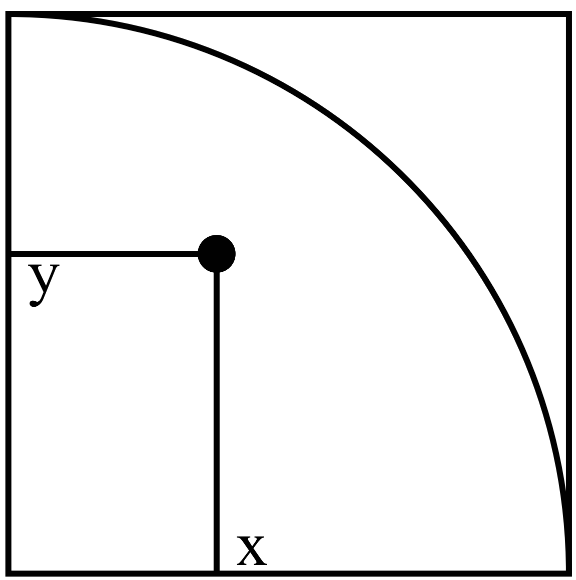
- Extremely flexible
- Can be noisy (or slow)

Today: Monte Carlo Ray Tracing

- Principle of Monte-Carlo Ray Tracing
- Monte Carlo integration
- Review of Rendering equation
- Advanced Monte Carlo Ray Tracing

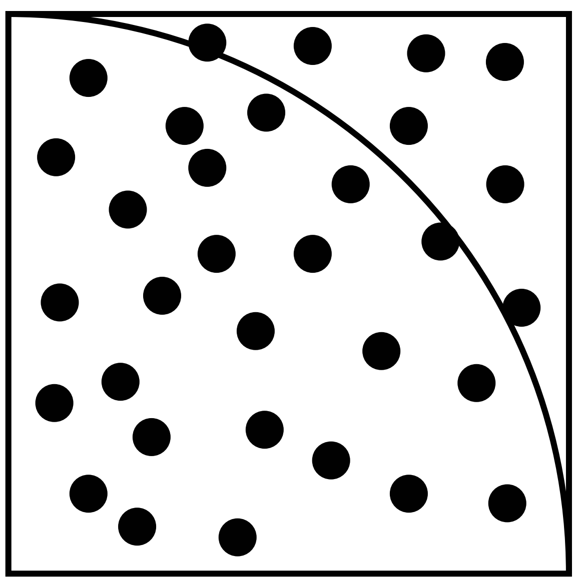
Monte-Carlo computation of π

- Take a square
- Take a random point (x,y) in the square
- Test if it is inside the ¼ disc ($x^2+y^2 < 1$)
- The probability is $\pi /4$



Monte-Carlo computation of π

- The probability is π /4
- Count the inside ratio n = # inside / total # trials
- $\pi \approx n * 4$
- The error depends on the number or trials

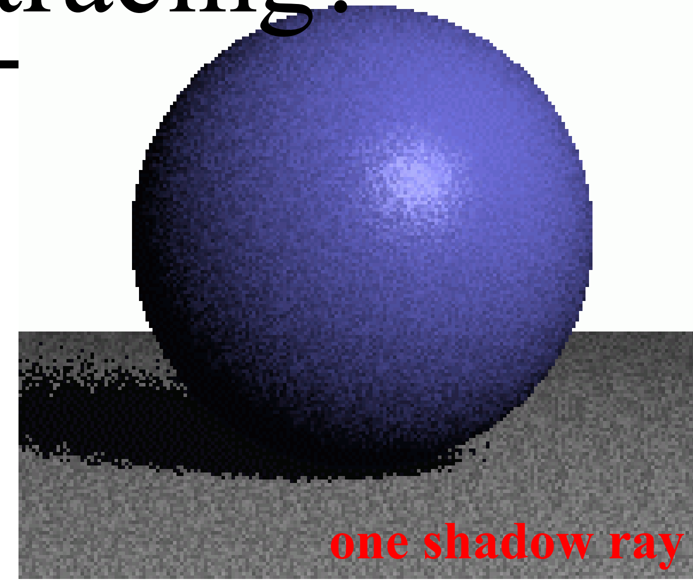


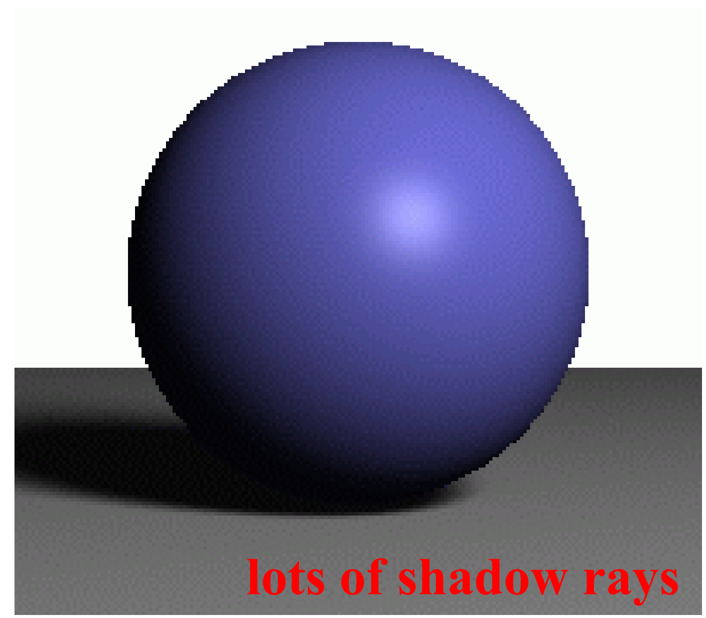
Why not use Simpson integration?

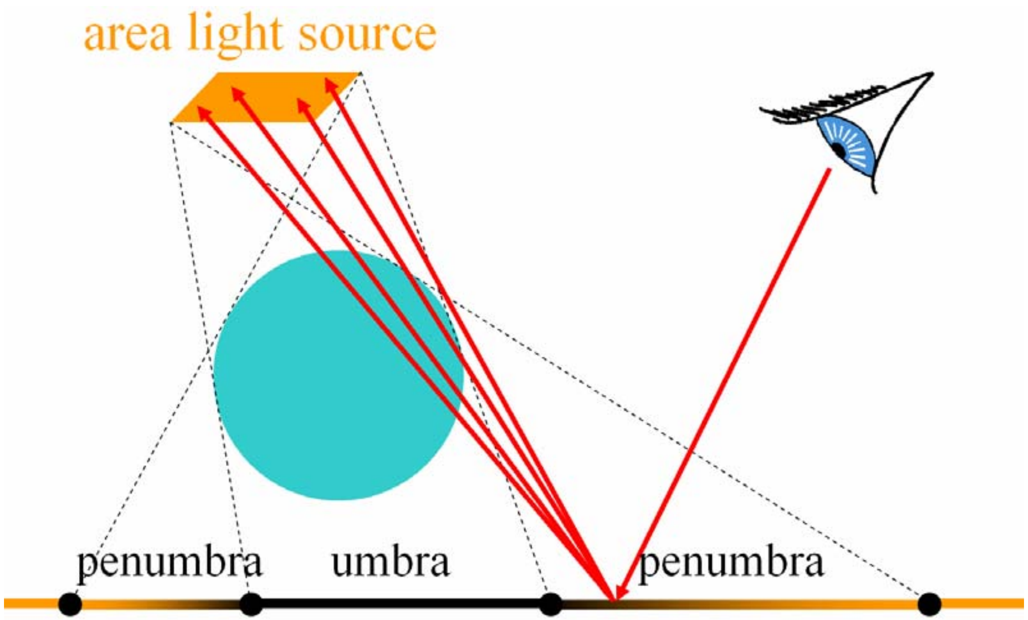
- Yeah, to compute π,
  Monte Carlo is not very efficient
- But convergence is independent of dimension
- Better to integrate high-dimensional functions
- For d dimensions, Simpson requires N^d domains

What's the link to ray tracing?

- Light source is square
- Occluder is sphere
- Some pixels have exactly $\pi/4$ visibility ;-)







Dumbest Monte-Carlo integration

- Compute 0.5 by flipping a coin
- 1 flip: 0 or 1=> average error =0.5
- 2 flips: 0, 0.5, 0.5 or 1 =>average error=0. 25
- 4 flips: 0 (*1),0.25 (*4), 0.5 (*6), 0.75(*4), 1(*1) => average error =0.1875

- Does not converge very fast
- Doubling the number of samples does not double accuracy

Convergence of Monte Carlo integration

- Variance decrease in 1/n
- Convergence is $\frac{1}{\sqrt{n}}$
- Independent of dimension

Monte Carlo integration

- Want to evaluate $$\int_a^b f(x)dx$$
- Use random variable x_i with uniform probability over the domain
$$\frac{1}{n}\sum_{i=1}^{n} f(x_i)$$
- Note that achieving uniform probability can be tough for complex domain (e.g. sphere)

Monte Carlo integration – smarter

- Want to evaluate $\int_a^b f(x)dx$
- Use random variable x_i **with probability p_i**

$$\frac{1}{n}\sum_{i=1}^{n}\frac{f(x_i)}{p_i}$$

- The whole trick is to choose the x_i and p_i

Sampling strategy

- Sample directions vs. sample light source

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Sampling strategies

(Images removed due to copyright considerations.)

Sampling more the light

Sampling more the BRDF

Monte Carlo recap

- Turn integral into finite sum
- Use random samples
- Convergence $\frac{1}{\sqrt{n}}$
- Independent of dimension
- Very flexible

- Tweak sampling/probabilities for optimal result
- A lot of integration and probability theory to get things right

What can we integrate?

- Pixel: antialiasing
- Light sources: Soft shadows
- Lens: Depth of field
- Time: Motion blur
- BRDF: glossy reflection
- Hemisphere: indirect lighting

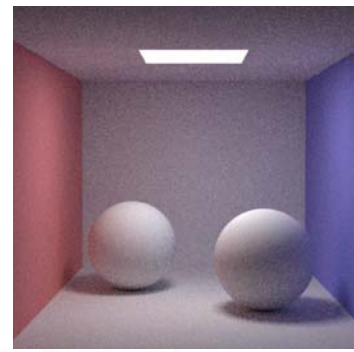
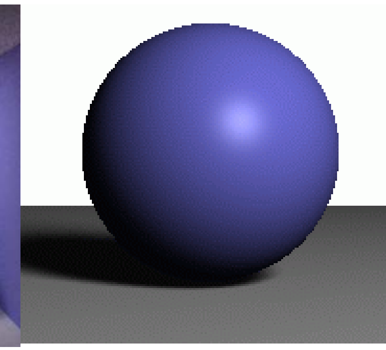
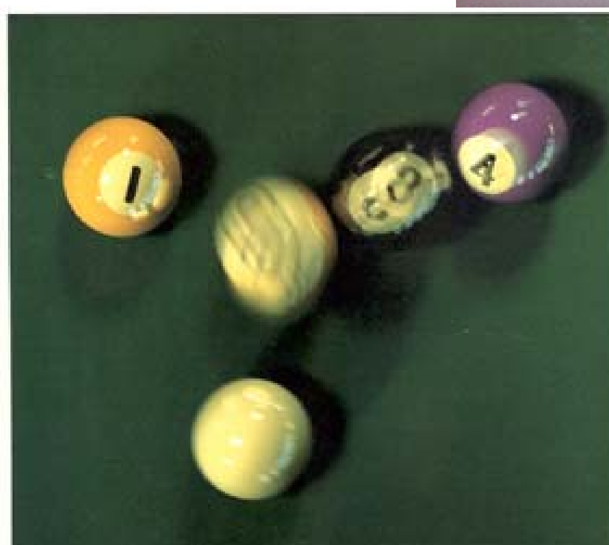
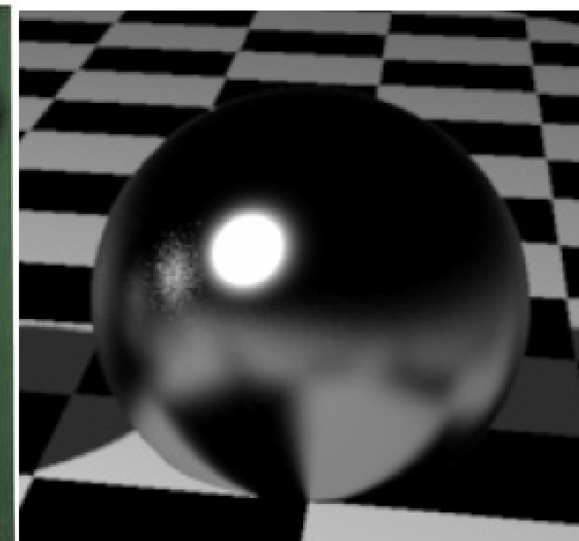



Image removed due to copyright considerations.
Please see *Computer Graphics* 25, no. 4
(July,1991): 157-164. ISBN: 0097-8930.

Questions?

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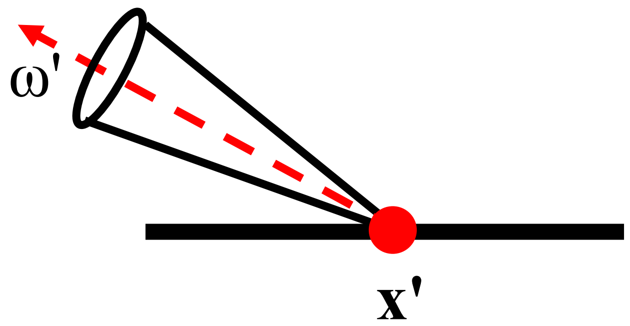
- Monte Carlo integration
- Review of Rendering equation
- Advanced Monte Carlo Ray Tracing

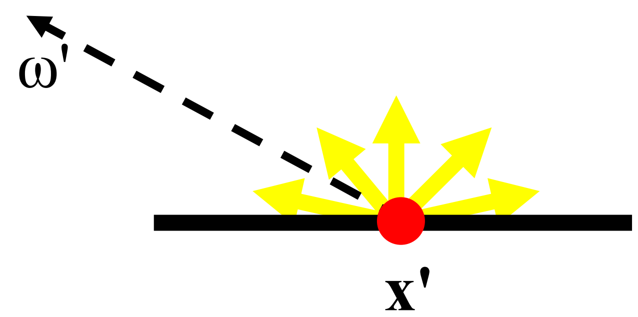
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\, dA$$

L (x',ω') is the radiance from a point on a surface in a given direction ω'

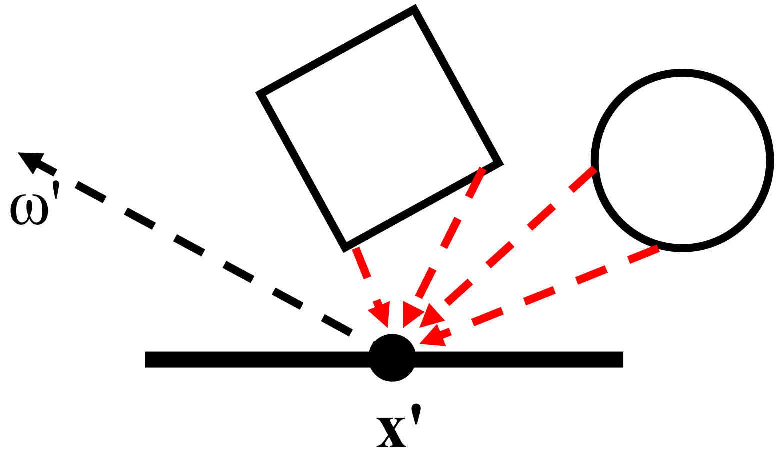
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

E(x',ω') is the emitted radiance from a point: E is non-zero only if x' is emissive (a light *source*)

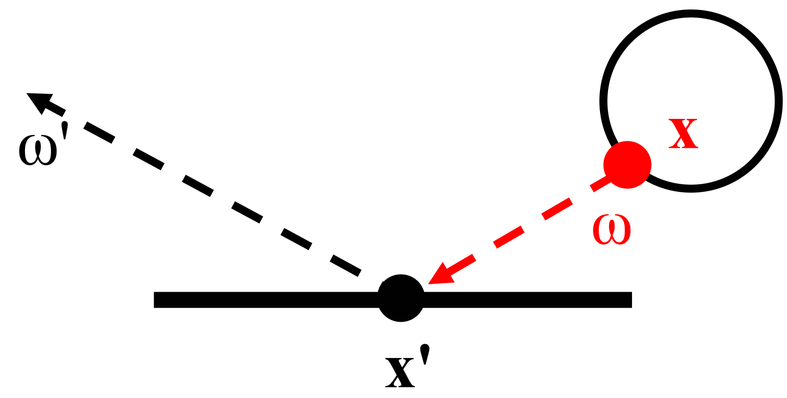
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \underbrace{\int \rho_{x'}(\omega,\omega') L(x,\omega) G(x,x') V(x,x')\, dA}$$

Sum the contribution from all of the other surfaces in the scene

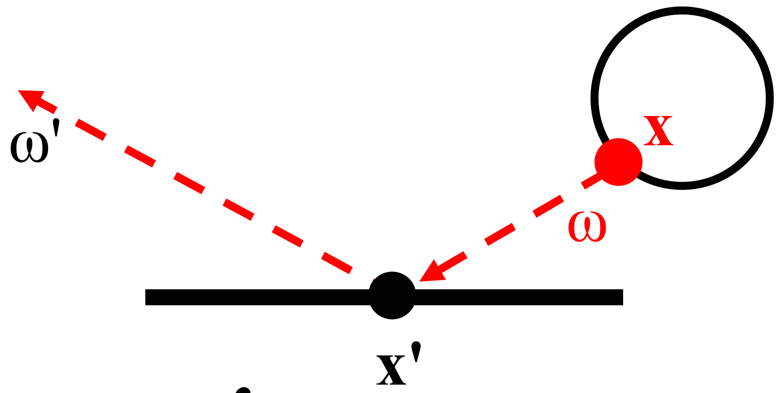
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega') L(x,\omega) G(x,x') V(x,x')\, dA$$

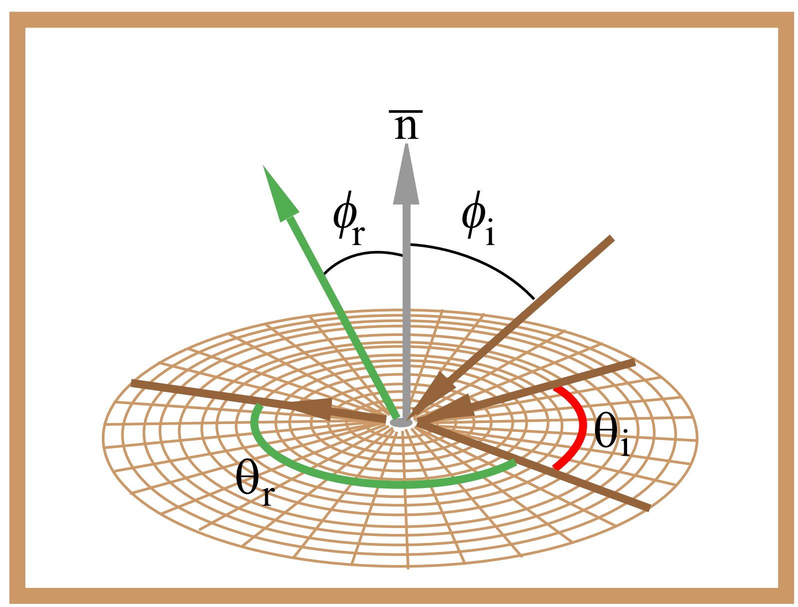
For each x, compute L(x, ω), the radiance at point x in the direction ω (from x to x')

The Rendering Equation

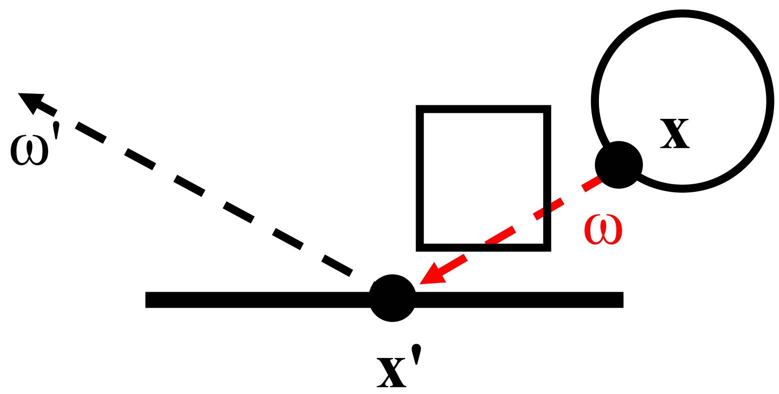


$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega') L(x,\omega) G(x,x') V(x,x')\, dA$$

scale the contribution by $\rho_{x'}(\omega,\omega')$, the reflectivity (BRDF) of the surface at x'



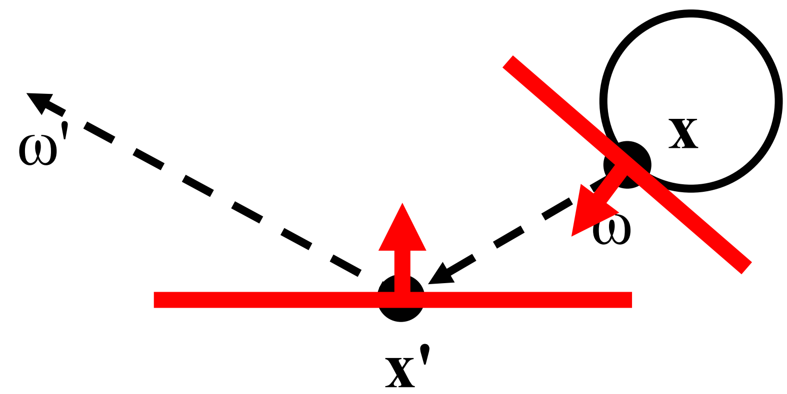
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

For each x, compute V(x,x'),
the visibility between x and x':
1 when the surfaces are unobstructed
along the direction ω, 0 otherwise

The Rendering Equation

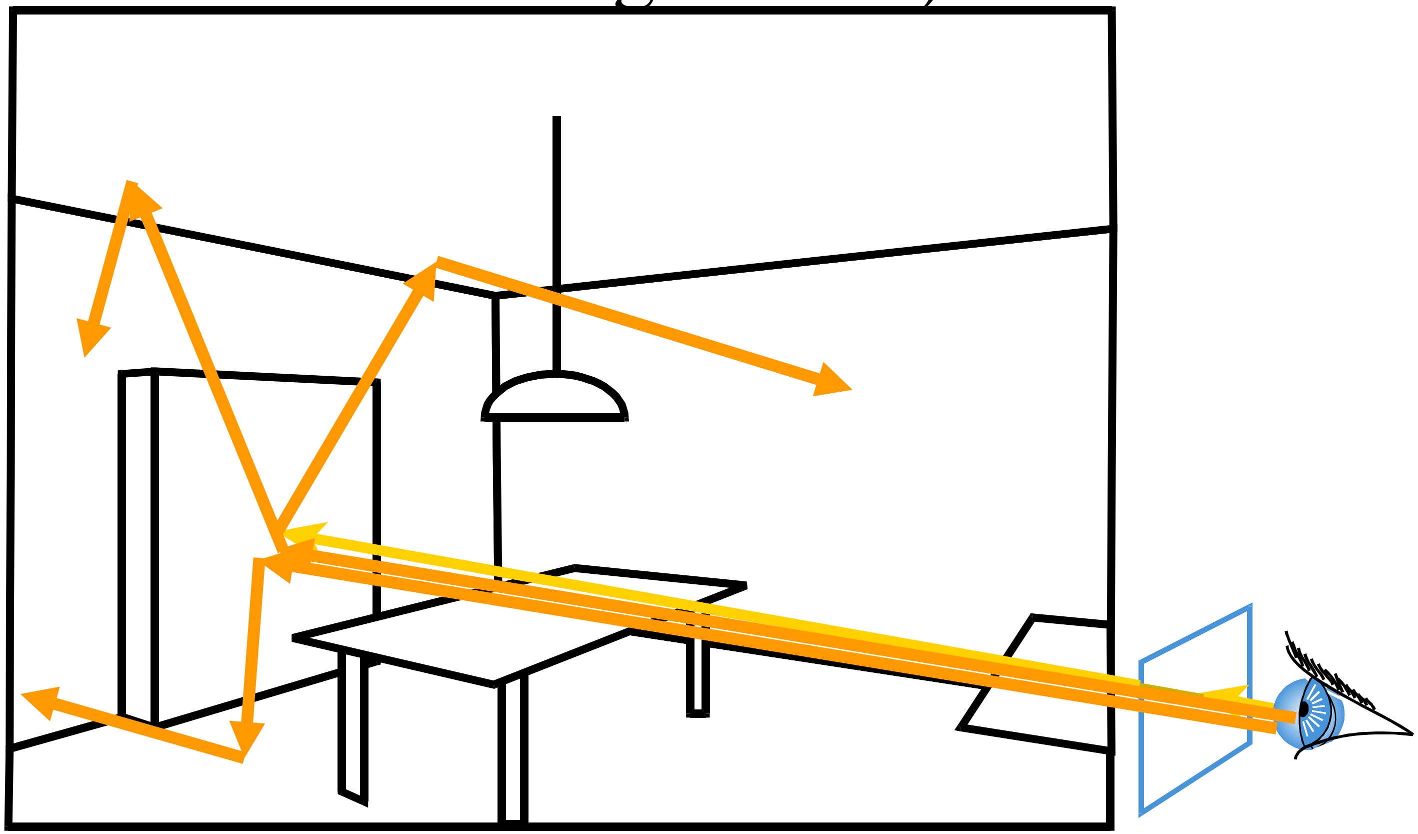


$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega') L(x,\omega) G(x,x') V(x,x')\, dA$$

For each x, compute G(x, x'), which describes the on the geometric relationship between the two surfaces at x and x’

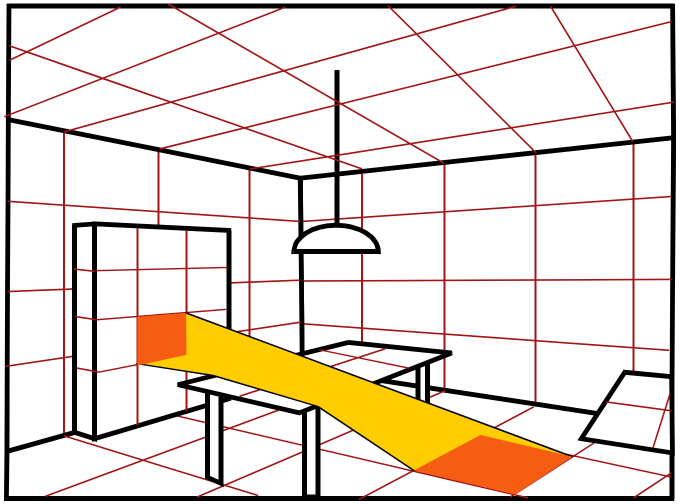
Monte Carlo Path Tracing

- Trace only one secondary ray per recursion
- But send many primary rays per pixel
- (performs antialiasing as well)

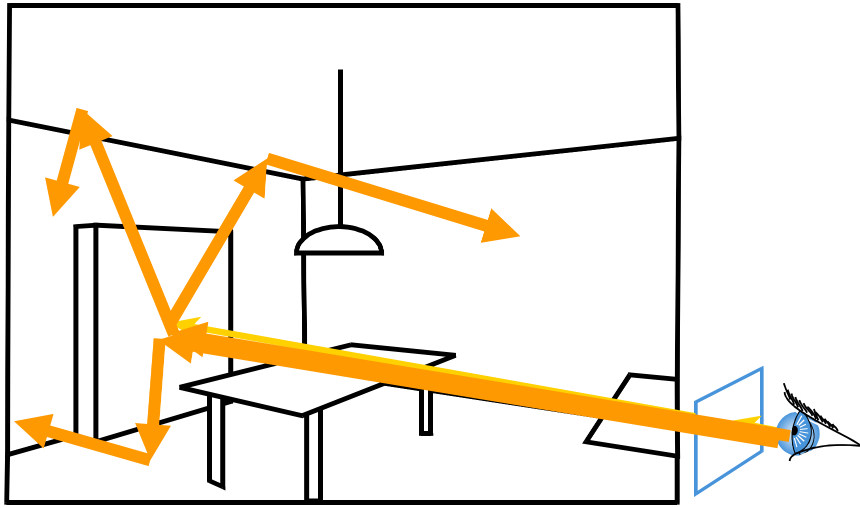


Radiosity vs. Monte Carlo

- We have an integral equation on an infinite space
- Finite elements (Radiosity)
  - Project onto finite basis of functions
  - Linear system

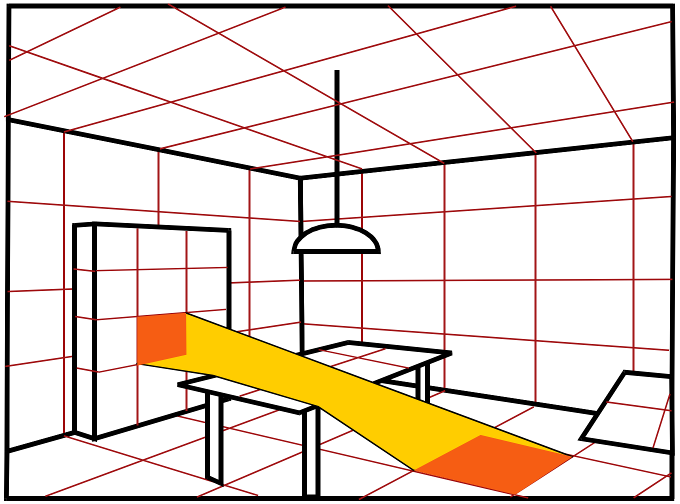


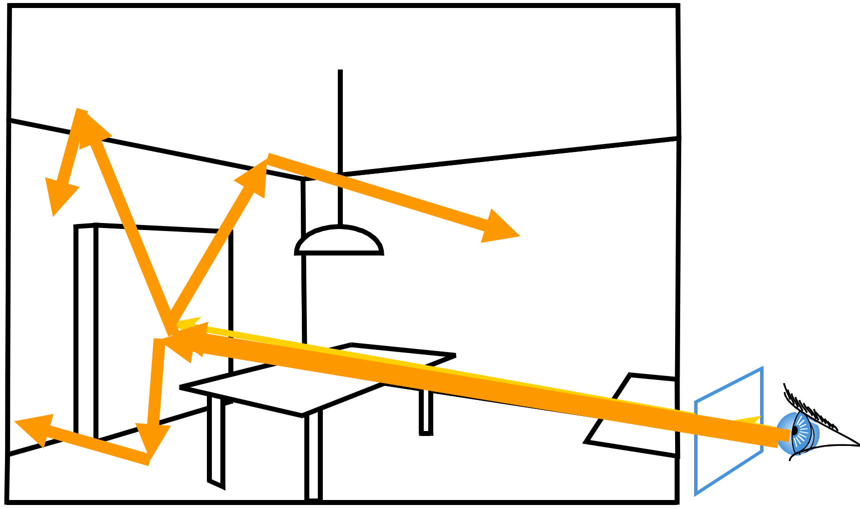
- Monte Carlo
  - Probabilistic sampling



Radiosity vs. Monte Carlo

- We have an integral equation on an infinite space
- Finite elements (Radiosity)
  - Project onto finite basis of functions
  - Linear system
  - View-independent (no angular information)
- Monte Carlo
  - Probabilistic sampling
  - View-dependent (but angular information)





References

- See also Eric Veach's PhD
  http://graphics.stanford.edu/papers/veach_thesis/

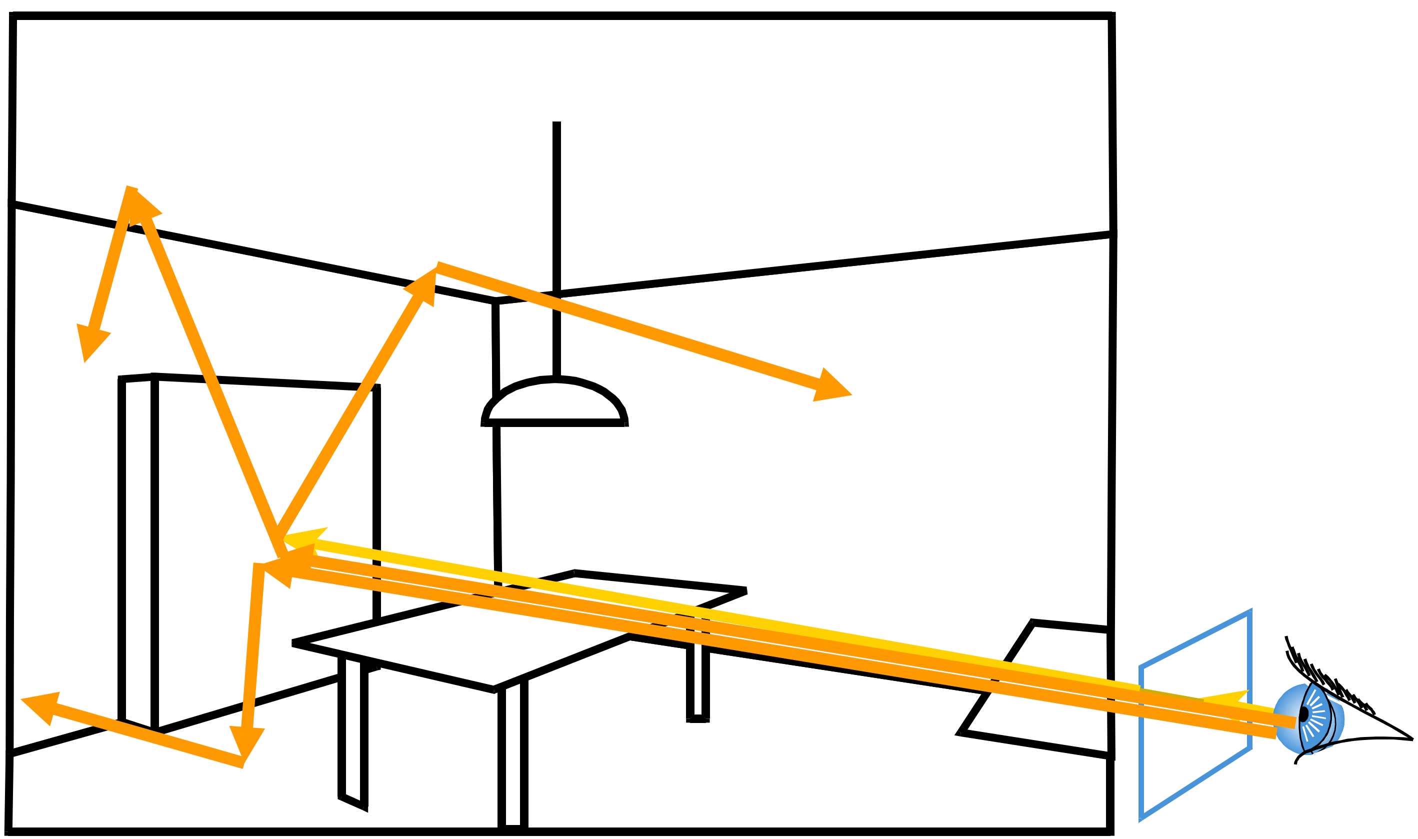
- Dutre, P., P. Bekaert and K. Bala. *Advanced Global Illumination*. AK Peters, Ltd. 2003. ISBN:1568811772.
- Shirley, Pete. *Realistic Ray Tracing*. AK Peters, Ltd. 1st. Ed. 2000. ISBN:1568811101.
- Jensen, Henrik Wann. *Realistic Image Synthesis Using Photon Mapping*. AK Peters, Ltd. 1st Ed. 2001. ISBN:1568811470.

Today: Monte Carlo Ray Tracing

- Principle of Monte-Carlo Ray Tracing
- Monte Carlo integration
- Review of Rendering equation
- Advanced Monte Carlo Ray Tracing
  - Irradiance caching
  - Photon mapping

Path Tracing is costly

- Needs tons of rays per pixel



Direct illumination

(Image removed due to copyright considerations.)

Global Illumination

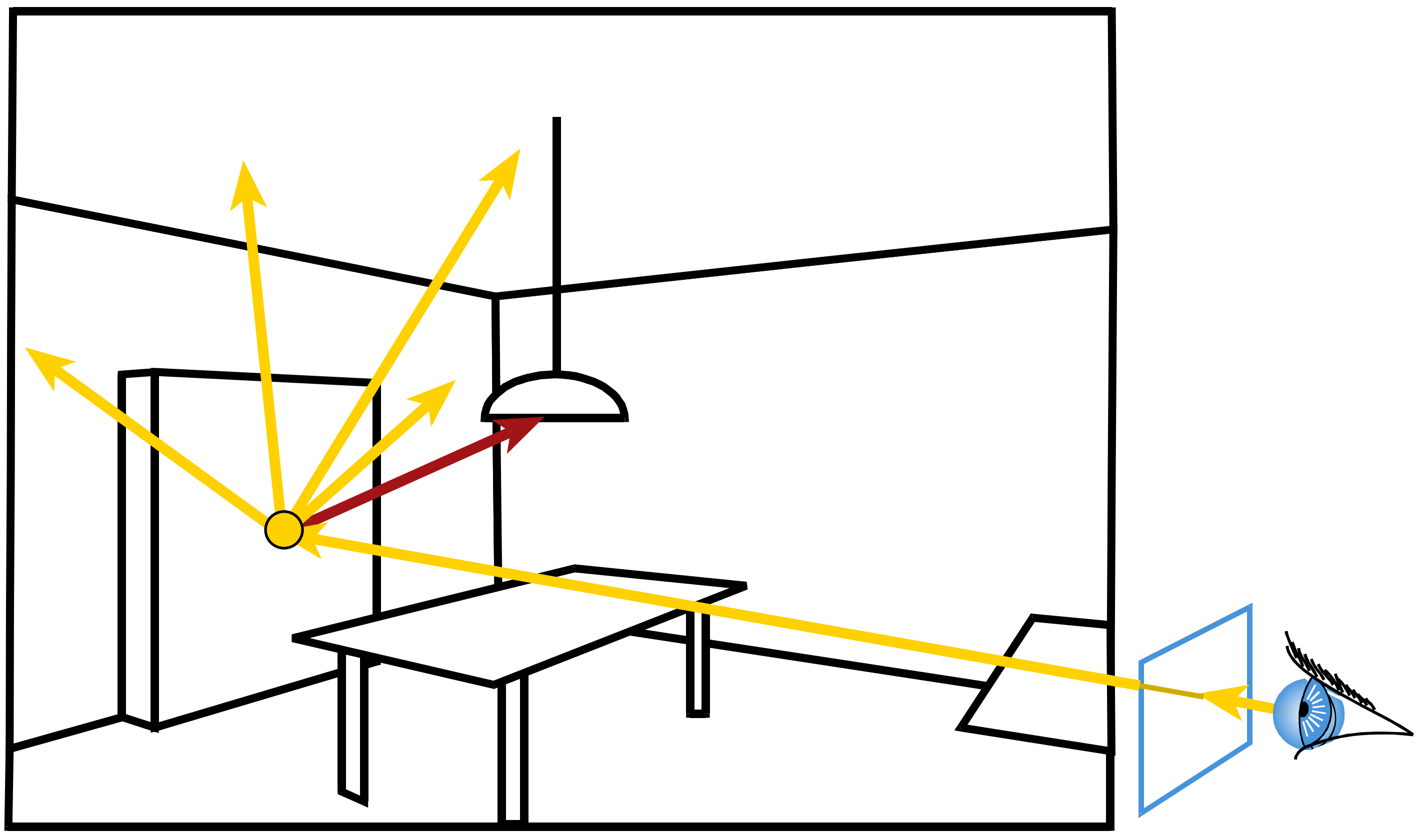
(Image removed due to copyright considerations.)

Indirect illumination: smooth

(Image removed due to copyright considerations.)

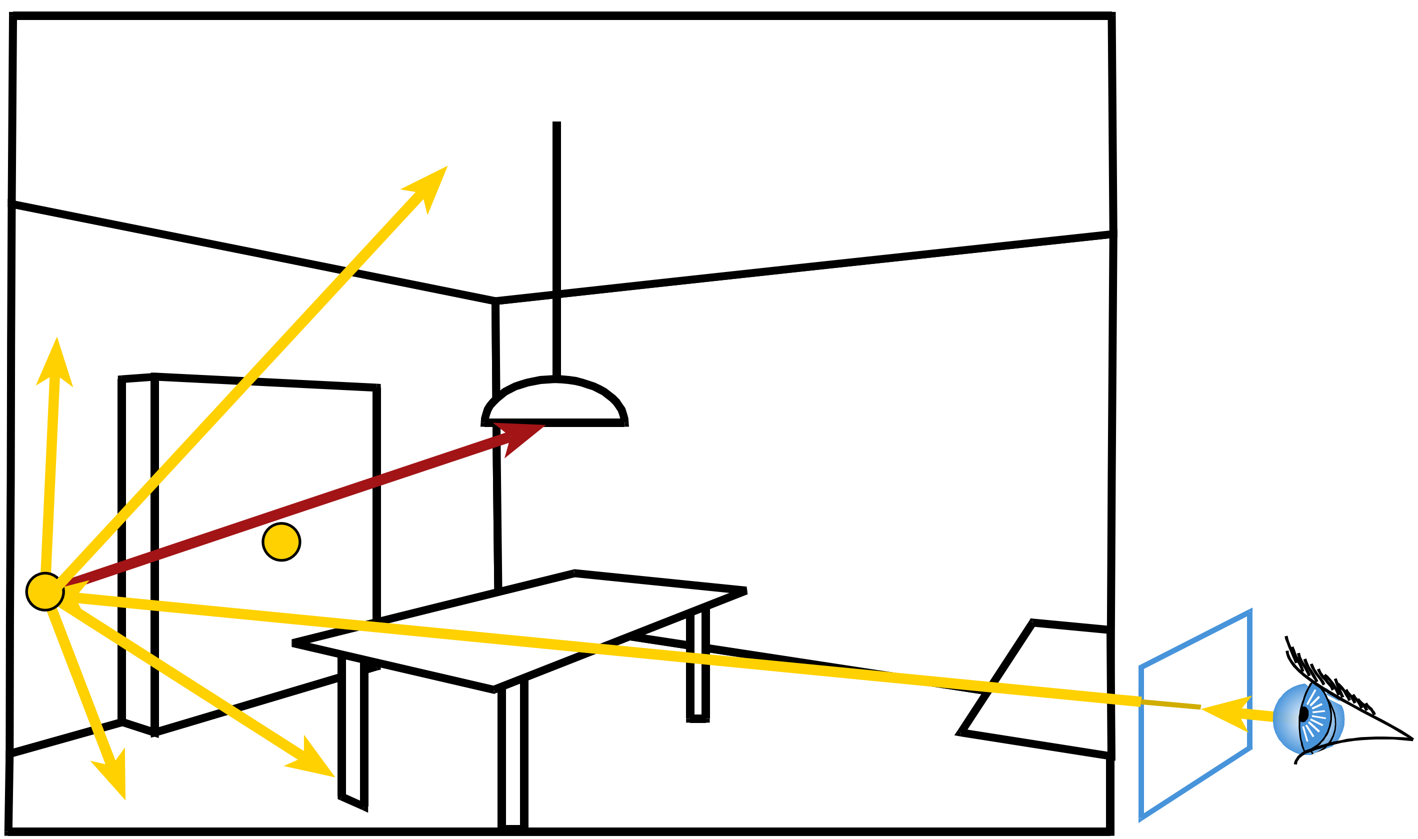
Irradiance cache

- The indirect illumination is smooth



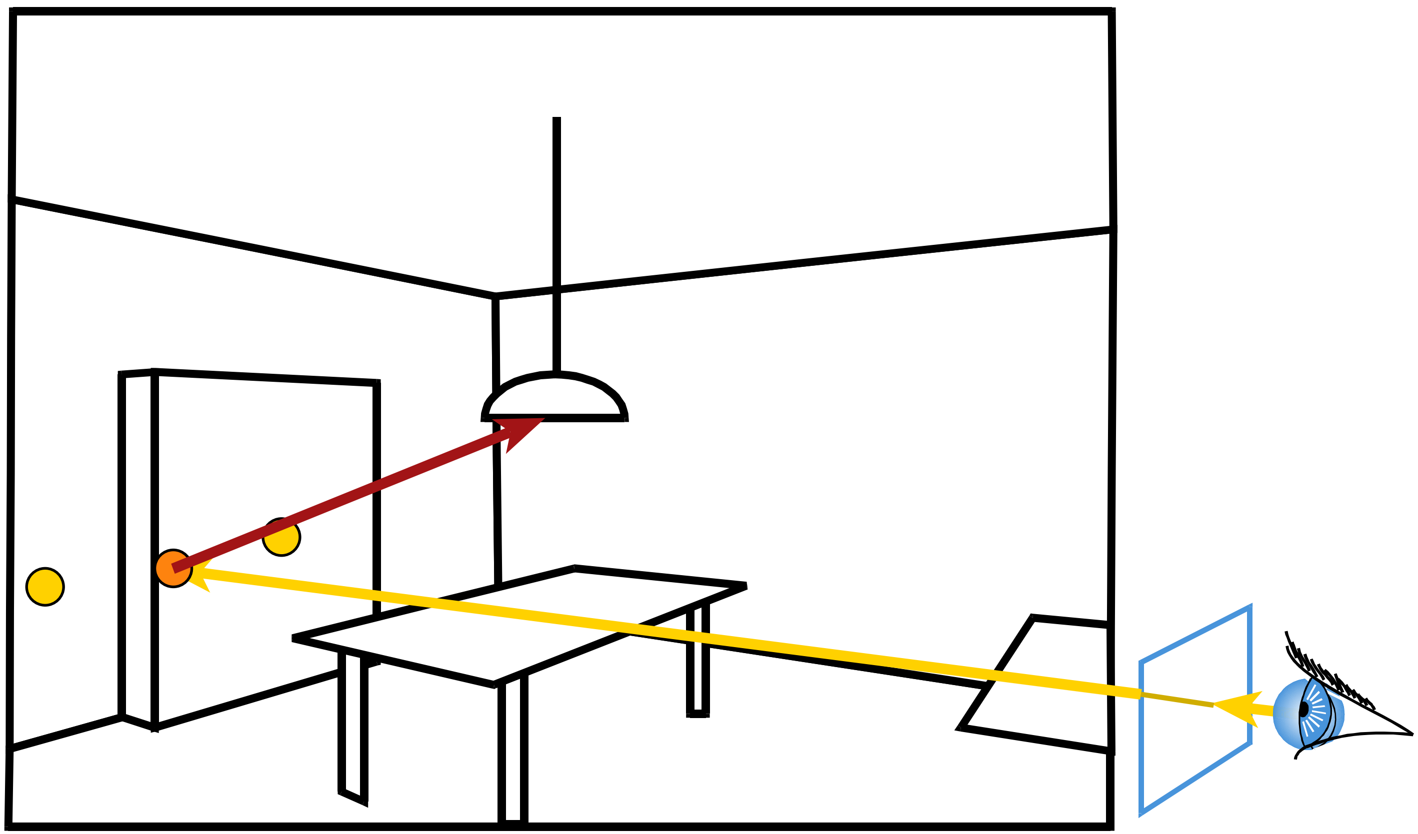
Irradiance cache

- The indirect illumination is smooth



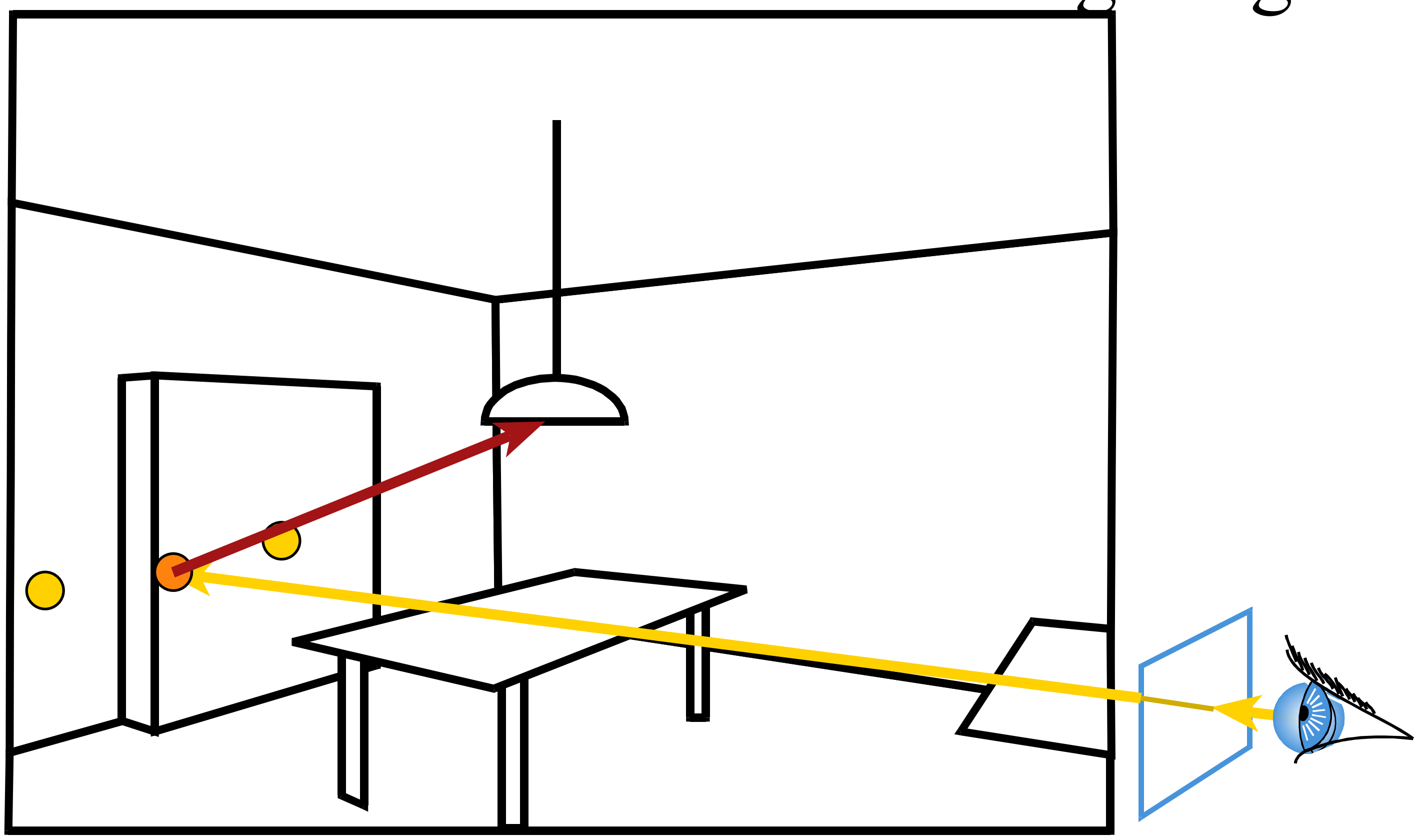
Irradiance cache

- The indirect illumination is smooth
- Interpolate nearby values



Irradiance cache

- Store the indirect illumination
- Interpolate existing cached values
- But do full calculation for direct lighting



Irradiance caching

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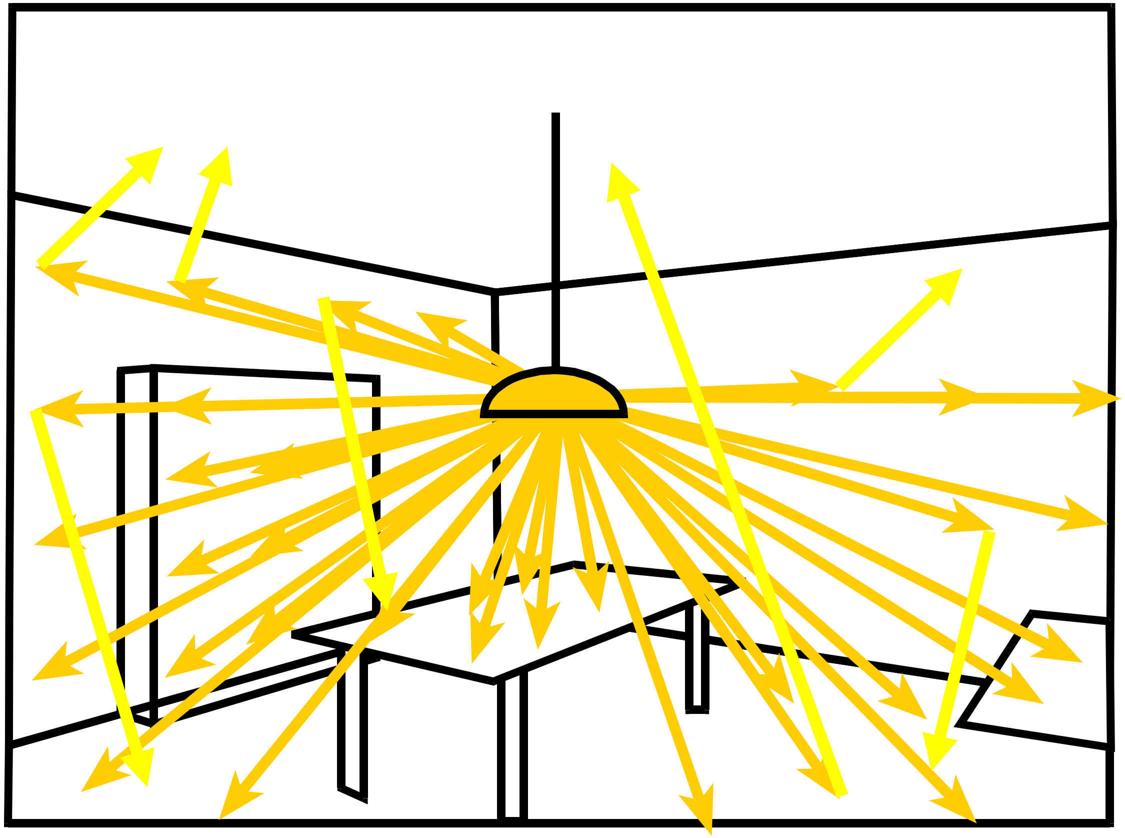
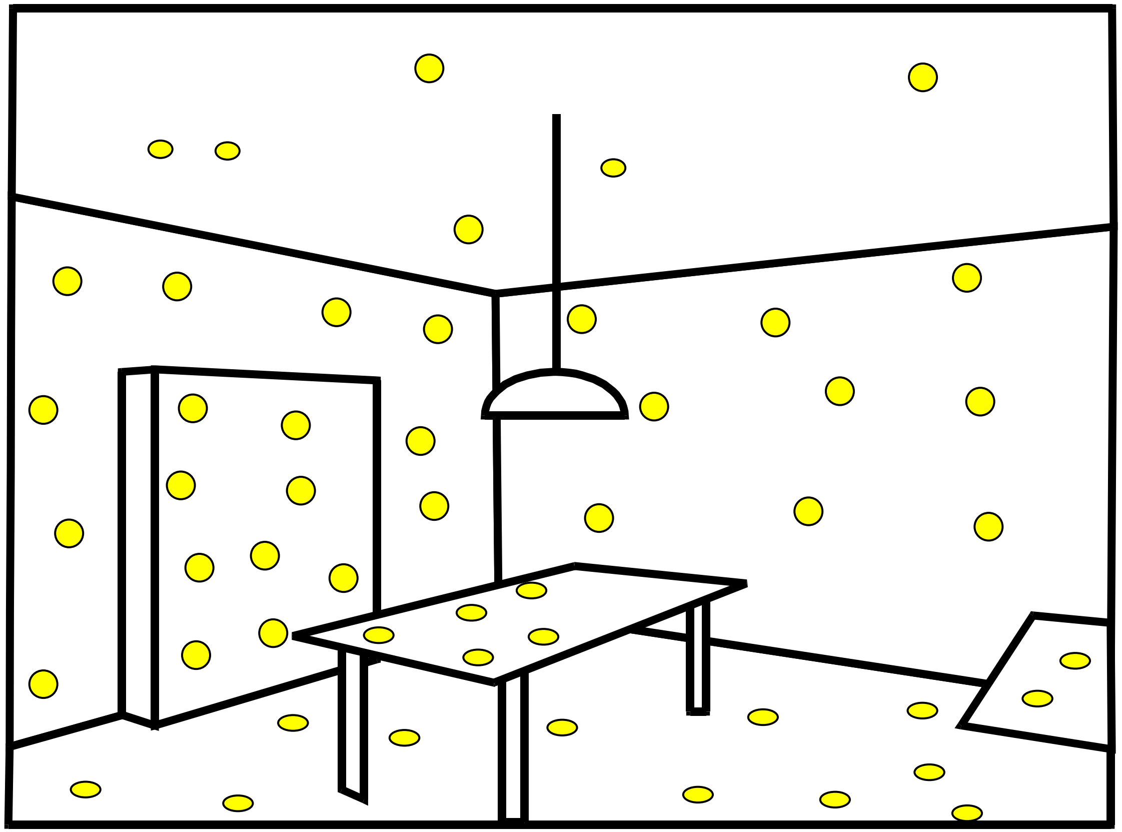
Photon mapping

- Preprocess: cast rays from light sources
- Store photons



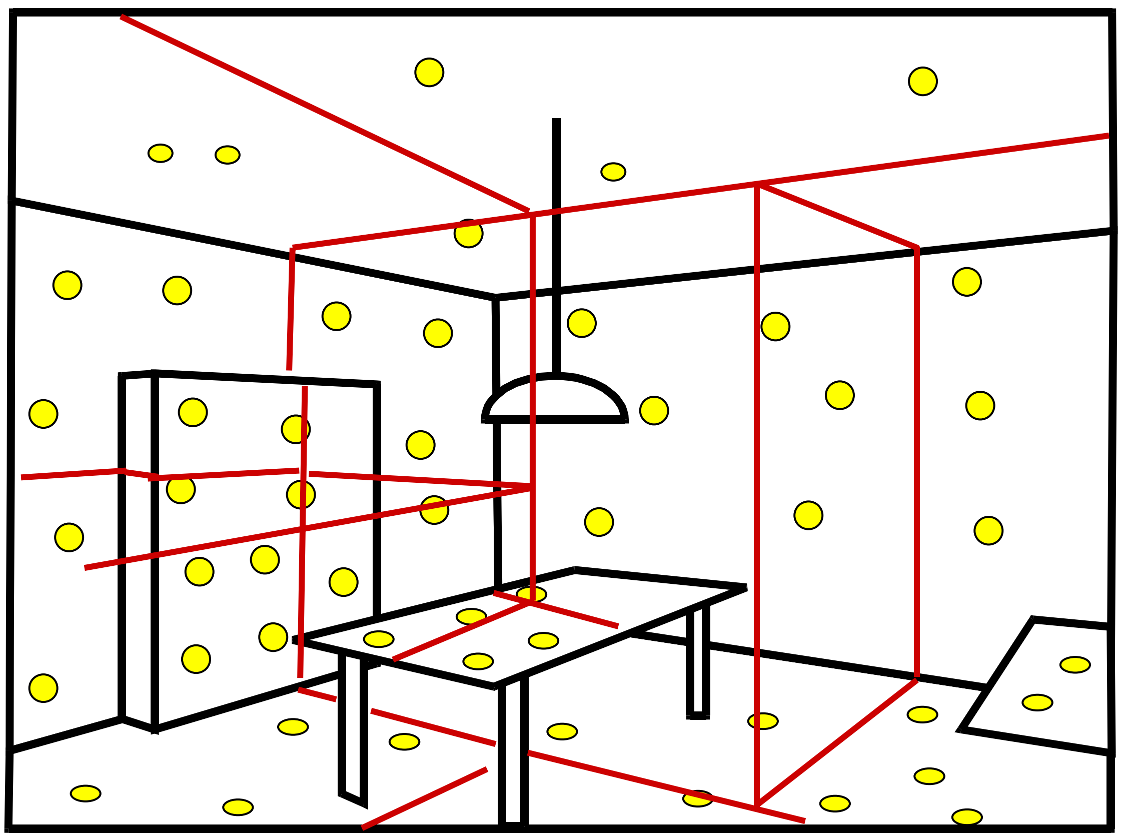
Photon mapping

- Preprocess: cast rays from light sources
- Store photons (position + light power + incoming direction)



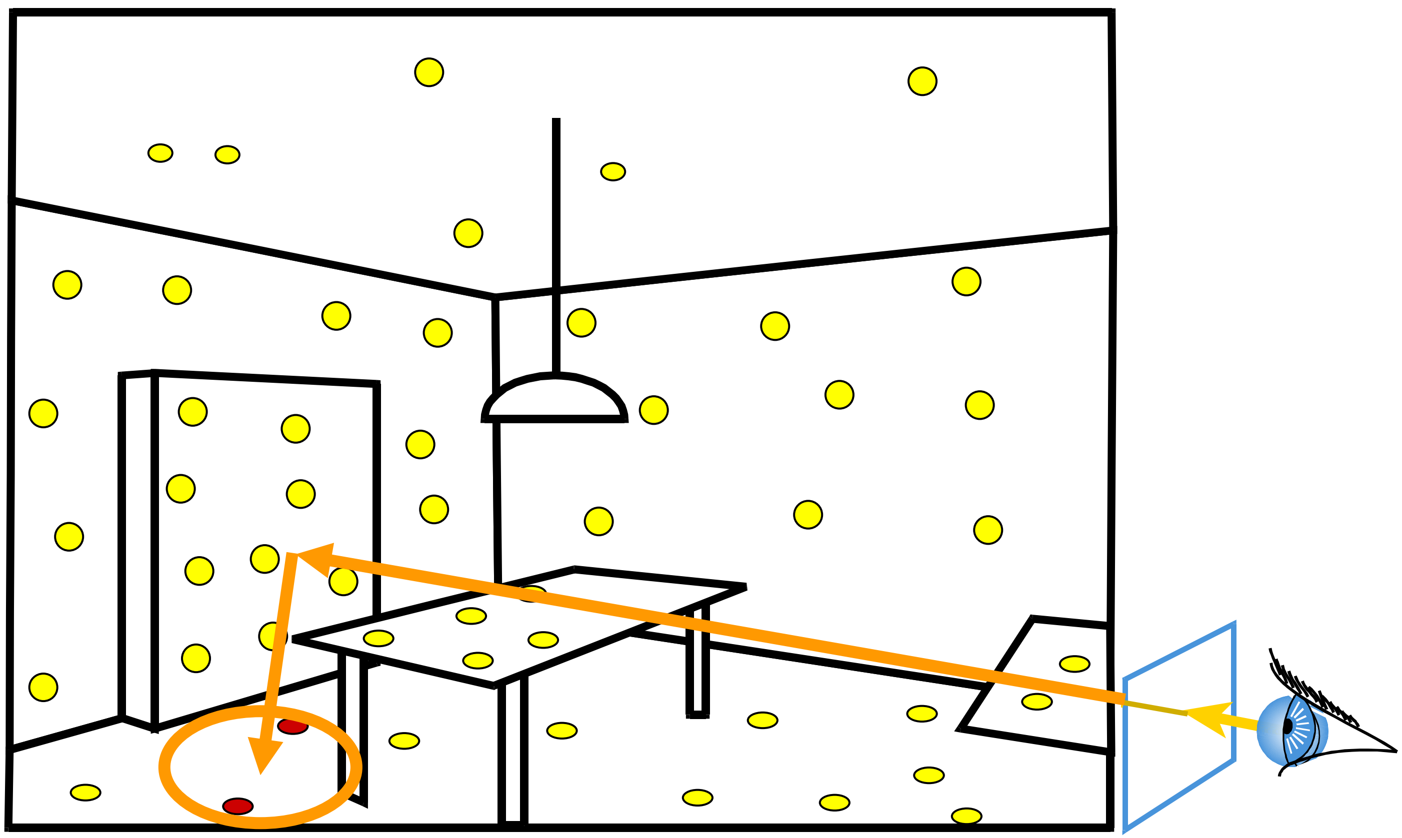
Photon map

- Efficiently store photons for fast access
- Use hierarchical spatial structure (kd-tree)



Photon mapping - rendering

- Cast primary rays
- For secondary rays
  - reconstruct irradiance using adjacent stored photon
  - Take the k closest photons
- Combine with irradiance caching and a number of other techniques



Photon map results

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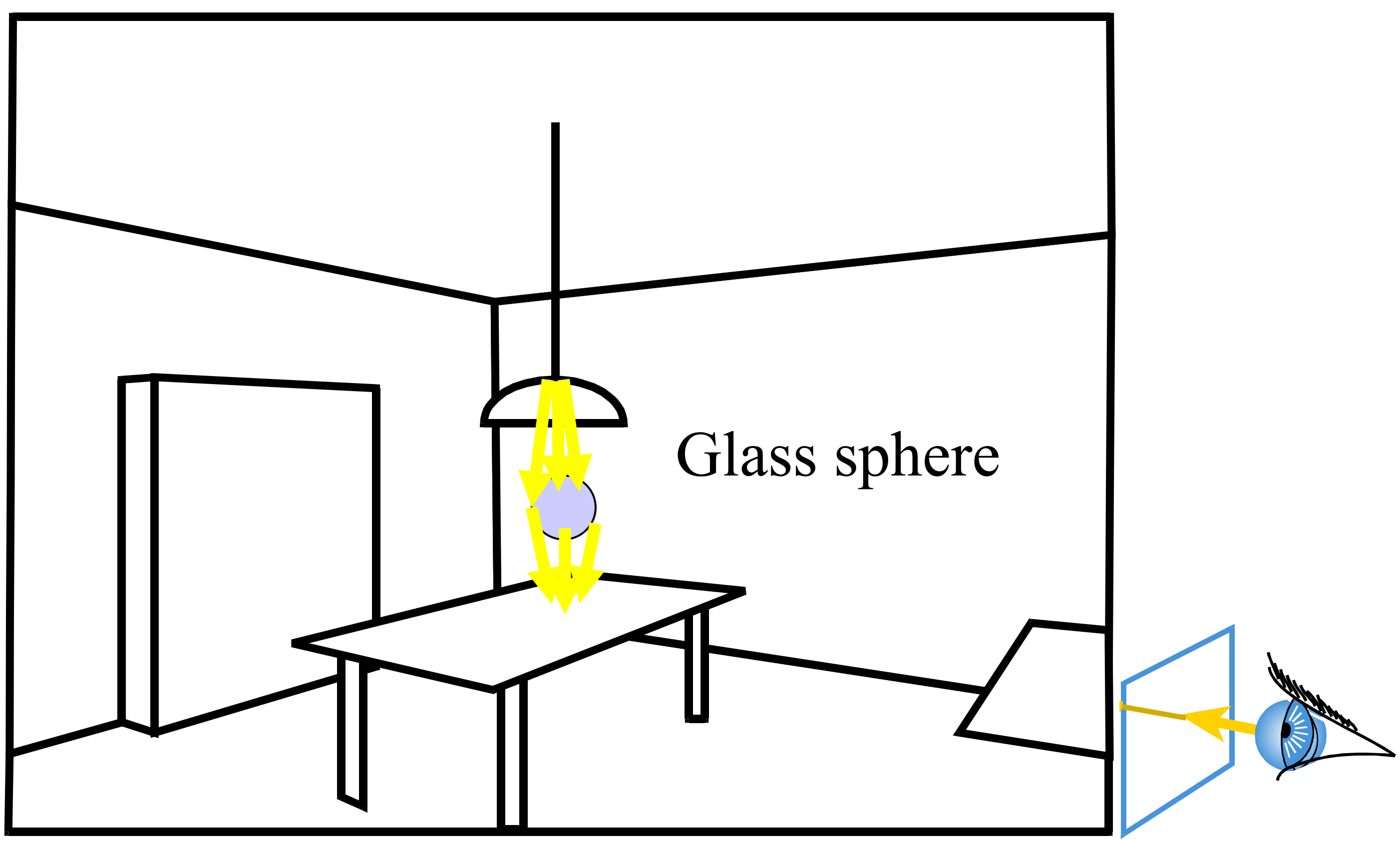
Photon mapping - caustics

- Special photon map for specular reflection and refraction



- 1000 paths/pixel

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- Photon mapping

(Image removed due to copyright considerations.)

Photon mapping recap

- Preprocess: Path tracing
  - Store photons
  - Special map for caustics (because not as uniform)
- Rendering
  - Primary rays
  - Direct lighting
  - Indirect lighting
    - Reconstruct irradiance using k nearest photons
    - Irradiance caching
  - Reconstruct caustics from caustic map

Questions?

(Image removed due to copyright considerations.)

Why is the sky blue?

(Image removed due to copyright considerations.)

Answer: because sunset is red

(Image removed due to copyright considerations.)

Why is the sky blue?

(Image removed due to copyright considerations.)

Sampling strategies

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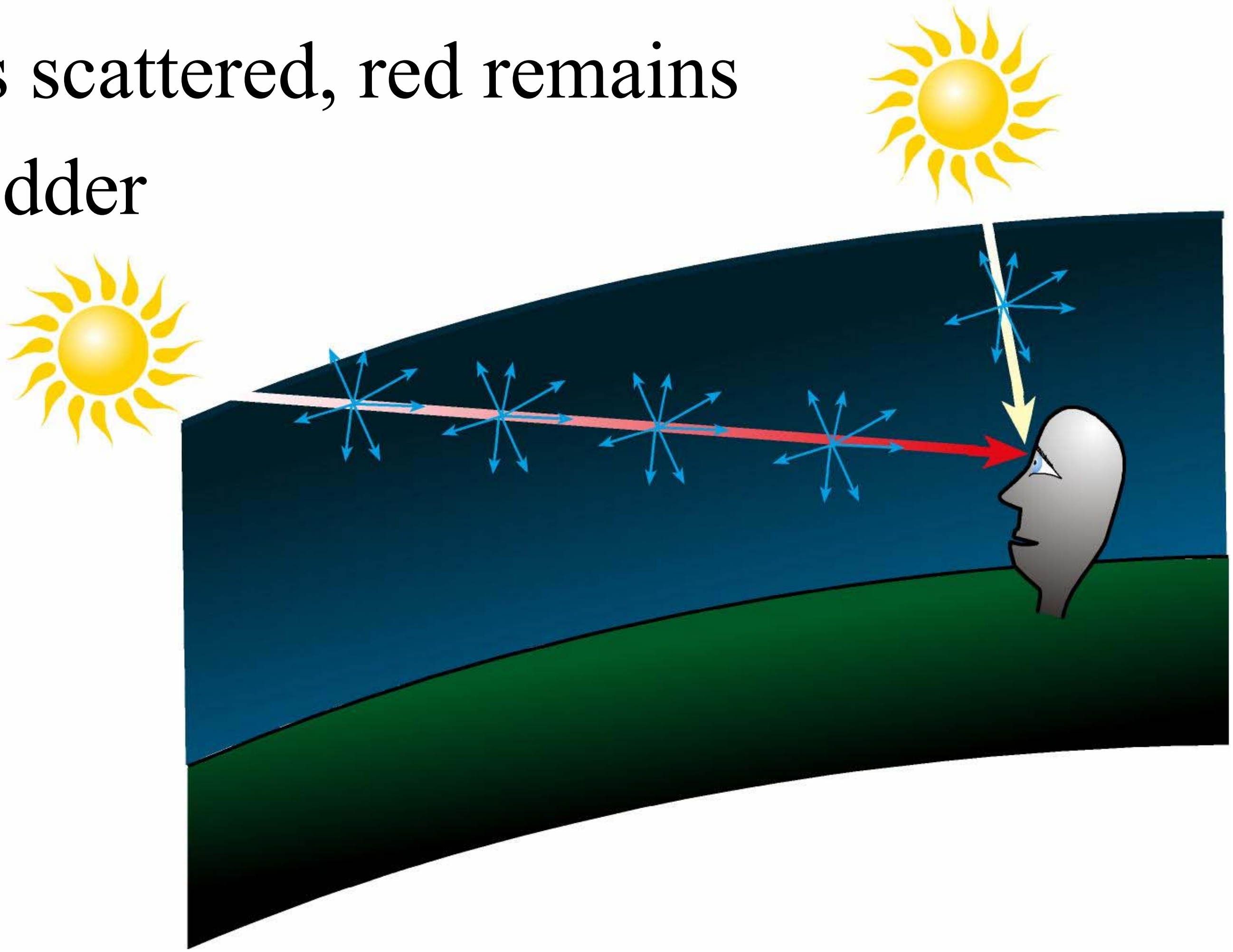
Sampling more the light

Sampling more the BRDF

Why is the sky blue?

Sun Color

- At sunset, longer traversal of atmosphere
- More blue is scattered, red remains
- Therefore redder



Computer graphics sky models

- E.g. Preetham et al. SIGGRAPH 99.

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Night Sky Model

- Jensen, Durand, Stark, Premoze, Dorsey, Shirley 2001

(Images removed due to copyright considerations.)

Questions?

(Image removed due to copyright considerations.)